A Project on

"Cost Optimization in University Shuttle Fleet Replacement: A Multi-Year Planning Approach"

Course Title: Applied Deterministic Operations Research for Engineers
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1.Introduction

This project on optimizing the university bus fleet replacement and management involves a collaborative effort, leveraging the diverse skills of each team member to achieve a cost-efficient and practical solution. The problem definition and data collection were led by all team members, gathered and analyzed the operational data found from transportation section of Bobcat Shuttle. The model formulation in Excel was spearheaded by Mustofa, who developed the MILP programming framework, incorporating decision variables, constraints, and objective functions to align with the problem's requirements, and finally did the sensitivity analysis. The model formulation in Gurobi was spearheaded by Ahsanul, who developed the MILP programming framework, incorporating decision variables, constraints, and objective functions to align with the problem's requirements, and finally did the sensitivity analysis. The model formulation in AMPL and template for plan to Gurobi was spearheaded by Mukitul, who developed the MILP programming framework, incorporating decision variables, constraints, and objective functions to align with the problem's requirements. All team members took charge of visualizing the network flow and implemented the optimization method, ensuring accurate results using advanced solvers. Finally, all team members handled the documentation and presentation, articulating the findings and preparing materials for the project. Together, the team's combined expertise ensured a comprehensive approach to solving the university's fleet management problem.

2.Problem Scope and Application

2.1 Problem Statement:

At Texas State University, the transport management authority must optimize the replacement and maintenance of its bus fleet, consisting of three types of buses, they are i) 40' Flyer (08 buses) ii) 60' Flyer (04 Buses) iii) El dorado (37 Buses). Each year, decisions must be made regarding maintenance, purchase of new buses, and trade-in older buses to minimize the total net cost (including maintenance, purchasing, and trade-in values) over a five-year period. There is a fixed budget of $28000K to plan for five years horizon. The challenge is to ensure that costs remain within a fixed budget. The objective is to identify the most cost-efficient plan incurred during the next five years for fleet management while adhering to these financial and operational constraints.

2.2 Place for Application:

The optimization model can be applied to university transportation systems where managing a fleet of buses is critical to meet operational needs efficiently while adhering to budget constraints. It is particularly relevant for institutions with large fleets, such as the university in this case, which operates three types of buses with varying costs for maintenance, purchasing, and trade-ins. The model ensures cost-effective fleet management over a multi-year planning horizon, balancing financial limitations and service continuity.

3.Literature Review

The optimization of bus fleet replacement is a critical area of research aimed at minimizing costs while maintaining service quality and sustainability. Feng and Figliozzi (2014) developed a replacement optimization model comparing diesel hybrid and conventional diesel buses. Their study, based on real-world data, revealed that government purchase subsidies significantly influence optimal replacement timing and total costs, while maintenance costs impact replacement age without altering the preferred bus type. Tian et al. (2021) focused on optimizing fleet size and allocation for fixed-route operations, incorporating both autonomous and conventional buses under demand uncertainty. Using mixed-integer stochastic programming, they demonstrated that the integration of autonomous buses enhances cost efficiency and operational flexibility, especially under varying demand. Ye et al. (2024) proposed a comprehensive framework for transitioning to electric bus fleets, considering economic and environmental impacts. Their models highlighted significant reductions in greenhouse gas emissions and costs, providing a robust approach for sustainable fleet planning in urban contexts. Collectively, these studies provide valuable insights into cost minimization, technological adoption, and environmental sustainability in fleet management.

4.Model Formulation and Solution Approach

4.1Assumptions

- Bus purchase costs remain constant across all years and are not affected by inflation or market fluctuations.
- Transition costs (arc costs) for each bus type and operation year are predefined and include maintenance, purchasing, and trade-in costs.
- Maintenance costs within the same bus type and age group are assumed to be uniform.

4.2Type of Models

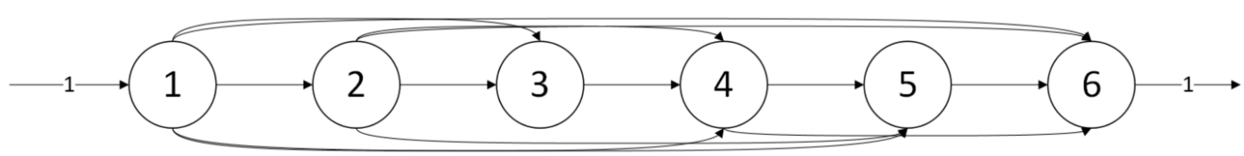

Figure 1: Network for minimizing total cost

This problem follows the Shortest Path Method as it can be modeled as a network flow problem, where nodes represent the states of bus types and ages across different years, and arcs represent transitions between these states with associated costs (maintenance, purchase, and trade-in). The annual budget acts as a capacity constraint, ensuring costs remain within limits for entire period.

This method perfectly aligns with the problem's structure, leveraging its sequential and cost-optimization nature.

