Applied Physics EE (117)

WEEK # 4

MOTION IN 2D/3D

TWO-DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

The position vector for a particle moving in the *xy* plane can be written

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j}$$

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \qquad \begin{cases} v_{xf} = v_{xi} + a_x t \\ v_{yf} = v_{yi} + a_y t \end{cases}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2 \qquad \begin{cases} x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \\ y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2 \end{cases}$$

Example

A particle starts from the origin at $t = 0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of acceleration only, given by $a_x = 4.0 \text{ m/s}^2$. (a) Determine the components of the velocity vector at any time and the total velocity vector at any time.

The equations of kinematics give

$$v_{xf} = v_{xi} + a_x t = (20 + 4.0t) \text{ m/s}$$

$$v_{yf} = v_{yi} + a_y t = -15 \text{ m/s} + 0 = -15 \text{ m/s}$$

Therefore,

$$\mathbf{v}_f = v_{xf}\mathbf{i} + v_{yf}\mathbf{j} = [(20 + 4.0t)\mathbf{i} - 15\mathbf{j}] \text{ m/s}$$

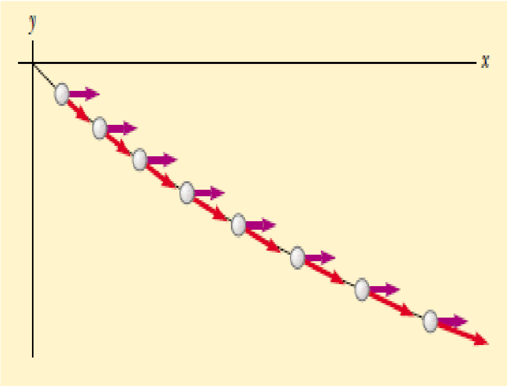


Figure 4.5 Motion diagram for the particle.

Example

(b) Calculate the velocity and speed of the particle at $t =$ 5.0 s.

$$\mathbf{v}_f = \{[20 + 4.0(5.0)]\mathbf{i} - 15\mathbf{j}\}\ \text{m/s} = \boxed{(40\mathbf{i} - 15\mathbf{j})\ \text{m/s}}$$

This result tells us that at $t = 5.0$ s, $v_{xf} = 40$ m/s and $v_{yf} = -15$ m/s. Knowing these two components for this two-dimensional motion, we can find both the direction and the magnitude of the velocity vector. To determine the angle θ that $\mathbf{v}$ makes with the x axis at $t = 5.0$ s, we use the fact that $\tan\theta = v_{yf}/v_{xf}$:

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-15\ \text{m/s}}{40\ \text{m/s}}\right) = \boxed{-21^\circ}$$

where the minus sign indicates an angle of 21° below the positive x axis. The speed is the magnitude of $\mathbf{v}_f$:

$$v_f = |\mathbf{v}_f| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(40)^2 + (-15)^2}\ \text{m/s} = \boxed{43\ \text{m/s}}$$

In looking over our result, we notice that if we calculate v_i from the x and y components of $\mathbf{v}_i$, we find that $v_f > v_i$. Does this make sense?

Example

(c) Determine the x and y coordinates of the particle at any time t and the position vector at this time.

Solution Because $x_i = y_i = 0$ at $t = 0$, Equation 2.11 gives

$$x_f = v_{xi}t + \tfrac{1}{2}a_x t^2 = \boxed{(20t + 2.0t^2)\text{ m}}$$

$$y_f = v_{yi}t = \boxed{(-15t)\text{ m}}$$

Therefore, the position vector at any time t is

$$\mathbf{r}_f = x_f\mathbf{i} + y_f\mathbf{j} = \boxed{[(20t + 2.0t^2)\mathbf{i} - 15t\mathbf{j}]\text{ m}}$$

(Alternatively, we could obtain $\mathbf{r}_f$ by applying Equation 4.9 directly, with $\mathbf{v}_i = (20\mathbf{i} - 15\mathbf{j})$ m/s and $\mathbf{a} = 4.0\mathbf{i}$ m/s^2. Try it!) Thus, for example, at $t = 5.0$ s, $x = 150$ m, $y = -75$ m, and $\mathbf{r}_f = (150\mathbf{i} - 75\mathbf{j})$ m. The magnitude of the displacement of the particle from the origin at $t = 5.0$ s is the magnitude of $\mathbf{r}_f$ at this time:

$$r_f = |\mathbf{r}_f| = \sqrt{(150)^2 + (-75)^2}\text{ m} = 170\text{ m}$$

Note that this is *not* the distance that the particle travels in this time! Can you determine this distance from the available data?

Equations for Motion with Constant Acceleration

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

Motion Equations

The kinematic equations can be used with any particle under uniform acceleration.

The kinematic equations may be used to solve any problem involving one-dimensional motion with a constant acceleration

You may need to use two of the equations to solve one problem

Many times there is more than one way to solve a problem

Kinematic Equations, specific

For constant *a*, $$v_{xf} = v_{xi} + a_x t$$

Can determine an object's velocity at any time *t* when we know its initial velocity and its acceleration

- Assumes $t_i = 0$ and $t_f = t$

Does not give any information about displacement

Kinematic Equations, specific

For constant acceleration,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Gives final position in terms of velocity and acceleration

Doesn’t tell you about final velocity

Kinematic Equations, specific

For constant *a*,

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Gives final velocity in terms of acceleration and displacement

Does not give any information about the time

Kinematic Equations, specific

For constant acceleration,

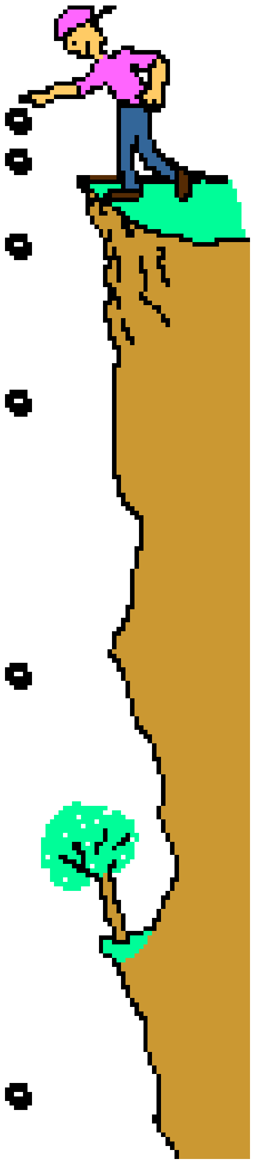
$$v_{x,avg} = \frac{v_{xi} + v_{xf}}{2}$$

The average velocity can be expressed as the arithmetic mean of the initial and final velocities

Free Fall and Gravitational Acceleration

A free-falling object is one which is falling under the sole influence of gravity. This definition of free fall leads to two important characteristics about a free-falling object:

- Free-falling objects do not encounter air resistance
- All free-falling objects (on Earth) accelerate downwards at a rate of -9.8 m/s^2
  - This rate is commonly referred to as *g*

