Optional Lab: Gradient Descent for Logistic Regression

Goals

In this lab, you will:

- update gradient descent for logistic regression.
- explore gradient descent on a familiar data set

In [1]:
```python
import copy, math
import numpy as np
%matplotlib widget
import matplotlib.pyplot as plt
from lab_utils_common import  dlc, plot_data, plt_tumor_data, sigmoid, compute_cost_logistic
from plt_quad_logistic import plt_quad_logistic, plt_prob
plt.style.use('./deeplearning.mplstyle')
```

Data set

Let's start with the same two feature data set used in the decision boundary lab.

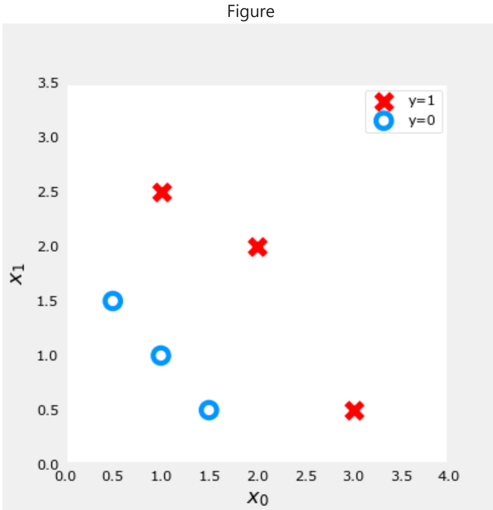
In [2]:
```python
X_train = np.array([[0.5, 1.5], [1,1], [1.5, 0.5], [3, 0.5], [2, 2], [1, 2.5]])
y_train = np.array([0, 0, 0, 1, 1, 1])
```

As before, we'll use a helper function to plot this data. The data points with label $y = 1$ are shown as red crosses, while the data points with label $y = 0$ are shown as blue circles.

In [3]:
```python
fig,ax = plt.subplots(1,1,figsize=(4,4))
plot_data(X_train, y_train, ax)

ax.axis([0, 4, 0, 3.5])
ax.set_ylabel('$x_1$', fontsize=12)
ax.set_xlabel('$x_0$', fontsize=12)
plt.show()
```

Figure

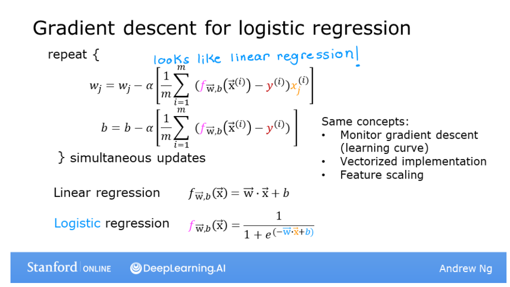


Logistic Gradient Descent

Recall the gradient descent algorithm utilizes the gradient calculation:

$$\begin{align*}
&\text{repeat until convergence:} \; \lbrace \\
&\; \; \;w_j = w_j - \alpha \frac{\partial J(\mathbf{w},b)}{\partial w_j} \tag{1} \; & \text{for j := 0..n-1} \\
&\; \; \; \; \;b = b - \alpha \frac{\partial J(\mathbf{w},b)}{\partial b} \\
&\rbrace
\end{align*}$$

Where each iteration performs simultaneous updates on w_j for all j, where



$$\frac{\partial J(\mathbf{w},b)}{\partial w_j} = \frac{1}{m} \sum\limits_{i = 0}^{m-1} (f_{\mathbf{w},b}(\mathbf{x}^{(i)}) - y^{(i)})x_{j}^{(i)} \tag{2}$$

$$\frac{\partial J(\mathbf{w},b)}{\partial b} = \frac{1}{m} \sum\limits_{i = 0}^{m-1} (f_{\mathbf{w},b}(\mathbf{x}^{(i)}) - y^{(i)}) \tag{3}$$

- m is the number of training examples in the data set
- $f_{\mathbf{w},b}(x^{(i)})$ is the model's prediction, while $y^{(i)}$ is the target
- For a logistic regression model
  $z = \mathbf{w} \cdot \mathbf{x} + b$
  $f_{\mathbf{w},b}(x) = g(z)$
  where $g(z)$ is the sigmoid function:
  $g(z) = \frac{1}{1+e^{-z}}$

Gradient Descent Implementation

The gradient descent algorithm implementation has two components:

- The loop implementing equation (1) above. This is `gradient_descent` below and is generally provided to you in optional and practice labs.
- The calculation of the current gradient, equations (2,3) above. This is `compute_gradient_logistic` below. You will be asked to implement this week's practice lab.