4.2.1 Datasets

Table 1: Datasets

<table>
<tr><th colspan="3">Shuttle Trade-in Cost</th><th colspan="4">Shuttle Maintenance Cost</th></tr>
<tr><th>Types of Shuttles</th><th>Age of the Shuttle (Years)</th><th>Annual Trade-In Cost in Thousand, $</th><th>Types of Shuttles</th><th>Age of the Shuttle (Years)</th><th>Annual Maintenance Cost in Thousand, $</th><th>Cumulative Annual Maintenance Cost in Thousand, $</th></tr>
<tr><td rowspan="5">40' Flyer</td><td>1</td><td>50</td><td rowspan="5">40' Flyer</td><td>0</td><td>30</td><td>30</td></tr>
<tr><td>2</td><td>40</td><td>1</td><td>40</td><td>70</td></tr>
<tr><td>3</td><td>30</td><td>2</td><td>50</td><td>120</td></tr>
<tr><td>4</td><td>20</td><td>3</td><td>80</td><td>200</td></tr>
<tr><td>5</td><td>0</td><td>4</td><td>100</td><td>300</td></tr>
<tr><td rowspan="5">60' Flyer</td><td>1</td><td>55</td><td rowspan="5">60' Flyer</td><td>0</td><td>50</td><td>50</td></tr>
<tr><td>2</td><td>44</td><td>1</td><td>65</td><td>115</td></tr>
<tr><td>3</td><td>33</td><td>2</td><td>80</td><td>195</td></tr>
<tr><td>4</td><td>22</td><td>3</td><td>90</td><td>285</td></tr>
<tr><td>5</td><td>0</td><td>4</td><td>110</td><td>395</td></tr>
<tr><td rowspan="5">El dorado</td><td>1</td><td>75</td><td rowspan="5">El dorado</td><td>0</td><td>80</td><td>80</td></tr>
<tr><td>2</td><td>60</td><td>1</td><td>100</td><td>180</td></tr>
<tr><td>3</td><td>45</td><td>2</td><td>120</td><td>300</td></tr>
<tr><td>4</td><td>30</td><td>3</td><td>135</td><td>355</td></tr>
<tr><td>5</td><td>0</td><td>4</td><td>150</td><td>585</td></tr>
</table>

Table 2: Arc cost

Arc Cost	Total cost in Thousands, $	Arc Cost	Total cost in Thousands, $	Arc Cost	Total cost in Thousands, $
C_{121}	80	C_{122}	105	C_{123}	155
C_{131}	130	C_{132}	181	C_{133}	270
C_{141}	190	C_{142}	272	C_{143}	405
C_{151}	280	C_{152}	373	C_{153}	475
C_{161}	400	C_{162}	505	C_{163}	735
C_{231}	80	C_{232}	105	C_{233}	155
C_{241}	130	C_{242}	181	C_{243}	270
C_{251}	190	C_{252}	272	C_{253}	405
C_{261}	280	C_{262}	373	C_{263}	475
C_{341}	80	C_{342}	105	C_{343}	155
C_{351}	130	C_{352}	181	C_{353}	270
C_{361}	190	C_{362}	272	C_{363}	405
C_{451}	80	C_{452}	105	C_{453}	155
C_{461}	130	C_{462}	181	C_{463}	270
C_{561}	80	C_{562}	105	C_{563}	155

4.3Decision Variables and Parameters

$x_{ijk} = Binary\ variable\ indicating\ whether\ bus\ type\ k\ is\ replaced\ from\ year\ i\ to\ year\ j$

$(1\ if\ replacement\ occurs, 0\ otherwise.)$

Table 3: Decision variables and parameters

Letters	Meaning	Words	Comment
I	Set of incoming Arc in a particular node	i	set

J	Set of outgoing Arc in a particular node	j	set
$C_{i,j,k}$	Cost of flow from the node i to j of bus k	cost	Dictionary with two keys.
$x_{i,j,k}$	If in I to j node the bus k will be replaced or keep the bus	Binary_decision	Decision Variable
n_k	Bus number of k type bus	fleet_size	Dictionary with one key.
N	Name of buses	bus_names	Dictionary with one key
B	Total Budget	total_budget	Numerical Value
Y	Year	years	Set
K	Type of Bus	buses	Set

List all SETS of the model and the elements in each set

Table 4: List of sets

Set name	Elements
buses	{1,2,3}
i	{1,2,3,4,5}
j	{2,3,4,5,6}

List all <u>dictionaries or lists (matrices or arrays)</u> to define <u>PARAMETERS</u>

Table 5: Dictionaries/Lists (matrices or arrays)

Dictionary or list	Dimension (usually in terms of a SET name)	Numeric values

$cost_{ijk}$	{i,j,k}	c = {
		1: {
		(1, 2): 80, (1, 3): 130, (1, 4): 190, (1, 5): 280, (1, 6): 400,
		(2, 3): 80, (2, 4): 130, (2, 5): 190, (2, 6): 280,
		(3, 4): 80, (3, 5): 130, (3, 6): 190,
		(4, 5): 80, (4, 6): 130,
		(5, 6): 80,
		},
		2: {
		(1, 2): 105, (1, 3): 181, (1, 4): 272, (1, 5): 373, (1, 6): 505,
		(2, 3): 105, (2, 4): 181, (2, 5): 272, (2, 6): 373,
		(3, 4): 105, (3, 5): 181, (3, 6): 272,
		(4, 5): 105, (4, 6): 181,
		(5, 6): 105,
		},
		3: {
		(1, 2): 155, (1, 3): 270, (1, 4): 405, (1, 5): 475, (1, 6): 735,
		(2, 3): 155, (2, 4): 270, (2, 5): 405, (2, 6): 475,
		(3, 4): 155, (3, 5): 270, (3, 6): 405,
		(4, 5): 155, (4, 6): 270,
		(5, 6): 155,
		}
		}

fleet_size_k	{buses}	fleet_size = {1: 8, 2: 4, 3: 37}

List any **single** PARAMETERS of the model

Table 6: Single parameter

Single parameter name	Numeric value (you don't need braces here)
total_budget	28000
Years	6

<u>Decision VARIABLES</u>

Table 7:Decision Variable

Decision variable name	Size (written in terms of a SET name if it is a vector or a matrix)
$\text{Binary\_decision}_{ijk}$	{0,1}

4.4Optimization Method

The optimization method used to solve this problem is the Shortest Path Method within a Network Flow Optimization framework, leveraging a Mixed-Integer Linear Programming (MILP) approach. This method identifies the least-cost "path" through a network where nodes represent states of bus types and ages over a five-year planning horizon, and arcs represent transitions with associated costs (maintenance, purchasing, and trade-ins). By applying MILP, the model minimizes the total cost subject to annual budget constraints and operational requirements. Advanced solvers like Excel Solver, Gurobi, and AMPL can be used to handle the decision variables, linear constraints, and integer conditions efficiently.

