Multi-criteria intuitionistic fuzzy Group Decision Making using GRA with A new approach in measuring distance from reference alternatives

Abstract:
In this paper, we will present a GRA method combines with IFS for MCGDM problems and use new approach to measuring distance from reference alternatives. In our representation, IFWA operator proposed by (Xu, 2007) is used for aggregating expert opinion to create decision making matrix, also weights of criteria achieved by intuitive fuzzy entropy. The advantage of this novel distance measure is its flexibility, which permits different fuzzy implications to be incorporated by extending its applicability to several applications where the most appropriate implication is used. In the end of paper a numerical example is presented to show the effectiveness of this method.

1 1. Introduction

Multiple-criteria decision making (MCDM) is a sub-discipline and full-grown branch of operations research that is concerned with designing mathematical and computational tools to support the subjective evaluation of a finite number of decision alternatives under a finite number of performance criteria by a single decision maker(DM) or by a group (Lootsma, 1999). Much of the decision-making in the real world, the decision information provided by a DM is often imprecise or uncertain due to time pressure, lack of data, or the DM's limited attention and information processing capabilities (Xu & Yager, 2006)
The theory of fuzzy set (Zadeh, 1965) is ideally suited for handling this ambiguity encountered in solving MCDM problems. Fuzzy set theory was first used to solve decision-making problems by Bellman and Zadeh (Bellman & Zadeh, 1970). Since then, many investigators such as Zimmerman (Zimmerman, 1987), Baas and (Baas & Kwakernaak, 1977), Yager (Yager , 1980), among many others, have proposed approaches to handle fuzzy optimization problems.
As an extension of the fuzzy set, the concept of intuitionistic fuzzy set (IFS) was introduced (Atanassov K. , 1986; Atanassov K. , 1989; Atanassov K. , 2000). It is characterized by two functions expressing the degree of membership and the degree of non-membership, respectively.
Since IFS is more suitable for dealing with fuzziness and uncertainty, they have received more and more attention. (Atanassov, Pasi, & Yager, 2005), presented a method to solve the multiple criteria group decision making problem where the attribute values are intuitionistic fuzzy numbers and the attribute weights are exact numerical values. (Xu Z. S., 2007), proposed approaches to the multiple attribute decision making problem with intuitionistic fuzzy preference information. The intuitionistic fuzzy ideal solution was defined in (Xu Z. S., 2007) and an optimization model is set up to solve the incomplete criteria weight information problem with the distance measure.
As a result, many decision making processes, in the real world, take place in group settings. To determine the weights of every DMs is a very important step in multiple attribute group decision making (MAGDM) (Yue, Jia, & Ye, 2009), (Yue Z. , 2011). Moreover, since in the group decision making, evaluation is resulted from different evaluator's view of linguistic variables, its evaluation must be conducted in an uncertain, fuzzy environment. Therefore, aggregation of expert opinions is very important to appropriately perform evaluation process. In group decision making process, intuitionistic fuzzy weighted averaging (IFWA) operator proposed by (Xu Z. S., 2007) is utilized to aggregate all individual DMs' opinions into a group opinion. Intuitionistic fuzzy entropy is used to obtain the entropy weights of the criteria.
In the type of fuzzy multi-criteria model, Grey relational analysis (GRA) is suggested as a tool for implementing a multiple criteria performance scheme, which is used to identify solutions from a finite set of alternatives (Tseng, 2010) (Lin, Lee, & Chang, 2009) (Li, Yamaguchi, & Nagai, 2007) (Wu, 2009) (Olson & Wu, 2006). Grey system theory (Deng, 1989) presents a grey relation space, and a series of

nonfunctional type models are established in this space so as to overcome the obstacles of needing a massive amount of samples in general statistical methods, or the typical distribution and large amount of calculation work.

Using GRA to solving MCDM problems with intuitionistic has been successfully applied in solving a variety of MCDM problems (Zhang & Liu , 2011) (Li, Yamaguchi, & Nagai, 2007) (CHEN, 2004) (Lin, Lee, & Chang, 2009).