Calculating the Gradient, Code Description

Implements equation (2),(3) above for all w_j and b. There are many ways to implement this. Outlined below is this:

- initialize variables to accumulate `dj_dw` and `dj_db`
- for each example
  - calculate the error for that example $g(\mathbf{w} \cdot \mathbf{x}^{(i)} + b) - \mathbf{y}^{(i)}$
  - for each input value $x_{j}^{(i)}$ in this example,
    - multiply the error by the input $x_{j}^{(i)}$, and add to the corresponding element of `dj_dw`. (equation 2 above)
  - add the error to `dj_db` (equation 3 above)
- divide `dj_db` and `dj_dw` by total number of examples (m)
- note that $\mathbf{x}^{(i)}$ in numpy `X[i,:]` or `X[i]` and $x_{j}^{(i)}$ is `X[i,j]`

In [9]:

```
def compute_gradient_logistic(X, y, w, b):
    """
    Computes the gradient for logistic regression

    Args:
      X (ndarray (m,n): Data, m examples with n features
      y (ndarray (m,)): target values
      w (ndarray (n,)): model parameters
      b (scalar)      : model parameter
    Returns
      dj_dw (ndarray (n,)): The gradient of the cost w.r.t. the parameters w.
      dj_db (scalar)      : The gradient of the cost w.r.t. the parameter b.
    """
    m,n = X.shape

    # compute hypothesis
    f_wb = sigmoid(np.dot(X,w) + b)

    # compute bias derivative
    dj_db = np.sum(f_wb-y) / m

    # compute weight derivative
    dj_dw = np.dot(np.transpose(X),(f_wb-y)) / m

    return dj_db, dj_dw
```

Check the implementation of the gradient function using the cell below.

In [10]:

```
X_tmp = np.array([[0.5, 1.5], [1,1], [1.5, 0.5], [3, 0.5], [2, 2], [1, 2.5]])
y_tmp = np.array([0, 0, 0, 1, 1, 1])
w_tmp = np.array([2.,3.])
b_tmp = 1.
dj_db_tmp, dj_dw_tmp = compute_gradient_logistic(X_tmp, y_tmp, w_tmp, b_tmp)
print(f"dj_db: {dj_db_tmp}" )
print(f"dj_dw: {dj_dw_tmp.tolist()}" )
```

```
dj_db: 0.49861806546328574
dj_dw: [0.498333393278696, 0.49883942983996693]
```

Expected output

```
dj_db: 0.49861806546328574
dj_dw: [0.498333393278696, 0.49883942983996693]
```

Gradient Descent Code

The code implementing equation (1) above is implemented below. Take a moment to locate and compare the functions in the routine to the equations above.

In [11]:
```
def gradient_descent(X, y, w_in, b_in, alpha, num_iters):
    """
    Performs batch gradient descent

    Args:
      X (ndarray (m,n)   : Data, m examples with n features
      y (ndarray (m,))   : target values
      w_in (ndarray (n,)): Initial values of model parameters
      b_in (scalar)      : Initial values of model parameter
      alpha (float)      : Learning rate
      num_iters (scalar) : number of iterations to run gradient descent

    Returns:
      w (ndarray (n,))   : Updated values of parameters
      b (scalar)         : Updated value of parameter
    """
    # An array to store cost J and w's at each iteration primarily for graphing later
    J_history = []
    w = copy.deepcopy(w_in)  #avoid modifying global w within function
    b = b_in

    for i in range(num_iters):
        # Calculate the gradient and update the parameters
        dj_db, dj_dw = compute_gradient_logistic(X, y, w, b)

        # Update Parameters using w, b, alpha and gradient
        w = w - alpha * dj_dw
        b = b - alpha * dj_db

        # Save cost J at each iteration
        if i<100000:      # prevent resource exhaustion
            J_history.append( compute_cost_logistic(X, y, w, b) )

        # Print cost every at intervals 10 times or as many iterations if < 10
        if i% math.ceil(num_iters / 10) == 0:
            print(f"Iteration {i:4d}: Cost {J_history[-1]}   ")

    return w, b, J_history         #return final w,b and J history for graphing
```

Let's run gradient descent on our data set.