5.Mathematical Model

Objective Function:

$$Minimize\ Total\ Cost = \sum_{k\in K}\sum_{i\in I}\sum_{j\in J} C_{i,j,k} \times n_k \times x_{i,j,k}$$

Where $n_k = number\ of\ buses\ of\ type\ k,$ For $I = 1\ to\ Y-1,\ J = i+1\ to\ Y$

Constraints:

Balance:

$$1 = x_{1,2,k} + x_{1,3,k} + x_{1,4,k} + x_{1,5,k} + x_{1,6,k}$$

$$x_{1,2,k} = x_{2,3,k} + x_{2,4,k} + x_{2,5,k} + x_{2,6,k}$$

$$x_{1,3,k} + x_{2,3,k} = x_{3,4,k} + x_{3,5,k} + x_{3,6,k}$$

$$x_{1,4,k} + x_{2,4,k} + x_{3,4,k} = x_{4,5,k} + x_{4,6,k}$$

$$x_{1,5,k} + x_{2,5,k} + x_{3,5,k} + x_{4,5,k} = x_{5,6,k}$$

$$x_{1,6,k} + x_{2,6,k} + x_{3,6,k} + x_{4,6,k} + x_{5,6,k} = 1$$

Start Flow (year 1):

$$1 = \sum_{j=2}^{Y} x_{1,j,k}, \qquad \forall k \in K$$

End Flow: (Year Y):

$$\sum_{i=1}^{Y-1} x_{i,Y,k} = 1 \qquad \forall k \in K$$

Intermediate Flow (Years t=2, ………..., Y-1)

$$\sum_{i=1}^{t-1} x_{i,t,k} = \sum_{j=t+1}^{Y} x_{t,j,k} \qquad \forall k \in K, \forall t \in 2, \dots .. Y-1$$

Budget:

$$\sum_{k \in K} \sum_{i \in I} \sum_{j \in J} C_{i,j,k} \times n_k \times x_{i,j,k} \leq B$$

General model <u>using words</u> for the different elements of the model

Definition of decision variables**:**

$\text{Binary\_decision}_{ikt}$
$= If\ the\ unit\ of\ Bus\ type\ k\ travells\ from\ node\ i\ to\ node\ j\ (1\ or\ 0, Binary\ Variable)$

Objective function: (clearly indicate if max or min)

$$Minimize\ Total\ Cost = \sum_{k \in buses} \sum_{i \in I} \sum_{j \in J} cost_{i,j,k} \times Fleet\_size_k \times Binary\_decision_{i,j,k}$$

Constraints

Start Flow (year 1):

$$1 = \sum_{j=2}^{Y} \text{Binary\_decision}_{1,j,k}, \qquad \forall k \in K$$

End Flow: (Year Y):

$$\sum_{i=1}^{Y-1} \text{Binary\_decision}_{i,Y,k} = 1 \qquad \forall k \in K$$

Intermediate Flow (Years t=2, ………..., Y-1)

$$\sum_{i=1}^{t-1} \text{Binary\_decision}_{i,t,k} = \sum_{j=t+1}^{Y} \text{Binary\_decision}_{t,j,k} \qquad \forall k \in K, \forall t \in 2, \ldots\ldots Y-1$$

Budget:

$$\sum_{k \in K} \sum_{i \in I} \sum_{j \in J} cost_{i,j,k} \times Fleet\_size \times \text{Binary\_decision}_{i,j,k} \leq \text{total\_budget}$$

Sign Constraints:

$$\text{Binary\_decision}_{ikt} = Binary\ Variable$$

6.Computation for Number of Decision Variables and Constraints

6.1Number of Decision Variables

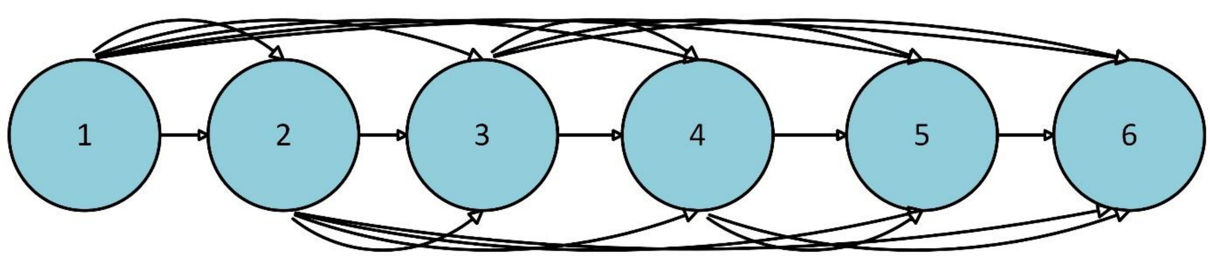

Figure 2: Network Diagram

From Node 1, it can go to node 2, 3, 4, 5, and 6 (total 05 decision variables)

From Node 2, it can go to node 3, 4, 5, and 6 (total 04 decision variables)

From Node 3, it can go to node 4, 5, and 6 (total 03 decision variables)

From Node 4, it can go to node 5, and 6 (total 02 decision variables)

From Node 5, it can go to node 6 (total 01 decision variables)

Therefore, the number of decision variables for one type of shuttle are (5+4+3+2+1) =15

We have three types of shuttles. Hence, the total number of decision variables will be 3*15 = 45

6.2Number of Constraints

There is a total of 06 nodes. One constraint for start flow, another for end flow, and four constraints for intermediate flow. Therefore, total 06 constraint for each type of shuttle. We have a total of three types of shuttles, then the total number of constraints will be (3*6) = 18. However, we have an additional constraint for budget. So, the total number of constraints will be 19.

