Accurate 100+ site Kagome Lattice Ground State Energy (GSE) by Perturbing Hamiltonian on the defect triangles

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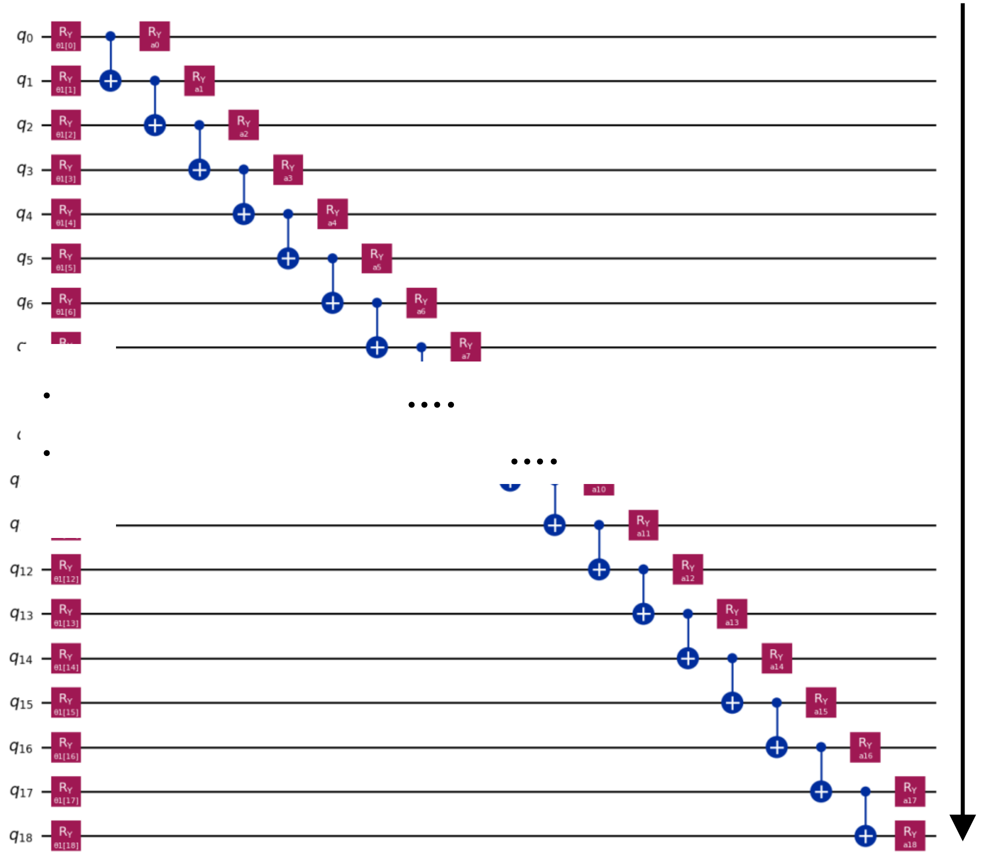
Key insights from the credits program experiment and study (1) Success of ODR Method for 100x100 circuit



Problem: Higher error in observable (qubits) at higher circuit depth !

Qiskit101-qubit Real Amplitude ansatz for KAFM Ground State Energy (GSE) problem

Increasing Circuit Depth

Truncated circuit qubits range [a-b]	Segmented original $\langle obs \rangle: x$	Segmented Ref $\langle obs \rangle: y$	Noise mitigated Segmented $\langle obs \rangle = 10x/r$
[1-20]	-6.50±0.15	8.45±0.10	-7.7
[21-40]	-6.64±0.27	6.6±0.42	-10.06
[41-60]	-5.58±0.36	3.83±0.52	-14.6
[61-80]	-3.99±0.06	3.16±0.16	-12.62
[81-100]	-3.64±0.30	3.09±0.32	-11.78
Complete Circuit	$\langle obs \rangle$ =-26.53 Original error in the $\langle obs \rangle$: **47%**	25.13	$\langle obs \rangle$ =-52.76 Mitigated Error in the $\langle obs \rangle$: **5.2%**

Torino jobs: crhgpgvut4r4kjefe2mg & crhgphaji5oi10mnb9ng

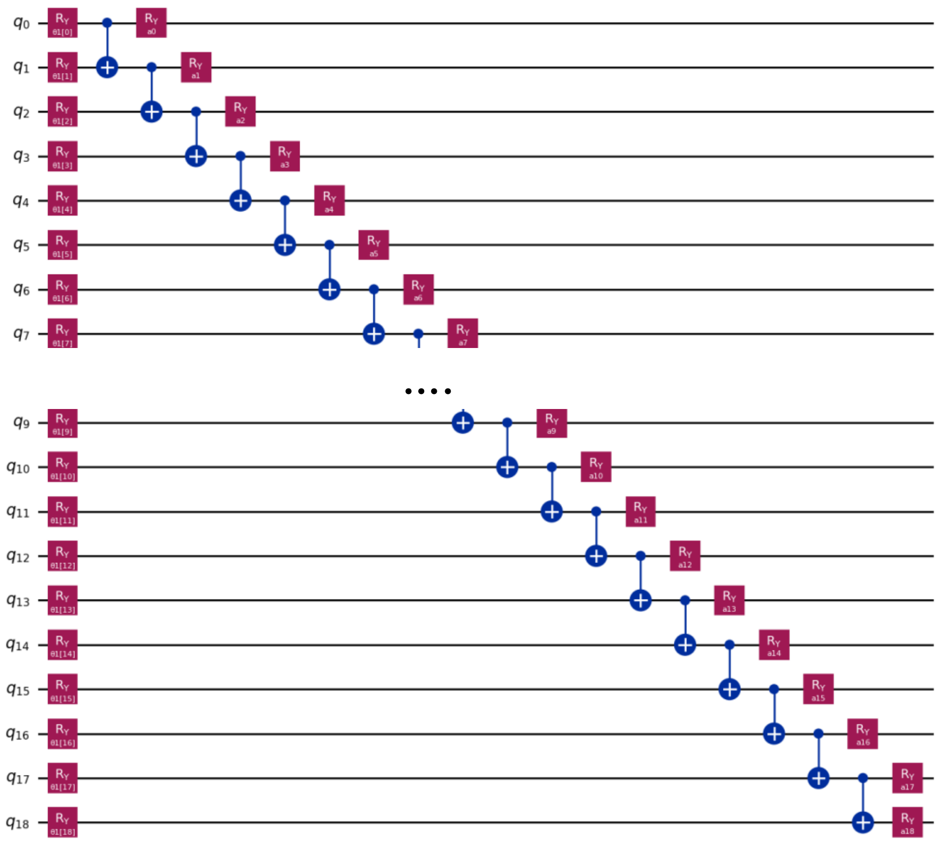
Solution: With operator Decoherence Renormalization (ODR) method, noise at higher depths was effectively mitigated !

