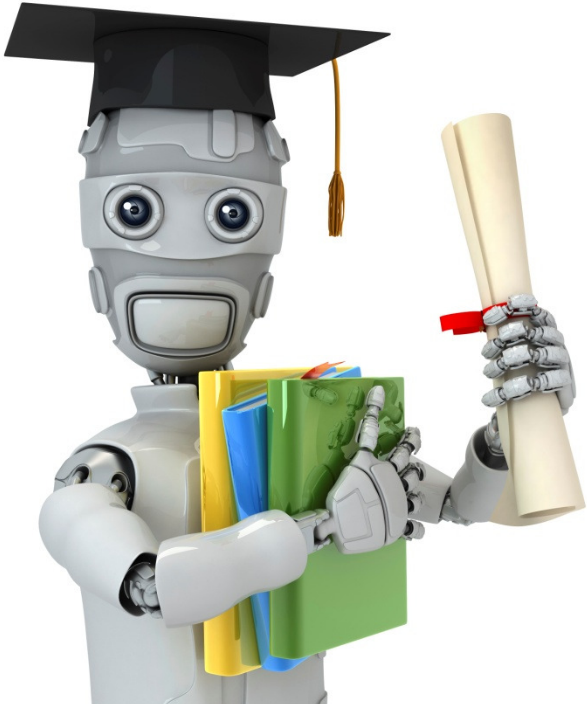

Machine Learning

Introduction

Welcome

Apple - iPhoto - New full-s
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iPod
iPhone
iPad
iTunes
Support
iLife '11
iPhoto
iMovie
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Video Showcase
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Upgrade Now
iPhoto '11
From your Facebook Wall to your coffee table to your best friend's inbox (or mailbox). Do more with your photos than you ever thought possible. And do it all in one place. iPhoto.
Watch the iPhoto video
MacBook Pro
What's New in iPhoto
What is iPhoto?


SPAM

Machine Learning

- Grew out of work in AI
- New capability for computers

Examples:

- Database mining

  Large datasets from growth of automation/web.

  E.g., Web click data, medical records, biology, engineering
- Applications can’t program by hand.

  E.g., Autonomous helicopter, handwriting recognition, most of Natural Language Processing (NLP), Computer Vision.

Machine Learning

- Grew out of work in AI
- [illegible]

Exam[illegible]

- [illegible]

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- [illegible]

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  E.g., Amazon, Netflix product recommendations

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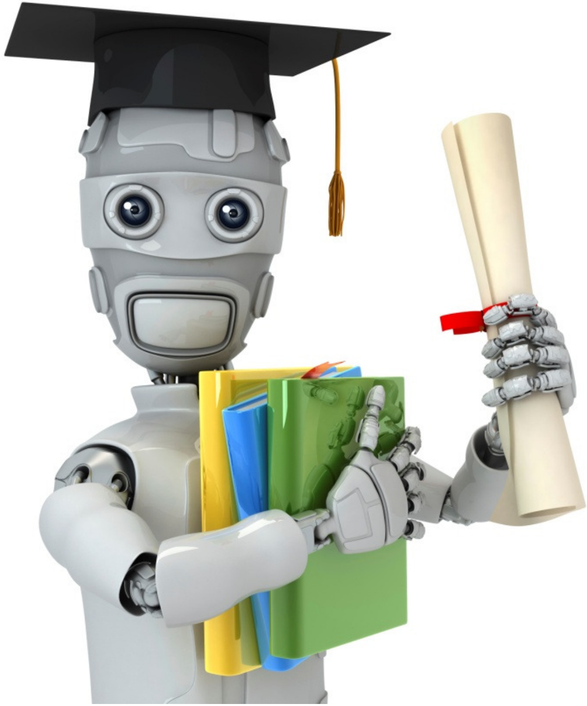
  E.g., Autonomous helicopter, handwriting recognition, most of Natural Language Processing (NLP), Computer Vision.
- Self-customizing programs
  E.g., Amazon, Netflix product recommendations
- Understanding human learning (brain, real AI).



Machine Learning

Introduction

What is machine learning

Machine Learning definition

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- Arthur Samuel (1959). Machine Learning: Field of study that gives computers the ability to learn without being explicitly programmed.

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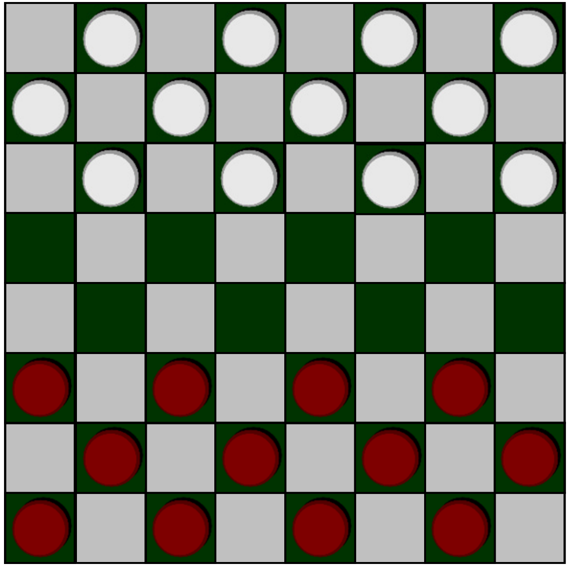

Machine Learning definition

- Arthur Samuel (1959). Machine Learning: Field of study that gives computers the ability to learn without being explicitly programmed.
- Tom Mitchell (1998) Well-posed Learning Problem: A computer program is said to *learn* from experience E with respect to some task T and some performance measure P, if its performance on T, as measured by P, improves with experience E.

"A computer program is said to *learn* from experience E with respect to some task T and some performance measure P, if its performance on T, as measured by P, improves with experience E."

Suppose your email program watches which emails you do or do not mark as spam, and based on that learns how to better filter spam. What is the task T in this setting?

- ◯ Classifying emails as spam or not spam. T
- ◯ Watching you label emails as spam or not spam. E
- ◯ The number (or fraction) of emails correctly classified as spam/not spam. P
- ◯ None of the above—this is not a machine learning problem.

“A computer program is said to *learn* from experience E with respect to

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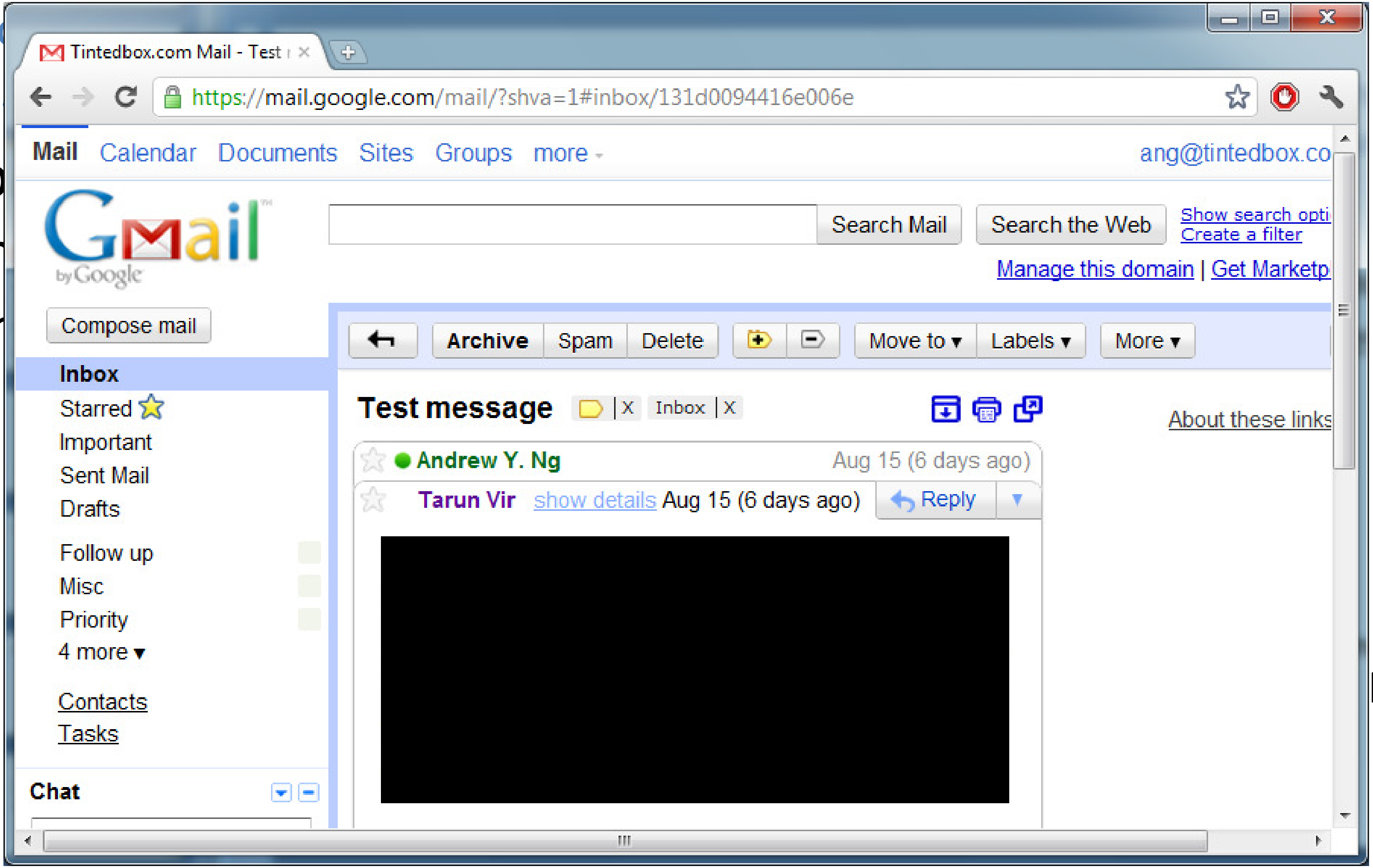

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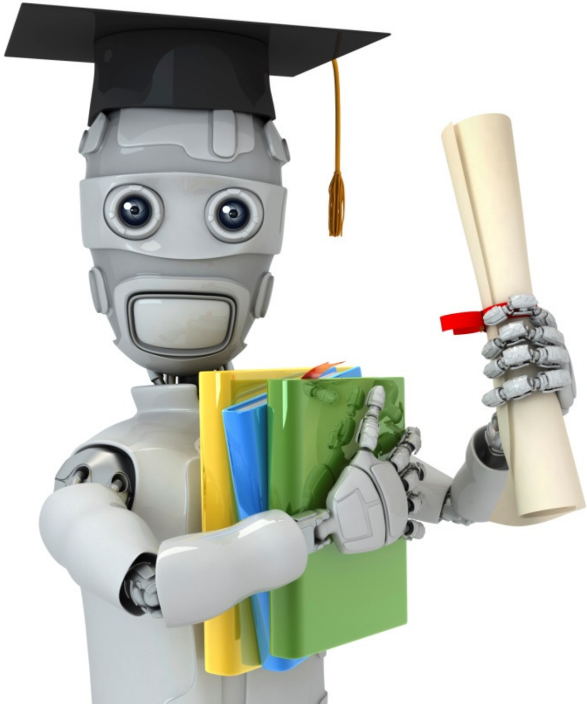
Machine learning algorithms:

- Supervised learning
- Unsupervised learning

Others: Reinforcement learning, recommender systems.

Also talk about: Practical advice for applying learning algorithms.

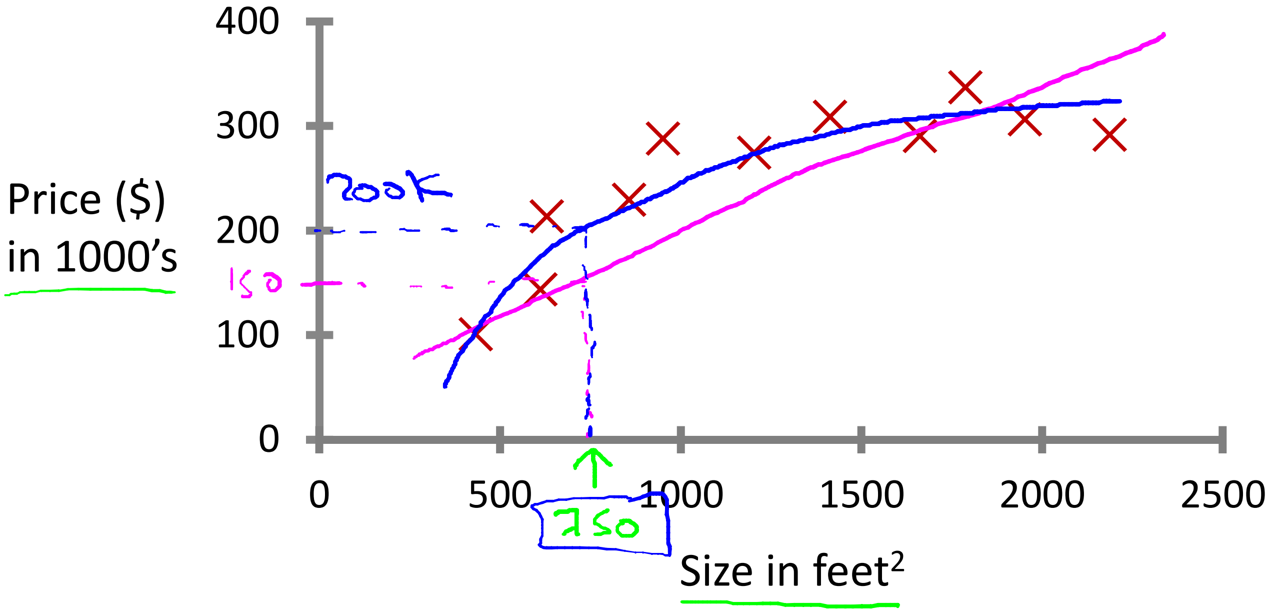


Machine Learning

Introduction

Supervised Learning

Housing price prediction.

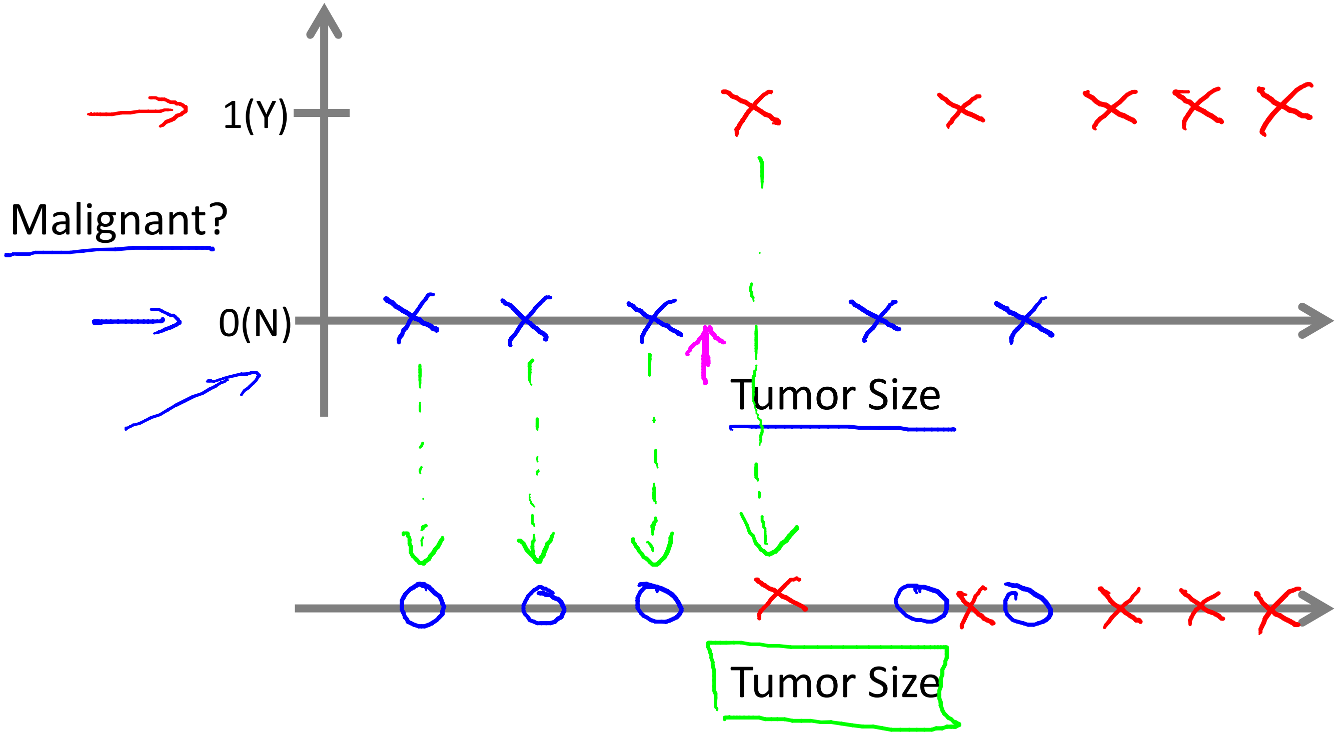


Supervised Learning

"right answers" given

Regression: Predict continuous valued output (price)

Breast cancer (malignant, benign)



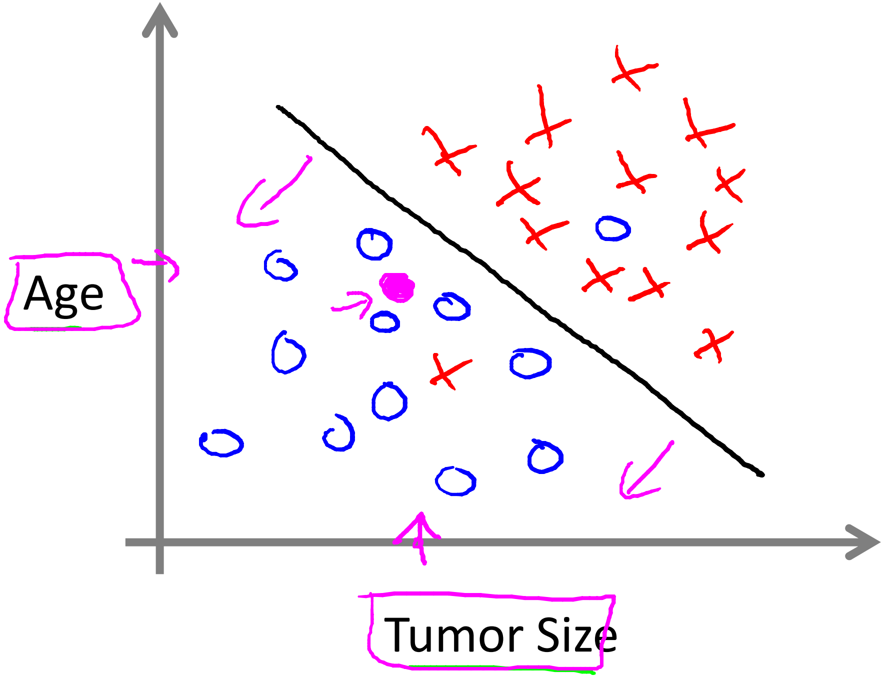
Classification

Discrete valued output (0 or 1)

0, 1, 2, 3

benign

type 1 cancer



- Clump Thickness
- Uniformity of Cell Size
- Uniformity of Cell Shape

…

You're running a company, and you want to develop learning algorithms to address each of two problems.

1000's

Problem 1: You have a large inventory of identical items. You want to predict how many of these items will sell over the next 3 months.

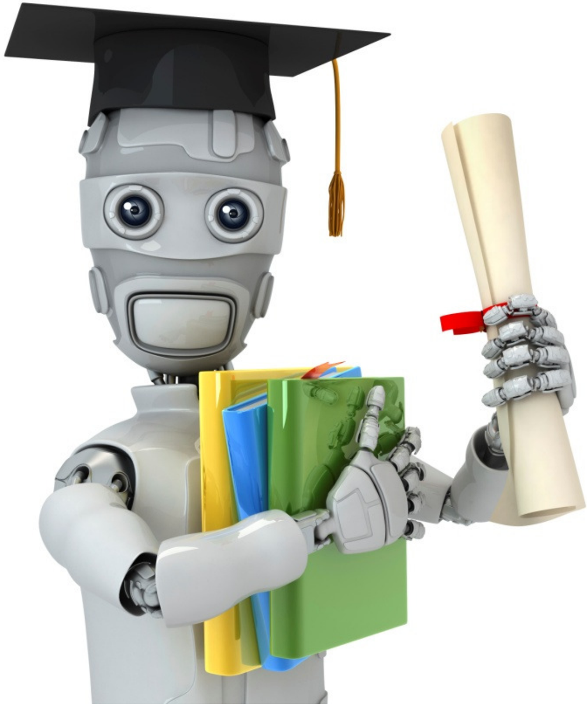
Problem 2: You'd like software to examine individual customer accounts, and for each account decide if it has been hacked/compromised.

0 - not hacked

1 - hacked

Should you treat these as classification or as regression problems?

- ○ Treat both as classification problems.
- ○ Treat problem 1 as a classification problem, problem 2 as a regression problem.
- ○ Treat problem 1 as a regression problem, problem 2 as a classification problem.
- ○ Treat both as regression problems.