In this paper, we will present a GRA method combines with IFS for MCGDM problems and use new approach to measuring distance from reference alternatives. The rest of this paper is organized as follows: After the introduction, some concepts and knowledge about IFSs are introduced in Section 2, in Section 3, we extend the GRA method to solve intuitionistic fuzzy MCGDM problems with incompletely known weight information. In Section 4, furthermore, we have extended the above results to an interval-valued intuitionistic fuzzy environment and developed modified GRA method for interval-valued intuitionistic fuzzy multiple attribute decision-making with incompletely known attribute weight information. In Section 5 we illustrate our proposed algorithmic method with an example. The final section concludes.

2 Intuitionistic fuzzy set

Intuitionistic fuzzy set introduced by (Atanassov K. , 1986) is an extension of classical Fuzzy set theory, which is suitable way to deal with vagueness. Firstly, let's introduce the concept of IFSs.

Definition 2.1 an intuitive fuzzy set A of reference set X is defined as:

$$A = \{(x, \mu_A(x), v_A(x)) | x \in X\} \qquad \text{Equation(1)}$$

Where $v_A(x) \rightarrow [0,\ 1]$, $\mu_A(x) \rightarrow [0,1]$,And $\forall x \in X: 0 \leq \mu_A(x) + v_A(x) \leq 1$. With this condition that Actual values $\mu_A(x) v_A(x)$ belonging to the interval [0, 1]. The numbers $\mu_A(x)$ and $v_A(x)$ denote the degree of membership and the degree of nonmember ship of the element $x \in X$ to the set A, respectively

Definition 2.2 for each IFS A in X, if

$\pi_A(x) = 1 - \mu_A(x) - v_A(x) \quad \forall x \in X$

Then $\pi_A(x)$ is called the degree of indeterminacy of x to A (Atanassov K. , 1986) (Atanassov K. , 1989) (Atanassov K. , 2000).or called the degree of hesitancy of x to A. If the $\pi_A(x)$ is small, knowledge about x is more certain. If $\pi_A(x)$ is great, knowledge about x is more uncertain. Obviously, when $\mu_A(x) = 1 - v_A(x)$ for all ellements of the universe, the ordinary fuzzy set concept is recovered (Shu, Cheng, & Chang, 2006).

For convenience of computation, let $\alpha = (\mu_A(x), v_A(x), \pi_A(x))$ be an intuitionistic fuzzy number (IFN) (Xu Z. S., 2007). For an IFN α^+ = (1, 0, 0) and α^- = (0, 1, 0) are the largest and the smallest IFNs, respectively.

Difination2.3: Distance between two IFNs

An important issue related with the representation of intuitionistic fuzzy sets is that of measuring distances (Yang & Chiclana, 2009).

In every IFS we have $\pi_A(x) + \mu_A(x) + v_A(x) = 1$ which can be equivalently transformed to $\boxed{a^2 + b^2 + c^2 = 1}$ With $a^2 = \pi_A(x), b^2 = \mu_A(x), c^2 = v_A(x)$However, as shown in the existing distances, there is no special reason to discriminate $\mu_A(x), v_A(x)$ and $\pi_A(x)$. Therefore, a simple non-linear transformation to the unit sphere is selected here. This last equality represents a unit sphere in a 3D Euclidean space as shown in Figure 2 (Yang & Chiclana, 2009)

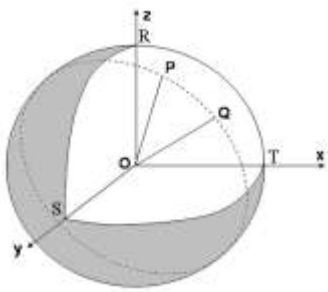

Figure 1 3D sphere representation of intuitionistic fuzzy sets (Yang & Chiclana, 2009)

This transformation allows us to interpret an intuitionistic fuzzy set as a restricted spherical surface. An immediate consequence of this interpretation is that the distance between two elements of an intuitionistic fuzzy set can be defined as the spherical distance between their corresponding points on its restricted spherical surface representation. This distance is defined as the shortest path between the two points, i.e. the length of the arc of the great circle passing through both points. For points P and Q in **Figure 1**, their spherical distance is (Yang & Chiclana, 2009)

$$d_{ns}(P,Q) = \frac{2}{\pi}\sum_{i=1}^{n} arccos\left(\sqrt{\mu_P(i)\mu_Q(i)} + \sqrt{\nu_P(i)\nu_Q(i)} + \sqrt{\pi_P(i)\pi_Q(i)}\right) \quad \text{Equation(2)}$$