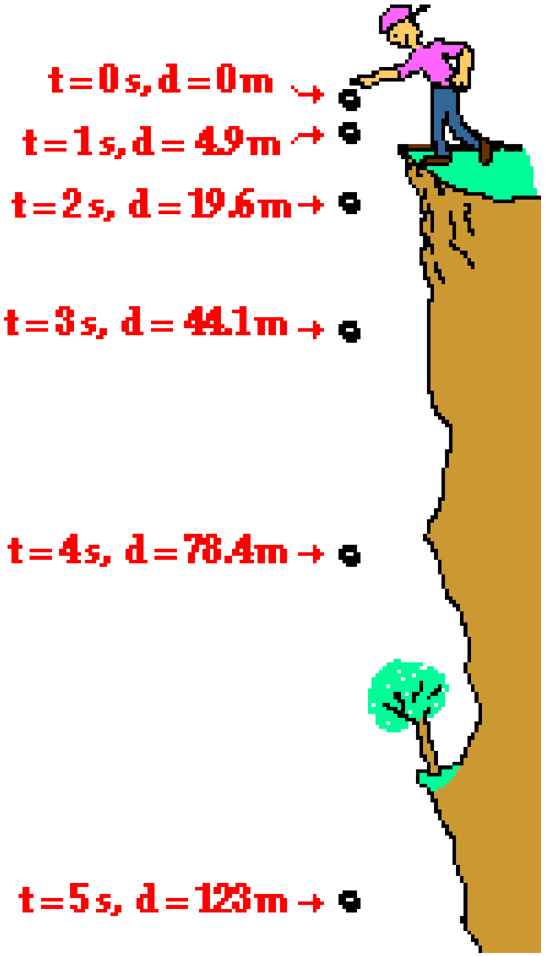


Free Fall and Gravitational Acceleration

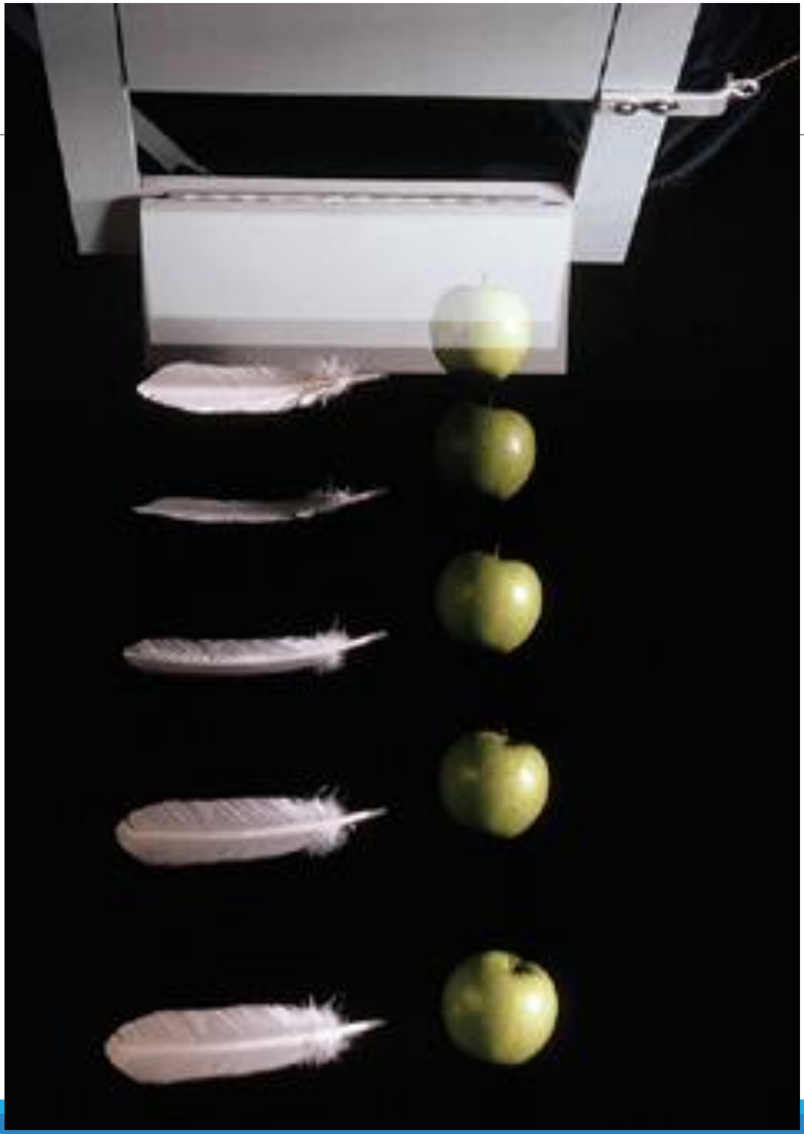
All of the equations that work for HORIZONTAL (x-direction) motion ALSO work for VERTICAL (y-direction) motion.

Simply substitute *g* for *a* and Δ*y* for Δ*x*

Generally, free falling objects start falling from rest, so it tends to simplify the equations by eliminating v_i



2.9 Free-Fall Acceleration



Freely Falling Objects

A ***freely falling object*** is any object moving freely under the influence of gravity alone.

It does not depend upon the initial motion of the object

- Dropped – released from rest
- Thrown downward
- Thrown upward

Free fall with initial velocity

The motion of an object in free fall is described by the equations for speed and position with constant acceleration.

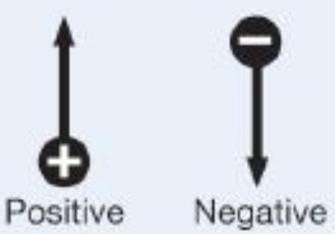
The acceleration (*a*) is replaced by the acceleration due to gravity (*g*) *and t*he variable (*x*) is replaced by (*y*).

FREE FALL MOTION FORMULAS

(choosing up as positive)

$$v = v_0 - gt$$

$$y = y_0 + v_0 t + \tfrac{1}{2} g t^2$$



v	Speed (m/s)
v_0	Initial speed (m/s)
g	9.8 (m/s^2)

y	Height (m)
y_0	Initial height (m)
t	Time (s)

Sample Problem

Time for full up-down flight, baseball toss

In Fig. 2-11, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?

KEY IDEAS

(1) Once the ball leaves the pitcher and before it returns to his hand, its acceleration is the free-fall acceleration $a = -g$. Because this is constant, Table 2-1 applies to the motion. (2) The velocity v at the maximum height must be 0.

Calculation: Knowing v, a, and the initial velocity $v_0 = 12$ m/s, and seeking t, we solve Eq. 2-11, which contains those four variables. This yields

$$t = \frac{v - v_0}{a} = \frac{0 - 12 \text{ m/s}}{-9.8 \text{ m/s}^2} = 1.2 \text{ s}. \qquad \text{(Answer)}$$

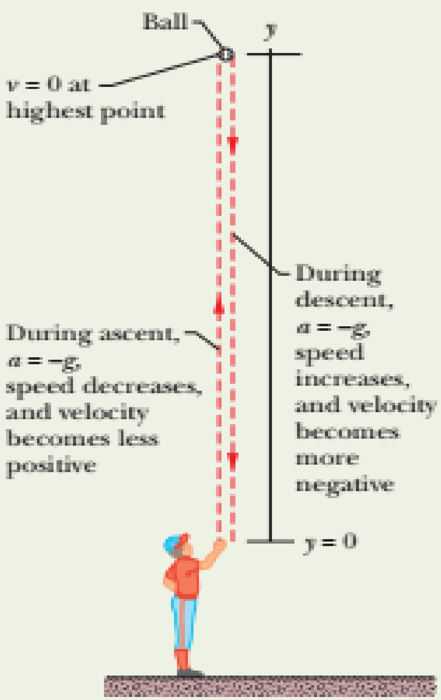


Fig. 2-11 A pitcher tosses a baseball straight up into the air. The equations of free fall apply for rising as well as for falling objects, provided any effects from the air can be neglected.