Key insights from the last year credits program experiments and study (2) Hamiltonian Perturbation method needs revision

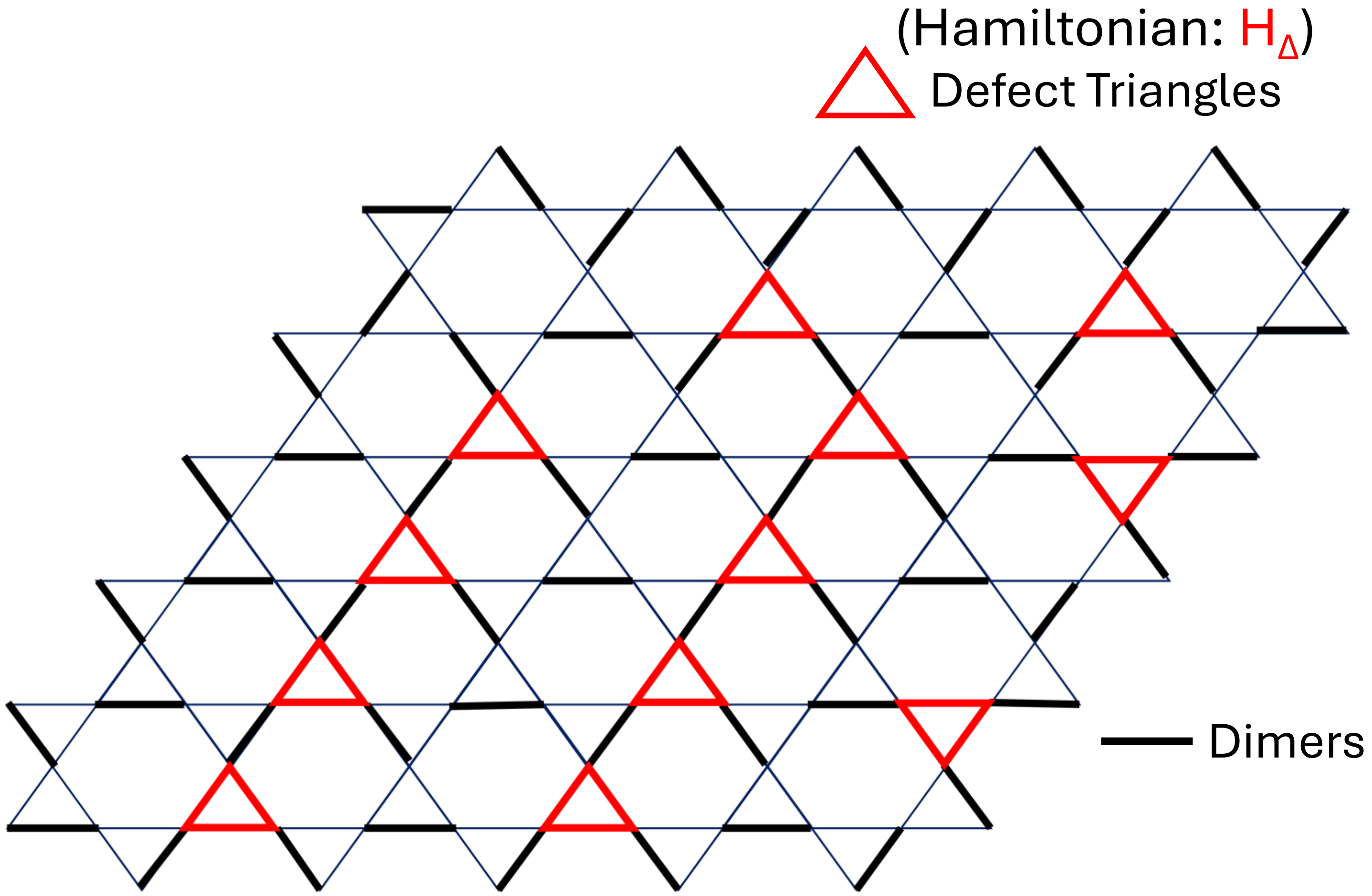
- Dimer covering: takes us close to the GSE, but not close enough. The gap can't be bridged without dealing with DEFECT TRIANGLES
- Qiskit 100x100 single layer Real Amplitude ansatz: is good at generating dimer cover, but can it address DEFECT TRIANGLE problem?
- Exploit Defect Triangles (new strategy):
  - Extract defect triangle from dimer covering generated by the ansatz
  - Focus Hamiltonian perturbation on defect triangle and update edge weights
  - Recompute GSE by VQE with the same ansatz

Existing IBM cloud quantum simulators can only simulate up to 64-qubit 100x100 real-amplitude ansatz !

