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CSE 331

Section - ~~10~~ 17

Assignment - 02

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(Answer to the Question-01)

Given, $L_1 = \{ w = 1^n : n \text{ is a power of } 2 \}$

so, $w = 1^{n^2}$,

for the sake of contradiction we assume, this Language is regular.

so, $s = 1^{p^2}$; $p^2 \geq p$

now, $|y| > 0$; $|xy| \leq p$ and xy^iz for any long number of i this Language will be regular. $i \geq 0$. if we consider,

$s = xy^2z = xyyz$ | Pumping 1 $|y|$ times

$\Rightarrow 1^{p^2 + |y|}$

the perfect square,

$1, 4, 9, 16, \ldots$

it has to be $1 \quad (1+1) = 2^2 \quad (1+2) = 3^2 \quad (1+3) = 4^2$

$(p+1)^2$, $p^2 + |y| \geq (p+1)^2$ $\boxed{\text{must be}}$

$\Rightarrow |y| \geq 2p + 1$, But $|xy| \leq p$

$\therefore$ so, $|y| \leq p$

which is a contradiction.

so, $w \notin L$, hence it is not a regular language.

(Answer to the Question - 02)

Given,

$L_2 = \{ w = 1^n : n \text{ is a perfect cube} \}$

For the sake of contradiction, lets assume,

$w = 1^{n^3}$ is a Legulagr language.

so, w has a ~~len~~ pumping length of P.

$\cdot S \to 1^{P^3}$ (1, 8, 27, 64 ...)

$\downarrow$ 1, 2^3, 3^3, 4

$\hookrightarrow (1+1)$ $\hookrightarrow (1+2)$ $(1+3)$

so, for $|y|>0$, $|xy| \leq P$ and xy^iz; $i \geq 0$ it will be a regular language.

if, $i=2$, $xy^2z \Rightarrow xyyz$

$\Rightarrow 1^{P^3+|y|}$

$\to P^3 + |y| \geq (P+1)^3$ Perfect cube.

$\Rightarrow P^3 + |y| \geq P^3 + 3P^2 + 3P + 1$

$\Rightarrow |y| \geq 3P^2 + 3P + 1$; but $|y| \leq P$ as $|xy| \leq P$ which is a contradiction.

so, $\cancel{w} \notin L_2$, Hence this is not a regular language.

(Answer to the Question - 03)

Given, $L_3 = \{ w w^R \mid w \in \Sigma^*$ and $w^R \}$
$(0,1)$

so, this ~~plc~~ a palindrom,

For the sake of contradiction, let's assume this is a regular language. So it will have a pumping length of P

now, $s \to \underbrace{0^P 1}_{w} \underbrace{1 0^P}_{w^R}$

here, As we know, $|xy| \le P$, so $|y|$ only consist of 0's in the string,

for, $i = 2$

$s \to xy^2z \to xyyz \to 0^{P+|y|} 1 \quad [|y| \le P]$

which is not equals to w^R

w contains more 0's the w^R.

now, if we ~~take~~ ~~$i=0$, the $xy^0z \to |xz|$~~ can say this ~~fune~~ Language is irregular.

(Answer to the Question - 04)

Given,

$L_4 = \{ w_1 \# w_2$ such that $|w_1| = |w_2| \}$

for the sake of ~~assumption~~ contradiction, let's consider this is a Regular Language. So this will have a pumping length of p.

$$S \to w_1^p \# w_2^p$$

so, $|y| > 0$, $|xy| \le p$ and $S = xy^iz$; $i \ge 0$ for any value of y it will be $\in L_4$.

now, $xy^iz \to i=2 \to xyyz$

$\Rightarrow p+|y|$, which is no longer equal to the length of w_2^p. which contradicts the Language.

so, $w_1 \# w_2 \notin L_4$

so, this language is not regular.

Answer to language (a 5)

$L_5 = \{ w \in \Sigma^* \mid w = 0^i 1^j \text{ where } \underline{i < 3j} \}$

for the sake of contradiction, lets assume this language is regular.

$w =$

$s \rightarrow 0^P 1^{3P+1}$

so, $|y| > 0$, $|xy| \leq P$, xy^iz for any $i \geq 0$ it should be a regular Language.

so, for any number of P ~~= 3P+1~~, if we consider $i \geq 3P+1$, the number of 0's will be greater than 1,

if $P = 1$, $i \geq 3 \cdot 1 + 1$

$i \geq 4 \rightarrow i = 5$

so, $xy^iz \rightarrow xy^5z = xyyyyyz$

$\Rightarrow P + 5|y| \geq 3P+1$, which contradicts the Language

so, $w_1 \notin L_5$

So, the language is not regular.