(b) What is the ball's maximum height above its release point?

Calculation: We can take the ball's release point to be $y_0 = 0$. We can then write Eq. 2-16 in y notation, set $y - y_0 = y$ and $v = 0$ (at the maximum height), and solve for y. We get

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = 7.3 \text{ m}. \qquad \text{(Answer)}$$

(c) How long does the ball take to reach a point 5.0 m above its release point?

Calculations: We know v_0, $a = -g$, and displacement $y - y_0 = 5.0$ m, and we want t, so we choose Eq. 2-15. Rewriting it for y and setting $y_0 = 0$ give us

$$y = v_0 t - \tfrac{1}{2} g t^2,$$

or

$$5.0 \text{ m} = (12 \text{ m/s})t - (\tfrac{1}{2})(9.8 \text{ m/s}^2)t^2.$$

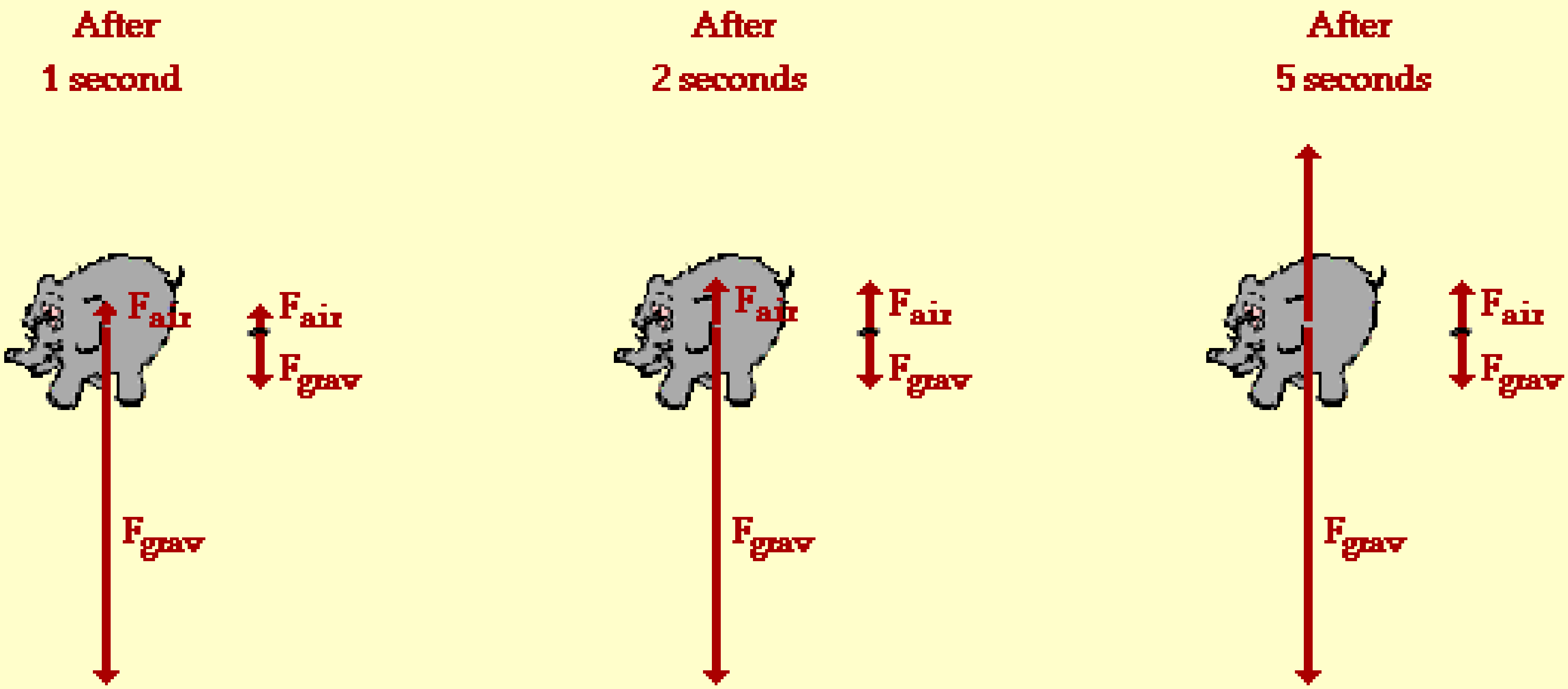
If we temporarily omit the units (having noted that they are consistent), we can rewrite this as

$$4.9t^2 - 12t + 5.0 = 0.$$

Solving this quadratic equation for t yields

$$t = 0.53 \text{ s} \quad \text{and} \quad t = 1.9 \text{ s}. \qquad \text{(Answer)}$$

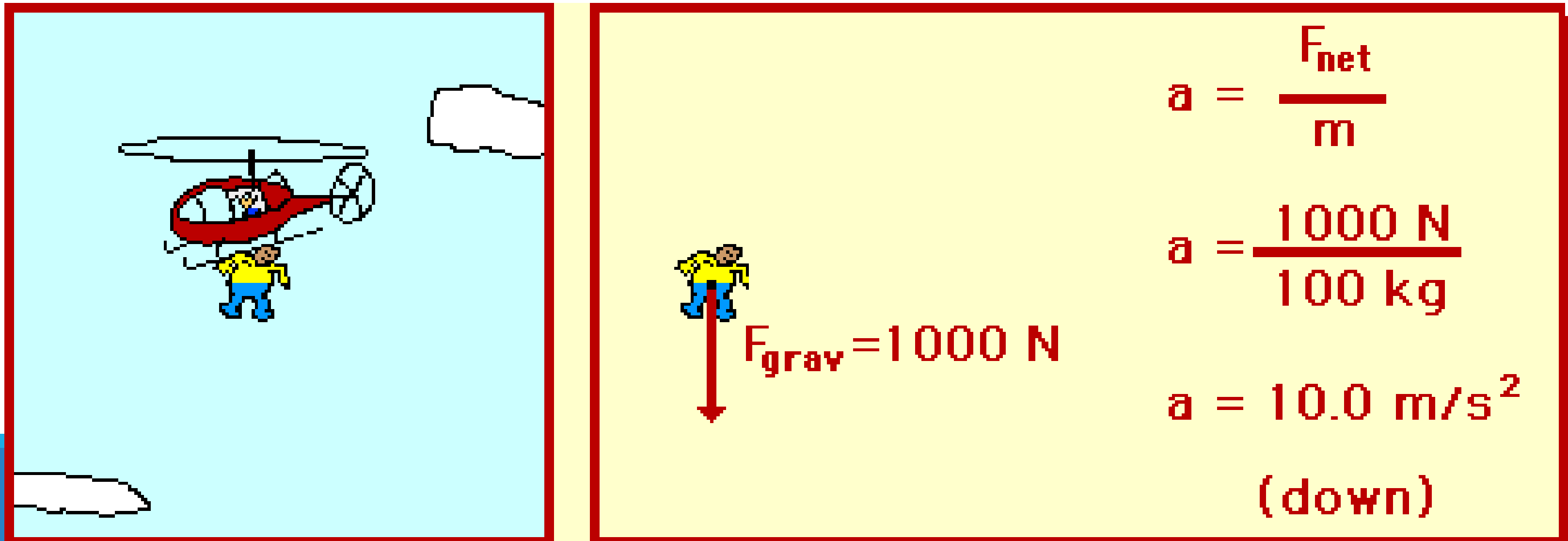
There are two such times! This is not really surprising because the ball passes twice through $y = 5.0$ m, once on the way up and once on the way down.