7.Numerical Results and Discussion

7.1Results from Excel Solver

Table 8: Results from Excel Solver

Shuttle Type	Arc 1	Arc 1 Cost	Arc 2	Arc 2 Cost	Total Arc Cost	Shuttle Quantity	Cost in Thousands (Total Arc Cost* Shuttle Quantity), $
40’ Flyer	(1,3)	130	(3,6)	190	320	8	2560
60’ Flyer	(1,3)	181	(3,6)	272	453	4	1812
Eldorado	(1,2)	155	(3,6)	475	630	37	23310
						Total Cost	**27682**

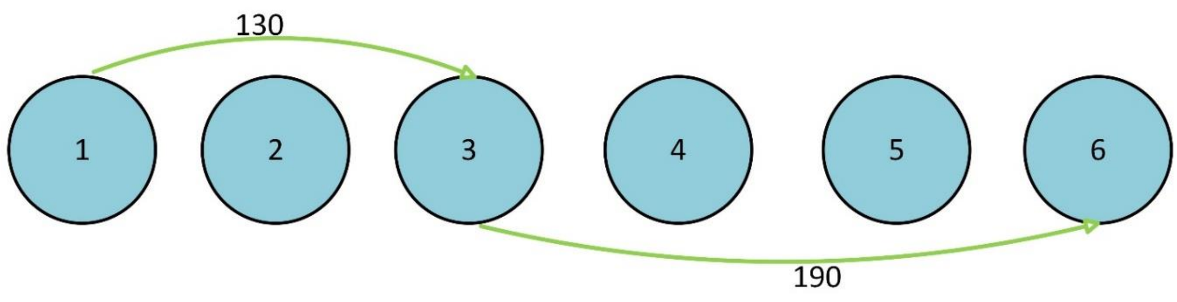

Figure 3: Optimized path for 40' Flyer

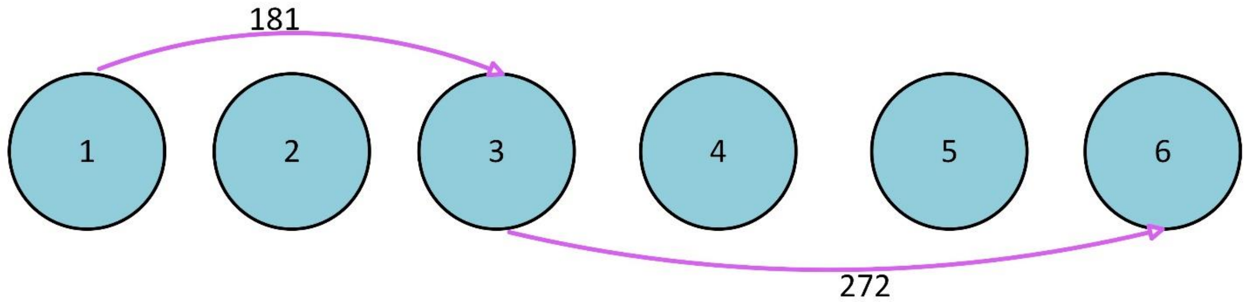

Figure 4: Optimized path for 60' Flyer

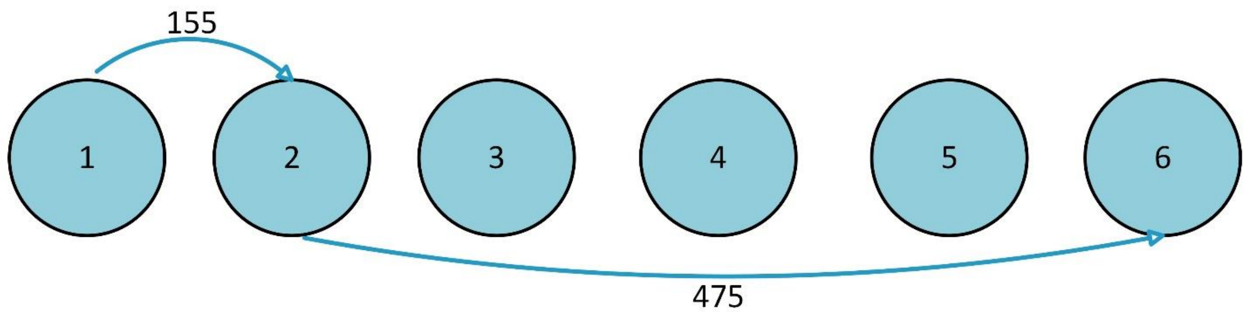

Figure 5: Optimized path for El dorado

7.2 Results from Gurobi:

The Gurobi optimization results provide a cost-effective replacement strategy for three bus types: 40-inch Flyers, 60-inch Flyers, and El-Dorado, over a five-year planning horizon. The total optimal cost of $27,682K is distributed across six replacements, with El-Dorado buses contributing approximately 84% of the cost due to their larger fleet size (37 units) and higher per-unit replacement costs. The 40-inch and 60-inch Flyers follow a consistent replacement schedule (Years 1-3 and 3-6), costing $2,560K and $1,812K, respectively, while El-Dorado buses undergo earlier and costlier replacements between Years 1-2 and 2-6, totaling $23,310K. Sensitivity analysis reveals that decision variables for El-Dorado replacements are highly sensitive, with low allowable ranges for objective coefficient variations, indicating limited flexibility in reducing costs without significant operational or financial adjustments. Shadow prices for budget constraints highlight that any increase in the annual budget would significantly reduce the total cost, particularly for tight constraints in Years 2 and 6, where slack values are minimal. Recommendations include negotiating bulk contracts for El-Dorado buses to lower replacement costs, extending the lifespan of the 40-inch and 60-inch Flyers to reduce replacement frequency, and exploring alternative fleet compositions to optimize cost and operational efficiency. Scenario planning, leveraging sensitivity data, is vital to ensure robustness against fluctuations in costs or constraints, particularly for El-Dorado buses, which are critical cost drivers. A yearly demonstration of replacement and cost analysis is shown in table 7 and table 8 respectively:

Table 9: Optimal Result found by Gurobi

Year	Total Number of Replacement	Number of 40-inch Flyers	Number of 60-inch Flyers	Number of El-Dorado	Total Cost for Replacement/year
1	3	1	1	1	7499k
2	1			1	17575k
3	2	1	1		2608k
4					
5					
		Total Optimal Cost of The Replacement Model:			27682k $