In [12]:
```
w_tmp  = np.zeros_like(X_train[0])
b_tmp  = 0.
alph = 0.1
iters = 10000

w_out, b_out, _ = gradient_descent(X_train, y_train, w_tmp, b_tmp, alph, iters)
print(f"\nupdated parameters: w:{w_out}, b:{b_out}")
```

```
Iteration    0: Cost 0.684610468560574
Iteration 1000: Cost 0.1590977666870457
Iteration 2000: Cost 0.08460064176930078
Iteration 3000: Cost 0.05705327279402531
Iteration 4000: Cost 0.04290759421682
Iteration 5000: Cost 0.03433847729884557
Iteration 6000: Cost 0.02860379802212006
Iteration 7000: Cost 0.02450156960879306
Iteration 8000: Cost 0.02142370332569295
Iteration 9000: Cost 0.019030137124109114

updated parameters: w:[5.28 5.08], b:-14.222409982019837
```

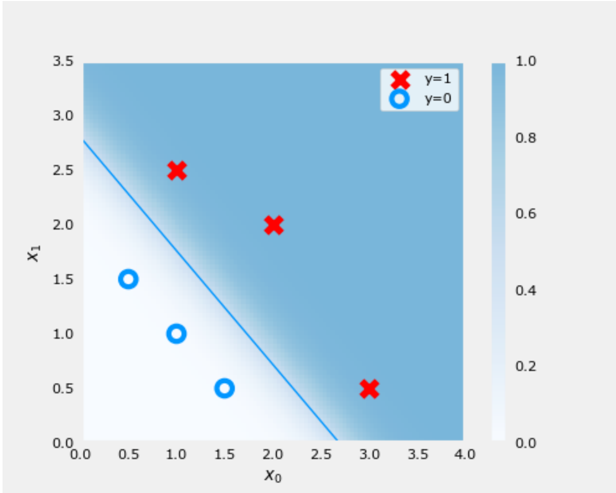
Let's plot the results of gradient descent:

In [13]:
```
fig,ax = plt.subplots(1,1,figsize=(5,4))
# plot the probability
plt_prob(ax, w_out, b_out)

# Plot the original data
ax.set_ylabel(r'$x_1$')
ax.set_xlabel(r'$x_0$')
ax.axis([0, 4, 0, 3.5])
plot_data(X_train,y_train,ax)

# Plot the decision boundary
x0 = -b_out/w_out[0]
x1 = -b_out/w_out[1]
ax.plot([0,x0],[x1,0], c=dlc["dlblue"], lw=1)
plt.show()
```

Figure



In the plot above:

- the shading reflects the probability y=1 (result prior to decision boundary)
- the decision boundary is the line at which the probability = 0.5

Another Data set

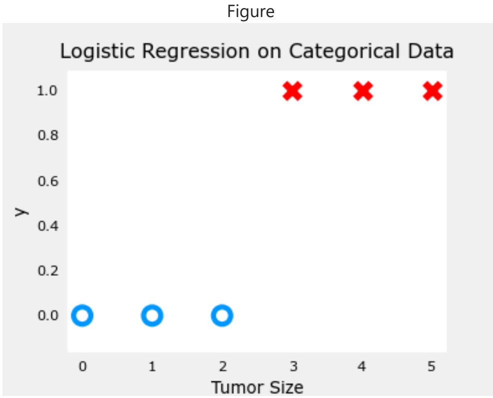
Let's return to a one-variable data set. With just two parameters, w, b, it is possible to plot the cost function using a contour plot to get a better idea of what gradient descent is up to.