Modified (new) proposal:
100-qubit quantum compute: $min_{\varphi}\langle\varphi|H_{pert}|\varphi\rangle$



100x100 real amplitude ansatz: $|\varphi\rangle$



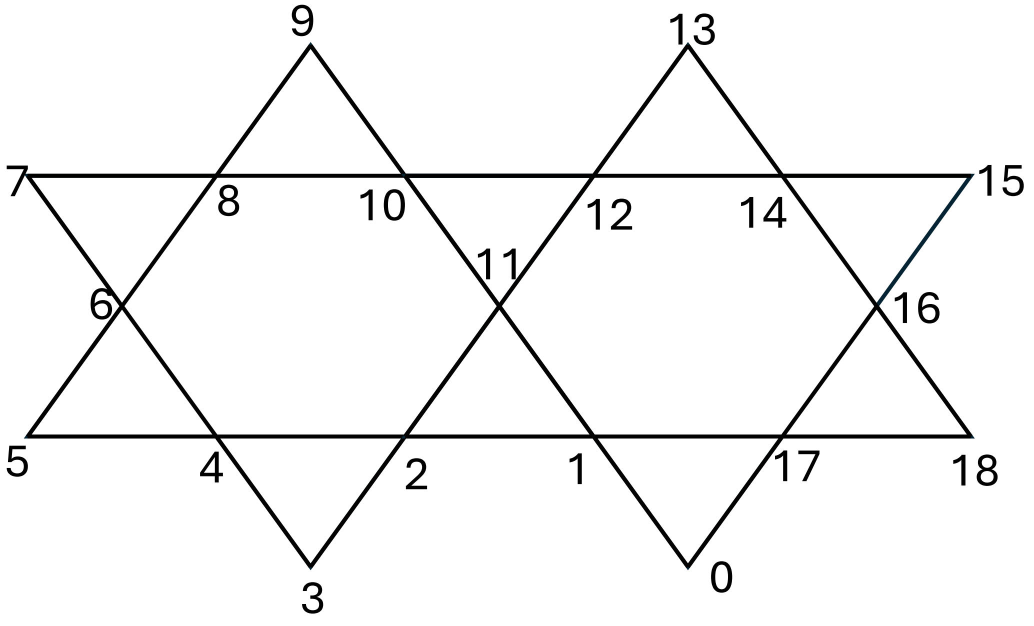
Hamiltonian perturbation on defect triangles