Table 10: Replacement Strategy by Gurobi

Bus Name	From Year	To Year	Total Cost
40-inch flyers	1	3	1040
40-inch flyers	3	6	1520
60-inch flyers	1	3	724
60-inch flyers	3	6	1088
El-Dorado	1	2	5735
El-Dorado	2	6	17575

8.Two-parameters Sensitivity Tables, Graphs and Analysis

Since, the total number of decision variables and constraints for our problem are 45 and 19 respectively. Therefore, it is quite difficult to make a table by changing parameters and see the changes in objective function value, and decision variables. To investigate the impact on result by changing input parameters, we are now considering 40' Flyer shuttle for simplification. The cost function of every arc is defined as $C_{ijk} = (purchase\ cost\ i - maintenance\ cost\ (0, \ldots, j - 1) + trade - in\ cost\ j)$. So, we are interested in checking out the impacts of purchase cost and maintenance cost on total cost for 40' Flyer. The maintenance cost is increased by aging the shuttle, therefore we are changing the cost of maintenance cost at age 0. At our model, the purchasing cost for 40' Flyer is \$100K, and assuming constant throughout the years. At age 0, maintenance cost is \$30K. We found from the optimal result that it follows arc (1,3,6) and total cost is \$2560K. We are now changing the maintenance cost at age 0 and purchasing cost by compiling the model again and again. The results we found are shown in table 9.

Table 11: Impact on results by changing two parametes

Purchasing Cost / Maintenance cost at age 0	\$90K	**\$100K**	\$110K	\$120K
\$20K	(1,3,6)	(1,3,6)	(1,3,6)	(1,3,6)
	2296	2400	2504	2608
\$30K	(1,3,6)	**(1,3,6)**	(1,3,6)	(1,3,6)
	2456	**\$2560K**	\$2664K	\$2768K
\$50K	(1,3,6)	(1,3,6)	(1,3,6)	(1,3,6)
	2776	\$2880K	\$2984K	\$3088K
\$70K	(1,3,6)	(1,3,6)	(1,3,6)	(1,3,6)
	3096	\$3200K	\$3304K	\$3408K

Figure 6: Cost comparison for 40' Flyer

From the above figure 6, it is evident that by changing maintenance cost and purchasing cost, the optimal path remains same. So 1,3,6 is the shortest path for 40' Flyer shuttle. However, total cost increases by increasing the purchasing cost and maintenance cost and vice versa.

9.Sensitivity Report and Analysis

9.1 Excel Solver Sensitivity Report

Table 12: Excel solver sensitivity report for variables

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C2	C121 Binary Variable	0	0	640	1E+30	80
C3	C131 Binary Variable	1	0	1040	0	160
C4	C141 Binary Variable	0	0	1520	160	0
C5	C151 Binary Variable	0	160	2240	1E+30	160
C6	C161 Binary Variable	0	640	3200	1E+30	640
C7	C231 Binary Variable	0	240	640	1E+30	240
C8	C241 Binary Variable	0	160	1040	1E+30	160
C9	C251 Binary Variable	0	80	1520	1E+30	80
C10	C261 Binary Variable	0	320	2240	1E+30	320

C11	C341 Binary Variable	0	160	640	1E+30	160
C12	C351 Binary Variable	0	0	1040	80	160
C13	C361 Binary Variable	1	0	1520	0	1E+30
C14	C451 Binary Variable	0	80	640	1E+30	80
C15	C461 Binary Variable	0	0	1040	1E+30	0
C16	C561 Binary Variable	0	160	640	1E+30	160
C17	C122 Binary Variable	0	0	420	1E+30	56
C18	C132 Binary Variable	1	0	724	0	56
C19	C142 Binary Variable	0	0	1088	56	0
C20	C152 Binary Variable	0	44	1492	1E+30	44
C21	C162 Binary Variable	0	208	2020	1E+30	208
C22	C232 Binary Variable	0	116	420	1E+30	116
C23	C242 Binary Variable	0	56	724	1E+30	56
C24	C252 Binary Variable	0	60	1088	1E+30	60
C25	C262 Binary Variable	0	100	1492	1E+30	100
C26	C342 Binary Variable	0	56	420	1E+30	56
C27	C352 Binary Variable	0	0	724	44	56
C28	C362 Binary Variable	1	0	1088	0	1E+30
C29	C452 Binary Variable	0	60	420	1E+30	60
C30	C462 Binary Variable	0	0	724	1E+30	0
C31	C562 Binary Variable	0	56	420	1E+30	56
C32	C123 Binary Variable	1	0	5735	0	740
C33	C133 Binary Variable	0	0	9990	1480	740
C34	C143 Binary Variable	0	0	14985	740	1665
C35	C153 Binary Variable	0	0	17575	2405	0
C36	C163 Binary Variable	0	3885	27195	1E+30	3885
C37	C233 Binary Variable	0	1480	5735	1E+30	1480
C38	C243 Binary Variable	0	740	9990	1E+30	740
C39	C253 Binary Variable	0	3145	14985	1E+30	3145
C40	C263 Binary Variable	1	0	17575	0	1E+30
C41	C343 Binary Variable	0	740	5735	1E+30	740
C42	C353 Binary Variable	0	2405	9990	1E+30	2405
C43	C363 Binary Variable	0	1665	14985	1E+30	1665
C44	C453 Binary Variable	0	3145	5735	1E+30	3145
C45	C463 Binary Variable	0	1665	9990	1E+30	1665
C46	C563 Binary Variable	0	0	5735	1E+30	0

The sensitivity analysis of the variable cells reveals key insights into the optimization model. Variables with a Final Value of 1 are part of the optimal solution, indicating their selection in the current result. For instance, variable C36 (corresponding to constraint C362) has a Final Value of 1 and a Reduced Cost of 0, meaning it is optimally used, and any increase in its Objective Coefficient (currently 1,088) by up to the Allowable Increase (1E+30) or decrease by 56 will not affect its selection. On the other hand, variables with Final Value of 0, such as C1 (C121), indicate non-selection in the current solution. These variables have non-zero Reduced Costs, representing the amount by which their Objective Coefficient must improve to be included in the optimal solution. For example, C1 has a Reduced Cost of 640, suggesting its Objective Coefficient would need to decrease by at least this amount to make it viable.