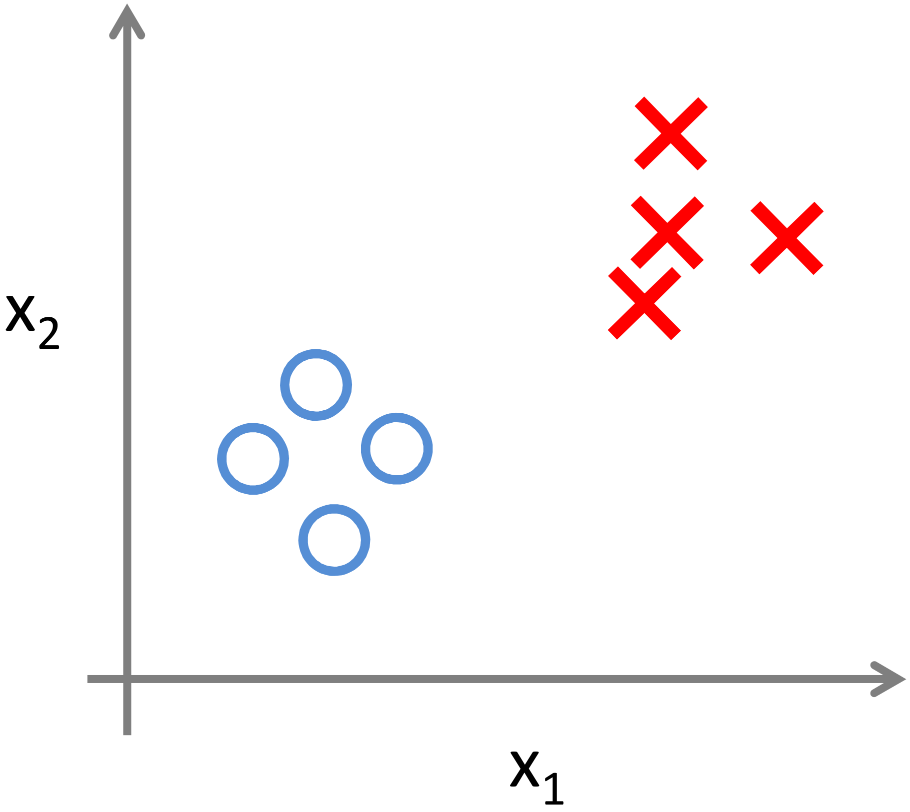


Machine Learning

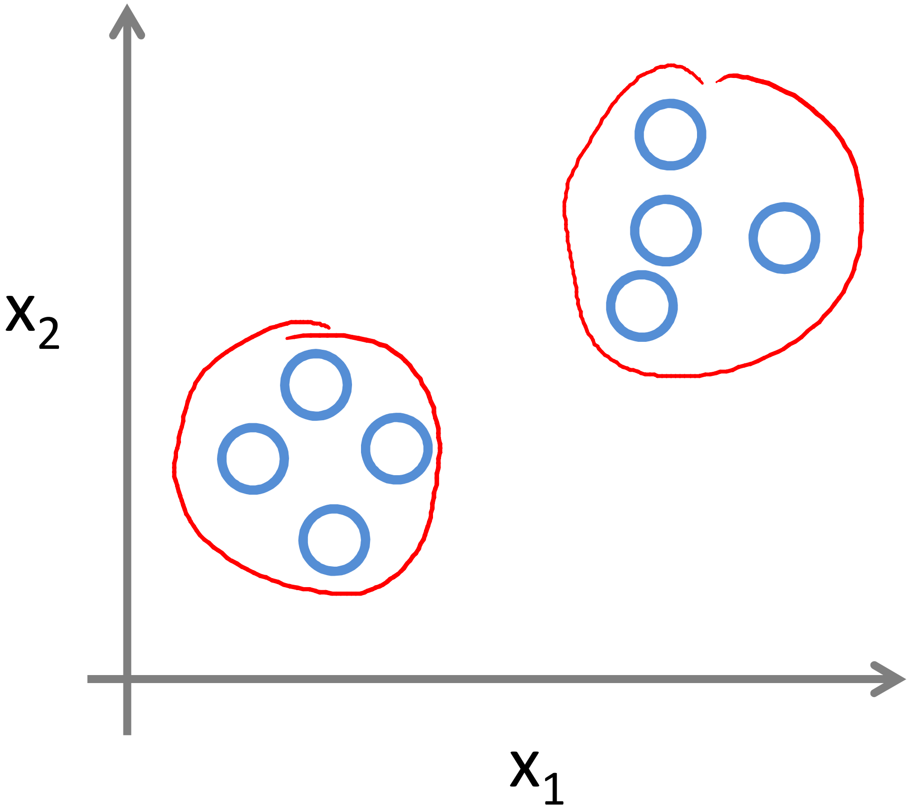
Introduction

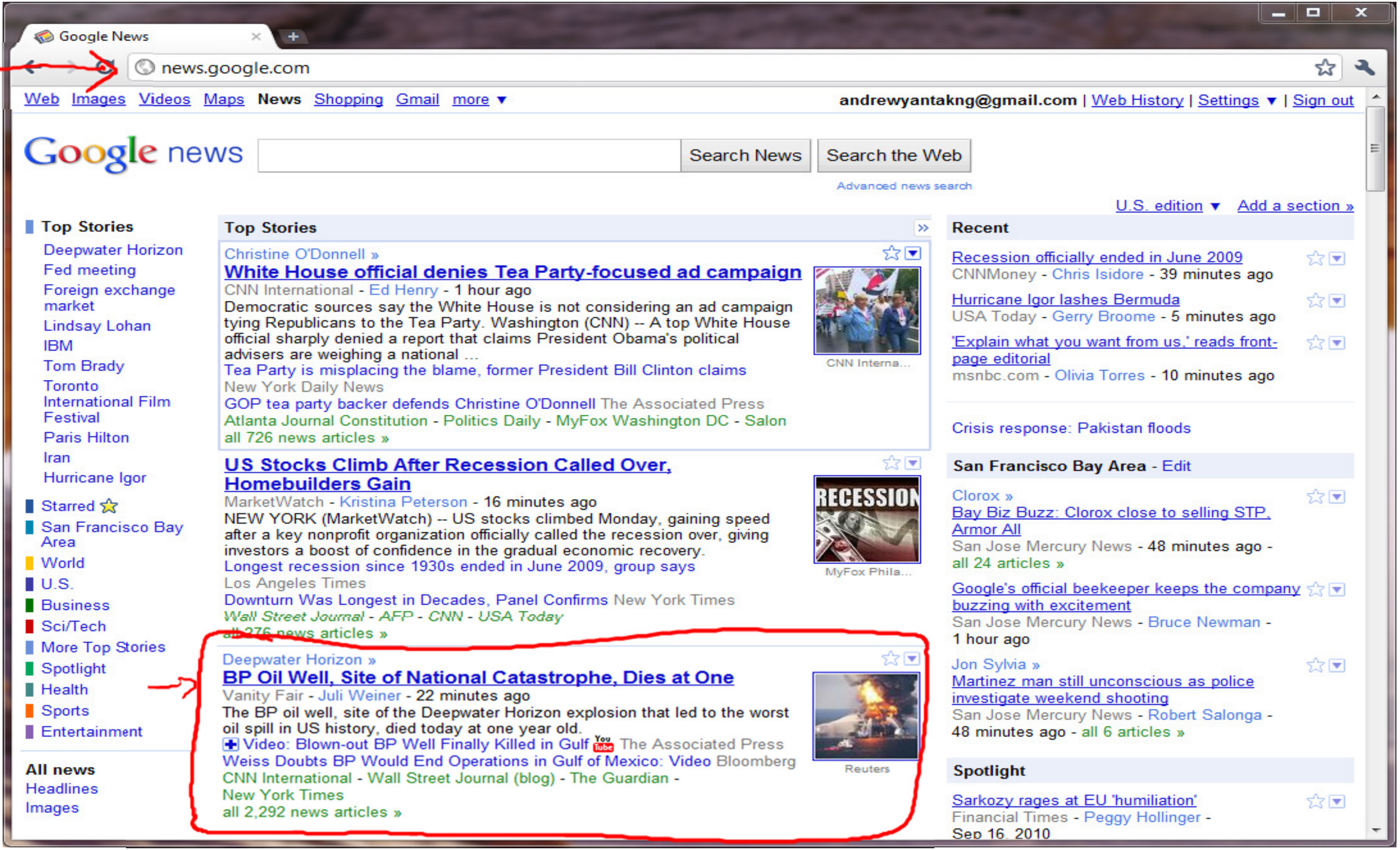
Unsupervised Learning

Supervised Learning



Unsupervised Learning

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Christine O'Donnell »
White House official denies Tea Party-focused ad campaign
CNN International - Ed Henry - 1 hour ago
Democratic sources say the White House is not considering an ad campaign tying Republicans to the Tea Party. Washington (CNN) -- A top White House official sharply denied a report that claims President Obama's political advisers are weighing a national ...
Tea Party is misplacing the blame, former President Bill Clinton claims
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GOP tea party backer defends Christine O'Donnell The Associated Press
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US Stocks Climb After Recession Called Over, Homebuilders Gain
MarketWatch - Kristina Peterson - 16 minutes ago
NEW YORK (MarketWatch) -- US stocks climbed Monday, gaining speed after a key nonprofit organization officially called the recession over, giving investors a boost of confidence in the gradual economic recovery.
Longest recession since 1930s ended in June 2009, group says
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Downturn Was Longest in Decades, Panel Confirms New York Times
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BP Oil Well, Site of National Catastrophe, Dies at One
Vanity Fair - Juli Weiner - 22 minutes ago
The BP oil well, site of the Deepwater Horizon explosion that led to the worst oil spill in US history, died today at one year old.
Video: Blown-out BP Well Finally Killed in Gulf The Associated Press
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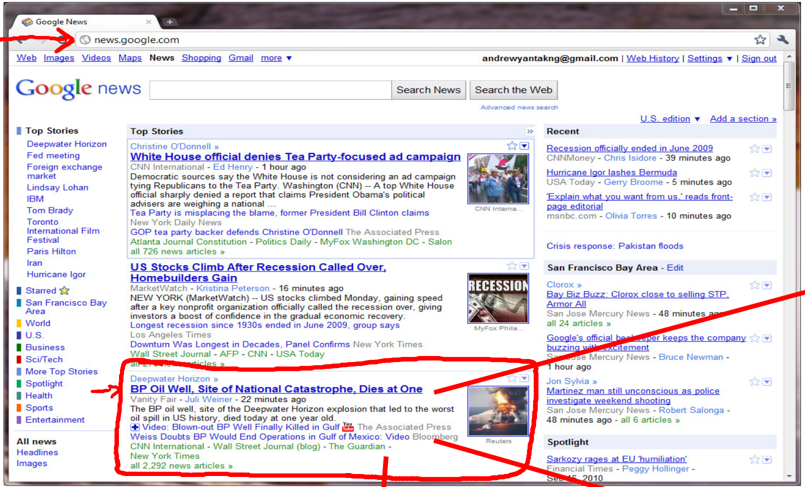
Spotlight
Sarkozy rages at EU 'humiliation'
Financial Times - Peggy Hollinger -
Sep 16, 2010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White House official denies Tea Party-focused ad campaign
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Democratic sources say the White House is not considering an ad campaign tying Republicans to the Tea Party. Washington (CNN) -- A top White House official sharply denied a report that claims President Obama's political advisers are weighing a national ...
Tea Party is misplacing the blame, former President Bill Clinton claims
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GOP tea party backer defends Christine O'Donnell The Associated Press
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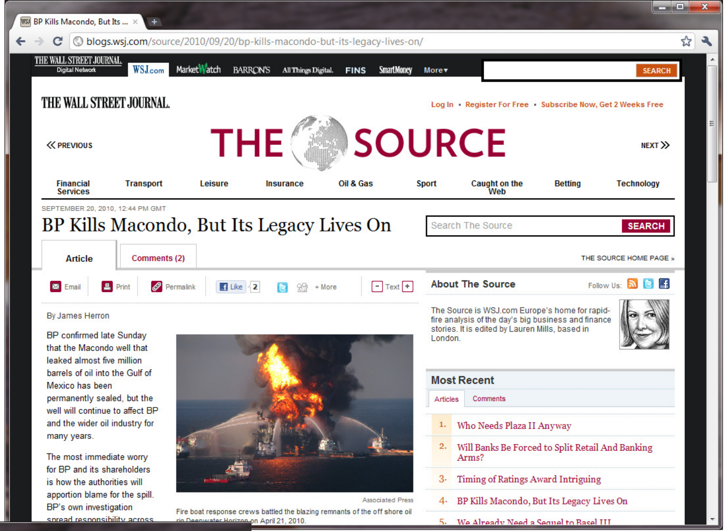
BP Kills Macondo, But Its ...
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BP Kills Macondo, But Its Legacy Lives On
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By James Herron
BP confirmed late Sunday that the Macondo well that leaked almost five million barrels of oil into the Gulf of Mexico has been permanently sealed, but the well will continue to affect BP and the wider oil industry for many years.
The most immediate worry for BP and its shareholders is how the authorities will apportion blame for the spill. BP's own investigation
Associated Press
Fire boat response crews battled the blazing remnants of the off shore oil rig Deepwater Horizon on April 21, 2010.
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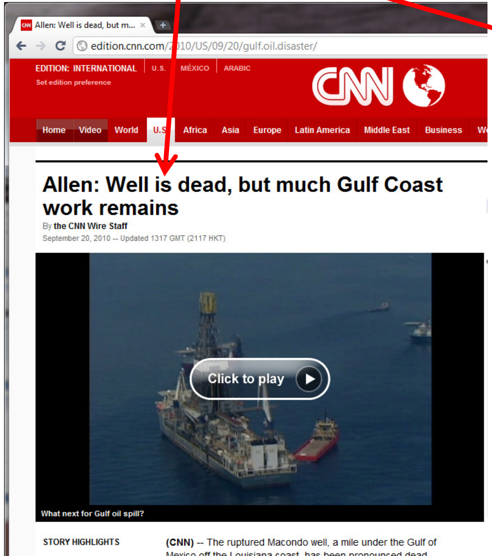
Allen: Well is dead, but m...
edition.cnn.com/2010/US/09/20/gulf.oil.disaster/
EDITION: INTERNATIONAL
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MÉXICO
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Set edition preference
CNN
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Allen: Well is dead, but much Gulf Coast work remains
By the CNN Wire Staff
September 20, 2010 -- Updated 1317 GMT (2117 HKT)
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What next for Gulf oil spill?
STORY HIGHLIGHTS
(CNN) -- The ruptured Macondo well, a mile under the Gulf of


BP oil spill cost hits nearl...
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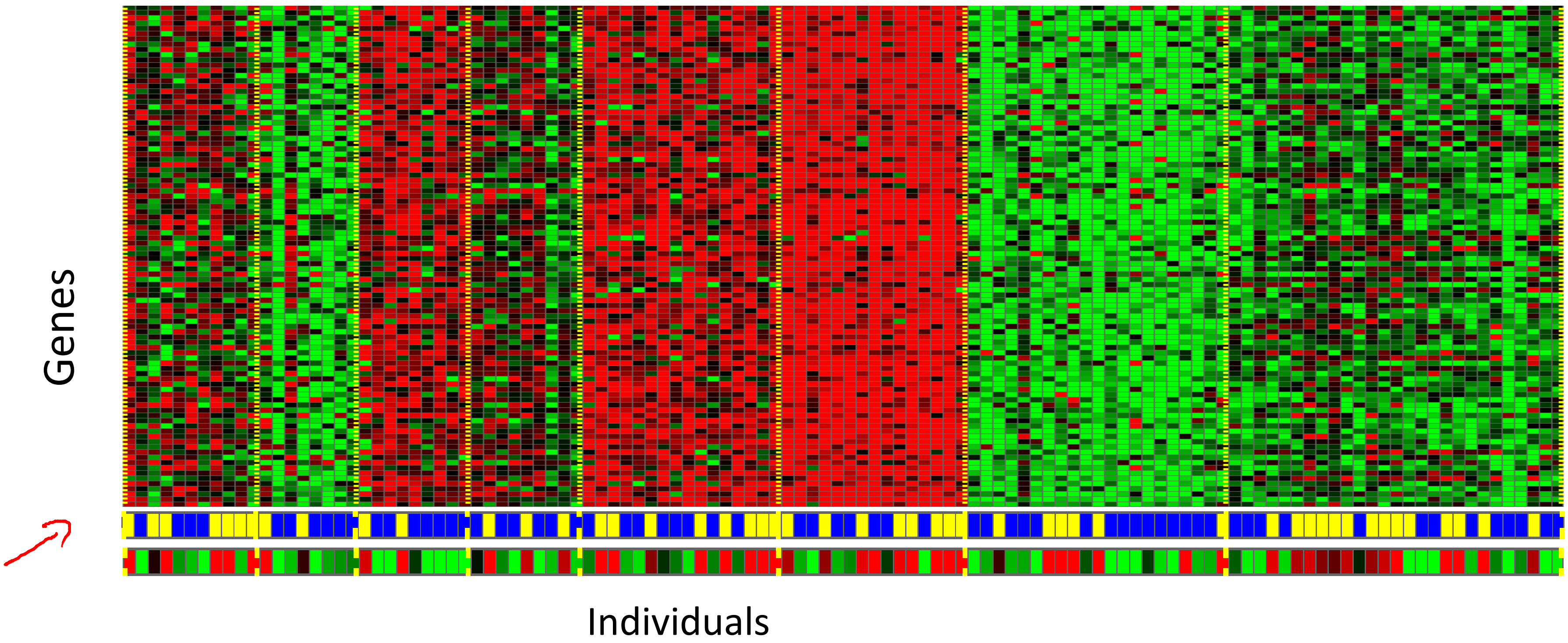
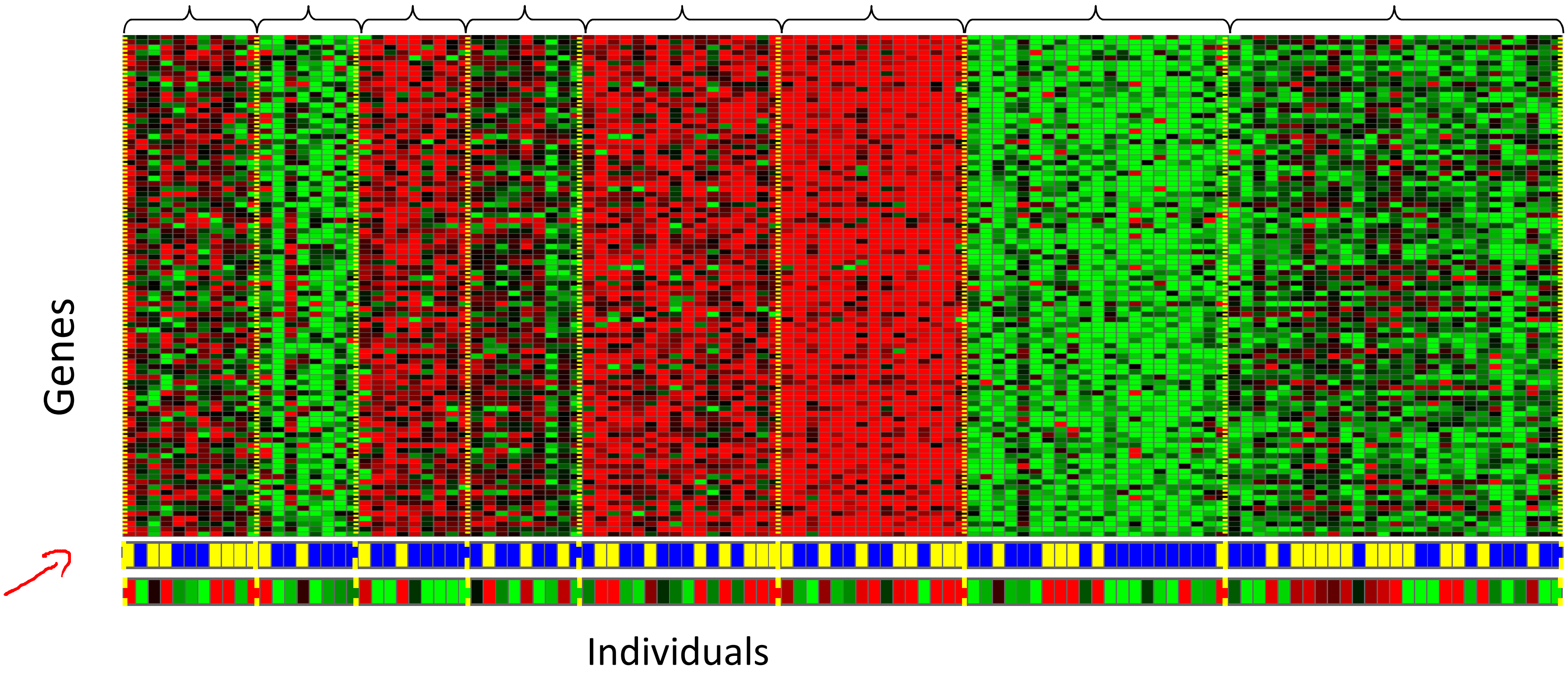
Business BP
BP oil spill cost hits nearly $10bn
BP has set up a $20bn compensation fund after the Deepwater Horizon disaster, which has so far paid out 19,000 claims totalling more than $240m
Julia Kollewe
guardian.co.uk, Monday 20 September 2010 08.33 BST
Article history
BP's costs for the Deepwater Horizon disaster have hit $10bn. Photograph: Ho/Reuters

Andrew Ng

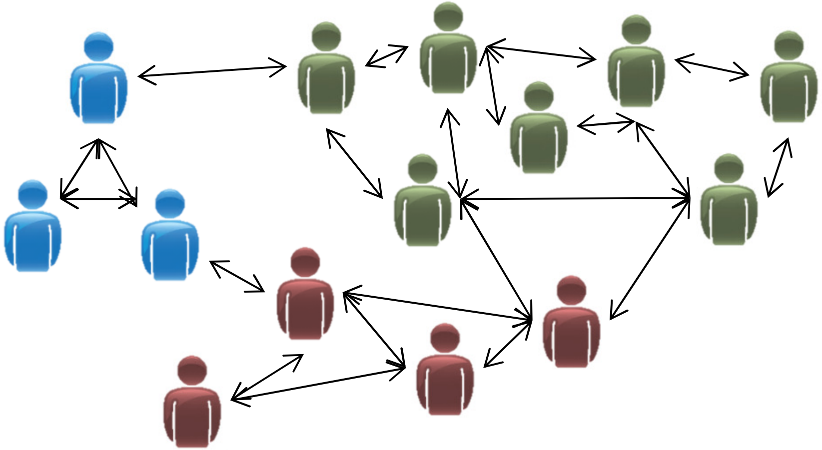


[Source: Daphne Koller]

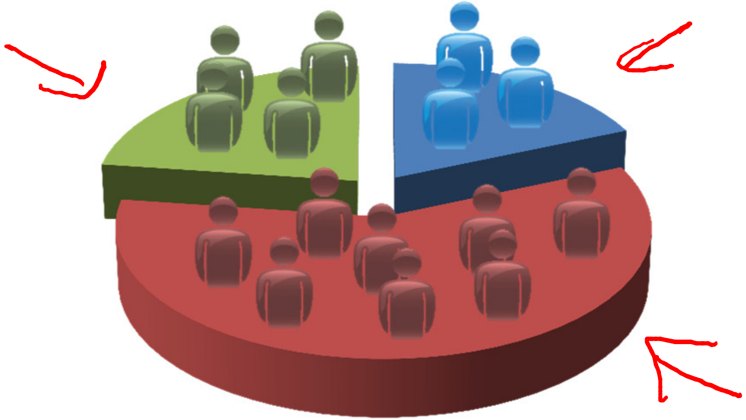


[Source: Daphne Koller]

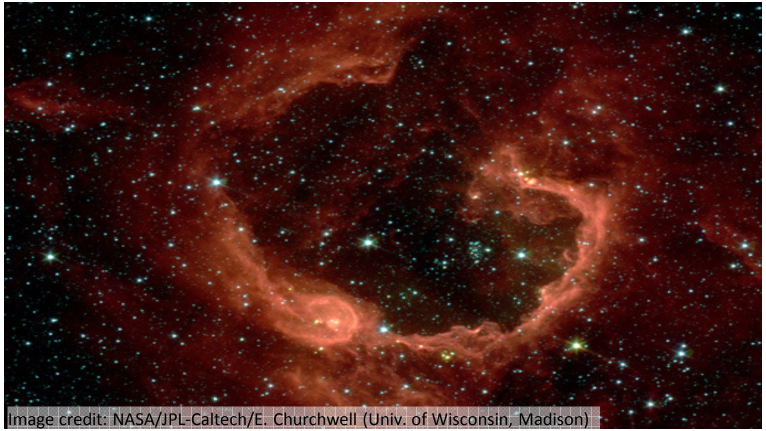
Organize computing clusters



Social network analysis

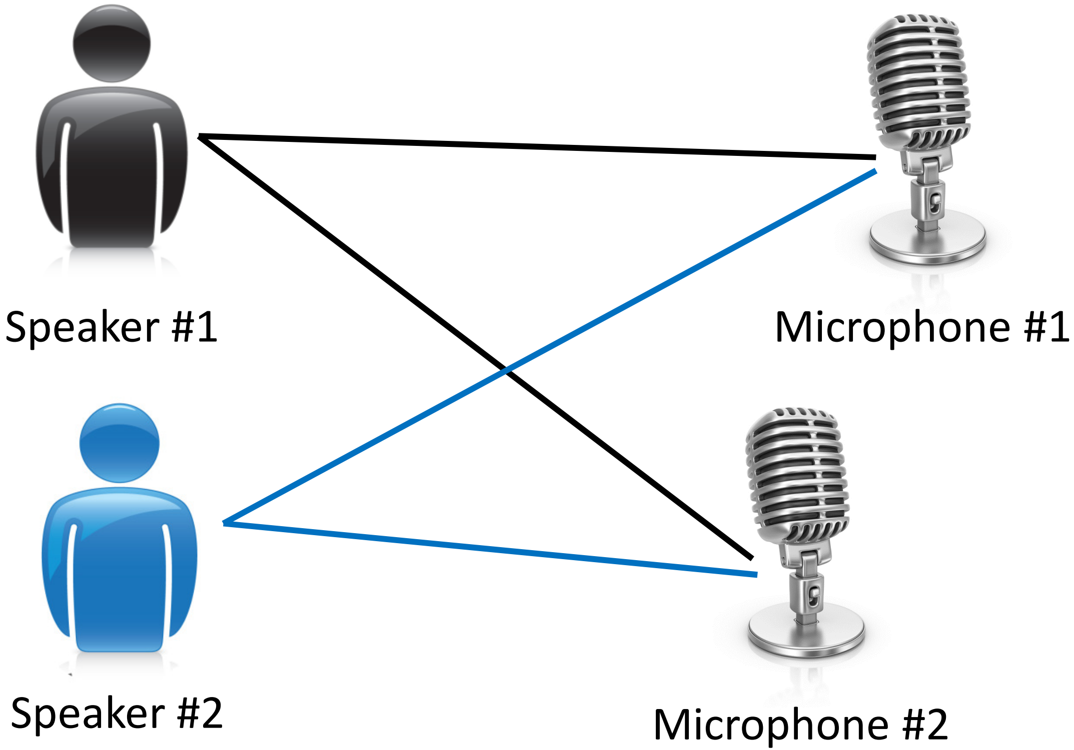


Market segmentation



Astronomical data analysis

Cocktail party problem



Microphone #1:

Output #1:

Microphone #2:

Output #2:

Microphone #1:

Output #1:

Microphone #2:

Output #2:

[Audio clips courtesy of Te-Won Lee.]