$$H_{pert} = H_{orig} + 0.5\,H_{\Delta}$$

Proof of concept

GSE with two-unit cell Kagome lattice

Two unit-cell Kagome Lattice GSE Example

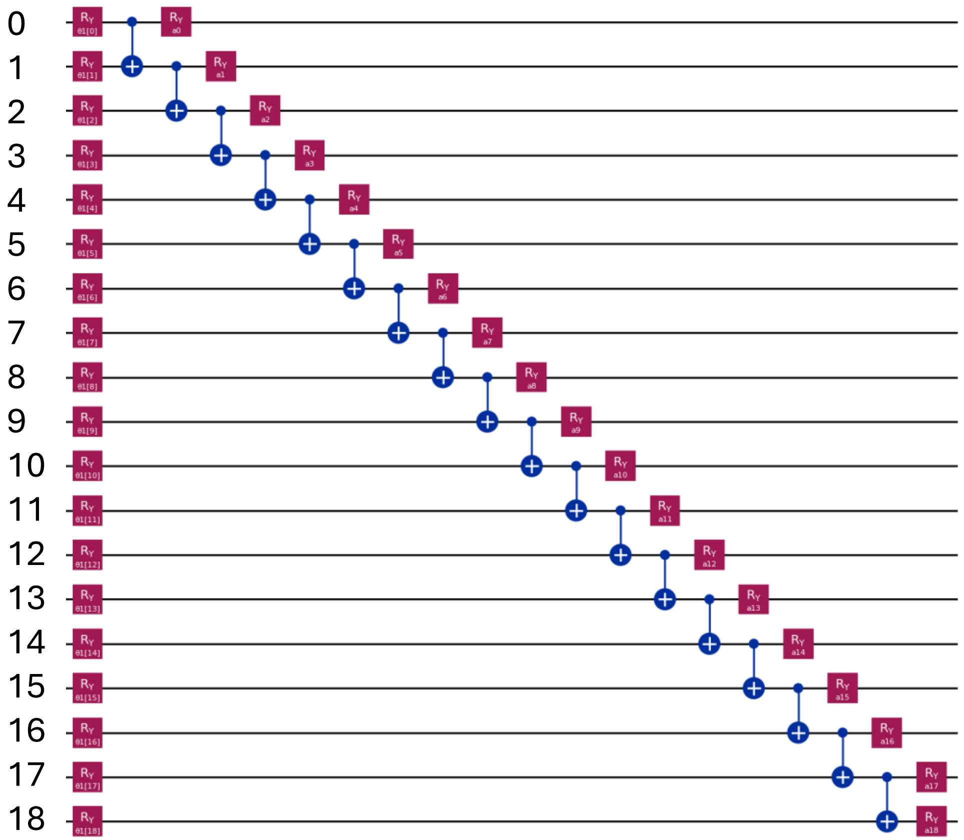


Two unit-cell Kagome lattice

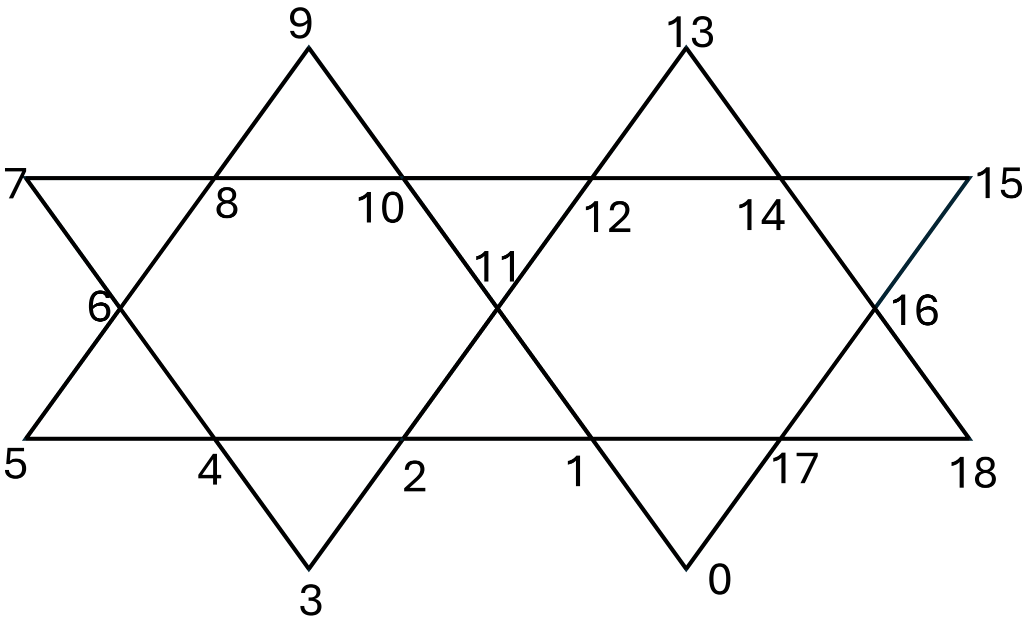
- 19 sites [0,1,2,..,18]
- 30 edges :

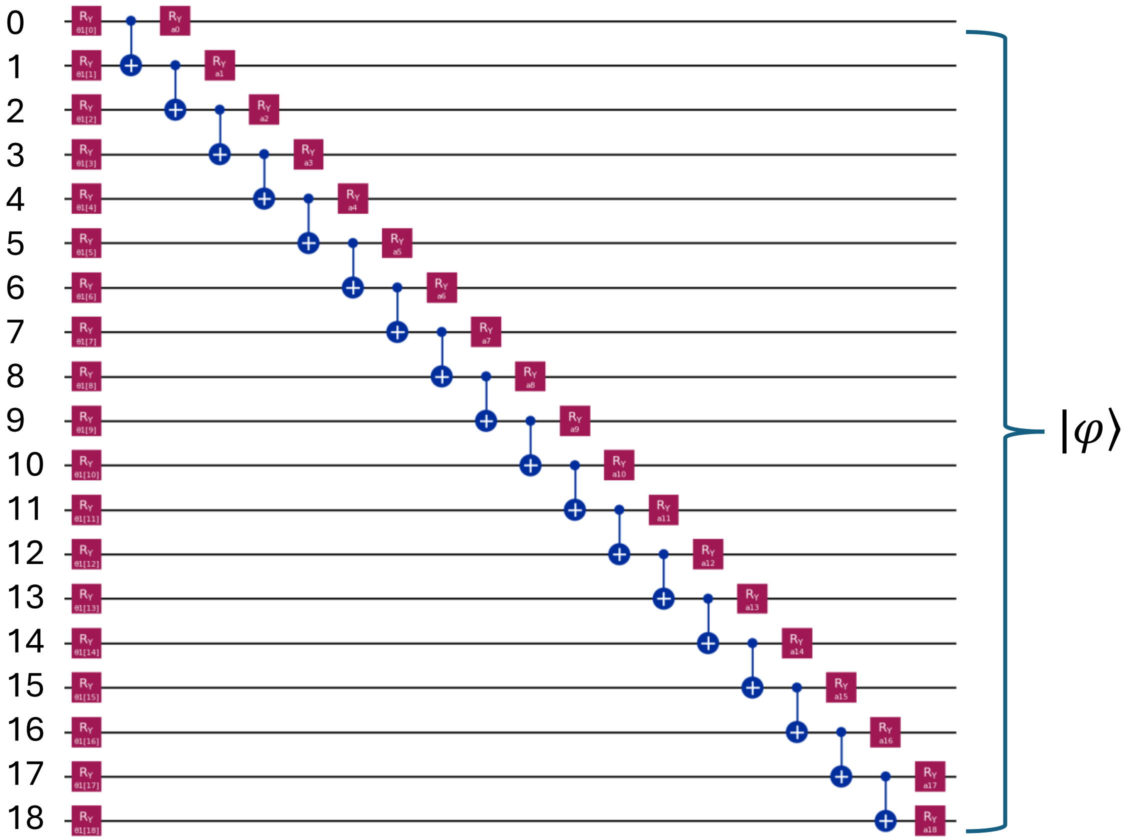
E={[0,1],[1,2],[1,11],[2,11],[2,3],[3,4],[2,4],[4,6],[4,5],[5,6],[6,7],[6,8],[7,8],[8,9],[9,10],[8,10],[10,11],[11,12],[10,12],[12,13],[13,14],[12,14],[14,15],[15,16],[14,1],[16,17],[17,1],[16,18],[17,0],[17,1]}

