COMP 775, Fall 2018 Assignment 3: **Registration**
Passed out on 16 October 2018. Due on Thursday, 1 November 2018 at class time

Let $G(x,y; \mu,\sigma)$ be a 2D Gaussian centered at $\mu$ and with standard deviation $\sigma$. Consider the $128 \times 128$ 2D image that I will call the Gaussian fan: It is $(1/(1+|\theta|)^2) \cos(2\pi k \theta)$ convolved with $G(x,y; \mu,\sigma)$, where $\theta$ is the angle between (x,y)-(64.5,64.5) and (1,0). Use k=4. This Gaussian fan will be your reference image.

Let the target image in a registration be the unblurred fan rotated by 0.25/k radians, then Gaussian blurred, and then multiplied in intensity by a constant A with the result added to a constant B. Let the transformation set be all rotations about (64.5, 64.5). Let the reference (moving) image be the unrotated Gaussian fan. It should be clear that the optimal registration transform between the two images is a rotation by 0.25/k radians. However, there will be a local optimum of certain image match metrics at a rotation angle of -0.75/k radians (off by one cycle from the correct rotation).

In this assignment, use $\sigma = 2$ pixels for the blurring, and when you do rotations in either forming the images or rotating them as part of the registration, rotate the unblurred image and then blur it.

You are to consider the catalogue of image match metrics given below and to compare the following three measures across the image match metrics. The first measure is the accuracy of the angle of rotation at which the metric is maximum. The second measure is the lowering of the image match measure (its value at the previous local maximum (at -0.75/k radians) minus its value at the right registration (0.25/k radians)) relative to its value at the correct answer. The second measure is the sharpness of the peak of the image match at the correct answer, namely the negative of its 2nd derivative wrt $\theta$, relative to the image match value at 0.25/k radians (the right registration). Better metrics would have a more accurate location, a larger relative difference between the match value at the right registration and the match value at the wrong registration, and a larger value (sharper decrease) of the relative negative 2nd derivative at the right answer.

The values of A and B you should use are as follows:
First, use A=1, B=0.
Then use A=1000, B=0
Then use A=1000, B=1000
Finally, use A=-500, B=1000.

The image match measures you should do this comparison on are as follows:

1) Normalized cross-correlation with a global mean and standard deviation
2) Mutual information
3) Quantile functions from histograms on image features that you define. Your features need to be somehow invariant between both positive and negative values of A. An example of a feature might be a normalized pixel intensity in a specified image region. If you have histograms from the target image for each of K features, e.g., normalized intensities from K image regions, and you have those histograms from the moving image at any candidate rotation, the image match would be the sum over the features of the Euclidean distances of the quantile functions for each feature between the quantile functions for the candidate moving image and those for the target image. Be careful with that your intensity normalizations for each feature produce results that are in the same units for each feature. (I remind you that when you build quantile functions you never actually build a histogram but rather sort the values of the features that would have been used to produce a histogram.)

What to pass in:
1) For each of the A,B tuples the values of each of the three measures of performance. Also comment on which measure seemed to perform best.

2) An example of a target image and the associated registered reference image (both as blurred) for each of the A,B tuples

3) A description of the features you used in the quantile function image match measure.