Hamming distance and Euclidean acceptable interpretation problems are linear and the only difference is the relative degree of membership, non-membership and hesitation reflects the intuitive fuzzy sets are However, the spherical distance between the good and acceptable and we distinguish Semantic gap between the good and obviously a complete and more (larger) than the mean is good and acceptable because it is expressed in the nonlinear

Difination2.4 Conversion between linguistic variables and IFNs

In the case of intuitionistic fuzzy sets, the description of linguistic variable is more realistic. A linguistic variable is a variable whose value is natural language phrase. The Table 1 Conversion between linguistic variables and IFNs. is used to converted linguistic variable to IFNs.

Table 1 Conversion between linguistic variables and IFNs.

Linguistic variables	IFNs
Extreme poor (EP)/Extreme low (EL)	(0.05, 0.95,0.00)
Very poor (VP)/Very Low (VL)	(0.15, 0.80, 0.05)
Poor (P)/Low (L)	(0.25, 0.65, 0.10)
Medium poor (MP)/Medium Low (ML)	(0.35, 0.55, 0.10)
Fair (F)/Medium (M)	(0.50, 0.40, 0.10)
Medium good (MG)/Medium High (MH)	(0.65, 0.25, 0.10)
Good (G)/High (H)	(0.75, 0.15, 0.10)

Very good (VG)/Very high (VH)	(0.85, 0.10, 0.05)
Extreme good (EG)/Extreme high (EH)	(0.95, 0.05, 0.00)

Difination2.5: Intuitionistic fuzzy entropy measure

Let A= { $\alpha_1, \alpha_2 \ldots \alpha_n$} is an IFS, $\alpha = (\mu_A(x), v_A(x), \pi_A(x)), i = 1,2 \ldots, n$, are intuitionistic fuzzy value in A. intuitive fuzzy entropy A is (Vlachos & Sergiadis, 2007)

$$Hj = -\frac{1}{nln2}\sum_{i=1}^{m}[\mu_i ln\mu_i + \vartheta_i ln\vartheta_i - (1-\pi_i)ln(1-\pi_i) - \pi_i ln2)] \qquad \text{Equation(3)}$$

3 An extended GRA method based IFMCGDM[1]Approach

In this section, we develop a procedure for intuitionistic fuzzy multi-criteria group decision making using GRA. let X ={$x_1, x_2, .., x_m$} be a set of alternatives , C={$c_1, c_2 \ldots c_n$} be a set of criteria and D={$d_1, d_2 \ldots d_t$} be a set of decision makers. The weight information of the criteria and the decision-makers are completely unknown. For simplicity, we denote M= {1, 2 . . . m}, N= {1, 2 . . . n}, T = {1, 2 . . . t}. The procedure for our method has been given as follows: (Zhang & Liu , 2011)

Step 1.Construct intuitionistic fuzzy decision matrices of decision makers. Assume that the rating of alternative x_i(i∈M) with respect to criterion c_j(j ∈ N) given by the k_{th} decision maker d_k(k ∈T) is linguistic variable, which can be expressed in IFS $r_{ij}^{(k)} = \left(\mu_{ij}^{(k)}, \vartheta_{ij}^{(k)}, \pi_{ij}^{(k)}\right)$ in Table 1

Step 2. Determination of the weights of the decision makers: We assume that the decision making group consists of p decision makers. The importance of the decision makers in the selection committee may not be equal. The importance of decision makers are considered as linguistic variables expressed by intuitionistic fuzzy numbers (IFNs). Let D_k = be an intuitionistic IFN that represents the rating of the k-th decision maker. Then the weight of the k-th decision maker can be determined as: (Boran, Genç, Kurt, & Akay, 2009)

$$\lambda_k = \frac{\left(\mu_\kappa + \pi_\kappa\left(\frac{\mu_\kappa}{\mu_\kappa + \nu_\kappa}\right)\right)}{\sum_{k=1}^{t}\left(\mu_\kappa + \pi_\kappa\left(\frac{\mu_\kappa}{\mu_\kappa + \nu_\kappa}\right)\right)}\ , \qquad \sum_{k=1}^{t}\lambda_\kappa = 1 \qquad \text{Equation(4)}$$

Step 3: Construct aggregated intuitionistic fuzzy decision matrix based on the opinions of decision makers.