All edges have weight = 1, i.e., $\forall_{i,j}\ weight[i,j] = 1$



Qiskit:19-qubit Real Amplitude Single-layer ansatz





Heisenberg KAFM Hamiltonian

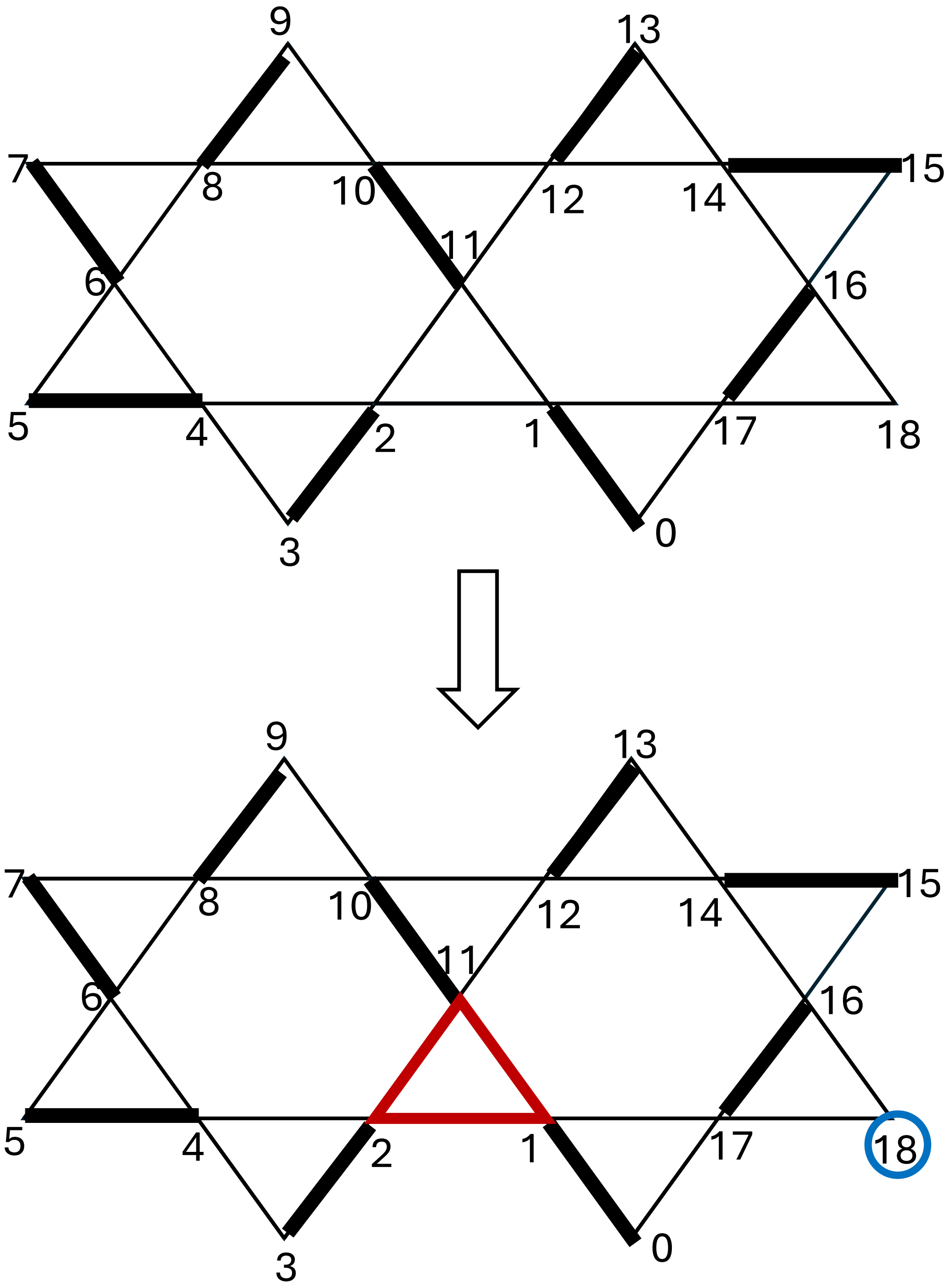
$$H = \sum_{[i,j]\in E} X_i X_j + Y_i Y_j + Z_i Z_j$$

True GSE = -29.14

Qiskit: 19-qubit Real Amplitude
Single-layer ansatz

The ansatz's lowest estimate of GSE: $\langle\varphi|H|\varphi\rangle$ = -27.2

Error = 6.7%



Lattice with dimer covering
C = {[0,1],[2,3],[4,5],[6,7],[8,9],
[10,11],[12,13],[14,15],[16,17]}

Dimer

Defect Triangle

Spinion

Dimer covering C resulting in
defect triangle and spinion

Perturbed Hamiltonian of the Defect Triangle

Original Hamiltonian: H

Perturbed Hamiltonian: H_{pert}

$$H_{orig} = X_aX_b + Y_aY_b + Z_aZ_b + X_bX_c + Y_bY_c + Z_bZ_c + X_cX_a + Y_cY_a + Z_cZ_a$$

$$H_{pert} = (1+wt)\,(X_aX_b + Y_aY_b + Z_aZ_b) + (1+wt)(X_bX_c + Y_bY_c + Z_bZ_c) + (1+wt)\,(X_cX_a + Y_cY_a + Z_cZ_a)$$

where $wt \approx 0.5$ (WHY?)