Free-body diagrams for the elephant and the feather at various times during the course of their fall reveal that the feather quickly reaches terminal velocity while the elephant continues to accelerate for the entire fall.

Terminal Velocity

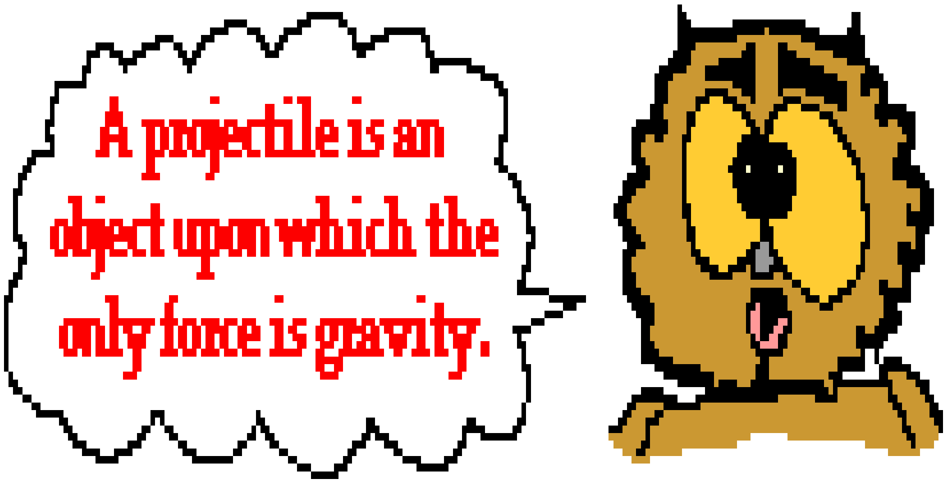
Forces cause objects to accelerate (2nd Law). When the force of gravity on a falling object equals the force of the air resistance going against gravity, the forces balance out and the object stops accelerating. The object will travel at a constant velocity – the terminal velocity.



Projectile Motion

Two-dimensional motion of an object

- Vertical
- Horizontal



4-4 Projectile Motion

Projectile motion is the motion of a particle that is launched with some initial velocity

In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other

During its flight,

- the particle's horizontal acceleration is zero and
- its vertical acceleration is the free-fall acceleration -g

Types of Projectile Motion

Horizontal

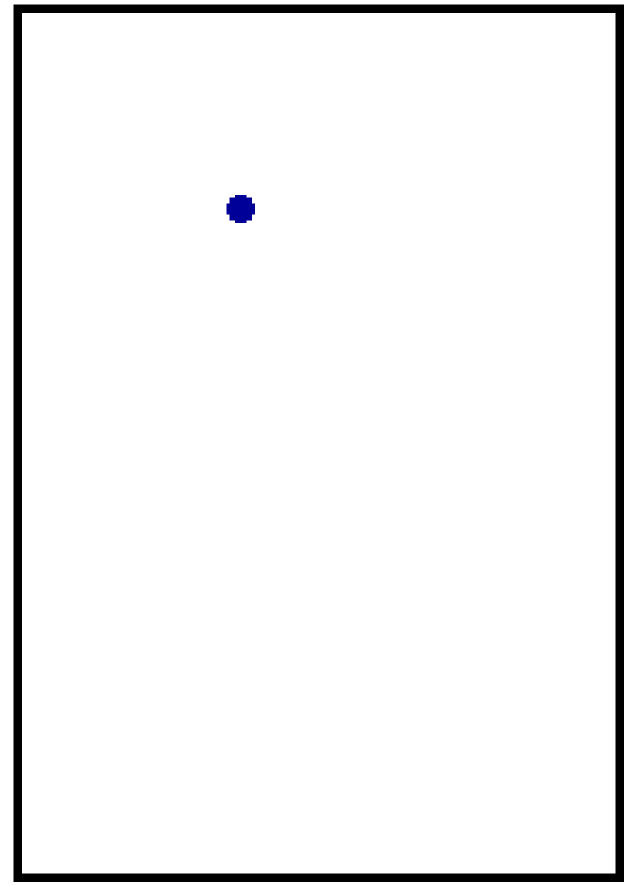
- Motion of a ball rolling freely along a level surface
- Horizontal velocity is ALWAYS constant