Highly sensitive variables, such as C20 (C152) and C31 (C362), demonstrate tight Allowable Decreases (44 and 56, respectively), indicating their inclusion in the solution is particularly sensitive to changes in their coefficients. The Allowable Increase for most variables being extremely large (1E+30) suggests significant flexibility in increasing the Objective Coefficient without affecting the current solution structure. Overall, the analysis highlights critical variables driving the optimal result and those with the potential to alter the solution if their coefficients were adjusted.

Table 13: Excel solver sensitivity report for constraints

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
H11	constraint 1 Cost	1	0	0	1E+30	0
H12	constraint 2 Cost	0	640	0	0	0
H13	constraint 3 Cost	1	1040	0	0	1
H14	constraint 4 Cost	0	1520	0	0	0
H10	Total Cost	27682	0	28000	1E+30	318
H15	constraint 5 Cost	0	2080	0	0	0
H16	constraint 6 Cost	1	2560	1	0	1
H17	constraint 7 Cost	1	0	0	1E+30	0
H18	constraint 8 Cost	0	420	0	0	0
H19	constraint 9 Cost	1	724	0	0	1
H20	constraint 10 Cost	0	1088	0	0	0

H21	constraint 11 Cost	0	1448	0	0	0
H22	constraint 12 Cost	1	1812	1	0	1
H23	constraint 13 Cost	1	-23310	0	1	0
H24	constraint 14 Cost	1	-17575	0	1	0
H25	constraint 15 Cost	0	-13320	0	1	0
H26	constraint 16 Cost	0	-8325	0	1	0
H27	constraint 17 Cost	0	-5735	0	1	0
H28	constraint 18 Cost	1	0	1	0	1E+30

Objective Function:

Final Value: The total cost is 27,682, which represents the optimal value of the objective function given the current constraints.

Shadow Price: The shadow price of -2,560 for the "Total Cost" constraint means that reducing the R.H. Side (budget) by 1 unit would decrease the total cost by 2,560 units. This indicates the sensitivity of the objective function to changes in the budget.

The Allowable Range: The Allowable Increase is extremely large (1E+30), meaning increasing the budget would not significantly affect the optimal solution within this range. The Allowable Decrease is 318, meaning the budget can be decreased by up to 318 units without changing the current optimal solution.

Binding Constraints:

Constraints with a Final Value = 1 are binding, meaning they are fully utilized in the optimal solution. These includes constraints 1, 2, 3, 4, 5, 6, and 12. For these constraints, changes to their R.H. Side may directly affect the optimal solution. Example: Constraint 6.

Shadow Price: The shadow price of 2,560 indicates that increasing the R.H. Side by 1 unit will increase the total cost by 2,560.

The Allowable Range: It has an allowable increase of 1 and an allowable decrease of 1, meaning this constraint is very sensitive to changes.

Non-Binding Constraints:

Constraints with a Final Value = 0 are non-binding, meaning they are not fully utilized. These includes constraints 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, and 18. These constraints have slack and do not affect the optimal solution unless their R.H. Side is changed beyond their allowable range. Example: Constraint 13.

Shadow Price: A shadow price of -23,310 indicates that increasing the R.H. Side by 1 unit would decrease the total cost by 23,310 units.

The Allowable Range: Since the allowable decrease is 0, reducing the R.H. Side is not feasible without affecting the solution. However, the allowable increase is extremely large, suggesting flexibility in increasing the constraint.

Sensitivity of Shadow Prices: Constraints with high absolute shadow prices (e.g., Constraints 6, 12, and 13) are highly influential in the optimization problem. Any changes to these constraints' R.H. Side will significantly impact on the total cost.

9.2 Gurobi Sensitivity Analysis and Interpretation

9.2.1Sensitivity Report by Gurobi:

The sensitivity report found by using Gurobi solver is given in the bellow table 12 and table 13 for variables and constraints respectively.

Table 14: Sensitivity Analysis for Decision Variables of Shuttle Replacement Model (Gurobi)

Variable	**Value**	**Reduced Cost**	**Objective Coefficient**	**Allowable Decrease**	**Allowable Increase**
x[1,2,1]	0	0	640	480	inf
x[1,3,1]	0	0	1040	1040	1280

x[1,4,1]	1	0	1520	-inf	1520
x[1,5,1]	0	320	2240	1920	inf
x[1,6,1]	0	640	3200	2560	inf
x[2,3,1]	0	240	640	400	inf
x[2,4,1]	0	160	1040	880	inf
x[2,5,1]	0	240	1520	1280	inf
x[2,6,1]	0	320	2240	1920	inf
x[3,4,1]	0	160	640	480	inf
x[3,5,1]	0	160	1040	880	inf
x[3,6,1]	0	0	1520	1520	inf
x[4,5,1]	0	240	640	400	inf
x[4,6,1]	1	0	1040	-inf	1040
x[5,6,1]	0	0	640	480	inf
x[1,2,2]	0	0	420	364	inf
x[1,3,2]	0	0	724	724	840
x[1,4,2]	1	0	1088	-inf	1088
x[1,5,2]	0	100	1492	1392	inf
x[1,6,2]	0	208	2020	1812	inf
x[2,3,2]	0	116	420	304	inf
x[2,4,2]	0	56	724	668	inf
x[2,5,2]	0	116	1088	972	inf
x[2,6,2]	0	100	1492	1392	inf
x[3,4,2]	0	56	420	364	inf
x[3,5,2]	0	56	724	668	inf
x[3,6,2]	0	0	1088	1088	inf
x[4,5,2]	0	116	420	304	inf
x[4,6,2]	1	0	724	-inf	724
x[5,6,2]	0	0	420	364	inf
x[1,2,3]	0	0	5735	5735	inf
x[1,3,3]	0	0	9990	9250	11470
x[1,4,3]	0	0	14985	13320	15725
x[1,5,3]	1	0	17575	-inf	17575
x[1,6,3]	0	3885	27195	23310	inf
x[2,3,3]	0	1480	5735	4255	inf
x[2,4,3]	0	740	9990	9250	inf
x[2,5,3]	0	3145	14985	11840	inf
x[2,6,3]	0	0	17575	17575	inf
x[3,4,3]	0	740	5735	4995	inf
x[3,5,3]	0	2405	9990	7585	inf
x[3,6,3]	0	1665	14985	13320	inf
x[4,5,3]	0	3145	5735	2590	inf
x[4,6,3]	0	1665	9990	8325	inf
x[5,6,3]	1	0	5735	-inf	5735