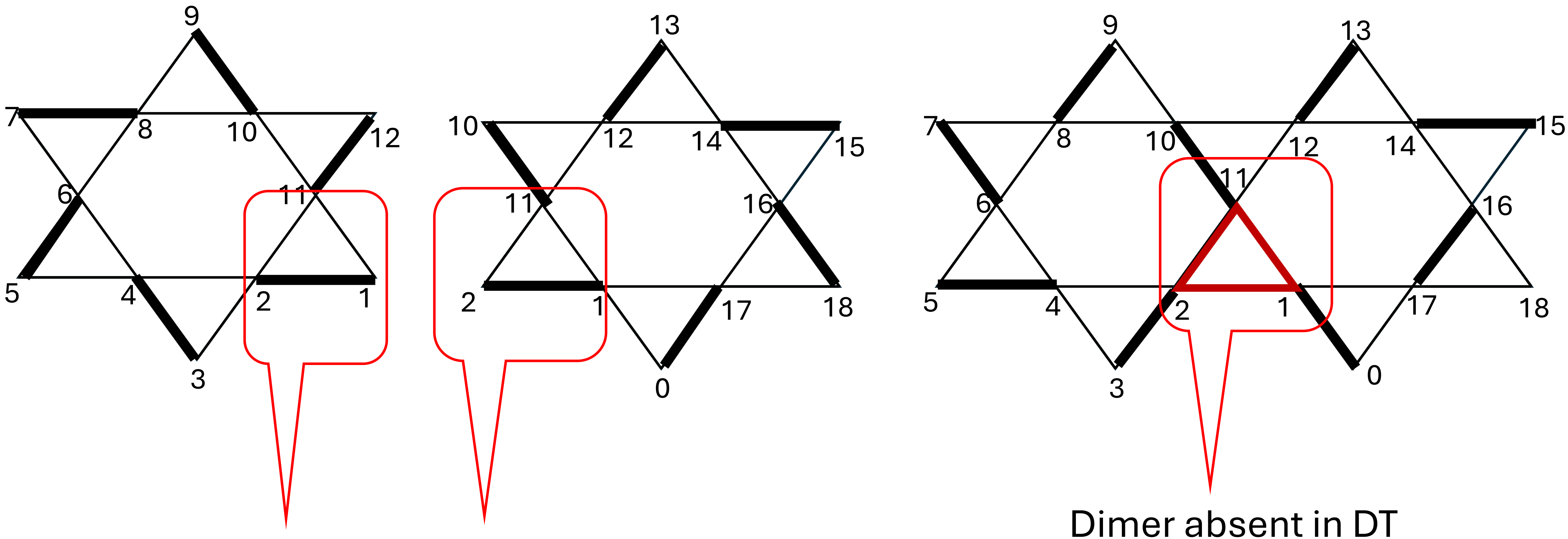
Calculating additional weight: wt

- N → Number of Kagome unit cells, mapped by the ansatz circuit, whose dimer covering creates the DEFECT TRIANGLE (DT)

- K (≤ 3) → Number of individual Kagome unit cells, each separately mapped by respective ansatz circuit, whose dimer covering places a dimer in DT

- $wt = N/(N + k)$

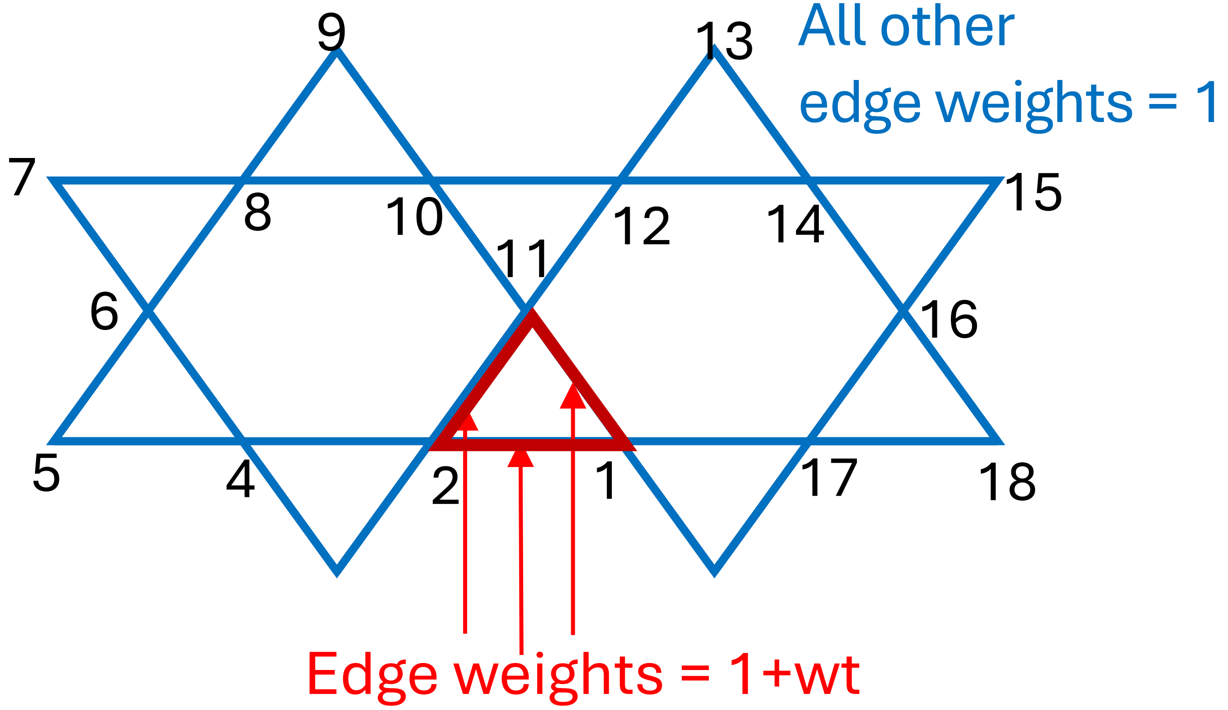
Example



Dimer present in DT
k = 1+1 = 2

Dimer absent in DT
N = 2

Example (contd...)



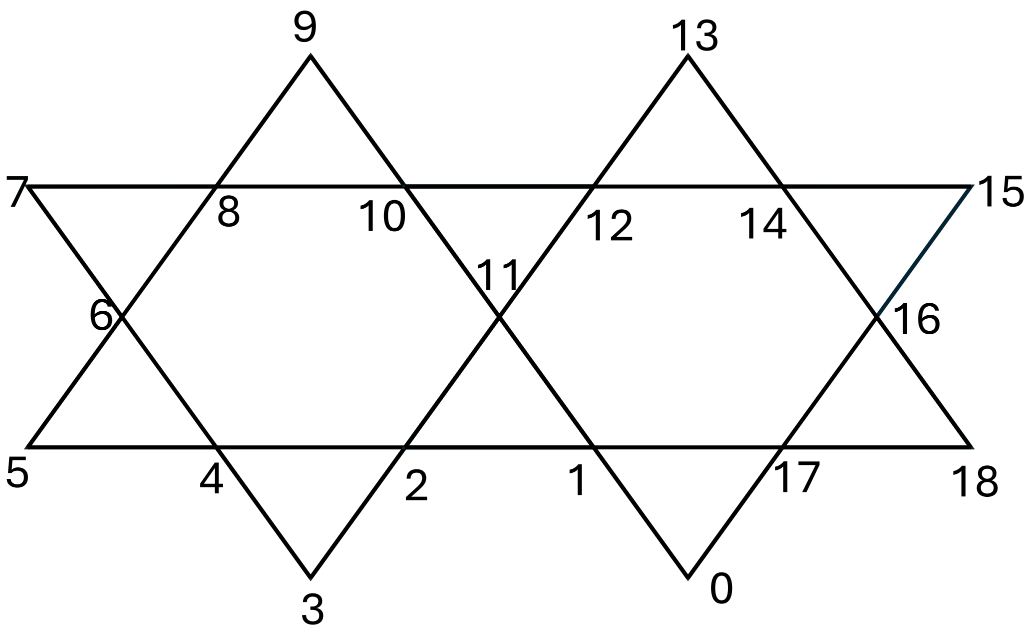
$$\underbrace{\texttt{U=\{[0,1],[2,3],[3,4],[2,4],[4,6],[4,5],[5,6],[6,7],[6,8],[7,8],[8,9],[9,10],[8,10],[10,11],[11,12],[10,12],[12,13],[13,14],[12,14],[14,15],[15,16],[14,1],[16,17],[17,1],[16,18],[17,0],[17,1]\}}}_{\text{weight = 1}}$$