In [14]:
```
x_train = np.array([0., 1, 2, 3, 4, 5])
y_train = np.array([0,  0, 0, 1, 1, 1])
```

As before, we'll use a helper function to plot this data. The data points with label $y = 1$ are shown as red crosses, while the data points with label $y = 0$ are shown as blue circles.

In [15]:
```
fig,ax = plt.subplots(1,1,figsize=(4,3))
plt_tumor_data(x_train, y_train, ax)
plt.show()
```

Figure

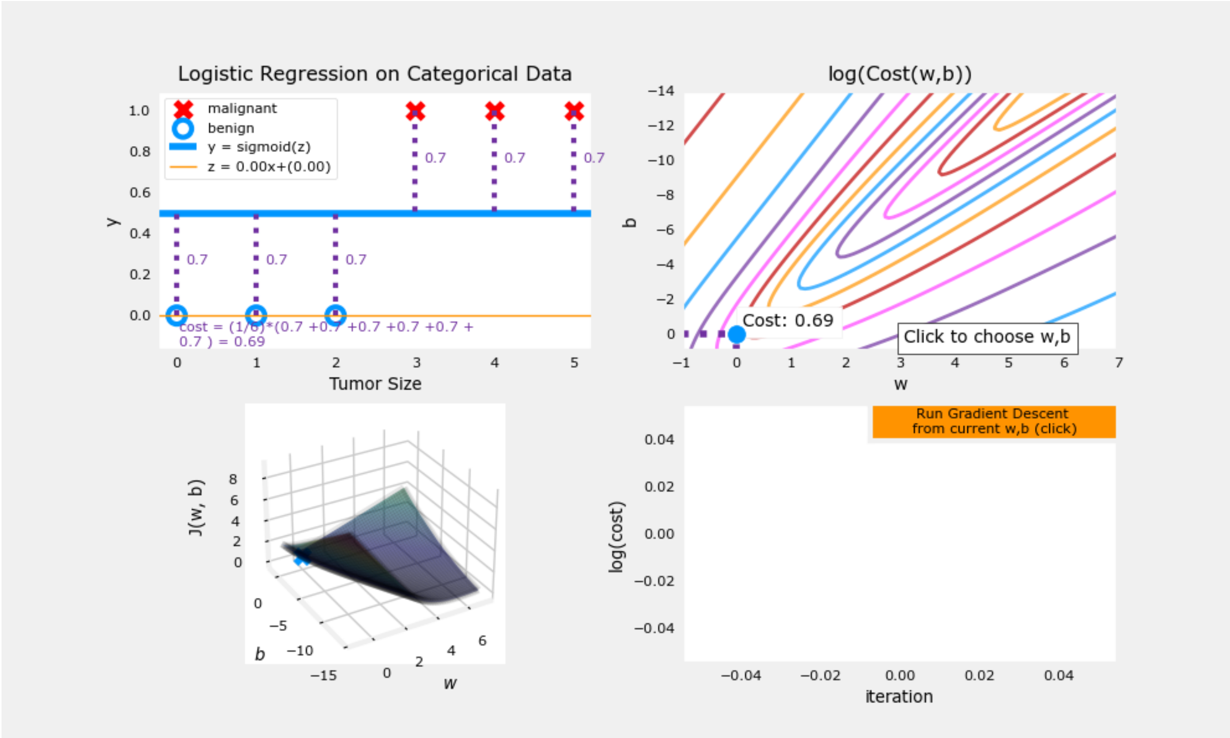


In the plot below, try:

- changing w and b by clicking within the contour plot on the upper right.
  - changes may take a second or two
  - note the changing value of cost on the upper left plot.
  - note the cost is accumulated by a loss on each example (vertical dotted lines)
- run gradient descent by clicking the orange button.
  - note the steadily decreasing cost (contour and cost plot are in log(cost)
  - clicking in the contour plot will reset the model for a new run
- to reset the plot, rerun the cell

In [16]:
```
w_range = np.array([-1, 7])
b_range = np.array([1, -14])
quad = plt_quad_logistic( x_train, y_train, w_range, b_range )
```

Figure



Congratulations!

You have:

- examined the formulas and implementation of calculating the gradient for logistic regression
- utilized those routines in
  - exploring a single variable data set
  - exploring a two-variable data set