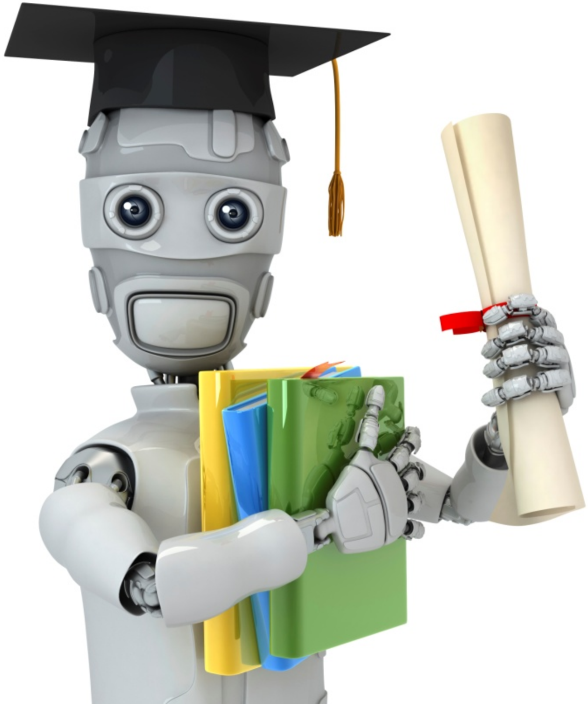
Cocktail party problem algorithm

```
[W,s,v] = svd((repmat(sum(x.*x,1),size(x,1),1).*x)*x');
```

[Source: Sam Roweis, Yair Weiss & Eero Simoncelli]

Of the following examples, which would you address using an <u>unsupervised</u> learning algorithm? (Check all that apply.)

- [ ] Given email labeled as spam/not spam, learn a spam filter.
- [ ] Given a set of news articles found on the web, group them into set of articles about the same story.
- [ ] Given a database of customer data, automatically discover market segments and group customers into different market segments.
- [ ] Given a dataset of patients diagnosed as either having diabetes or not, learn to classify new patients as having diabetes or not.

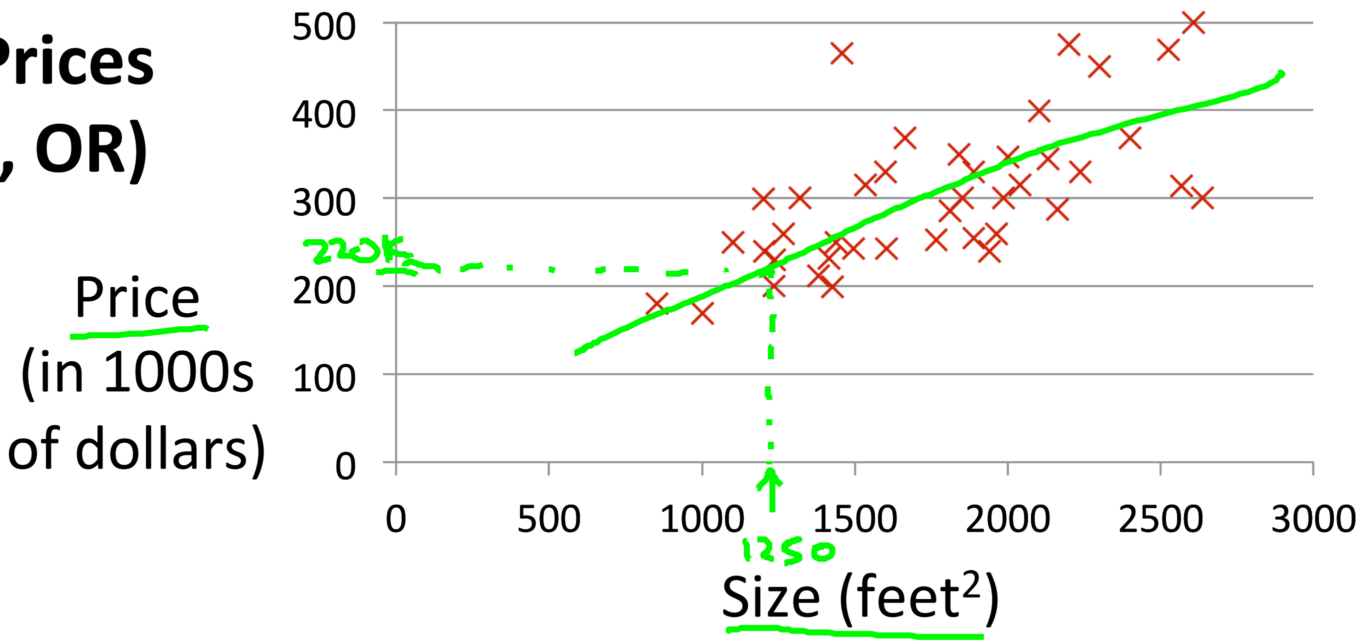


Machine Learning

Linear regression with one variable

Model representation

Housing Prices (Portland, OR)



Supervised Learning

Given the "right answer" for each example in the data.

Regression Problem

Predict real-valued output

Classification: Discrete-valued output

Training set of housing prices (Portland, OR)

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

m = 47

Notation:

- **m** = Number of training examples
- **x**'s = "input" variable / features
- **y**'s = "output" variable / "target" variable

(x, y) - one training example

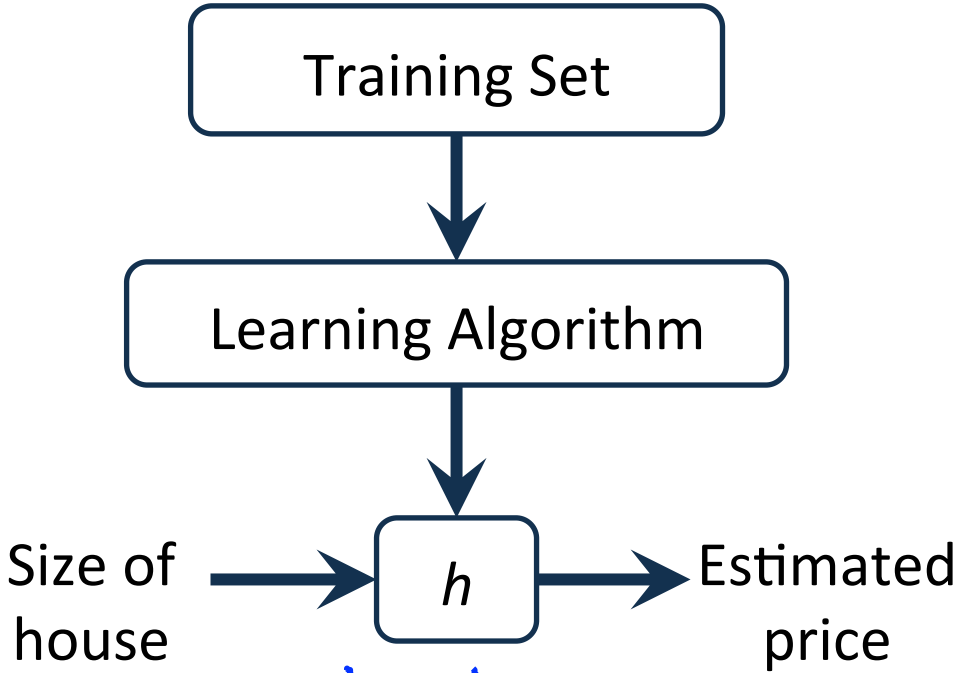
$(x^{(i)}, y^{(i)})$ - i^{th} training example

$x^{(1)} = 2104$

$x^{(2)} = 1416$

$y^{(1)} = 460$

Andrew Ng



x

hypothesis

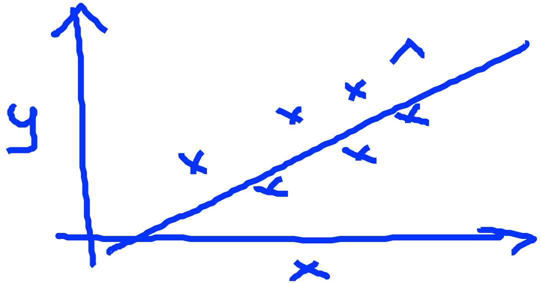
(estimated value of y)

h maps from x's to y's.

How do we represent *h* ?

$h_\theta(x) = \underline{\theta_0 + \theta_1 x}$

Shorthand: $h(x)$



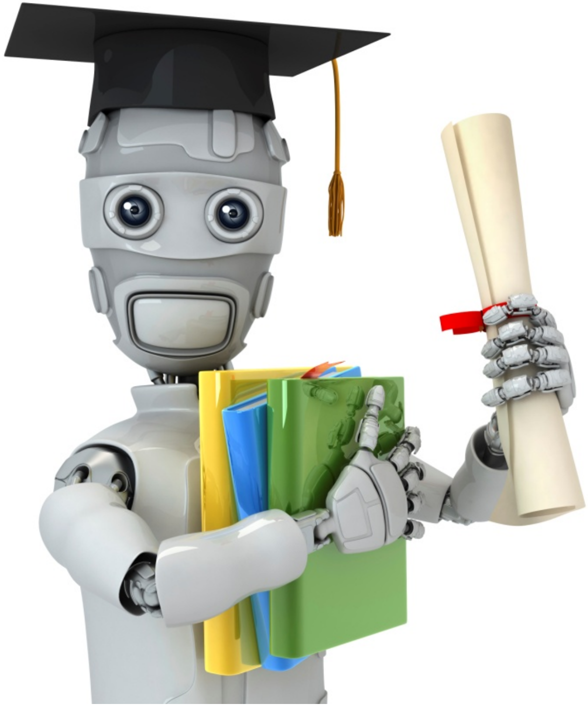
$h(x) = \theta_0 + \theta_1 x$

Linear regression with one variable. (x)

Univariate linear regression.

└ one variable

Andrew Ng



Machine Learning

Linear regression with one variable

Cost function

Training Set

Size in feet^2 (x)	**Price ($) in 1000's (y)**
2104	460
1416	232
1534	315
852	178
…	…

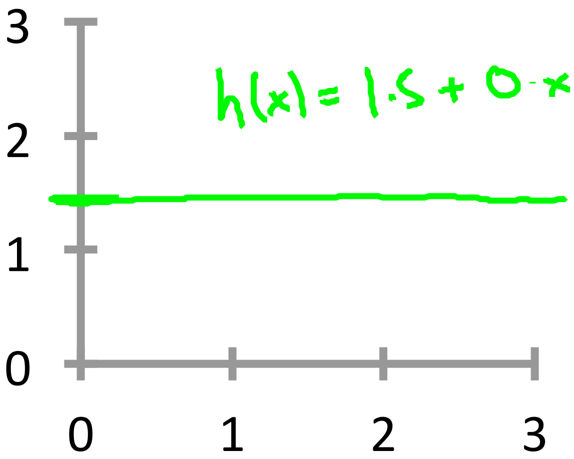
$m = 47$

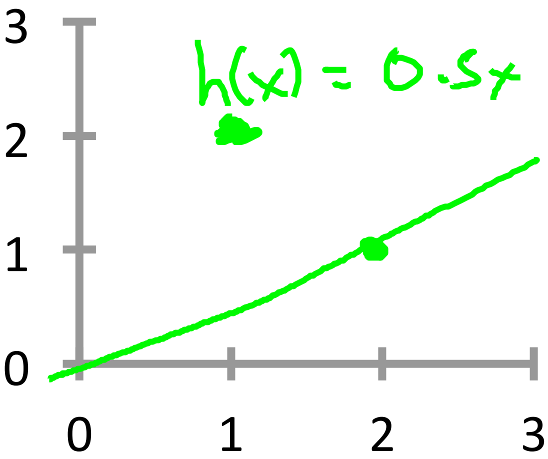
Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

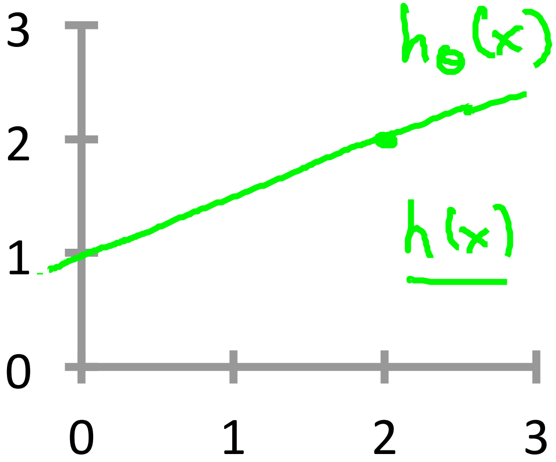
θ_i's: Parameters

How to choose θ_i's ?

$$h_\theta(x) = \theta_0 + \theta_1 x$$



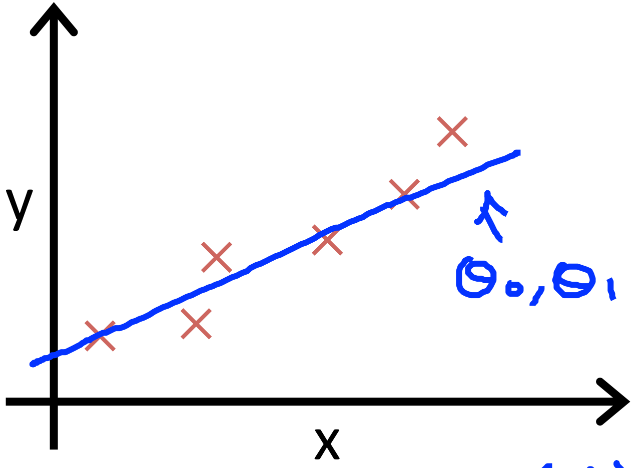




$\theta_0 = 1.5$
$\theta_1 = 0$

$\theta_0 = 0$
$\theta_1 = 0.5$

$\theta_0 = 1$
$\theta_1 = 0.5$



$(x^{(i)}, y^{(i)})$

training examples

$$\boxed{\underset{\theta_0, \theta_1}{\text{minimize}}} \ \frac{1}{2m} \sum_{i=1}^{m} \underbrace{\left(h_\theta(x^{(i)}) - y^{(i)} \right)^2}$$

$$h_\theta(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$$

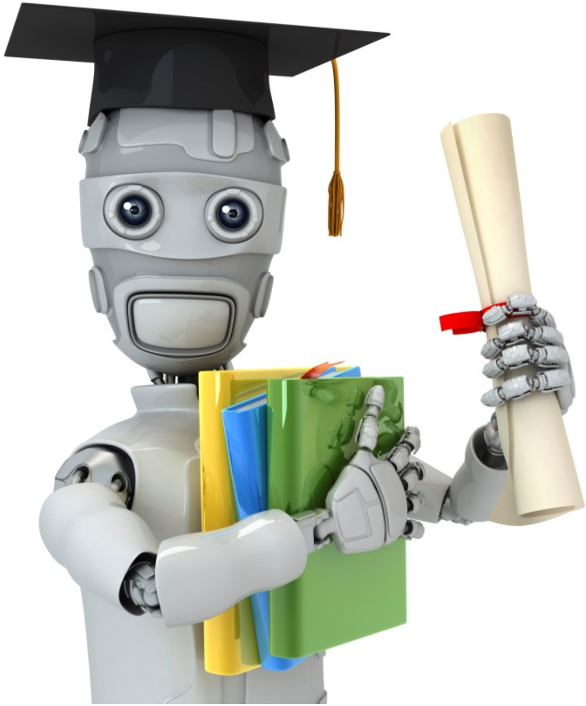
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$\underset{\theta_0, \theta_1}{\text{minimize}} \ \underbrace{J(\theta_0, \theta_1)}_{\text{Cost function}}$$

Squared error function

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ is close to y for our training examples (x, y)

x, y



Machine Learning

Linear regression with one variable

Cost function intuition I

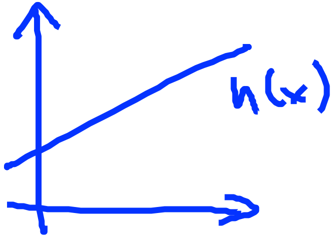
Simplified

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters:

$$\theta_0, \theta_1$$



Cost Function:

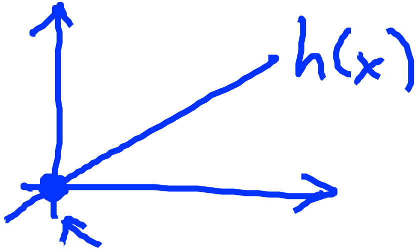
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

$$h_\theta(x) = \theta_1 x$$

$\theta_0 = 0$

$$\theta_1$$



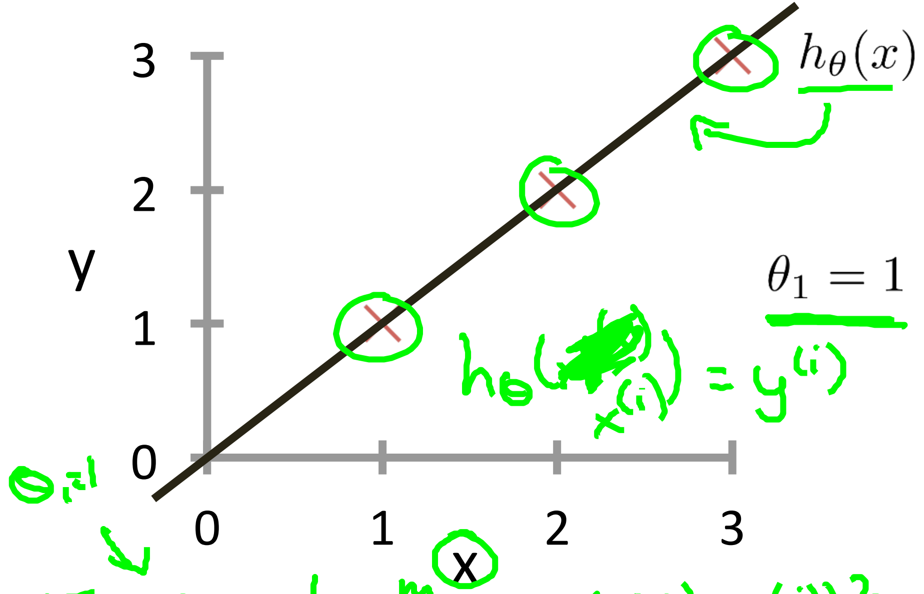
$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$\theta, x^{(i)}$

$$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$$

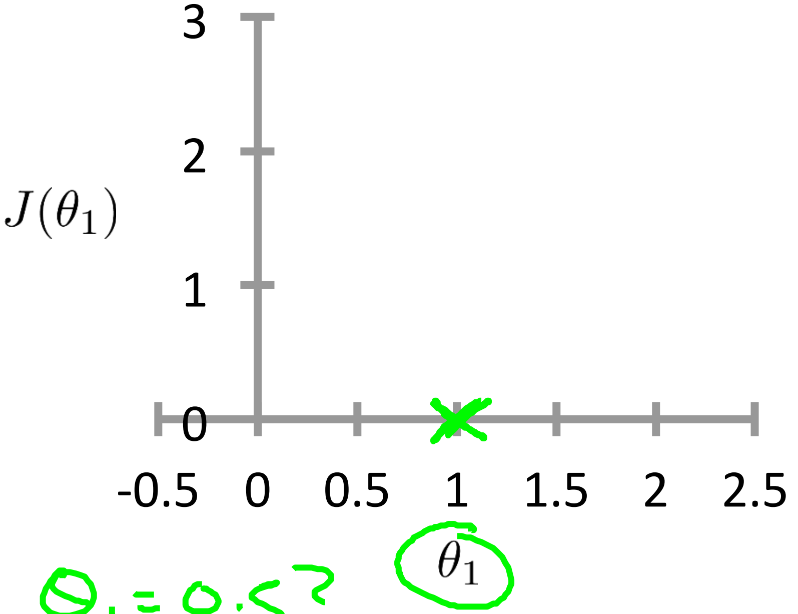
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameter θ_1)



$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$= \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_1 x^{(i)} - y^{(i)}\right)^2 = \frac{1}{2m}\left(0^2 + 0^2 + 0^2\right) = 0$$

$\theta_1 = 0.5$?