$$\underbrace{\texttt{S = \{[1,2],[1,11],[2,11]\}}}_{\text{weight = 1+wt}}$$

$$H_{\text{pert}} = \sum\nolimits_{[i,j]\in U} X_i X_j + Y_i Y_j + Z_i Z_j + (1+wt)\sum_{[i,j]\in S} X_i X_j + Y_i Y_j + Z_i Z_j$$

← H_{pert}: Perturbed Hamiltonian

Example (contd...)



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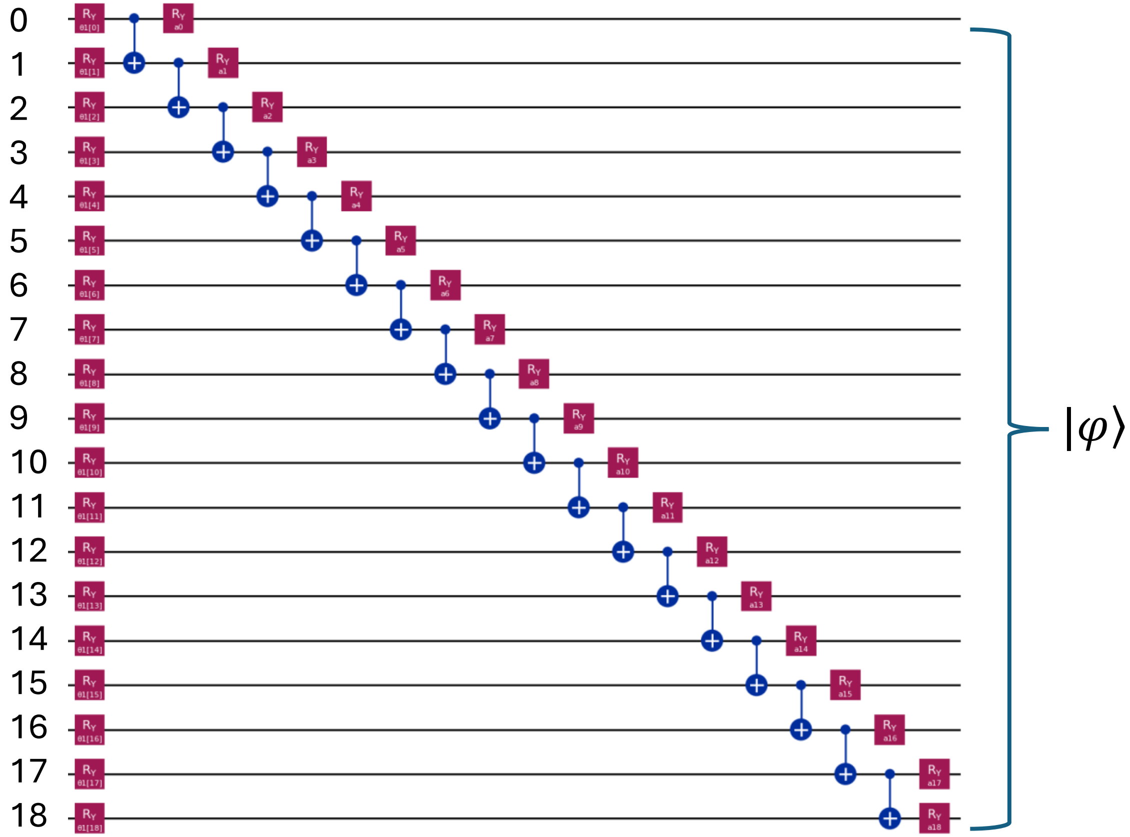
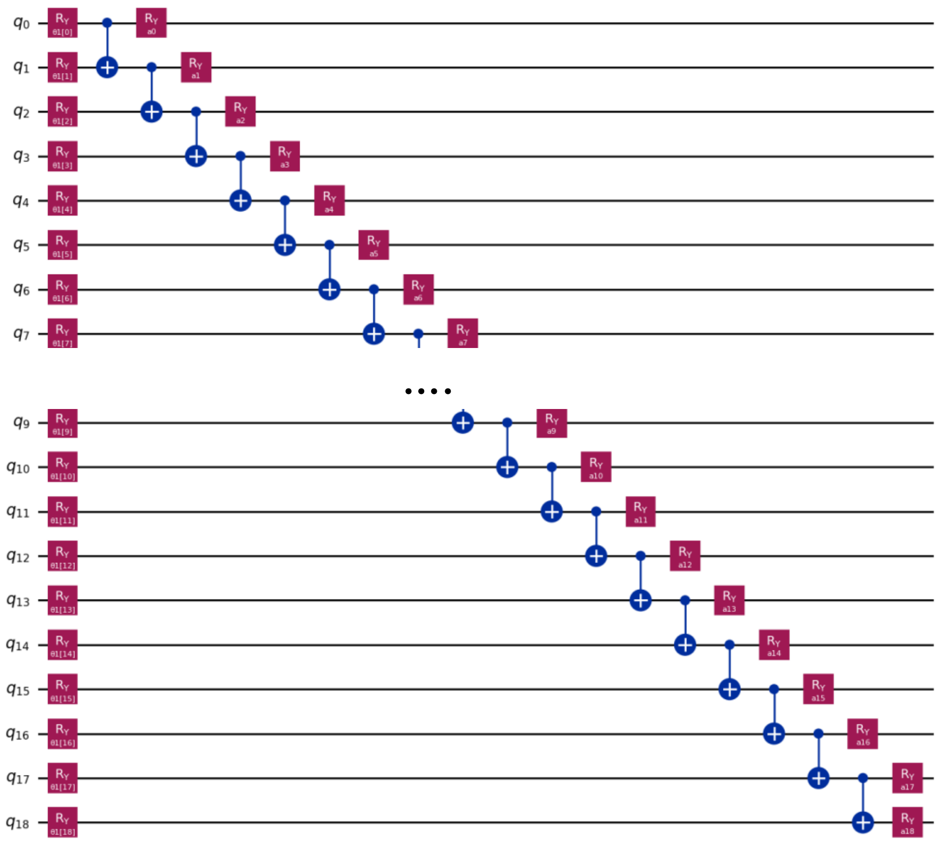