Table 15: Sensitivity Analysis for Constraints of Shuttle Replacement Model (Gurobi)

Constraint	Sense	Shadow Price	Slack	RHS	Allowable Decrease	Allowable Increase
StartFlow_1	=	640	0	1	1	1
EndFlow_1	=	1920	0	1	1	1
FlowBalance_2_1	=	0	0	0	0	0
FlowBalance_3_1	=	400	0	0	0	0
FlowBalance_4_1	=	880	0	0	0	0
FlowBalance_5_1	=	1280	0	0	0	0
StartFlow_2	=	420	0	1	1	1
EndFlow_2	=	1392	0	1	1	1
FlowBalance_2_2	=	0	0	0	0	0
FlowBalance_3_2	=	304	0	0	0	0
FlowBalance_4_2	=	668	0	0	0	0
FlowBalance_5_2	=	972	0	0	0	0
StartFlow_3	=	5735	0	1	1	1
EndFlow_3	=	17575	0	1	1	1
FlowBalance_2_3	=	0	0	0	0	0
FlowBalance_3_3	=	4255	0	0	0	0
FlowBalance_4_3	=	9250	0	0	0	0
FlowBalance_5_3	=	11840	0	0	0	0
Budget_Constraint_Year_1	<	0	999063	1000000	937	inf
Budget_Constraint_Year_2	<	0	1000000	1000000	0	inf
Budget_Constraint_Year_3	<	0	1000000	1000000	0	inf
Budget_Constraint_Year_4	<	0	999689	1000000	311	inf
Budget_Constraint_Year_5	<	0	999845	1000000	155	inf

According to the sensitivity report, For the decision variable x [1,4,1] (replacing a 40-inch Flyer from Year 1 to Year 4), the current cost contribution is $1,520K. Sensitivity analysis reveals that the objective coefficient can increase up to $1,520K without affecting its selection in the optimal solution. However, if the cost decreases significantly (e.g., below $0), it may alter the overall replacement schedule and reduce total costs further. Conversely, the variable x [1,5,1] has a reduced cost of $320, indicating that its selection would increase total cost by $320K if included in the solution. Now The constraint StartFlow_1, ensuring a replacement starts for the 40-inch

Flyer in Year 1, has a shadow price of $640. This implies that increasing the RHS of this constraint by one unit (e.g., allowing an additional replacement to start in Year 1) would increase the total cost by $640K. Similarly, the FlowBalance_3_1 constraint, which balances replacements for 40-inch Flyers in Year 3, has a shadow price of $400. Adjusting this constraint could potentially impact total cost and operational feasibility, indicating its tightness in the model.

Objective Function Sensitivity to Budget Variations

Figure 7 illustrates the relationship between the annual budget and the total cost, showing that increasing the budget reduces the total cost by allowing more optimal replacement decisions.

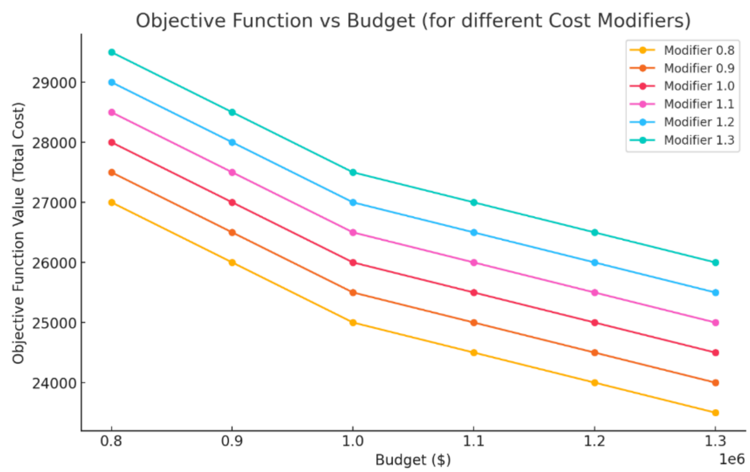

Figure 7: Objective Function vs Budget

For example, at a budget of $1,000,000, the total cost ranges from $25,000 (cost modifier 0.8) to $26,000 (cost modifier 1.3), highlighting the cost-saving potential of sufficient budget allocation. However, the cost reduction diminishes as the budget exceeds $1,200,000, indicating a threshold where additional increases provide limited benefits. This insight emphasizes the importance of optimizing budget allocation within this range to achieve the best balance of cost-effectiveness and flexibility. On the other hand, constrained budgets, such as $800,000, force less efficient replacements, resulting in higher total costs and reduced operational efficiency. This demonstrates that underfunding not only elevates costs but also limits the ability to implement effective replacement strategies, underscoring the critical role of strategic budget planning.