Vertical

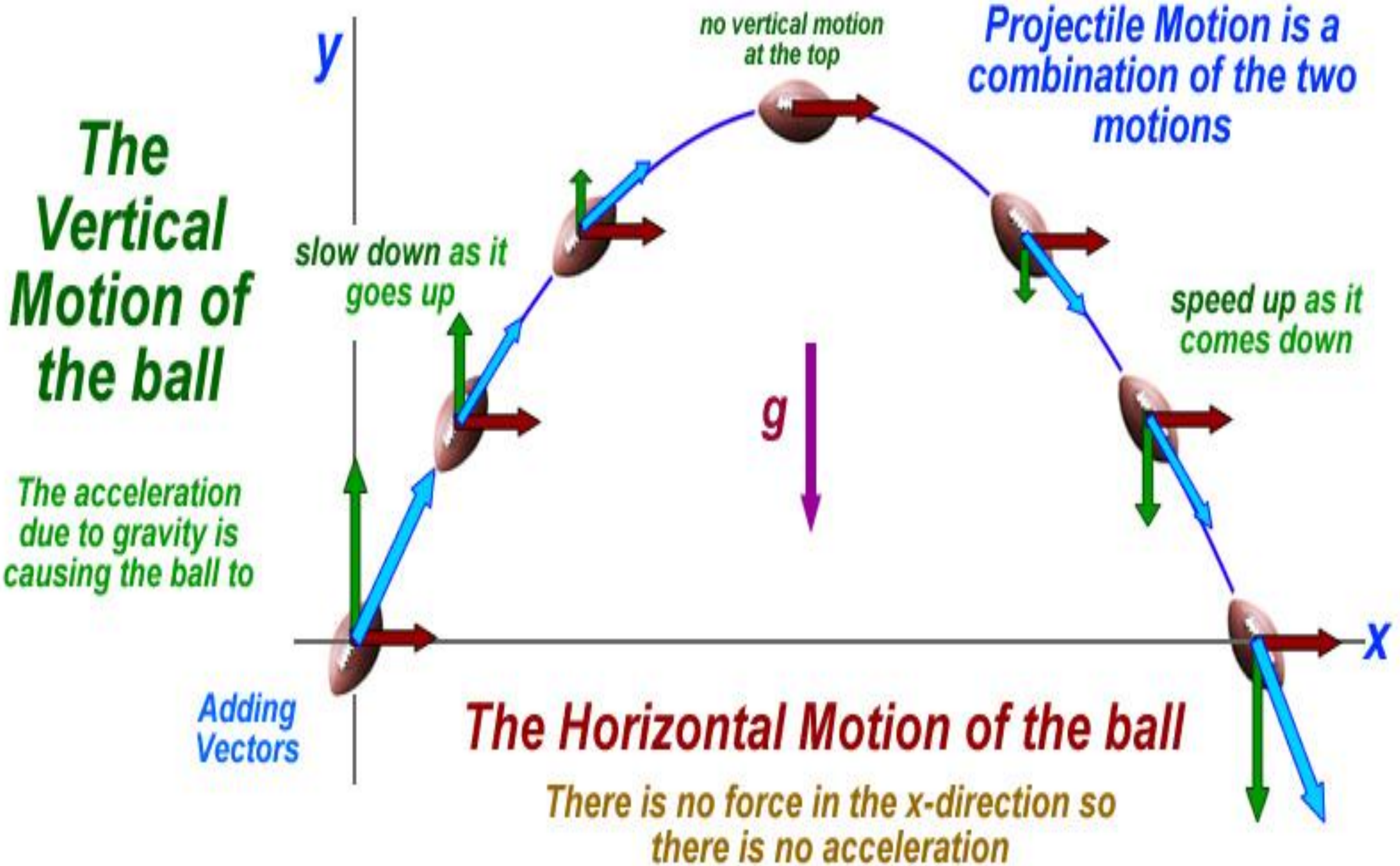
- Motion of a freely falling object
- Force due to gravity
- Vertical component of velocity changes with time

Parabolic

- Path traced by an object accelerating only in the vertical direction while moving at constant horizontal velocity