Let $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m\times n}$ is an intuitionistic fuzzy decision matrix of each decision maker.$\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_k\}$is the weight of each decision maker and $\sum_{i=0}^{k}\lambda_i = 1$, . In group decision-making process, all the individual decision opinions need to be fused into a group opinion to construct aggregated intuitionistic fuzzy decision matrix. In order to do, we use intuitionistic fuzzy weighted average (IFWA) operator due to (Xu Z. S., 2007) as.، $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m\times n}$ *where*

[1] - **Intuitionistic Fuzzy Multi Criteria Group Decision Making**

$$r_{ij} = IFWA_\lambda\left(r_{ij}^1, r_{ij}^2, \ldots, r_{ij}^{(t)}\right) = \lambda_1 r_{ij}^{(1)} \oplus \lambda_2 r_{ij}^{(2)} \oplus \ldots \oplus \lambda_t r_{ij}^{(t)}$$
$$= \left(1 - \prod_{k=1}^{t}\left(1-\mu_{ij}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{t}\left(\left(v_{ij}^{(k)}\right)^{\lambda_k}\right), \prod_{k=1}^{t}\left(1-\mu_{ij}^{(k)}\right)^{\lambda_k} - \prod_{k=1}^{t}\left(\left(v_{ij}^{(k)}\right)^{\lambda_k}\right)\right) \quad \textbf{Equation(5)}$$

The aggregate intuitionistic fuzzy decision matrix then can be defined as:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ & \cdots & \cdots & \vdots \\ r_{21} & \cdots & \cdots & \vdots \\ \vdots & & & \\ r_{m1} & \cdots & \cdots & r_{mn} \end{bmatrix}$$

Where $r_{ij} = \left(\mu_{ij}, \vartheta_{ij}, \pi_{ij}\right)$ *and* $\mu_{ij} = 1 - \prod_{k=1}^{t}\left(1-\mu_{ij}^{(k)}\right)^{\lambda_k}$,

$\boldsymbol{\pi_{ij}} = \prod_{k=1}^{t}\left(1-\mu_{ij}^{(k)}\right)^{\lambda_k} - \prod_{k=1}^{t}\left(\left(v_{ij}^{(k)}\right)^{\lambda_k}\right)$, $\boldsymbol{\vartheta_{ij}} = \prod_{k=1}^{t}\left(\left(v_{ij}^{(k)}\right)^{\lambda_k}\right)$

Step 4: Determine the weights of criteria.
All criteria may not be assumed to be equal importance. *W* represents a set of grades of importance. In order to obtain *W*, all the individual decision maker opinions for the importance of each criteria need to be fused. In order to obtain W, intuitionistic fuzzy entropy (Vlachos & Sergiadis, 2007) will be used to obtain the entropy weights of the criteria, we use the first:

$$Hj = -\frac{1}{nln2}\sum_{i=1}^{m}\left[\mu_{ij}ln\mu_{ij} + \vartheta_{ij}ln\vartheta_{ij} - \left(1-\pi_{ij}\right)ln(1-\pi_{ij}) - \pi_{ij}ln2\right] \quad \textbf{Equation(6)}$$

Here, though $\mu_{ij} = 0, \vartheta_{ij} = 0, \pi_{ij} = 1$ is when respectively $\mu_{ij}ln\mu_{ij} = 0$, ، $\vartheta_{ij}ln\vartheta_{ij} = 0, \left(1-\pi_{ij}\right)ln\left(1-\pi_{ij}\right) = 0$.The entropy weight of the j_{th} criterion is defined as follows:

$$W_j = \frac{1 - H_j}{n - \sum_{i=1}^{n} H_j} \quad \textbf{Equation(7)}$$

Step5: Obtain intuitionistic fuzzy positive-ideal solution.
Since the aspired level of the membership value, non-membership value and indeterminacy value are 1, 0, 0 respectively, the point consisting of highest membership value, minimum non-membership value and minimum indeterminacy value would represent the reference value or ideal point or utopia point. The maximum value