$J(1) = 0$

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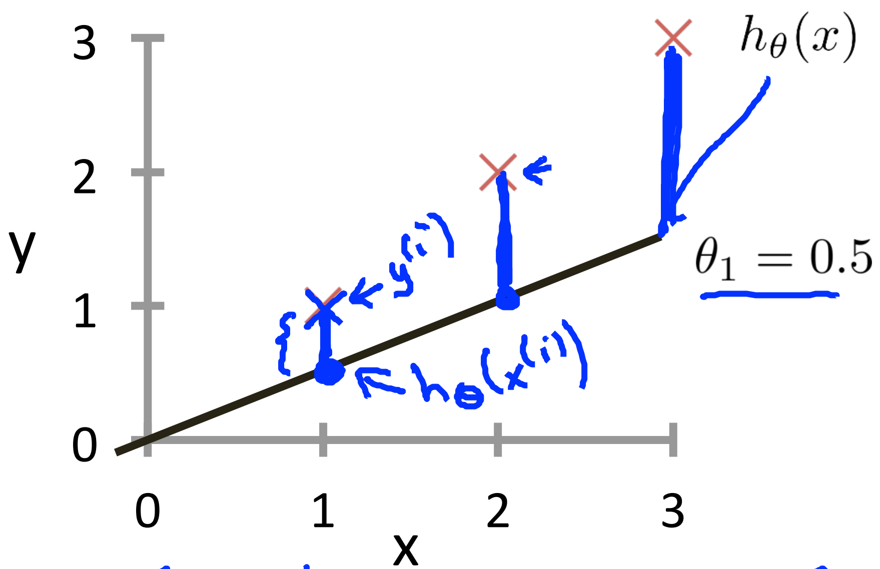
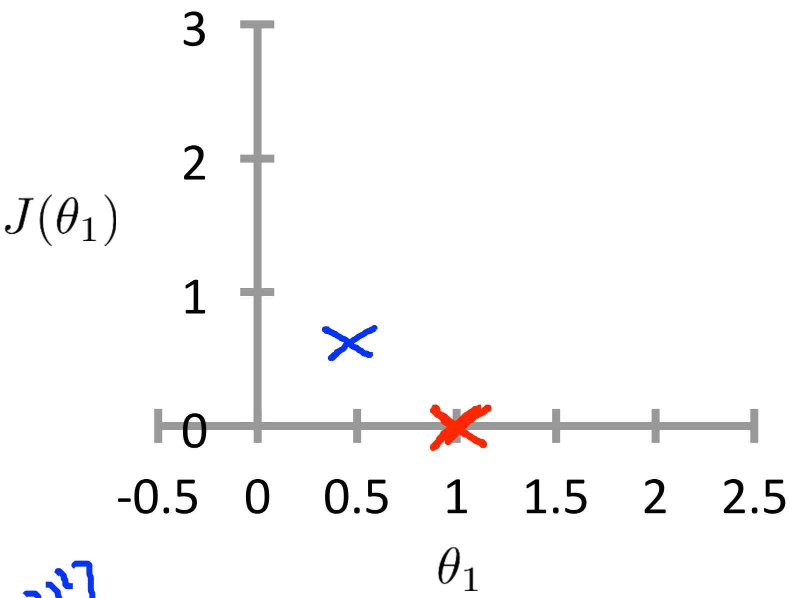
$h_\theta(x)$	$J(\theta_1)$
(for fixed θ_1, this is a function of x)	(function of the parameter θ_1)





$$J(0.5) = \frac{1}{2m}\left[(0.5-1)^2 + (1-2)^2 + (1.5-3)^2\right]$$

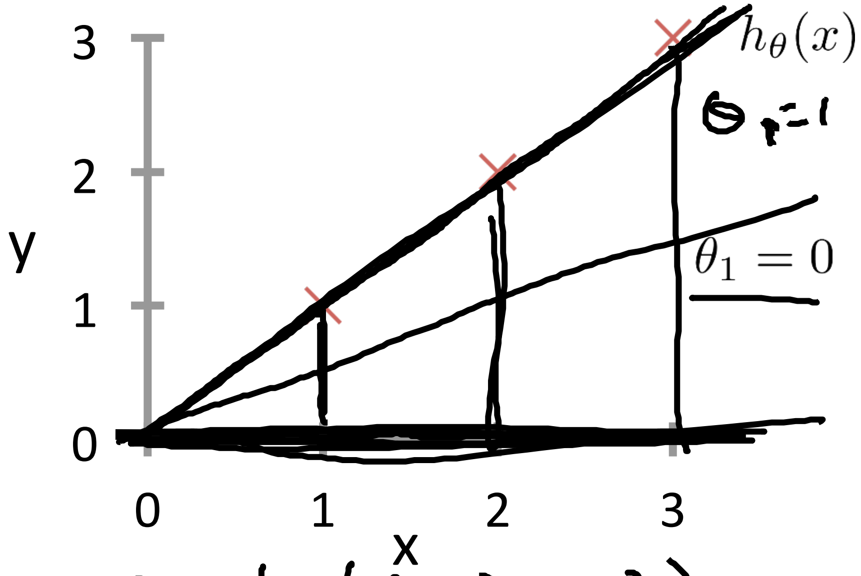
$$= \frac{1}{2\times 3}(3.5) = \frac{3.5}{6} \approx 0.58$$

$\theta_1 = 0$?

$J(0) = ?$

$h_\theta(x)$

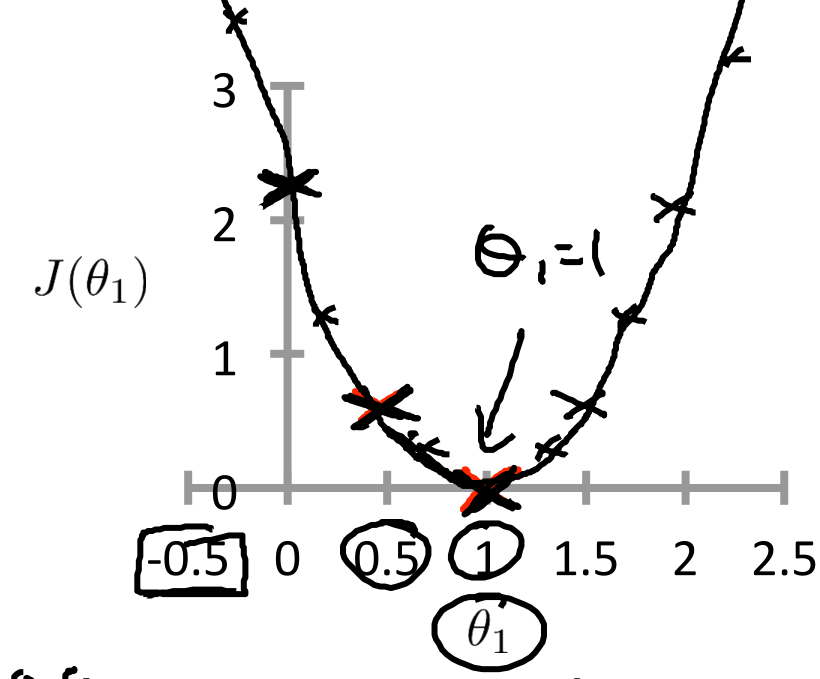
(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameter θ_1)

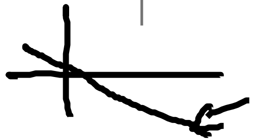
5.25



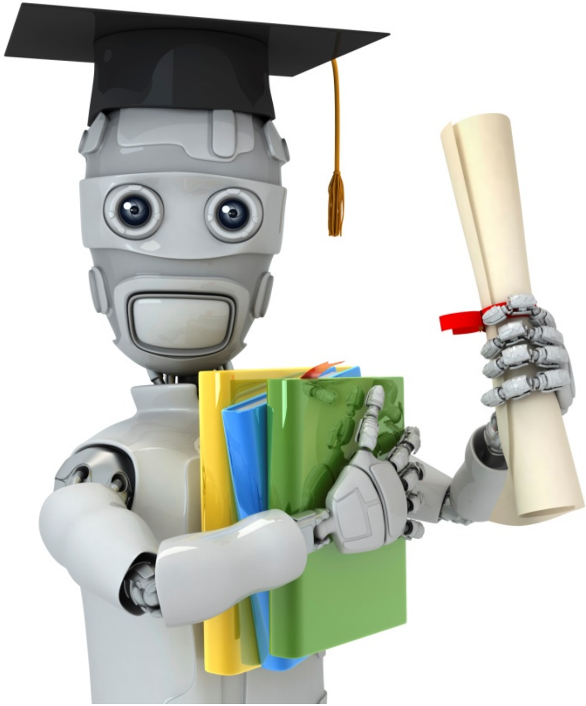
$J(0) = \frac{1}{2m}(1^2 + 2^2 + 3^2)$

$= \frac{1}{6} \cdot 14 \approx 2.3$

$h(x) = -0.5x$



$\boxed{\underset{\theta_1}{\text{minimize}}\ J(\theta_1)}$



Machine Learning

Linear regression with one variable

Cost function intuition II

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

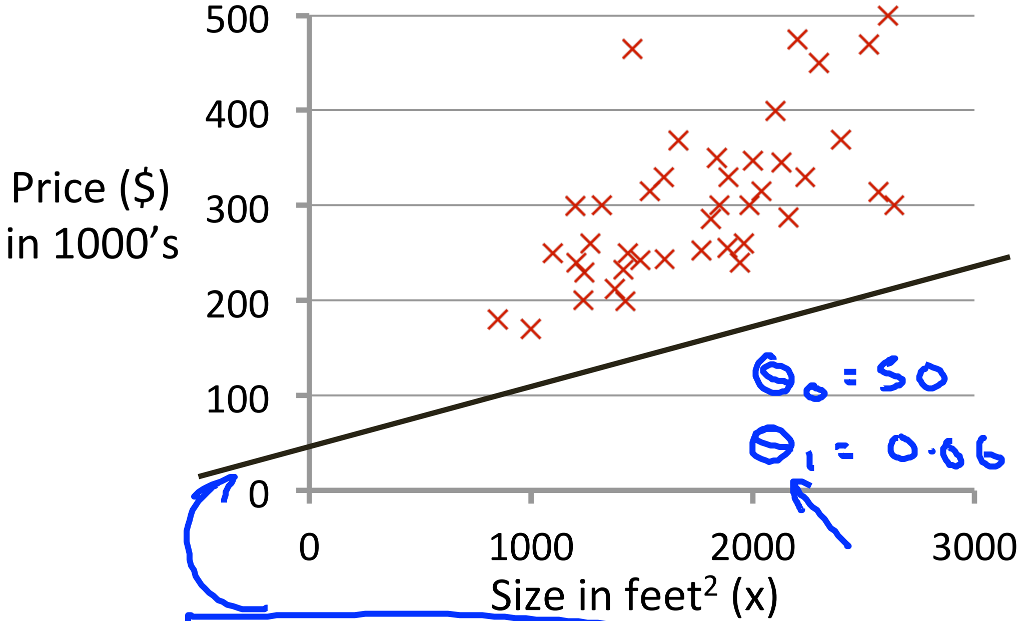
Parameters: $\underline{\theta_0, \theta_1}$

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

$h_\theta(x)$

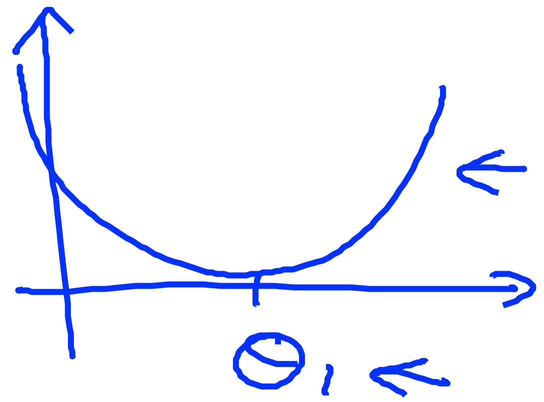
(for fixed θ_0, θ_1, this is a function of x)



$$\boxed{h_\theta(x) = 50 + 0.06x}$$

$J(\theta_0, \theta_1)$

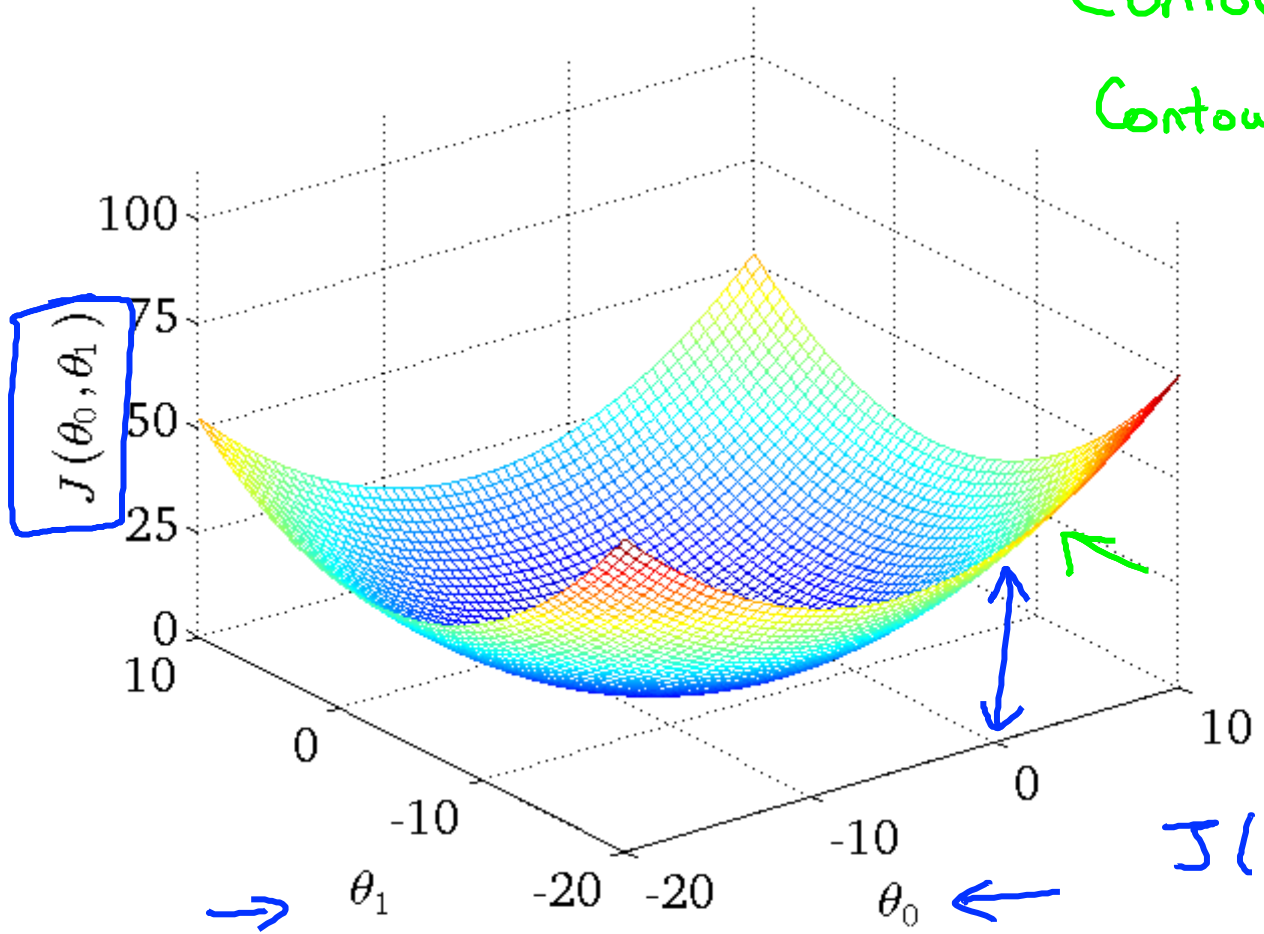
(function of the parameters θ_0, θ_1)



$$\boxed{\theta_0, \theta_1}$$

Contour plots

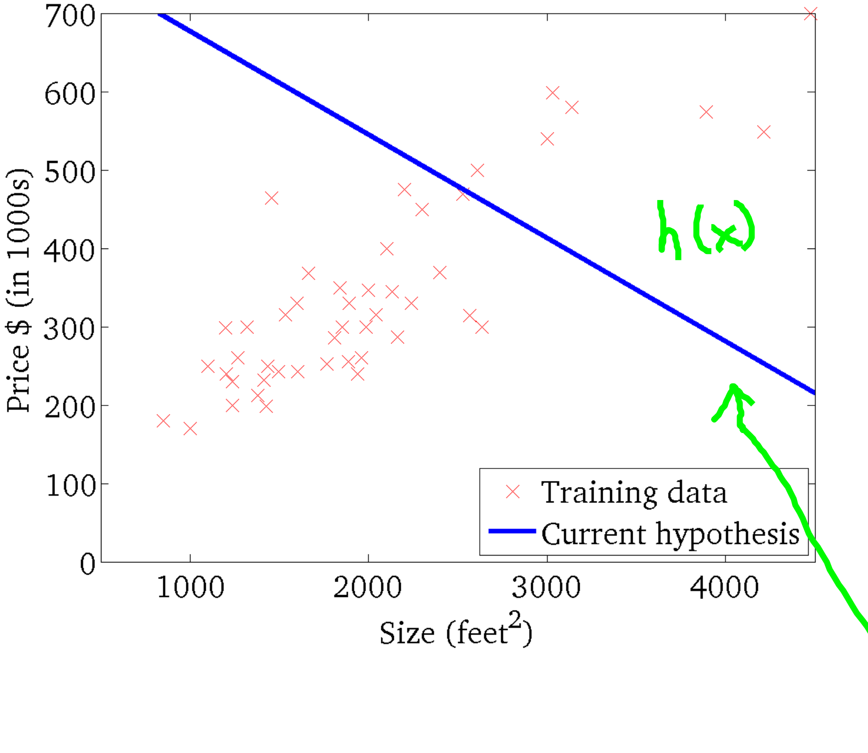
Contour figures.



$J(\theta_0, \theta_1)$

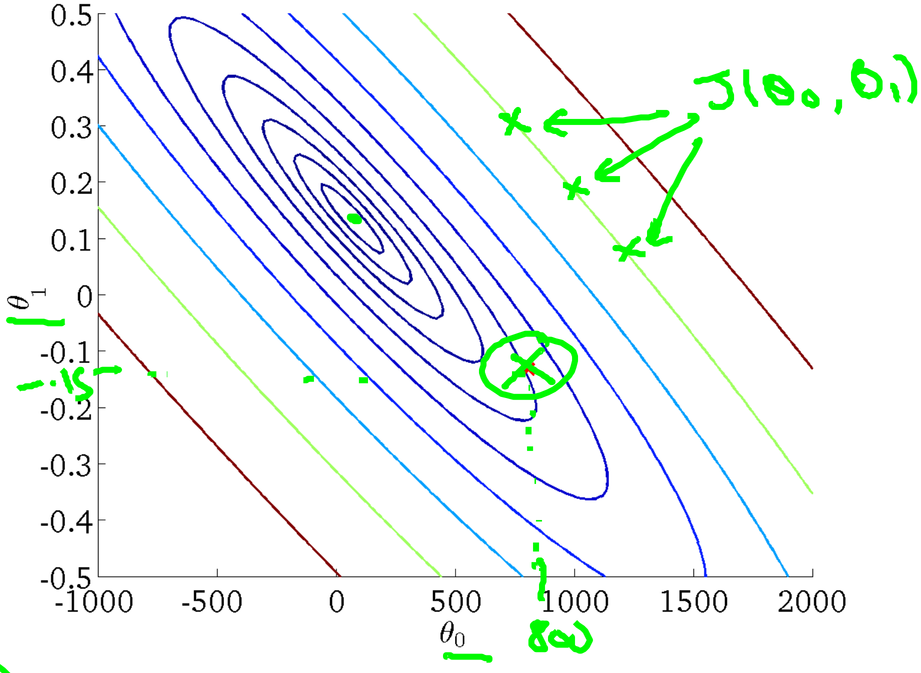
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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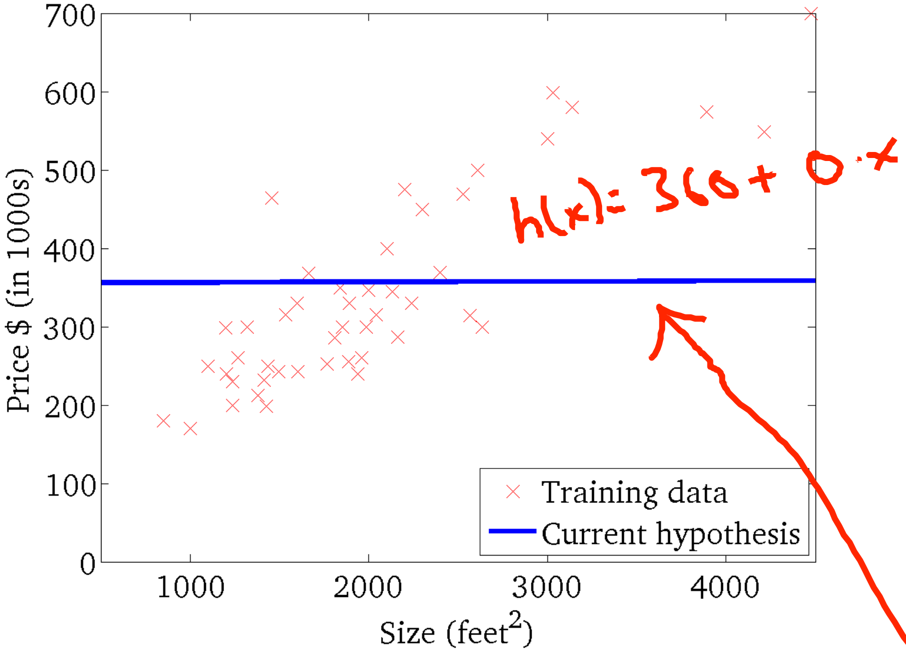
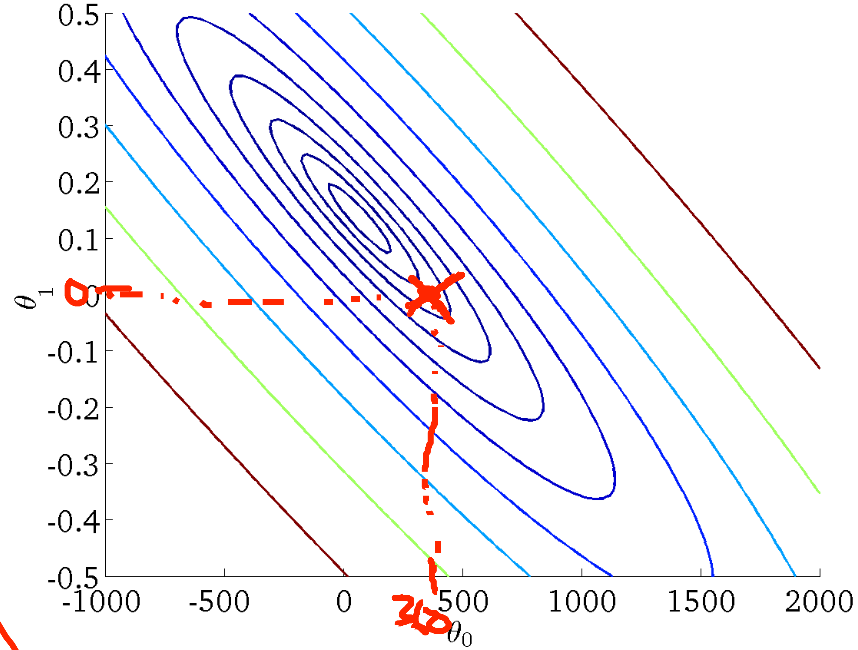
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