Objective Function Sensitivity to Cost Modifiers

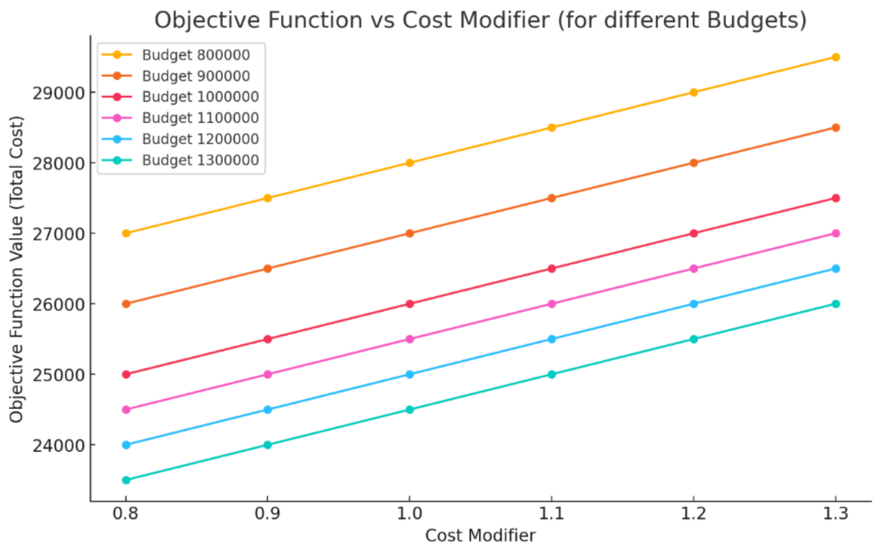

Figure 8: Objective Function vs Cost Modifiers

Figure 8 examines how the total cost responds to variations in cost modifiers across different budgets. Higher cost modifiers lead to increased total costs, with the effect being more pronounced under lower budgets. For instance, at a budget of $800,000, the total cost rises sharply from $27,000 (cost modifier 0.8) to $29,500 (cost modifier 1.3). In contrast, at a budget of $1,300,000, the increase is less significant due to greater budget flexibility.

Key Insights

1. **Budget Optimization**: Increasing the budget to approximately $1,200,000 achieves significant cost savings. Further increases provide diminishing returns, emphasizing the importance of efficient budget allocation.

2. **Cost Efficiency**: Lower cost modifiers result in substantial savings, particularly under constrained budgets. Strategies to negotiate lower costs for replacements can significantly improve outcomes.

3. **Robust Decision-Making**: Higher budgets reduce sensitivity to cost changes, offering greater flexibility in replacement planning and mitigating the impact of cost variations.

10. Answers to Some Questions from Result Obtained

The results we have obtained from Excel Solver, Gurobi, AMPL make practical sense. This is because we have focused our problem on the shortest path, by choosing optimal routes that meets all the constraint and minimize total cost. So, our optimal total cost is $27682K and routes are (1,3,6), (1,3,6), and (1,2,6) for 40' Flyer, 60' Flyer, and Eldorado respectively. Except this result, all possible combination for every shuttle is possible this will not guarantee minimum cost. Also, our total budget for entire period is $28000K, which slightly above the minimum total cost that is $27682K. So, optimal results satisfy the budget constraint as well. Our model lies on combinatorial optimization, and the decision variable is binary, therefore no intermediate data between 0 to 1 is possible.

For our problem, currently no optimal path has been adopted. This is why we are interested in finding the optimal path for the next five years that will minimize the total cost. In table 9, and fig 6, we have analyzed that if purchase and maintenance cost get increased, total cost will also increase by maintaining identical optimal path. Similarly, lowering the cost of purchase and maintenance, lowering the total cost by maintaining identical optimal path. The advantage of this comparison is that we know the optimal path prior next five years. So, if management tries to lower the total cost, then they should focus on lowering the maintenance and purchase cost if possible.

11.Conclusions

Our decision variable is binary. We have solved our problem with Excel, Gurobi, and AMPL by keeping this binary variable. The results we have found are practical. For every shuttle type, keeping or replacing will be implemented on fleet size of this type of shuttle. The fleet size is 8,6, and 37 respectively. For 40' Flyer, optimal path is (1,3,6), this means that management should keep 08 buses from start of year 1 to start of year 3. Then, they trade these shuttles and purchase a new fleet which will run through the start of year 6. This will lower the cost for 40' Flyer that is $2560K. For 60' Flyer, and Eldorado, the path are (1,3,6), and (1,2,6). Hence, the minimum total cost will be ($2560K+ $1812K+ $23310K) = $27682K.

For the purpose of doing sensitivity analysis, we relaxed the decision variable to make this problem is LP. Because, keeping the binary variable, the excel solver cannot perform sensitivity analysis. However, by changing the decision variable from binary to continuous, it did not change our optimal result. We did the same approach for Excel Solver and Gurobi sensitivity analysis.

12.Recommendations and/or Further Steps

The bus fleet optimization model provides actionable insights for effective decision-making in fleet replacement and management. To maximize its benefits, it is recommended to adopt a data-driven approach, reassessing fleet strategies annually based on updated information on costs, operational needs, and demand. Fleet modernization should be prioritized by transitioning to cost-efficient and environmentally friendly technologies, such as hybrid or electric buses, which can reduce long-term expenses and emissions. Additionally, budgets should be adjusted in line with inflation and service demands to maintain operational continuity. Leveraging government subsidies and incentives for sustainable vehicles is also advised to offset initial costs and enhance economic feasibility.

For future development, the model can be extended to address variability in transportation demand, making it more applicable to dynamic and uncertain environments. Environmental metrics, such as detailed emissions and energy consumption, can be integrated to align fleet decisions with sustainability objectives. A stochastic version of the model could also be developed to account for uncertainties in costs, budgets, and service requirements. Furthermore, adapting the model for broader transportation systems, such as urban transit networks, can expand its utility. Experimenting with advanced solvers and optimization algorithms can further improve efficiency in handling large-scale fleet management challenges, ensuring the model's long-term relevance and adaptability.

13.Acknowledgements

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