The ansatz's lowest estimate of GSE: $\langle\varphi|H_{pert}|\varphi\rangle$ = -29.2

With perturbed Hamiltonian same ansatz lowers the GSE error to 0.9% (<< 6.7%)

Modified (new) proposal:
To quantum compute: $min_\varphi \langle \varphi | H_{pert} | \varphi \rangle$



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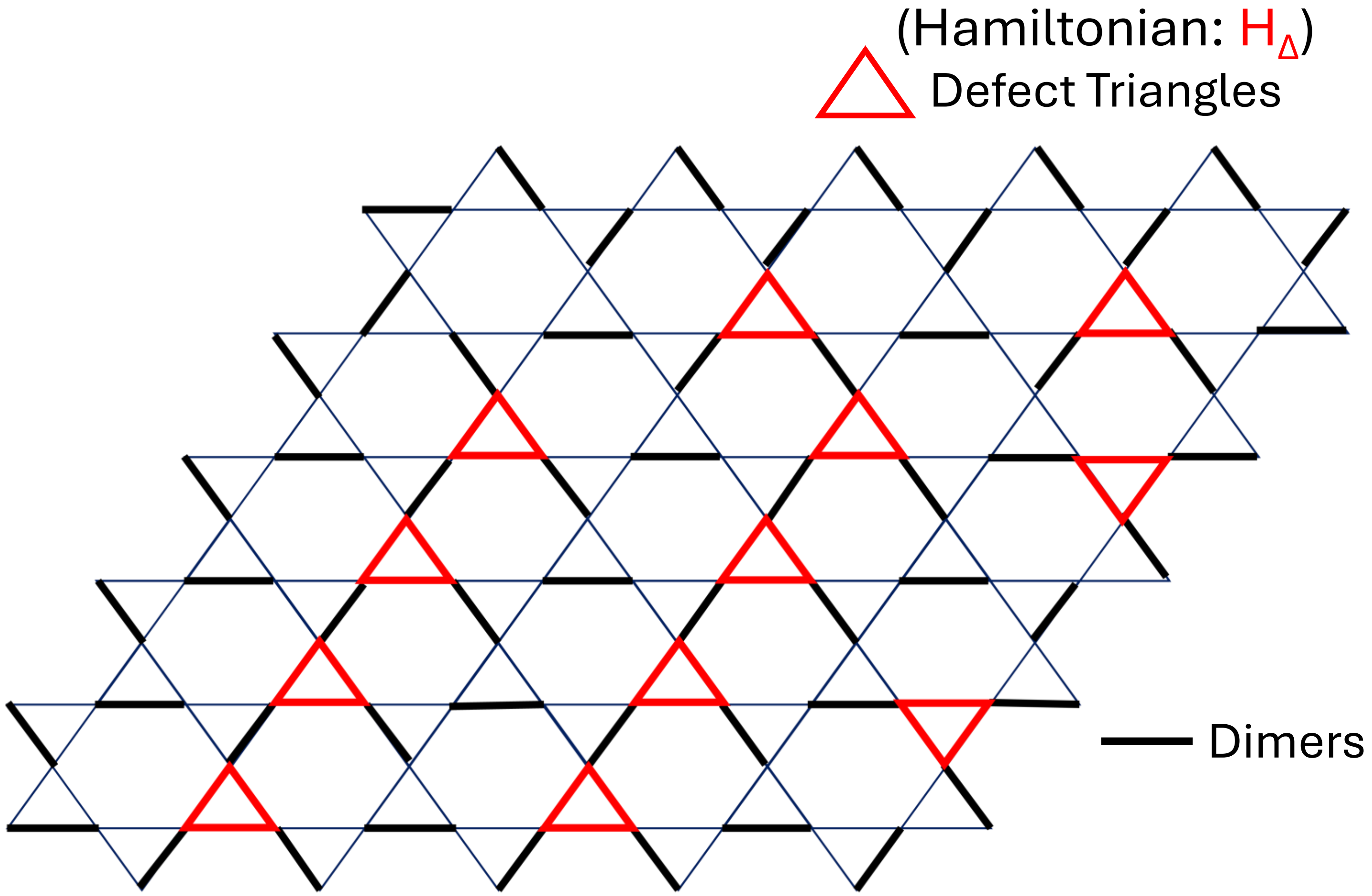

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