$\theta_0 = 360$
$\theta_1 = 0$

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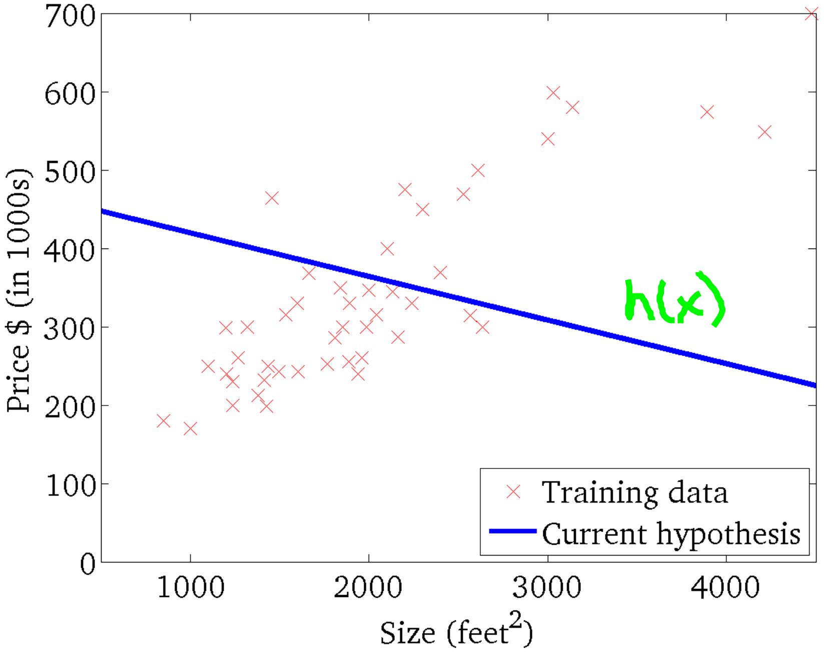
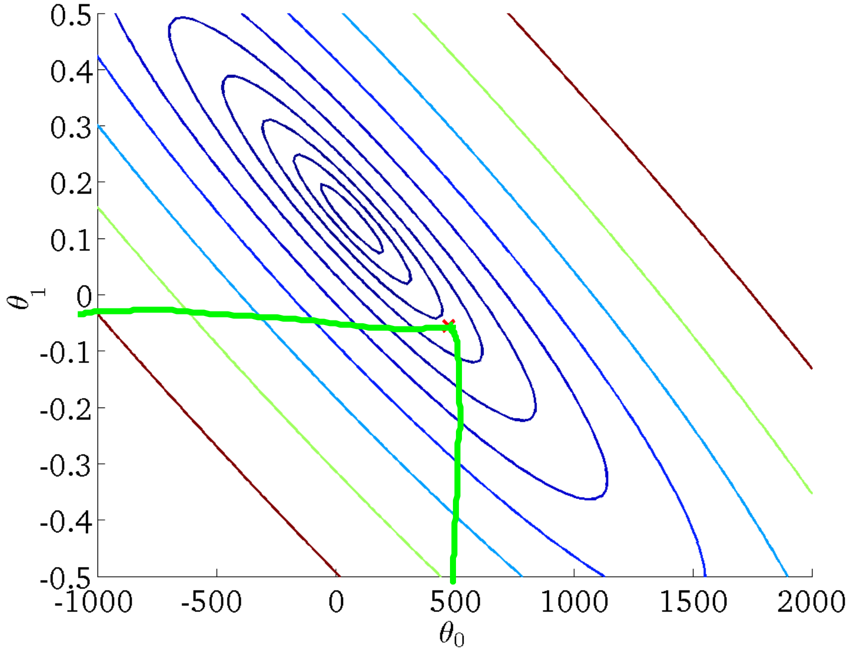
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



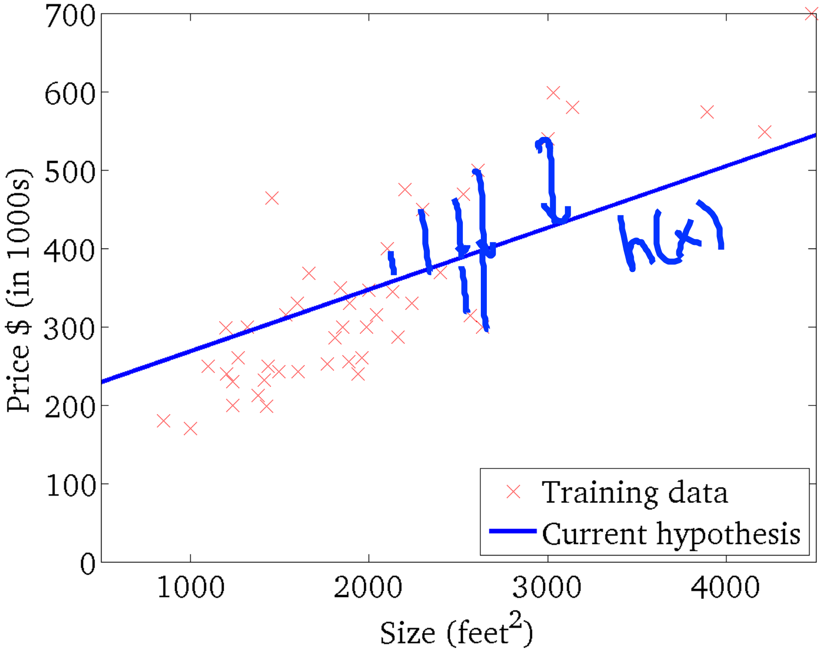
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



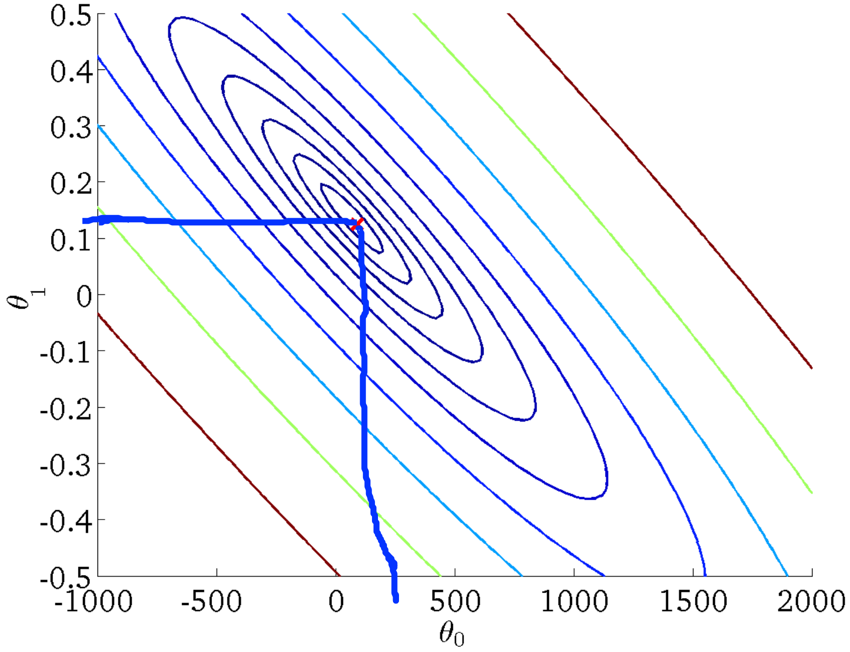
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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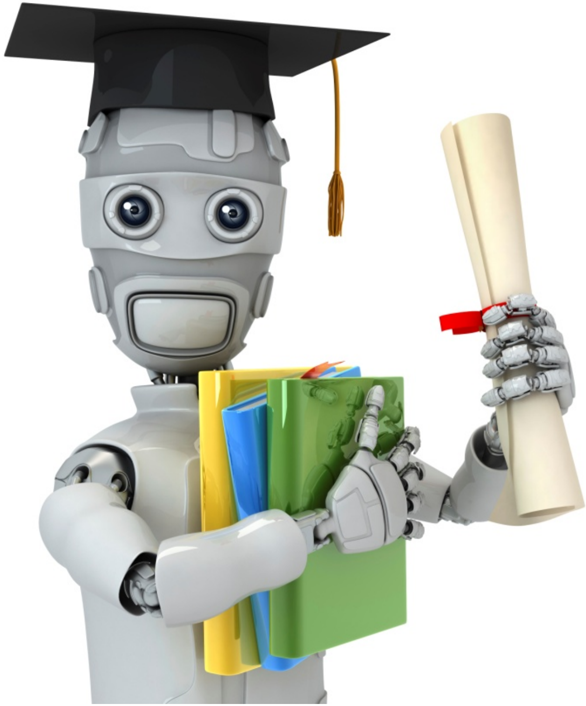


Machine Learning

Linear regression with one variable

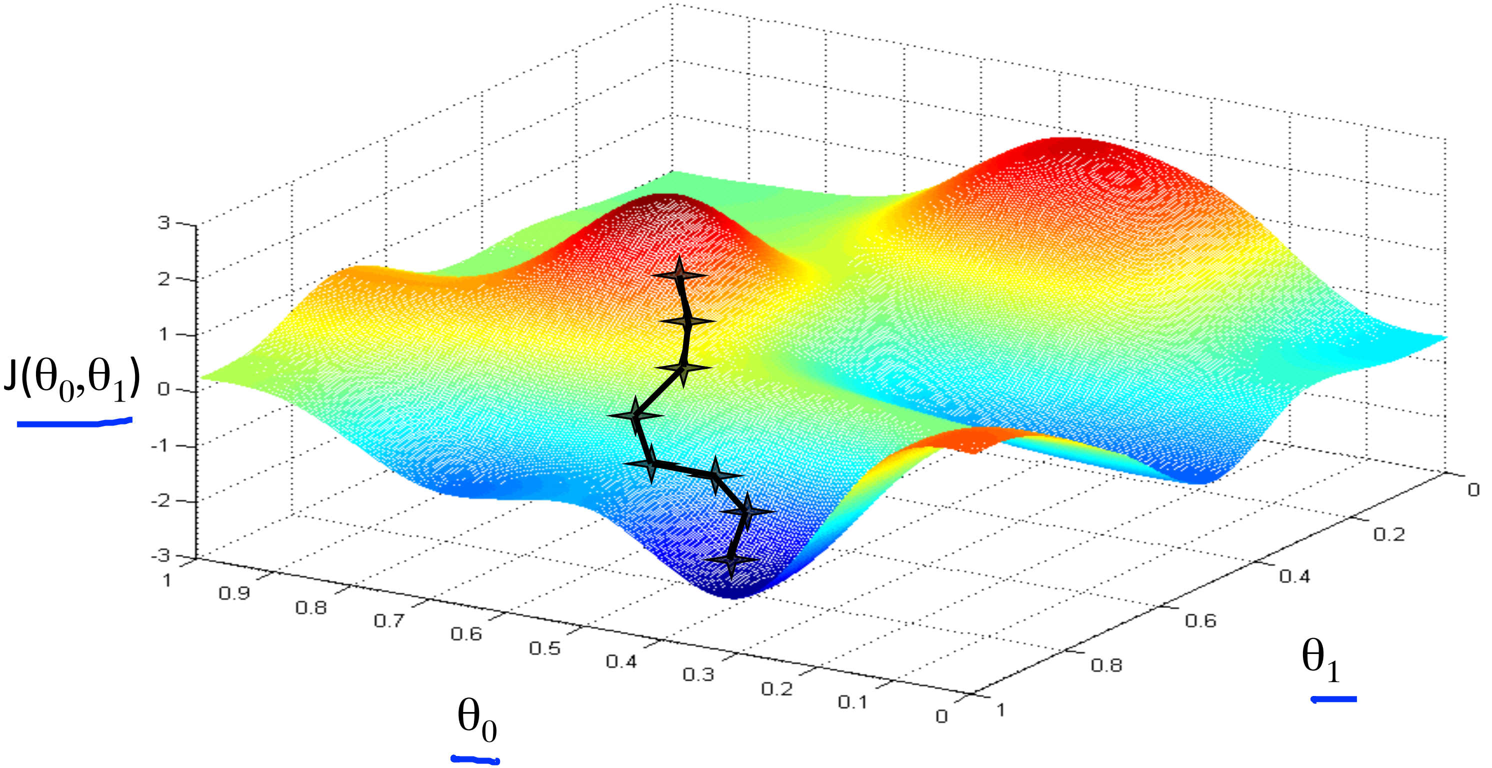
Gradient descent

Have some function $J(\theta_0, \theta_1)$ $J(\theta_0, \theta_1, \theta_2, \ldots, \theta_n)$

Want $\min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$ $\min_{\theta_0, \ldots, \theta_n} J(\theta_0, \ldots, \theta_n)$

Outline:

- Start with some θ_0, θ_1 (say $\theta_0 = 0, \theta_1 = 0$)
- Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$ until we hopefully end up at a minimum



$J(\theta_0,\theta_1)$
θ_0
θ_1
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1

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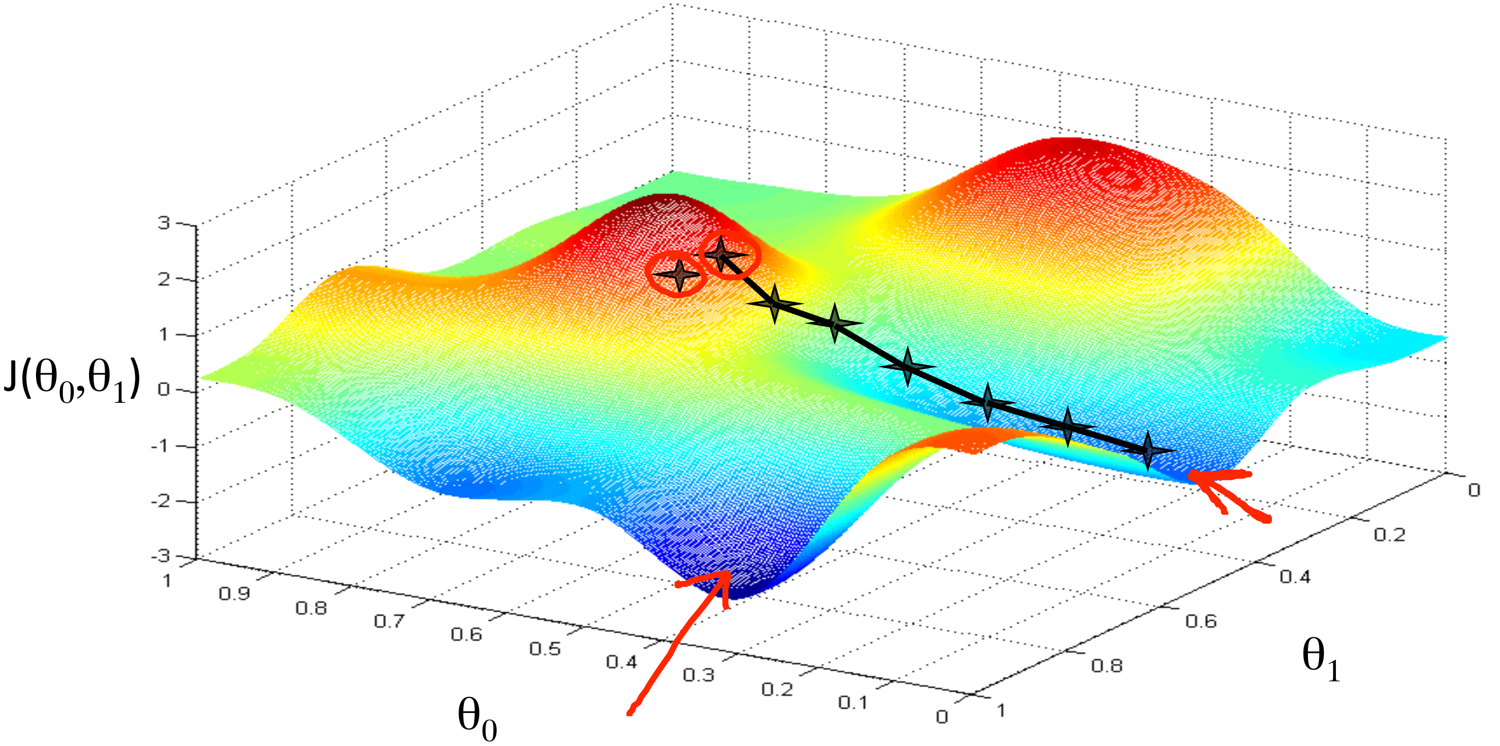

3
2
1
0
-1
-2
-3
$J(\theta_0,\theta_1)$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1

Gradient descent algorithm

Assignment

$a := b$

$a := a+1$

Truth assertion

$a = b$

$a = a+1$ X

θ_0, θ_1

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad (\text{for } j = 0 \text{ and } j = 1)$$

}

learning rate

Simultaneously update θ_0 and θ_1

Correct: Simultaneous update

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\theta_1 := \text{temp1}$$

Incorrect:

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_1 := \text{temp1}$$

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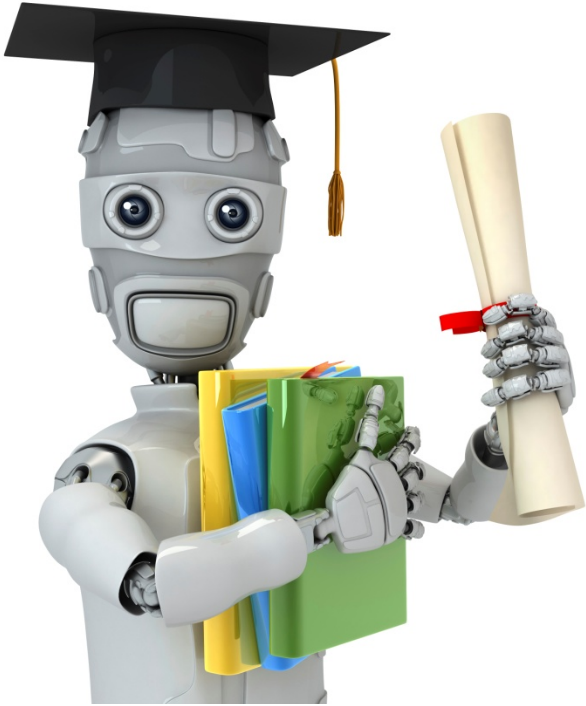


Machine Learning

Linear regression with one variable

Gradient descent intuition

Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)} \quad \text{(simultaneously update } j = 0 \text{ and } j = 1)$$

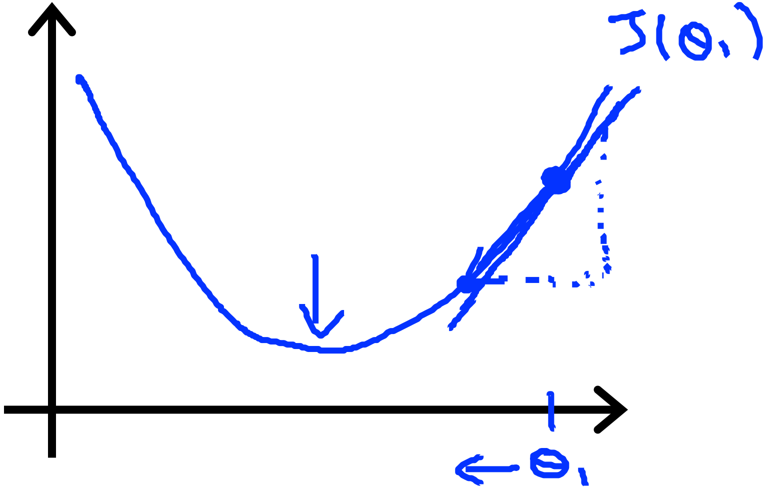
}

learning rate

derivative

$\min_{\theta_1} J(\theta_1)$

$\theta_1 \in \mathbb{R}$.

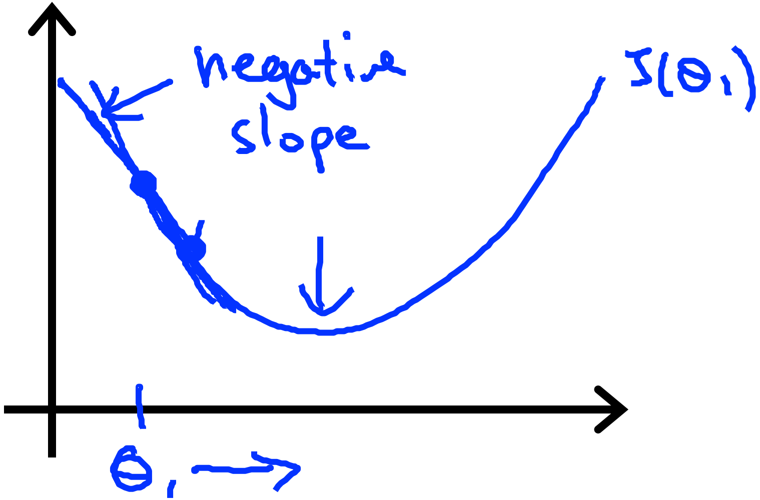


$(\theta_1 \in \mathbb{R})$

$\frac{\partial}{\partial\theta_1}$

$$\theta_1 := \theta_1 - \alpha \underbrace{\boxed{\frac{d}{d\theta_1} J(\theta_1)}}_{\geq 0}$$

$$\theta_1 := \theta_1 - \underline{\alpha} \cdot (\text{positive number})$$

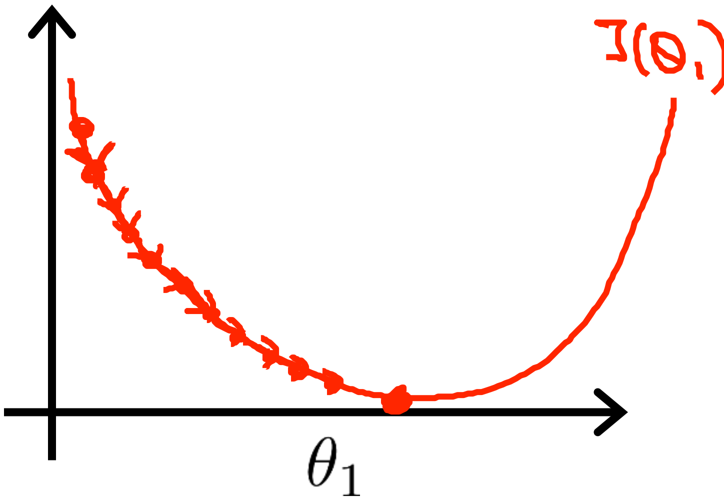


$$\underbrace{\frac{d}{d\theta_1} J(\theta_1)}_{\leq 0}$$

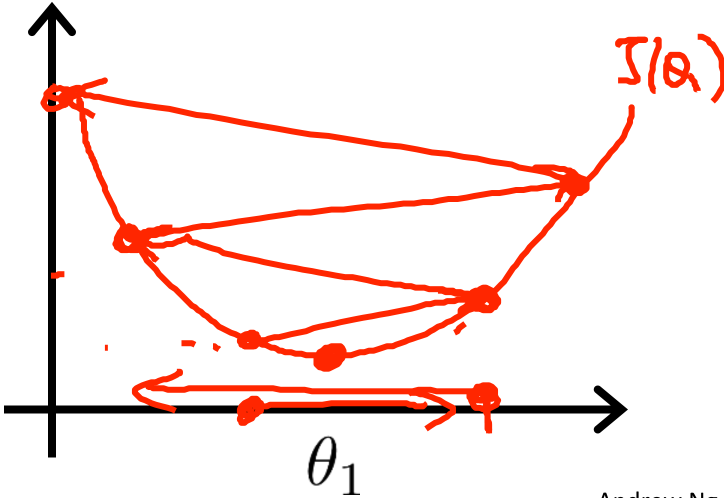
$$\theta_1 := \underline{\theta_1} - \alpha\,(\text{negative number})$$

$$\boxed{\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)}$$

If α is too small, gradient descent can be slow.