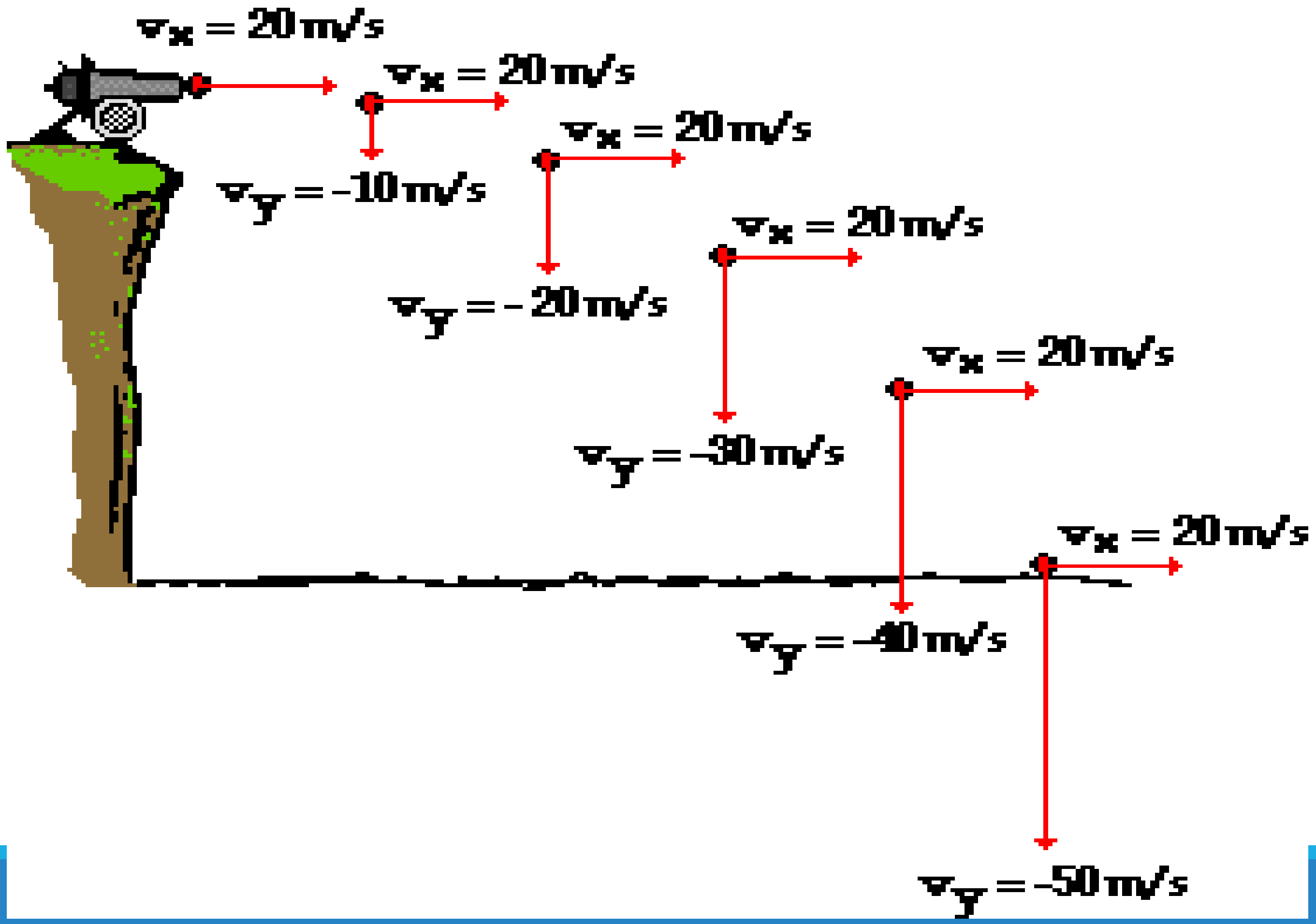


Projectile Motion - A Vector Perspective



Examples of Projectile Motion

Launching a Cannon ball



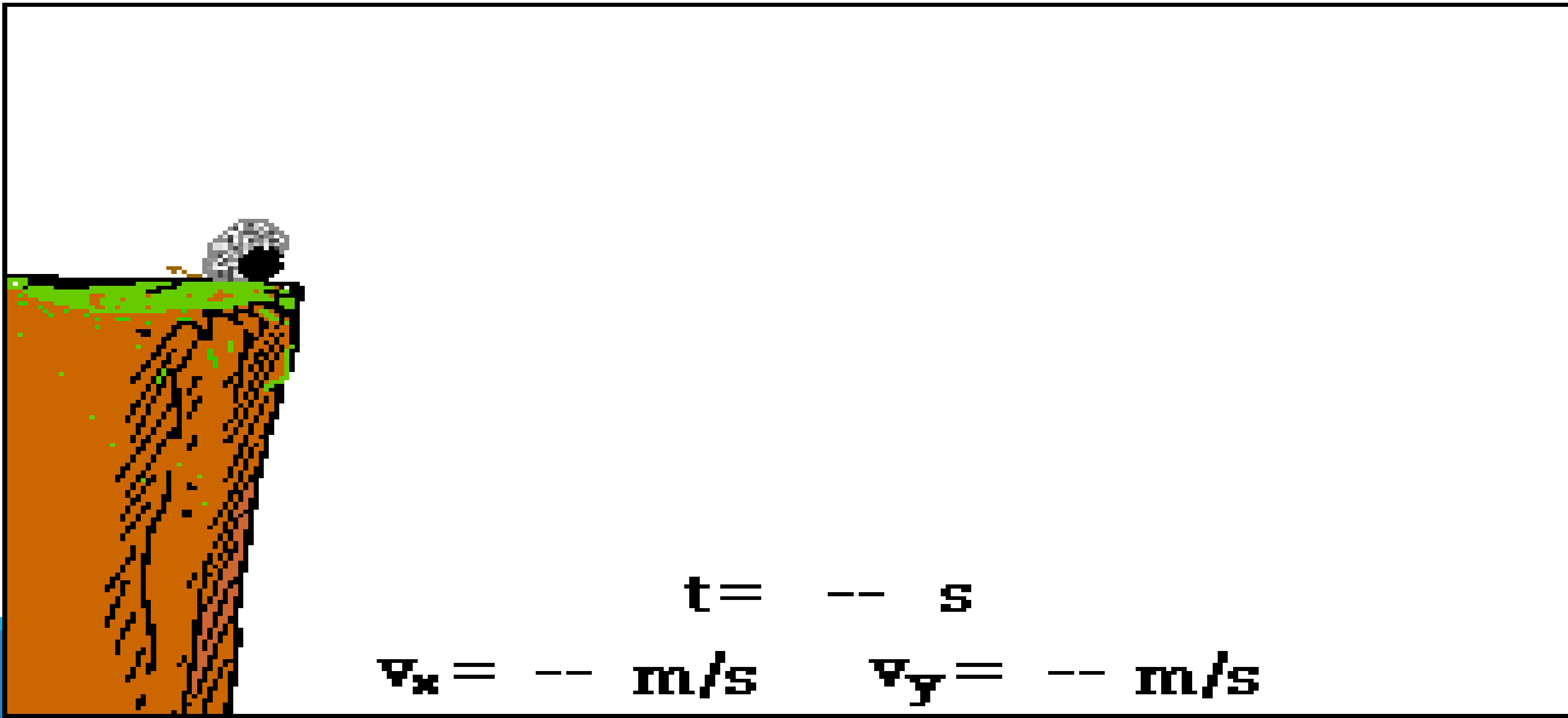
Final Horizontal and Vertical component of velocity:

$$v x_f = v \, Cos\theta$$
$$v y_f = v \, Sin\theta - gt$$

The projectile motion is the superposition of two motions:

(1) constant velocity motion in the horizontal direction and

(2) free-fall motion in the vertical direction.



Equations

X- Component

$$x_f = x_i + v_{xi} t$$

Y- Component

$$y_f = y_i + v_{yi} t - \frac{1}{2} g t^2$$

Note: g= 9.8 m/s^2

$$v_{yf}^{\ 2} = v_{yi}^{\ 2} - 2g\Delta y$$

$$v_{yf} = v_{yi} - gt$$

Vectors

$$v_{xi} = v_i \cos(\theta)$$

$$v_{yi} = v_i \sin(\theta)$$

The Horizontal Motion

There is *no acceleration* in the horizontal direction, the horizontal component v_x of the projectile's velocity remains unchanged from its initial value v_{0x} throughout the motion

a = 0

the initial x and y components of velocity are

$$v_{xi} = v_i \cos\theta_i \qquad v_{yi} = v_i \sin\theta_i$$

$$x - x_0 = v_{0x}t.$$

Because $v_{0x} = v_0 \cos\theta_0$, this becomes

$$x - x_0 = (v_0 \cos\theta_0)t.$$

The Vertical Motion

In vertical the acceleration is constant i.e

a = $-g$

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

$$= (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2,$$

$$v_y = v_0 \sin\theta_0 - gt$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$$

The Equation Path

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2} \qquad \text{(trajectory).}$$

The path of a projectile, which we call its *trajectory, is always a* parabola

The Horizontal Range

The *horizontal range R* of the projectile is the *horizontal* distance the projectile has traveled when it returns to its initial height (the height at which it is launched). To find range *R*, let us put $x - x_0 = R$ and $y - y_0 = 0$

$$R = (v_0 \cos \theta_0)t$$

and

$$0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2.$$

Eliminating t between these two equations yields

$$R = \frac{2v_0^2}{g} \sin \theta_0 \cos \theta_0.$$

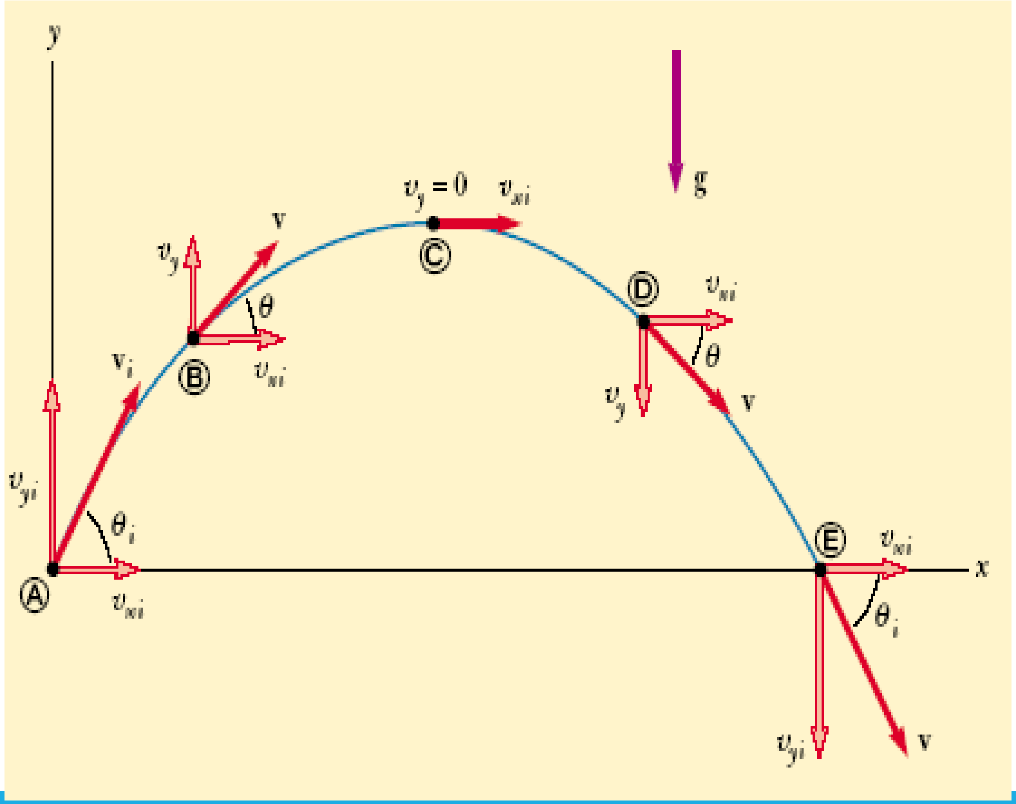
Using the identity $\sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0$ (see Appendix E), we obtain

$$R = \frac{v_0^2}{g} \sin 2\theta_0.$$

The horizontal range R is maximum for a launch angle of 45°.