$r^{+} = (\mu_{max}, \vartheta_{min}, \pi_{min})$ **Equation(8)**

Can be used as positive-ideal solution value, so the largest value in the intuitionistic fuzzy space is equal to:

r^{*}=(١.٠.٠)

Step 6: Calculate the distance from ideals: this distance can be calculated as Spherical distance method that was introduced in **Difination2.3**

$$d_{ns}(A^*, A) = \frac{2}{\pi} \sum_{i=1}^{n} arccos\left(\sqrt{\mu_{A^*}(i)\mu_A(i)} + \sqrt{\nu_{A^*}(i)\nu_A(i)} + \sqrt{\pi_{A^*}(i)\pi_A(i)}\right)$$

Step 7: Calculation of the grey relational coefficient of each alternative from positive ideal solution (PIS) : (CHEN, 2004)using the following equation:

$$\mathrm{r}\left(\mathrm{A}^+(j), \mathrm{A_i}(j)\right) = \frac{\mathrm{min_i min_j}|\mathrm{A}^+(\mathrm{j}) - \mathrm{A_i}(\mathrm{j})| + \zeta \mathrm{max_i max_j}|\mathrm{A}^+(\mathrm{j}) - \mathrm{A_i}(\mathrm{j})|}{|\mathrm{A}^+(\mathrm{j}) - \mathrm{A_i}(\mathrm{j})| + \zeta \mathrm{max_i max_j}|\mathrm{A}^+(\mathrm{j}) - \mathrm{A_i}(\mathrm{j})|} \qquad \textbf{Equation(9)}$$

Where

$$|\mathrm{A}^+(\mathrm{j}) - \mathrm{A_i}(\mathrm{j})| = d_{ns}(A^*, A)$$

ζ Is the distinguishing coefficient or the identification coefficient, [0,1]. Smaller value of distinguishing coefficient will yield in large range of grey relational coefficient. Generally, = 0.5 is considered for decision- making situation. [0, 1] is the distinguishable coefficient used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients. When =1, the comparison environment is unaltered; when =0, the comparison environment disappears. □

Step 8.Calculating the degree of gray relational coefficient of each alternative from PIS using the following equation:

$$\mathrm{r}\left(\mathrm{A}^+(j), \mathrm{A_i}(j)\right) = \sum_{\mathrm{j}=1}^{\mathrm{n}} w_{\mathrm{j}} \mathrm{r}\left(\mathrm{A}^+(j), A_i(j)\right) \qquad \textbf{Equation(10)}$$

$$\sum_{i=0}^{n} w_i = 1$$

Step 9. Rank all the alternatives $\mathrm{A_i}$ (i = 1,2,. . . ,m) and select the best one(s) in accordance with $\mathrm{r}\left(\mathrm{A}^+(j), \mathrm{A_i}(j)\right)$ If any alternative has the highest r value, then, it is the most important alternative.

4 Example illustration

Suppose that a software company need to hire a system analyst. After initial screening, four candidates (i.e., alternatives) x1, x2, x3 and x4 remain for further evaluation. On the other hand, interviews were conducted by a committee of three decision makers' i.e. D1, D2 and D3 to select the most suitable candidates. The selections of the candidates are based on five benefit criteria which are: (Chen, 2000) (Li, Yamaguchi, & Nagai, 2007)and (Zhang & Liu , 2011)

1. Emotional steadiness (C1)
2. Oral communication skill (C2)
3. Personality (C3)
4. Past experience (C4)
5. Self-confidence (C5)

Respectively, the decision matrices $f^{(k)}=(f_{ij}^{(k)})_{4*5}$ (k=1,2,3) listed in Tables 2–4.

Table-2
Decision matrices $F^{(1)}$.

	C_1	C_2	C_3	C_4	C_5
X_1	VG	MG	G	VG	EG
X_2	MG	G	VG	G	G
X_3	M	MG	M	M	MG
X_4	M	MG	M	G	G

Table-3
Decision matrices $F^{(2)}$.