If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



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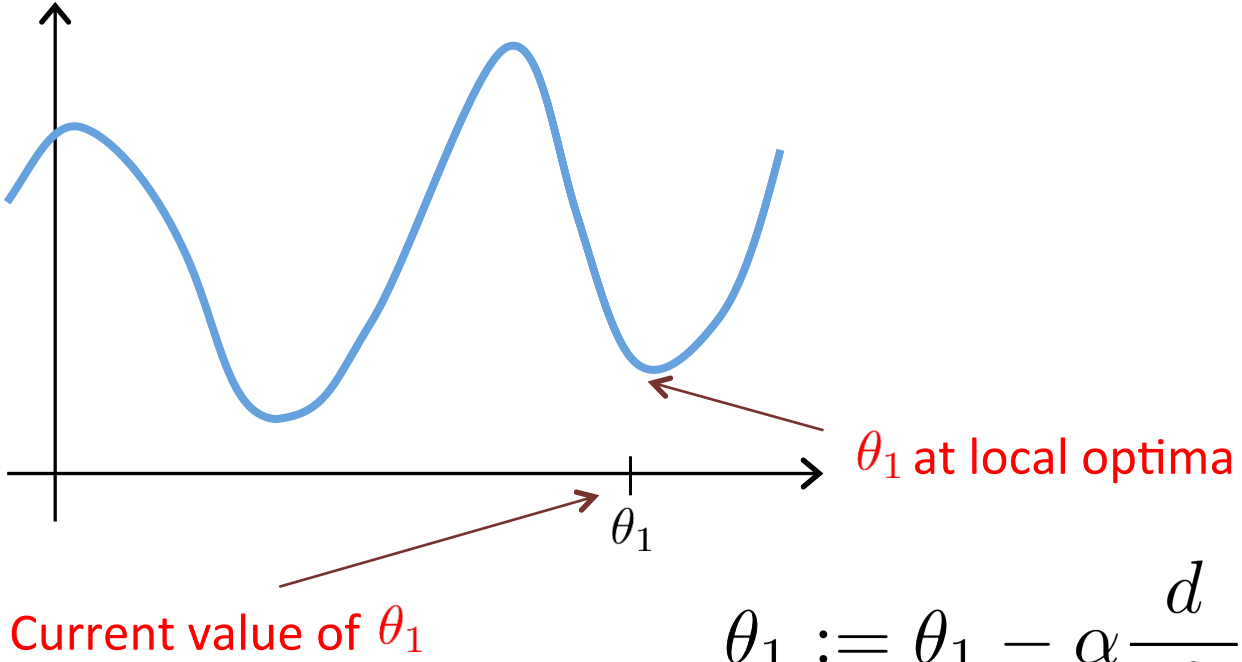
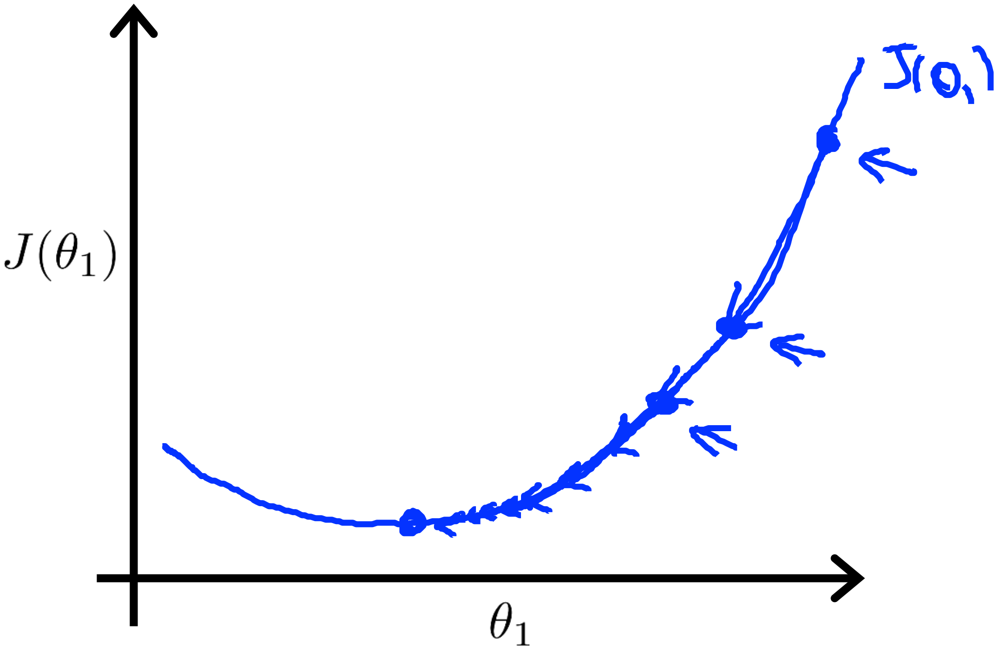


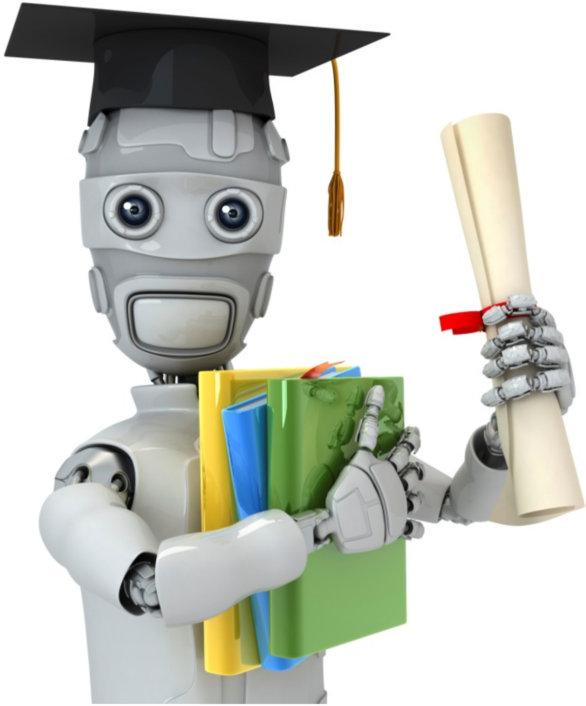
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

Gradient descent can converge to a local minimum, even with the learning rate α fixed.

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.





Machine Learning

Linear regression with one variable

Gradient descent for linear regression

Gradient descent algorithm

$$\text{repeat until convergence } \{$$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

$$(\text{for } j = 1 \text{ and } j = 0)$$

$$\}$$

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$= \frac{\partial}{\partial \theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)}\right)^2$$

$$j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)$$

$$j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right) \cdot x^{(i)}$$

Gradient descent algorithm

$$\frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

repeat until convergence {

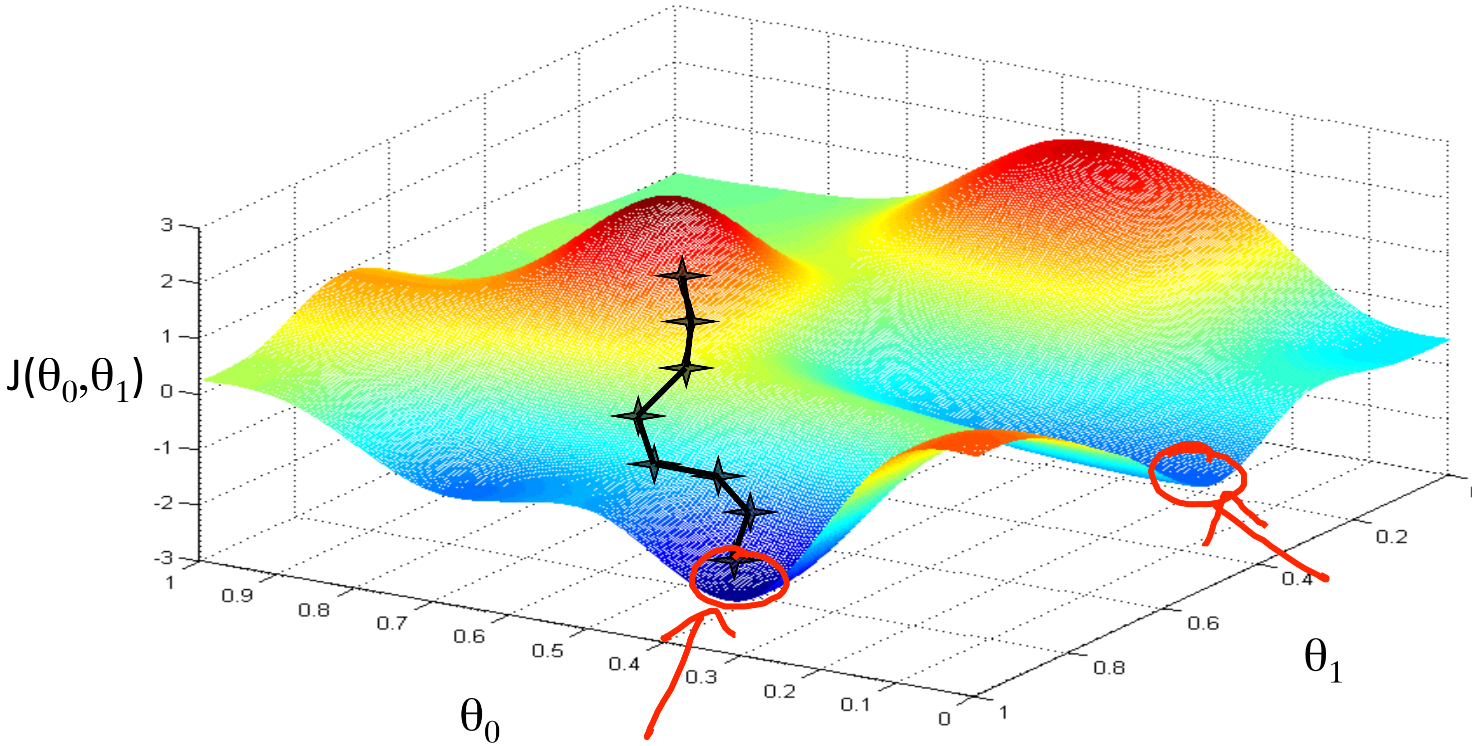
$$\theta_0 := \theta_0 - \alpha \boxed{\frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)}$$

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)}}$$

}

$$\frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

update θ_0 and θ_1 simultaneously



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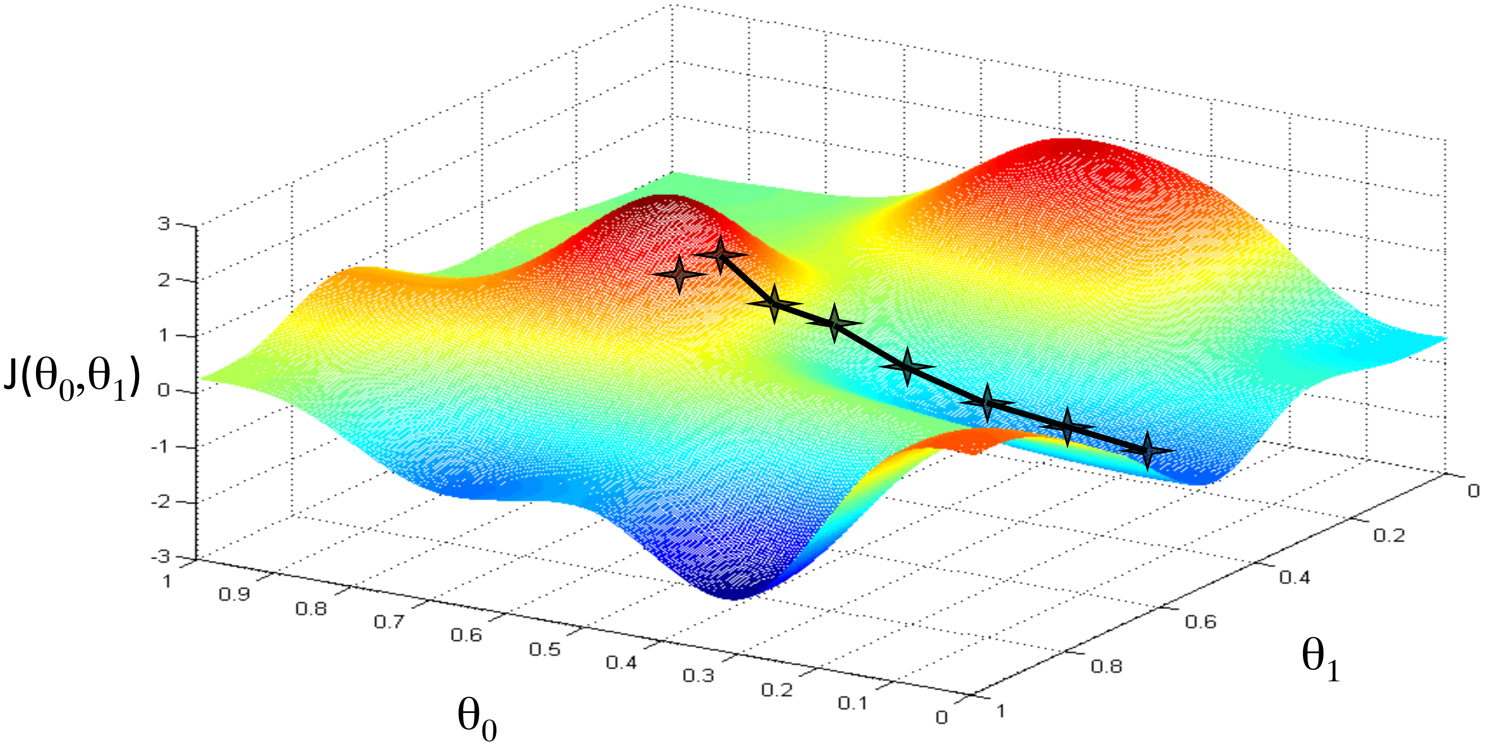
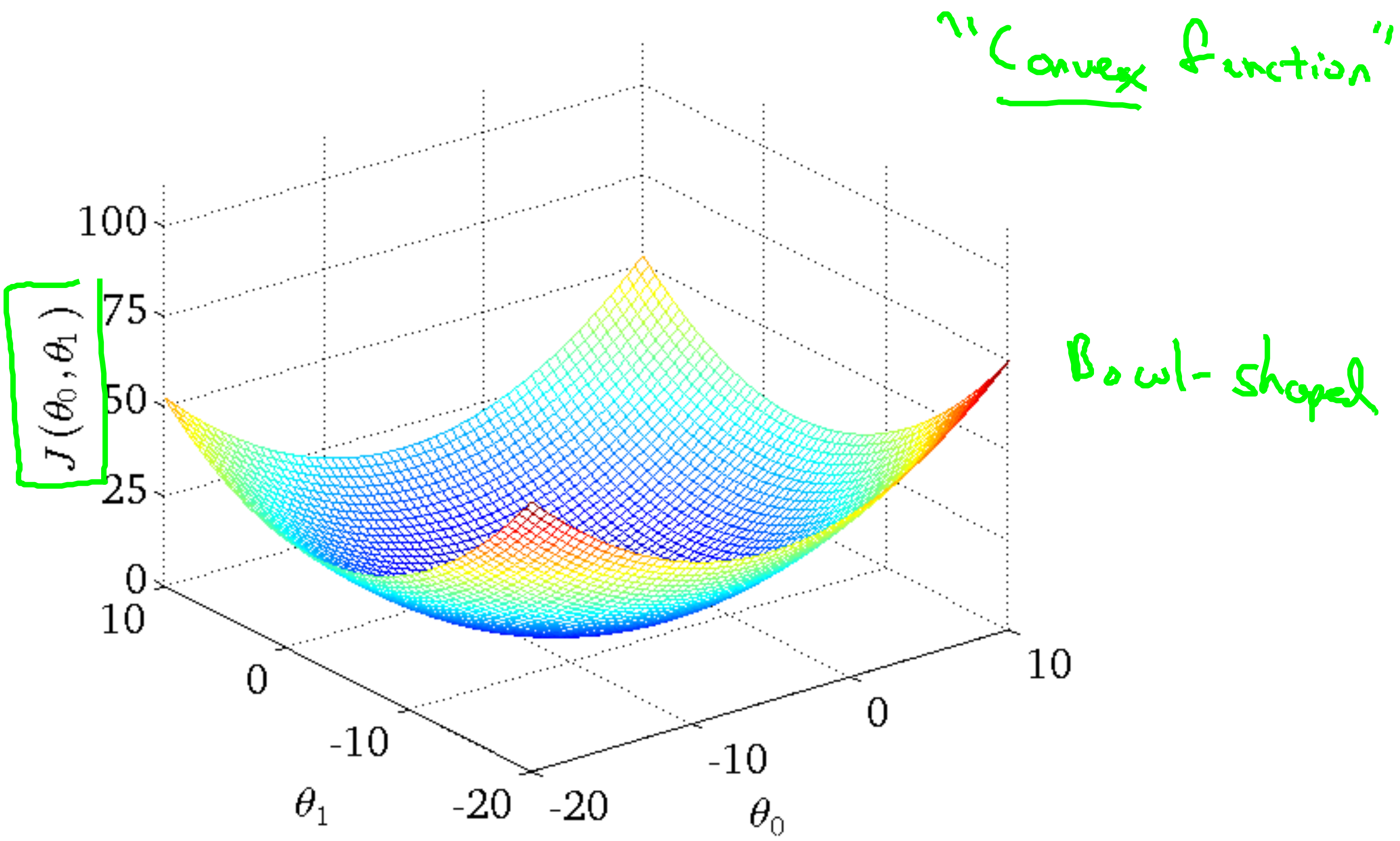


$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



"Convex function"
Bowl-shaped
$J(\theta_0, \theta_1)$
100
75
50
25
0
10
0
-10
-20
θ_1
-20
-10
0
10
θ_0

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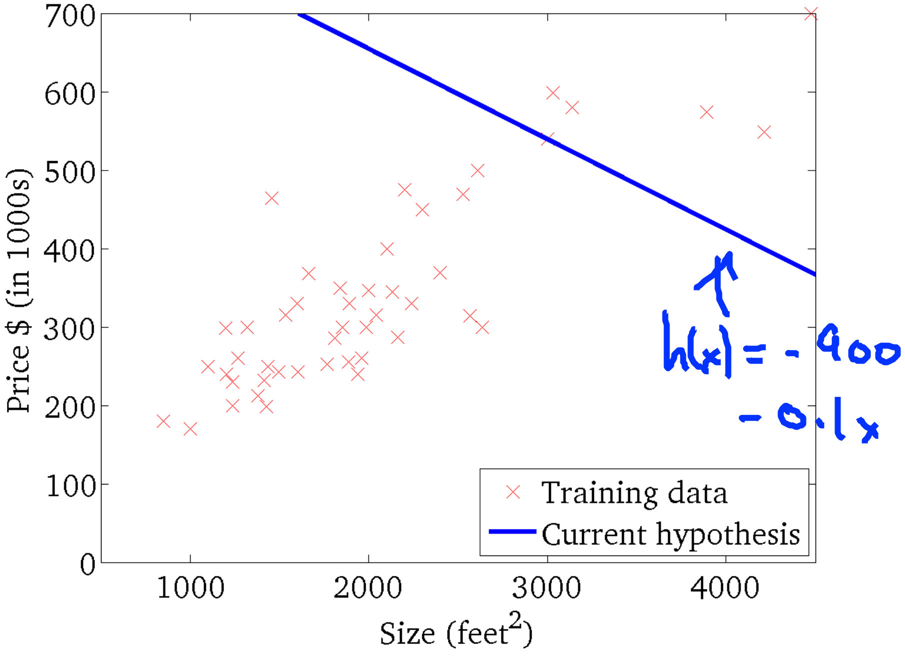
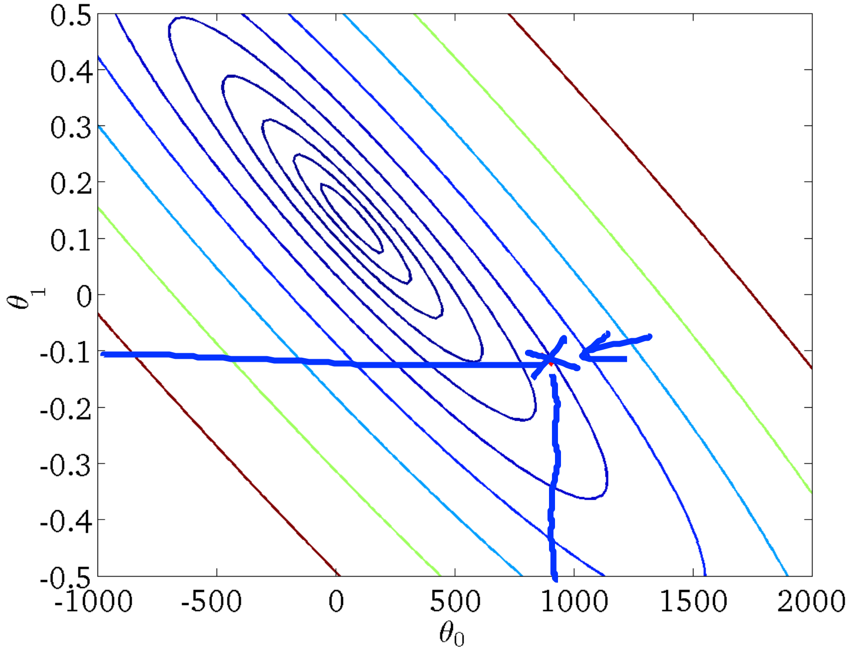
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