Maximum Height

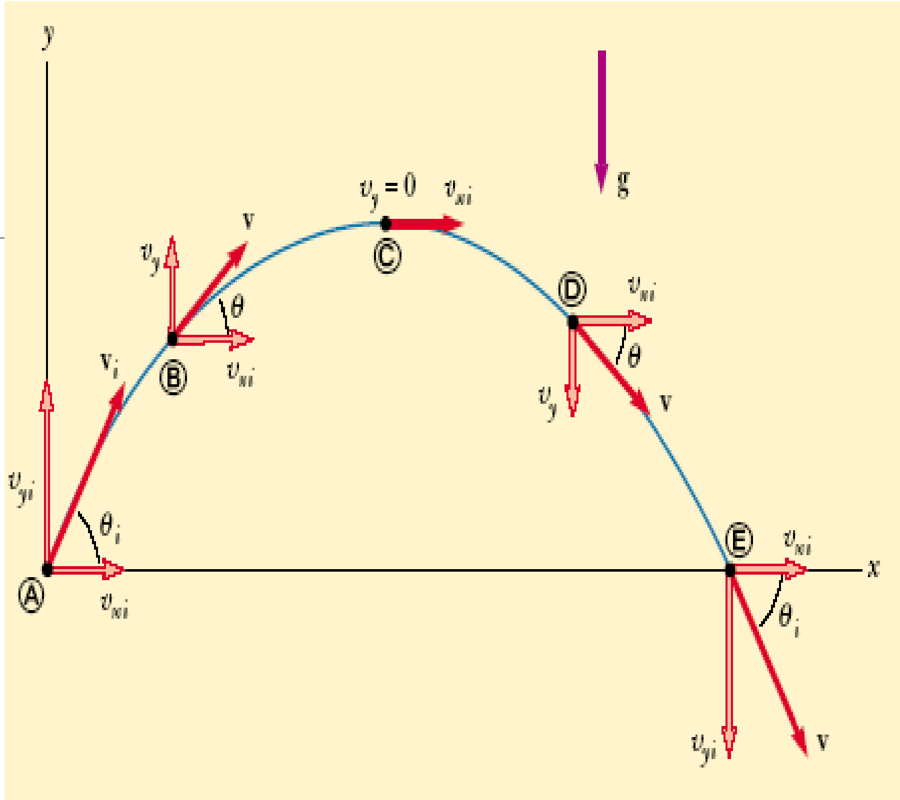
$$h = \frac{v_i^2 \sin^2 \theta_i}{2g}$$



PROJECTILE MOTION

the initial x and y components of velocity are

$$v_{xi} = v_i \cos \theta_i \qquad v_{yi} = v_i \sin \theta_i$$



Horizontal position component

$$x_f = v_{xi}t = (v_i \cos \theta_i)t$$

Vertical position component

$$y_f = v_{yi}t + \tfrac{1}{2}a_y t^2 = (v_i \sin \theta_i)t - \tfrac{1}{2}gt^2$$

the path of a projectile, which we call its *trajectory, is always a* parabola

$$y = (\tan \theta_i)x - \left(\frac{g}{2{v_i}^2 \cos^2 \theta_i}\right)x^2$$

Maximum height of projectile

$$h = \frac{{v_i}^2 \sin^2 \theta_i}{2g}$$

Range of projectile

$$R = \frac{{v_i}^2 \sin 2\theta_i}{g}$$

Example

A long-jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s. (a) How far does he jump in the horizontal direction? (Assume his motion is equivalent to that of a particle.)

(b) What is the maximum height reached?

Exercise To check these calculations, use Equations 4.13 and 4.14 to find the maximum height and horizontal range.

$$x_f = x_B = (v_i \cos \theta_i) t_B = (11.0 \text{ m/s})(\cos 20.0°) t_B$$

The value of x_B can be found if the total time of the jump is known. We are able to find t_B by remembering that $a_y = -g$ and by using the y part of Equation 4.8a. We also note that at the top of the jump the vertical component of velocity v_{yA} is zero:

$$v_{yf} = v_{yA} = v_i \sin \theta_i - g t_A$$

$$0 = (11.0 \text{ m/s}) \sin 20.0° - (9.80 \text{ m/s}^2) t_A$$

$$t_A = 0.384 \text{ s}$$

This is the time needed to reach the *top* of the jump. Because of the symmetry of the vertical motion, an identical time interval passes before the jumper returns to the ground. Therefore, the *total time* in the air is $t_B = 2t_A = 0.768$ s. Substituting this value into the above expression for x_f gives

$$x_f = x_B = (11.0 \text{ m/s})(\cos 20.0°)(0.768 \text{ s}) = \boxed{7.94 \text{ m}}$$

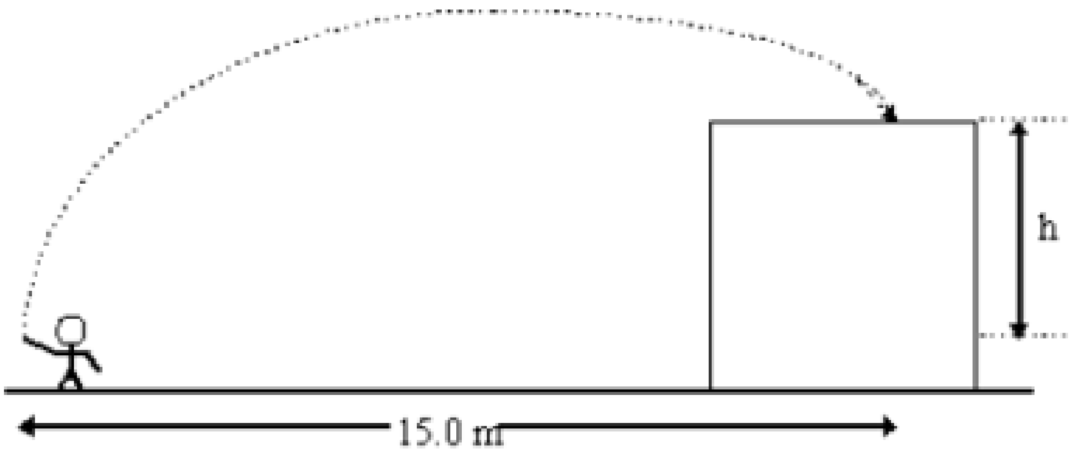
$$y_{max} = y_A = (v_i \sin \theta_i) t_A - \tfrac{1}{2} g t_A^2$$
$$= (11.0 \text{ m/s})(\sin 20.0°)(0.384 \text{ s})$$
$$-\tfrac{1}{2}(9.80 \text{ m/s}^2)(0.384 \text{ s})^2$$

$$= \boxed{0.722 \text{ m}}$$

Treating the long-jumper as a particle is an oversimplification. Nevertheless, the values obtained are reasonable.