X_1	G	MG	G	G	VG
X_2	M	G	G	VG	G
X_3	M	MG	MP	M	M
X_4	G	MG	MG	M	G

Table-4
Decision matrices $F^{(3)}$.

X_1	G	G	VG	EG	VG
X_2	MG	VG	VG	MG	G
X_3	G	G	MP	VG	MG
X_4	VG	G	M	VG	MG

Procedure for selection of supplier contains the following steps:

Step 1.The decision makers use the linguistic weighting variables to assess the importance of the criteria as shown in Table 1. Then the intuitionistic fuzzy decision matrix based on the opinions of decision makers is constructed as follows:

$$\boldsymbol{R}^{(1)} = \begin{bmatrix} (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.65, 0.25, 0.10}) & (\mathbf{0.75, 0.15, 0.10}) & \mathbf{0.95, 0.05, 0.00} & \mathbf{0.95, 0.05,} \\ (\mathbf{0.65, 0.25, 0.10}) & (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.75, 0.15,} \\ (\mathbf{0.50, 0.40, 0.10}) & (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.85, 0.10,} \\ (\mathbf{0.50, 0.40, 0.10}) & (\mathbf{0.65, 0.25, 0.10}) & (\mathbf{0.50, 0.40, 0.10}) & (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.75, 0.15,} \end{bmatrix}$$

$$\boldsymbol{R}^{(2)} = \begin{bmatrix} (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.65, 0.25, 0.10}) & (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.85, 0.10,} \\ (\mathbf{0.50, 0.40, 0.10}) & (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.75, 0.15,} \\ (\mathbf{0.50, 0.40, 0.10}) & (\mathbf{0.65, 0.25, 0.10}) & (\mathbf{0.35, 0.55, 0.10}) & (\mathbf{0.50, 0.40, 0.10}) & (\mathbf{0.50, 0.40,} \\ (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.65, 0.25, 0.10}) & (\mathbf{0.65, 0.25, 0.10}) & (\mathbf{0.50, 0.40, 0.10}) & (\mathbf{0.75, 0.15,} \end{bmatrix}$$

$$\boldsymbol{R}^{(3)} = \begin{bmatrix} (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.95, 0.05, 0.10}) & (\mathbf{0.85, 0.10,} \\ (\mathbf{0.65, 0.25, 0.10}) & (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.65, 0.25, 0.10}) & (\mathbf{0.75, 0.15,} \\ (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.35, 0.55, 0.10}) & (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.65, 0.25,} \\ (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.75, 0.15, 0.10}) & (\mathbf{0.50, 0.40, 0.10}) & (\mathbf{0.85, 0.10, 0.05}) & (\mathbf{0.65, 0.25,} \end{bmatrix}$$

Step 2. The weights of decision makers are determined. The importance degree of each decision makers on group decision is shown as Table 5 as follows:

Table 5 the importance of decision makers and their weights

	DM_1	DM_2	DM_3
Linguistic terms	Very important	Medium	Important
Weight	0.406	0.238	0.356

In order to obtain the weights of the decision makers, **Error! Reference source not found.** were utilized:

$$\boldsymbol{\lambda_1} = \frac{\mathbf{0.75+0.05}\left(\frac{\mathbf{0.75}}{\mathbf{0.75+0.20}}\right)}{\left(\mathbf{0.75+0.05}\left(\frac{\mathbf{0.75}}{\mathbf{0.75+0.20}}\right)\right)+\left(\mathbf{0.90+0.05}\left(\frac{\mathbf{0.90}}{\mathbf{0.90+0.05}}\right)\right)+\left(\mathbf{0.50+0.10}\left(\frac{\mathbf{0.50}}{\mathbf{0.50+0.40}}\right)\right)} = 0.3488$$

$$\boldsymbol{\lambda_2} = \frac{\mathbf{0.90+0.05}\left(\frac{\mathbf{0.90}}{\mathbf{0.90+0.05}}\right)}{\left(\mathbf{0.75+0.05}\left(\frac{\mathbf{0.75}}{\mathbf{0.75+0.20}}\right)\right)+\left(\mathbf{0.90+0.05}\left(\frac{\mathbf{0.90}}{\mathbf{0.90+0.05}}\right)\right)+\left(\mathbf{0.50+0.10}\left(\frac{\mathbf{0.50}}{\mathbf{0.50+0.40}}\right)\right)} = 0.4186$$