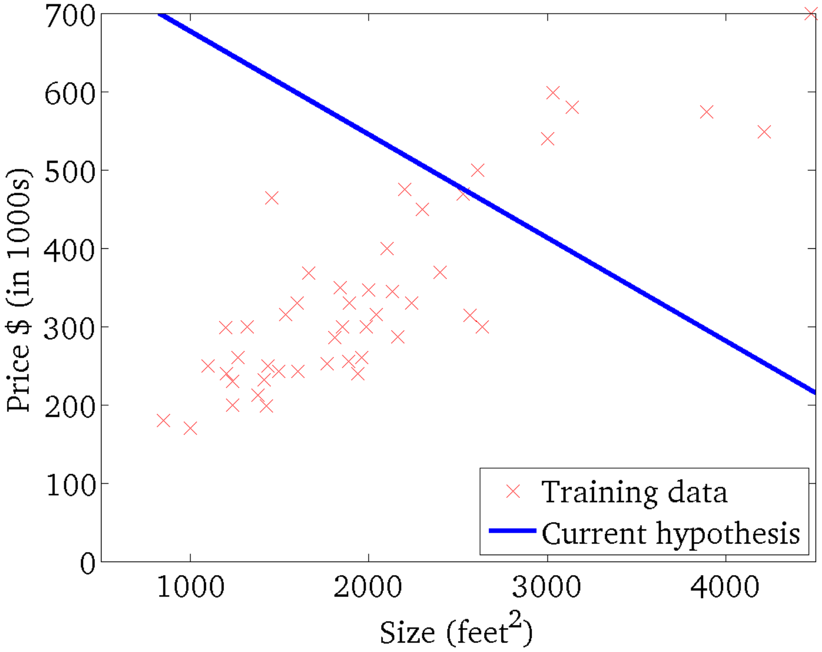
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



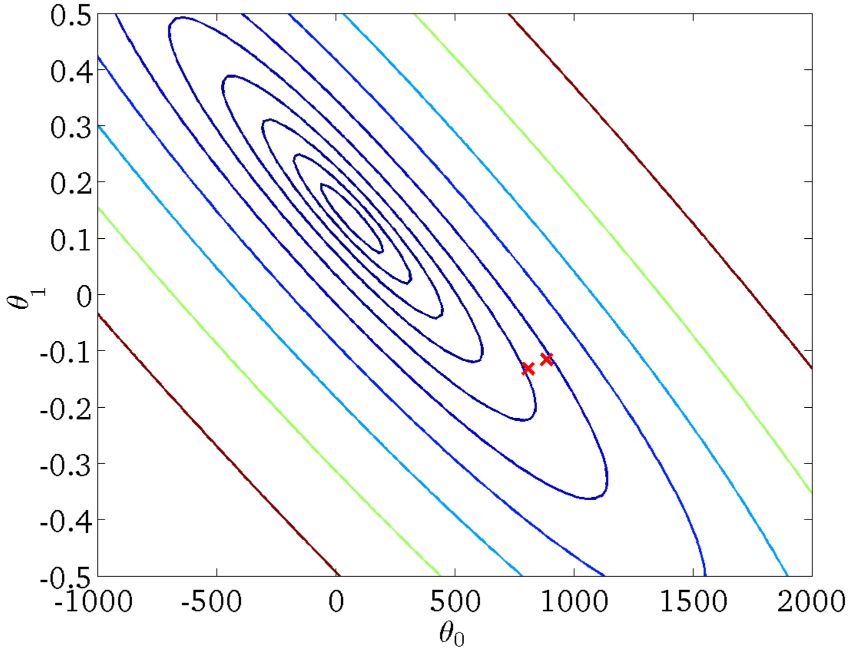
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



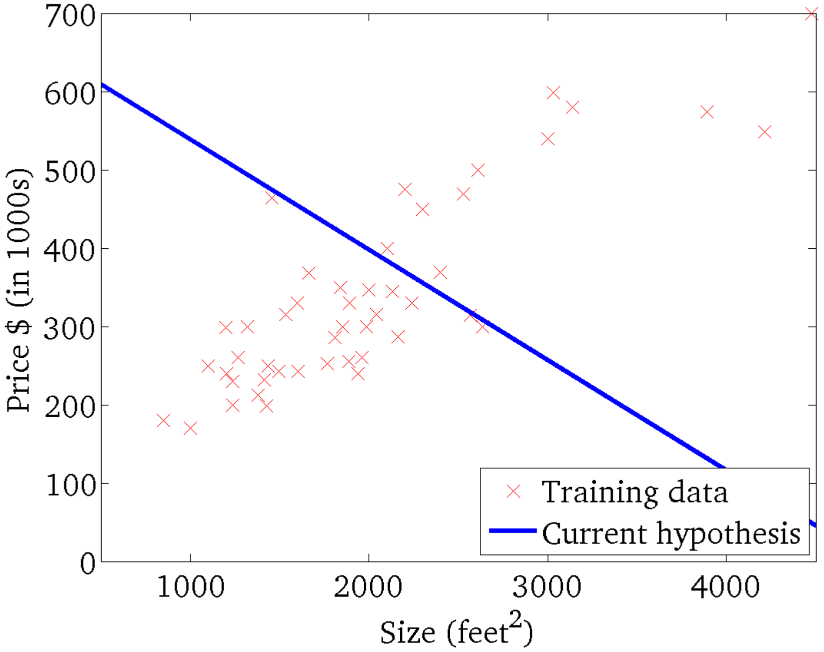
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



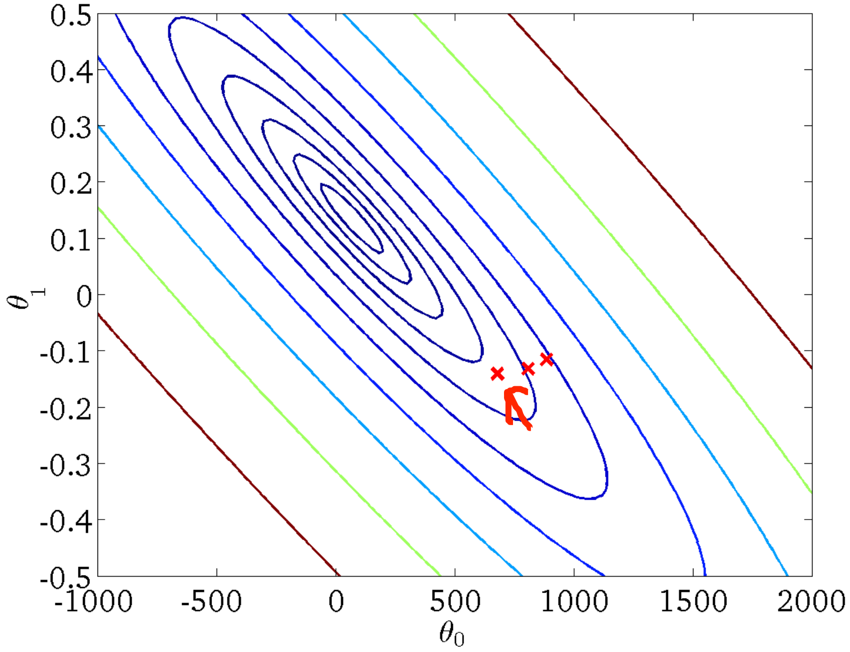
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



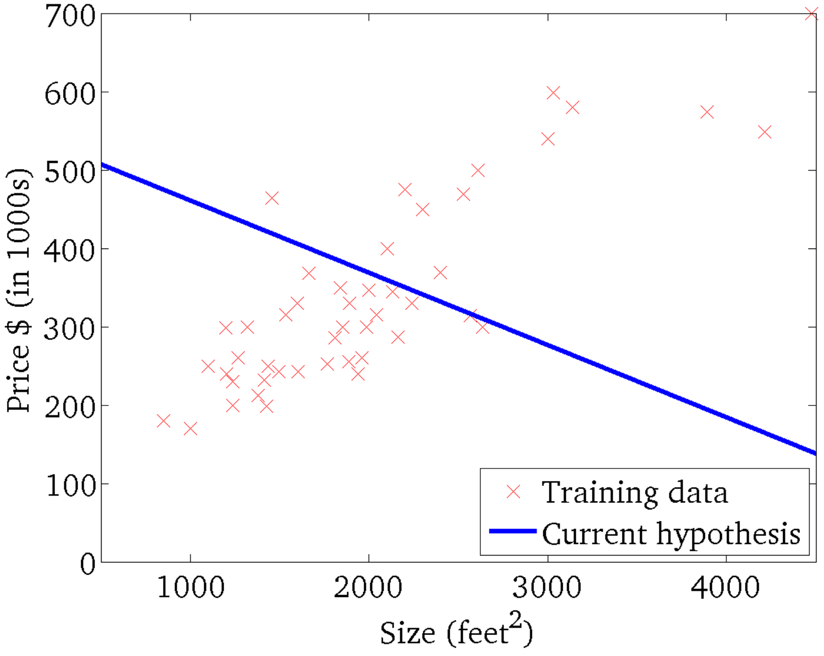
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



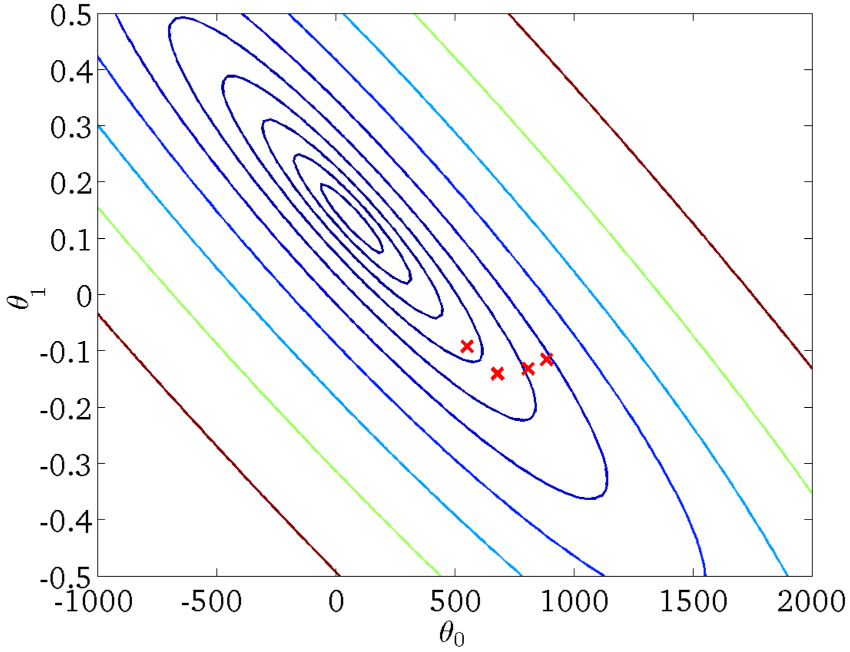
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



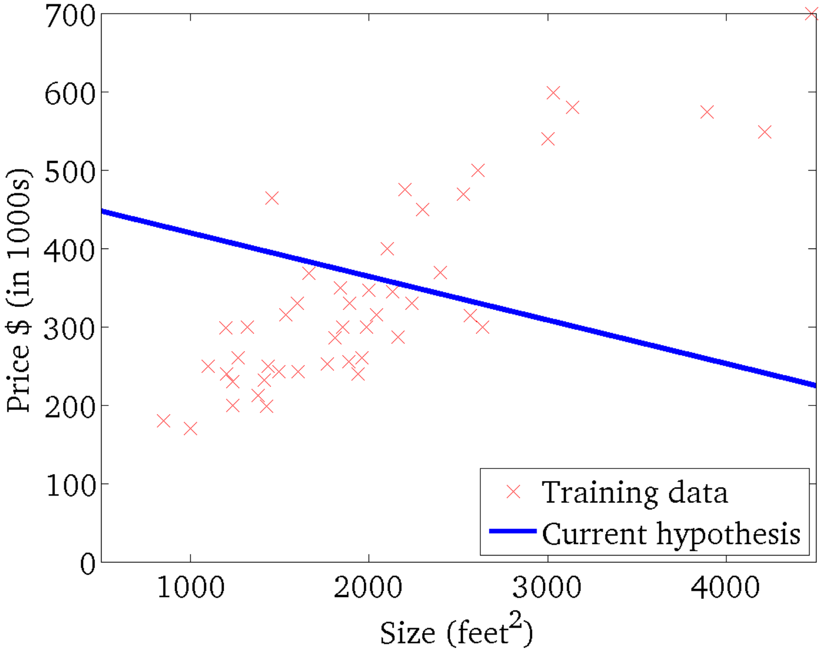
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

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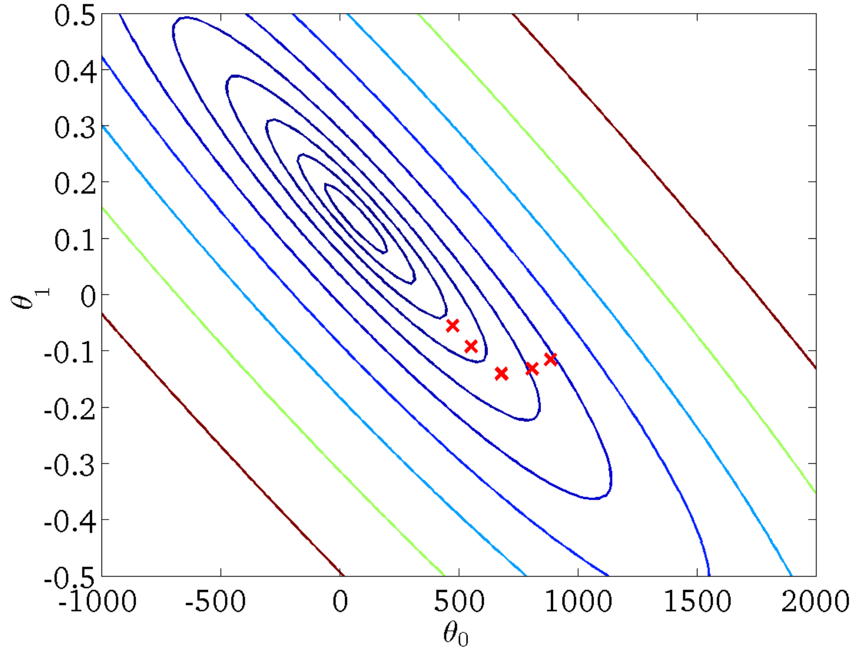

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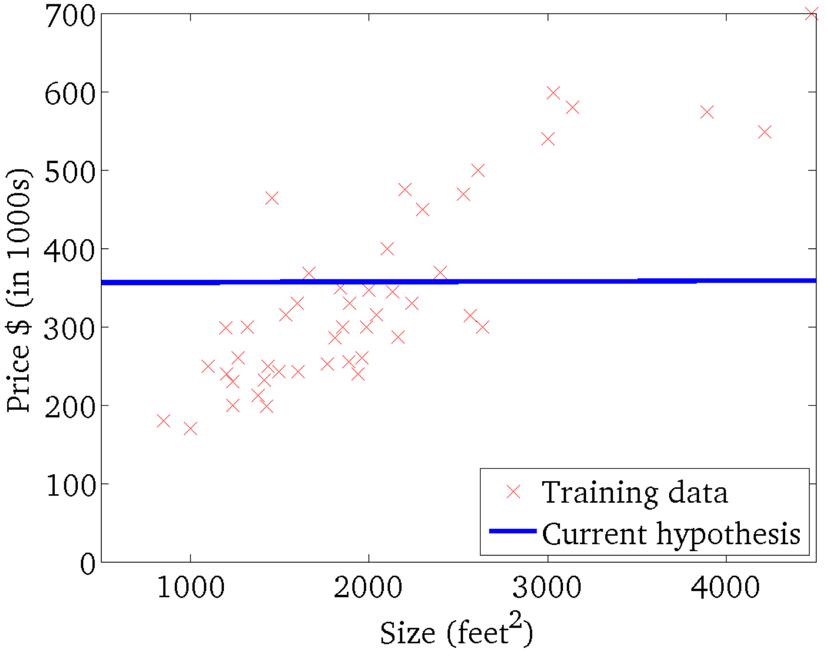

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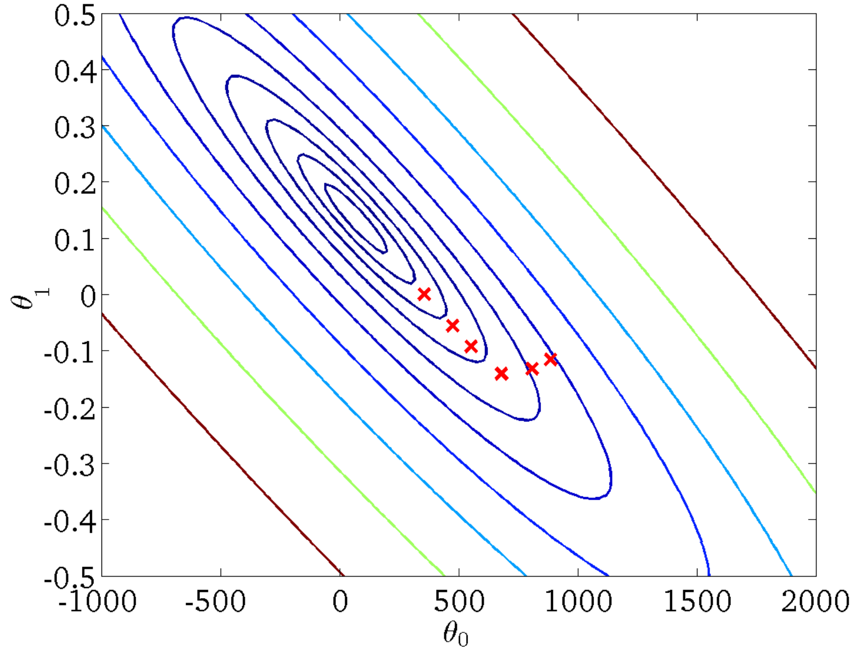

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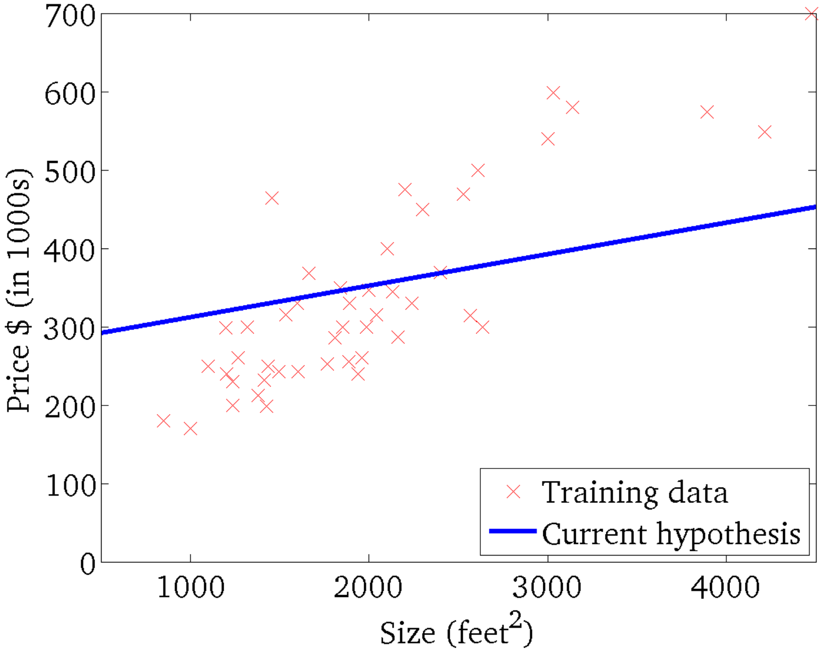

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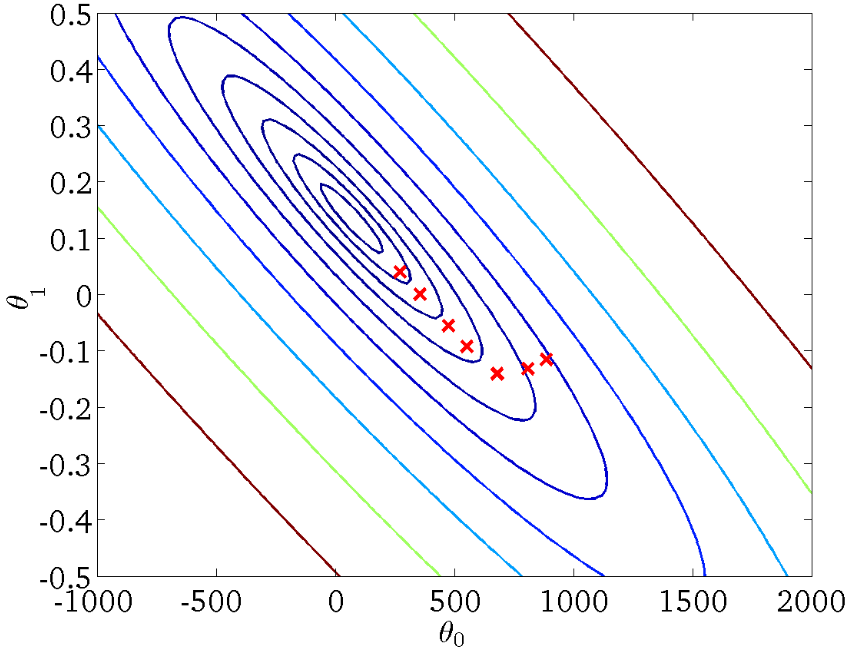

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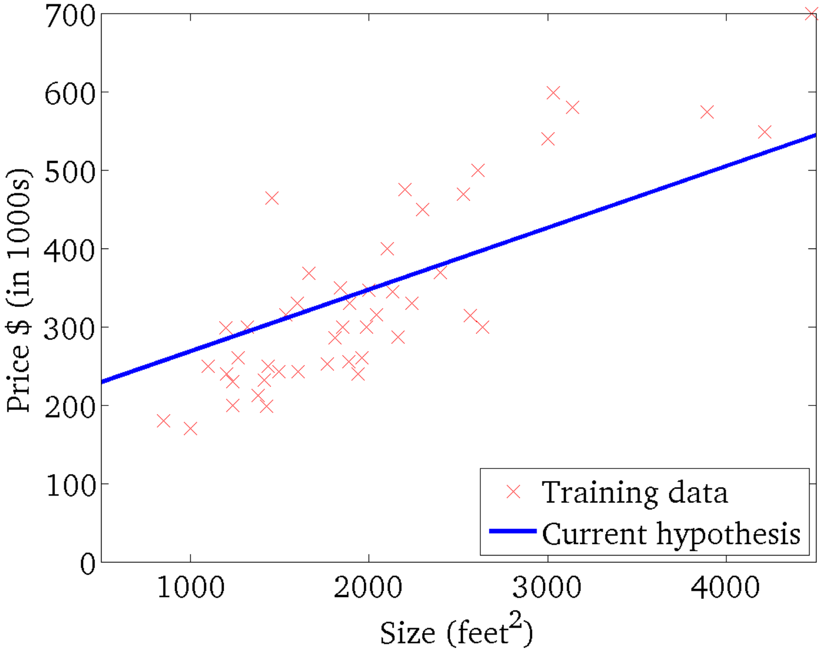
(for fixed θ_0, θ_1, this is a function of x)



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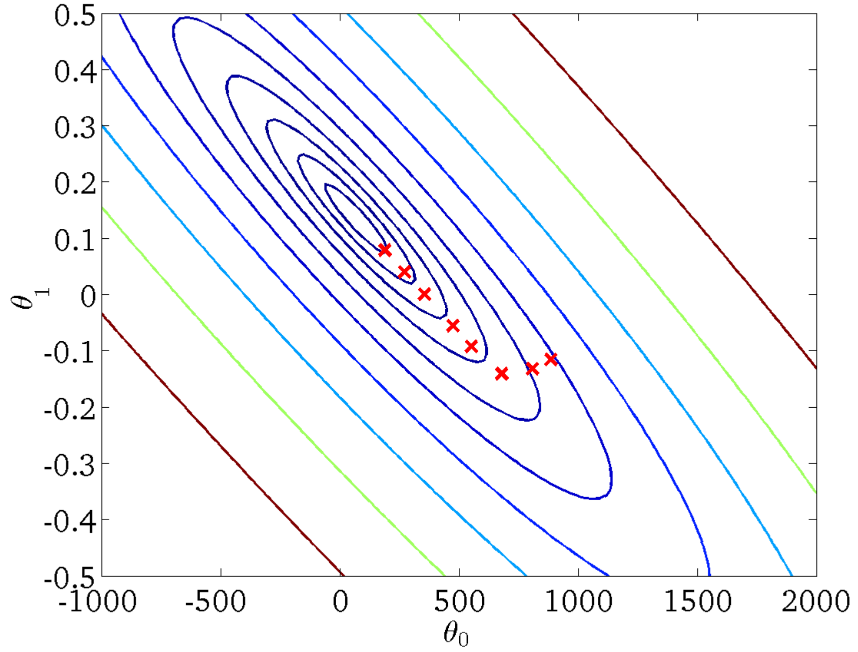

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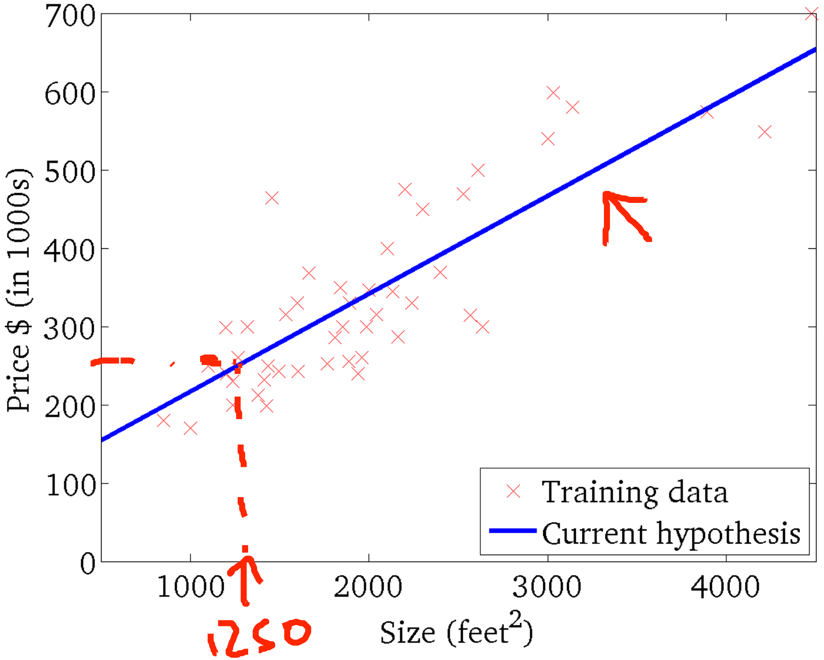
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

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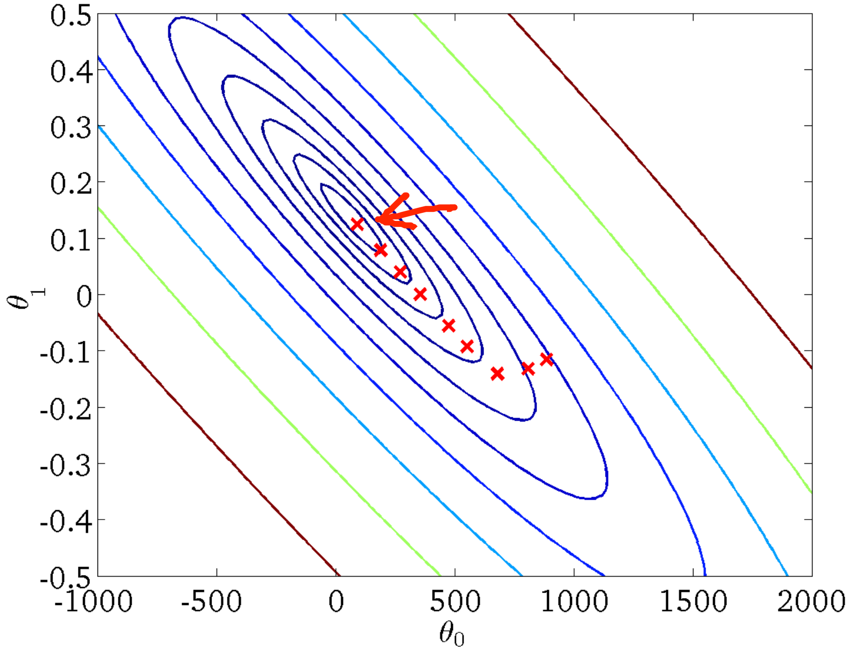
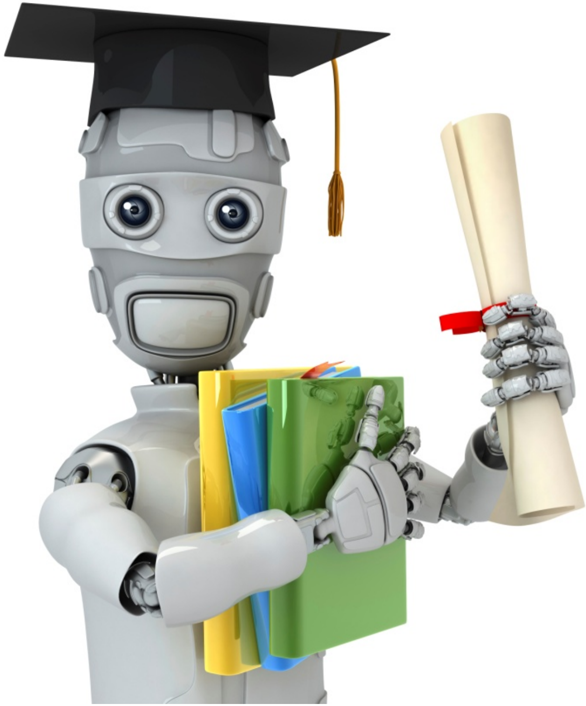


"Batch" Gradient Descent

"Batch": Each step of gradient descent uses all the training examples.

$$\sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)$$



Machine Learning

Linear Algebra review (optional)

Matrices and vectors

Matrix: Rectangular array of numbers:

$$\begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

4 x2 matrix

$\boxed{\mathbb{R}^{4\times 2}}$

$$2 \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

3

2 x3 matrix

$\boxed{\mathbb{R}^{2\times 3}}$

Dimension of matrix: number of rows x number of columns

Matrix Elements (entries of matrix)

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

$A_{ij} =$ "i, j entry" in the i^{th} row, j^{th} column.

$A_{11} = 1402$

$A_{12} = 191$

$A_{32} = 1437$

$A_{41} = 147$

$\cancel{A_{43}}$ = undefined (error)

Vector: An n x 1 matrix.

$\mathbb{R}^{3\times 2}$

n= 4

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$ ← 4-dimensional vector. $\mathbb{R}^4$

$y_i = i^{th}$ element

$y_1 = 460$
$y_2 = 232$
$y_3 = 315$

A, B, C, X

a, b, x, y

1-indexed vs 0-indexed:

y[1]

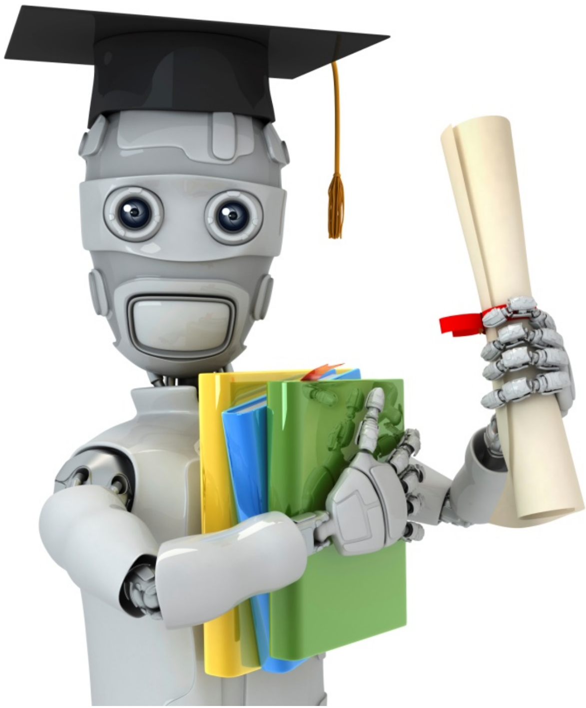
$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

1-indexed

$$y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

y[0]

0-indexed



Machine Learning

Linear Algebra review (optional)

Addition and scalar multiplication

Matrix Addition

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix}}_{\substack{3\times 2 \\ \text{matrix}}} + \underbrace{\begin{bmatrix} 4 & 0.5 \\ 2 & 5 \\ 0 & 1 \end{bmatrix}}_{3\times 2} = \underbrace{\begin{bmatrix} 5 & 0.5 \\ 4 & 10 \\ 3 & 2 \end{bmatrix}}_{3\times 2}$$

$$\cancel{\underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix}}_{3\times 2} + \underbrace{\begin{bmatrix} 4 & 0.5 \\ 2 & 5 \end{bmatrix}}_{2\times 2}} = \text{error}$$

Scalar Multiplication

real number

$$3 \times \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} \times 3$$

3×2 3×2

$$\boxed{\begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} / 4} = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & \frac{3}{4} \end{bmatrix}$$

Combination of Operands

$$3 \times \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} / 3$$

Scalar multiplication

Scalar division

$$= \begin{bmatrix} 3 \\ 12 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ \frac{2}{3} \end{bmatrix}$$

matrix subtraction / vector subtraction

matrix addition / vector addition

$$= \begin{bmatrix} 2 \\ 12 \\ 10\frac{1}{3} \end{bmatrix}$$

3x1 matrix

3-dimensional vector

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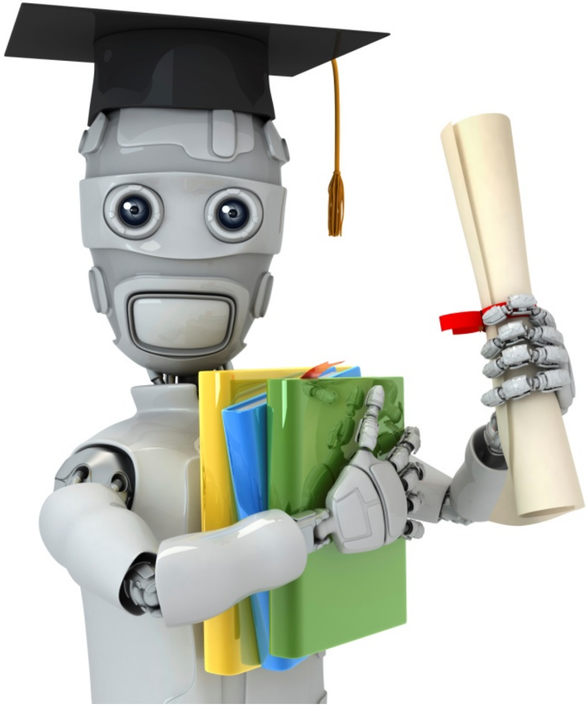


Linear Algebra review (optional)

Matrix-vector multiplication

Machine Learning

Example

$$\underbrace{\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix}}_{3\times 2} \underbrace{\begin{bmatrix} 1 \\ 5 \end{bmatrix}}_{2\times 1} = \underbrace{\begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix}}_{3\times 1}$$

matrix

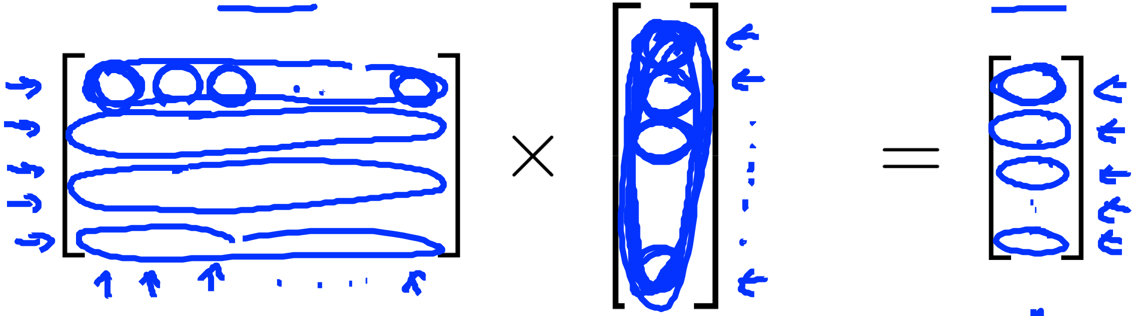
$1\times 1 + 3\times 5 = 16$

$4\times 1 + 0\times 5 = 4$

$2\times 1 + 1\times 5 = 7$

Details:

$$A \times x = y$$



m x n matrix (m rows, n columns) | n x 1 matrix (n-dimensional vector) | m-dimensional vector

To get y_i , multiply A's i^{th} row with elements of vector x, and add them up.

Example

$$\underset{\boxed{3}\times 4}{\begin{bmatrix}1&2&1&5\\0&3&0&4\\-1&-2&0&0\end{bmatrix}}\underset{4\times 1}{\begin{bmatrix}1\\3\\2\\1\end{bmatrix}} = \underset{3\times 1}{\begin{bmatrix}\boxed{14}\\\boxed{13}\\\boxed{-7}\end{bmatrix}} = \begin{bmatrix}14\\13\\-7\end{bmatrix}$$

$$1\times 1 + 2\times 3 + 1\times 2 + 5\times 1 = 14$$

$$0\times 1 + 3\times 3 + 0\times 2 + 4\times 1 = 13$$

$$-1\times 1 + (-2)\times 3 + 0\times 2 + 0\times 1 = -7$$

House sizes:

→ 2104
→ 1416
→ 1534
→ 852

$$h_\theta(x) = -40 + 0.25x$$

$h_\theta(x)$

Matrix 4x2

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} -40 \\ 0.25 \end{bmatrix} = \begin{bmatrix} -40 \times 1 + 0.25 \times 2104 \\ -40 \times 1 + 0.25 \times 1416 \\ \\ \\ \end{bmatrix}$$

2x1 Vector

4x1 matrix

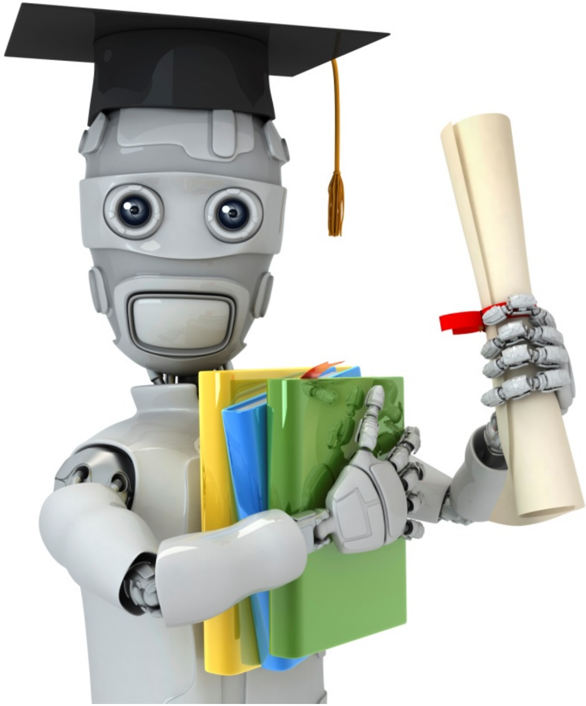
$h_\theta(2104)$

$h_\theta(1416)$

prediction = DataMatrix × parameters

4x1

for i = 1:1,000,
prediction (i) = ...



Machine Learning

Linear Algebra review (optional)

Matrix-matrix multiplication

Example

$$\underbrace{\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix}}_{2 \times 3} \underbrace{\begin{bmatrix} \boxed{\begin{matrix}1\\0\\5\end{matrix}} & \boxed{\begin{matrix}3\\1\\2\end{matrix}} \end{bmatrix}}_{3 \times 2} = \underbrace{\begin{bmatrix} 11 & 10 \\ 9 & 14 \end{bmatrix}}_{2 \times 2}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

Details:

$$A \times B = C$$

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \times \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} = \begin{bmatrix} & & \cdots & \\ & & & \\ & & & \end{bmatrix}$$

$n \times 1$

m x n matrix
(m rows,
n columns)

n x o matrix
(n rows,
o columns)

m x o
matrix

The i^{th} column of the matrix C is obtained by multiplying A with the i^{th} column of B. (for i = 1,2,…,o)

Example

$$\underset{2\times 2}{\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} 9 & 7 \\ 15 & 12 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\times 0 + 3\times 3 \\ 2\times 0 + 5\times 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\times 1 + 3\times 2 \\ 2\times 1 + 5\times 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

House sizes:

$$\left\{\begin{matrix} 2104 \\ 1416 \\ 1534 \\ 852 \end{matrix}\right.$$

Have 3 competing hypotheses:

1. $h_\theta(x) = -40 + 0.25x$
2. $h_\theta(x) = 200 + 0.1x$
3. $h_\theta(x) = -150 + 0.4x$

Matrix

Matrix

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix} \times \begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix} = \begin{bmatrix} 486 & 410 & 692 \\ 314 & 342 & 416 \\ 344 & 353 & 464 \\ 173 & 285 & 191 \end{bmatrix}$$

Prediction of first h_θ

Predictions of 2^{nd} h_θ

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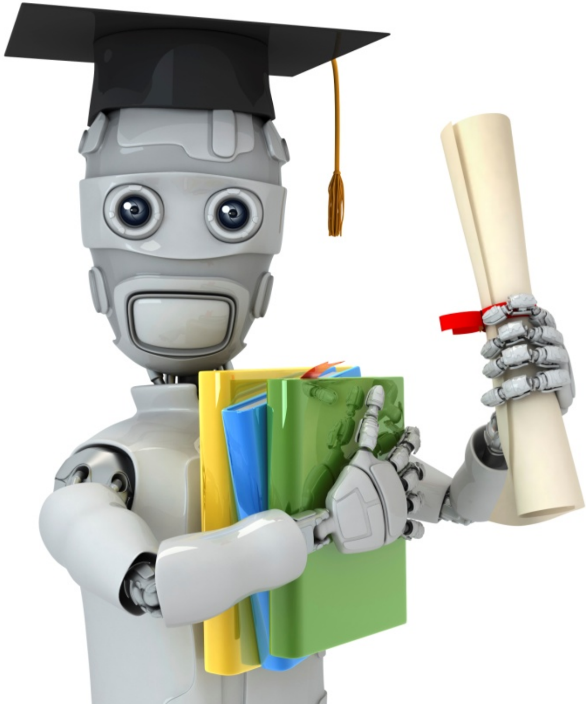


Linear Algebra review (optional)

Matrix multiplication properties

Machine Learning

$3 \times 5 = 5 \times 3$ "Commutative"

Let A and B be matrices. Then in general,

$A \times B \neq B \times A$. (not commutative.)

E.g.

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$\neq$

$$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$\underset{m \times n}{A} \times \underset{n \times m}{B}$

$A \times B$ is $m \times m$

$B \times A$ is $n \times n$

$3 \times 5 \times 2$

$3 \times 10 = 30 = 15 \times 2$

$3 \times (5 \times 2) = (3 \times 5) \times 2$

$\boxed{\text{"Associative"}}$

$A \times (B \times C)$

$(A \times B) \times C$

$$A \times B \times C.$$

Let $D = B \times C$. Compute $A \times D$.

Let $E = A \times B$. Compute $E \times C$.

$A \times (B \times C)$

$(A \times B) \times C$

Same answer.

Identity Matrix

1 is identity

$\boxed{1 \times z = z \times 1 = z}$

for any z

Denoted I (or $\boxed{I_{n \times n}}$).

Examples of identity matrices:

$[1]$

1×1

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 x 2   3 x 3   4 x 4

Informally:

$$\begin{bmatrix} 1 & & 0 \\ & 1 & \\ & & 1 \\ 0 & & \ddots & \\ & & & 1 \end{bmatrix}$$

For any matrix A,

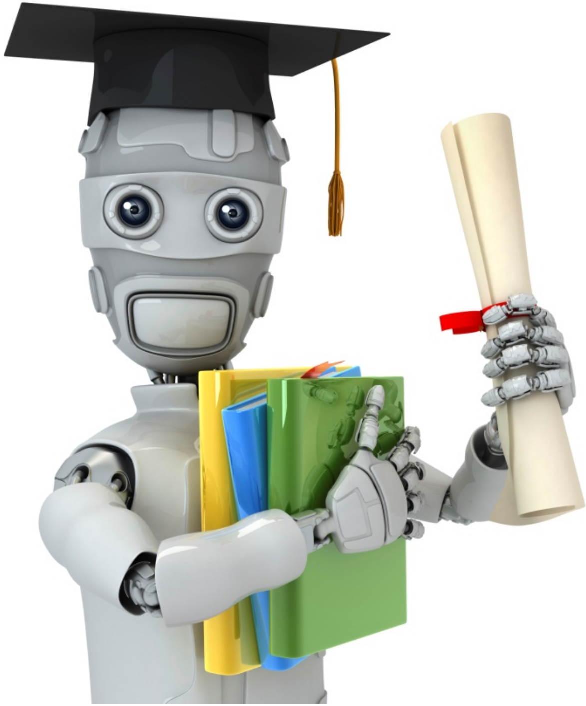
$$\underset{m \times n}{A} \cdot \underset{n \times n}{\boxed{I}} = \underset{m \times m}{\boxed{I}} \cdot \underset{m \times n}{A} = \underset{m \times n}{A}$$

$I_{n \times n}$

Note:

$AB \neq BA$ in general

$AI = IA$ ✓



Machine Learning

Linear Algebra review (optional)

Inverse and transpose

1 = "identity"

$3 \boxed{(3^{-1})} = 1$ $\quad \frac{1}{3}$

$12 \times (12^{-1}) = 1$ $\quad \frac{1}{12}$

Not all numbers have an inverse.

$0 (0^{-1})$ undefined

Matrix inverse:

square matrix (#rows = #columns)

A^{-1}

If A is an m x m matrix, and if it has an inverse,

$$A(A^{-1}) = A^{-1}A = I.$$

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

E.g.

$$\underbrace{\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}}_{\substack{2\times 2 \\ A}} \underbrace{\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}}_{A^{-1}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$$

$A^{-1}A$

Matrices that don't have an inverse are "singular" or "degenerate"

Matrix Transpose

Example: $\underline{A} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix}$ 2 x 3

$B = \underline{A^T} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$ 3 x 2

Let A be an m x n matrix, and let $B = A^T$.
Then B is an n x m matrix, and

$$B_{\underline{ij}} = A_{\underline{ji}}.$$

$B_{12} = A_{21} = 2$

$B_{32} = 9$ $\quad A_{23} = 9.$