$$\boldsymbol{\lambda_3} = \frac{\mathbf{0.50+0.10}\left(\frac{\mathbf{0.50}}{\mathbf{0.50+0.40}}\right)}{\left(\mathbf{0.75+0.05}\left(\frac{\mathbf{0.75}}{\mathbf{0.75+0.20}}\right)\right)+\left(\mathbf{0.90+0.05}\left(\frac{\mathbf{0.90}}{\mathbf{0.90+0.05}}\right)\right)+\left(\mathbf{0.50+0.10}\left(\frac{\mathbf{0.50}}{\mathbf{0.50+0.40}}\right)\right)} = 0.2326$$

Step3.Construct the aggregated intuitionistic fuzzy decision matrix based on the opinions of decision makers. Utilizing the IFWA operator **Error! Bookmark not defined.** to aggregate all the intuitionistic fuzzy decision matrices R^k (k=1, 2, 3) into a complex intuitionistic fuzzy decision matrix R:

$$\boldsymbol{R} = \begin{bmatrix} (0.79,0.13,0.08) & (0.68,0.22,0.10) & (0.78,0.14,0.08) & (0.9,0.08,0.02) & 0.9,0.08,0.02 \\ (0.59,0.3,0.11) & (0.78,0.14,0.08) & (0.81,0.12,0.07) & (0.78,0.14,0.08) & (0.75,0.15,0.10) \\ (0.57,0.32,0.11) & (0.76,0.16,0.08) & (0.61,0.3,0.09) & (0.75,0.18,0.07) & (0.7,0.22,0.08) \\ (0.72,0.19,0.09) & (0.68,0.22,0.10) & (0.57,0.33,0.10) & (0.7,0.21,0.09) & (0.73,0.17,0.1) \end{bmatrix}$$

Step 4. Obtain the entropy weights of the criteria. Utilizing **Error! Reference source not found.**:

$$w_1 = 0.166936, w_2 = 0.19374, w_3 = 0.181827\ w_4 = 0.231145, w_5 = 0.226$$

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Step 5: Obtain intuitionistic fuzzy positive-ideal solution. The reference sequence should be the optimal sequence of the criteria values. We are using **Error! Bookmark not defined.**

Step 6. Calculate the distance from ideals: this distance can be calculated as Spherical distance method that was introduced in **Difination2.3**

Step 7. Estimate the grey relational grade. Utilizing **Error! Reference source not found.** to calculate the grey relational grade ci(i = 1,2,3,4)

$$r(x^+, x_1) = 0.876843\ , r(x^+, x_2) = 0.776276$$

$$r(x^+, x_3) = 0.719021\,, r(x^+, x_4) = 0.71369$$

Step 8. Rank all the alternatives. According to descending order of c_i(i = 1,2,3,4):

$$C_1 > C_2 > C_3 > C_4$$

5 Conclusions

Grey relational analysis based intuitionistic fuzzy multi-criteria group decision- making approach is a practical, versatile and powerful tool that identifies the criteria and offers a consistent structure and process for evaluating candidates by employing the concept of acceptance, rejection and indeterminacy simultaneously. In this study, we have looked at the issue of 3D representation of intuitionistic fuzzy sets in Grey relational analysis based intuitionistic fuzzy multi-criteria group decision- making. We aggregate the IFNs by IFWA operator into the collective values, based on which, the alternatives are ranked, and the most desirable one is chosen. To the best of our knowledge, this is the first time that GRA using spherical distance measuring distance from reference alternatives. Spherical distance rather than Euclidean distance or Hamming helps us have more precise rankings, because cooperation of linguistic variable is not necessarily linear. For example, the distance between good and excellent cannot be equal with good and moderate, if we use the linear measurement it considered equal. The advantage of spherical distance measure is its flexibility, which permits different fuzzy implications to be incorporated by extending its applicability to several applications where the most appropriate implication is used.

6 Bibliography

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