Example

A boy throws a rock with speed $v = 18.3$ m/s at an angle of $\theta = 57.0°$ over a building. The rock lands on the roof 15.0 m in the x direction from the boy. How long was the rock in the air? How much taller, height h, is the building than the boy? Ignore air resistance.



The rock is a projectile. We solve projectile motion problems by considering the x and y components separately, keeping in mind that the time in air is common. We write out the ***i*** and ***j*** information in separate columns including the information that we can infer or that we are supposed to know. We see from the sketch that $h = \Delta y$, the vertical displacement. Note that the initial velocity is broken into components.

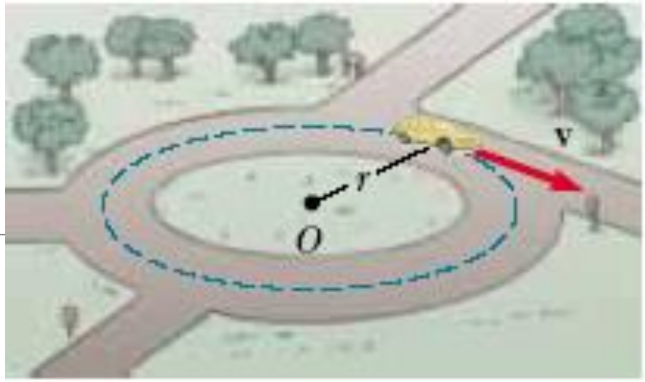
i		***j***	
$\Delta x = 15.0$ m		$\Delta y = h$	
$a_x = 0$	No x component for projectiles	$a_y = -g = -9.81$ m/s^2	gravity acts down
$v_{0x} = v_0 \cos\theta$ $= 18.3 \times \cos(57°)$ $= 9.9669$ m/s		$v_{0y} = v_0 \sin\theta$ $= 18.3 \times \sin(57°)$ $= 15.3477$ m/s	
$t_{air} = ?$	common	$t_{air} = ?$	common

Looking at the x information, we see that we have enough data to find t_{air}. The kinematics equation that has all four quantities is $\Delta x = v_{0x}t + ½a_xt^2$. Since $a_x = 0$ for a projectile, this equation become $\Delta x = v_{0x}t$. Solving for t, we get

$$t = \Delta x / v_{0x} = (15.0 \text{ m}) / (9.9669 \text{ m/s}) = 1.505 \text{ s} .$$

Looking at the y information, we see that we now have enough data to find h. The kinematics equation that has all four quantities is $\Delta y = v_{0y}t + ½a_yt^2$. Since $\Delta y = h$ and $a_y = -g$, this equation become $h = v_{0y}t - ½gt^2$. Substituting in the appropriate numbers reveals that h = 12.0 m. The building is 12.0 m taller that the

UNIFORM CIRCULAR MOTION

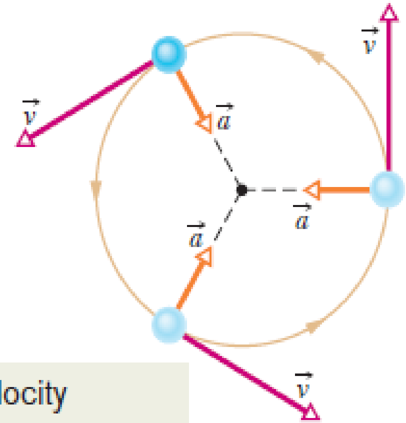


The acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the center of the circle. An Acceleration of this nature is called a centripetal (center-seeking) acceleration, and its magnitude is

$$a = \frac{v^2}{r} \quad \text{(centripetal acceleration)}, \qquad T = \frac{2\pi r}{v} \quad \text{(period)}.$$

Consider a particle moving along a curved path where the velocity changes both in direction and in magnitude

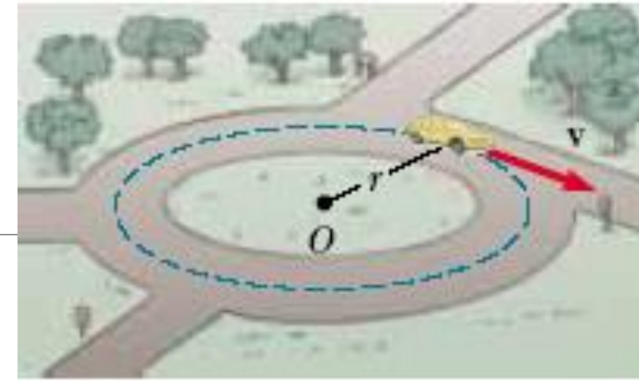
The acceleration vector always points toward the center.



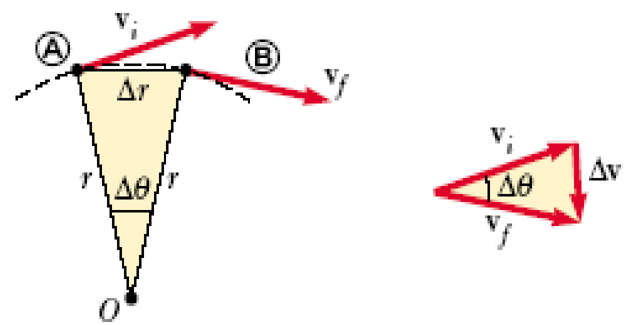
The velocity vector is always tangent to the path.

UNIFORM CIRCULAR MOTION

The acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the center of the circle. An Acceleration of this nature is called a centripetal (center-seeking) acceleration, and its magnitude is



$$a_r = \frac{v^2}{r}$$



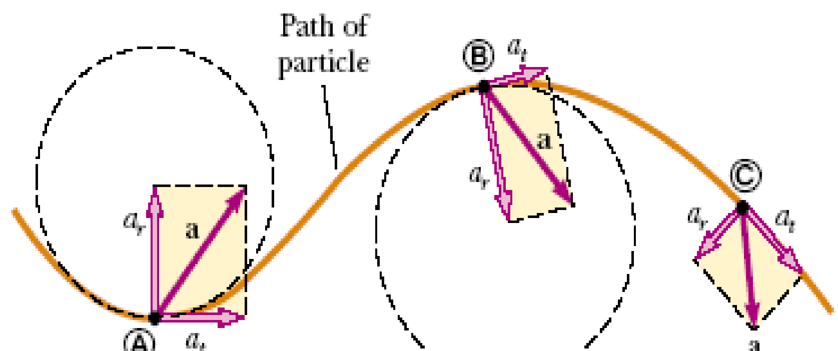
Consider a particle moving along a curved path where the velocity changes both in direction and in magnitude

TANGENTIAL AND RADIAL ACCELERATION

Total acceleration $\quad \mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$

Tangential acceleration $\quad a_t = \dfrac{d|\mathbf{v}|}{dt}$

Radial acceleration $\quad a_r = \dfrac{v^2}{r}$



Example(Homework)

A tile, initially at rest, slides down a roof for a distance of 3.75m before falling off the roof. The height of the building from ground to eave is 8.40 m. The acceleration of the tile as it slides is 2.10 m/s^2.

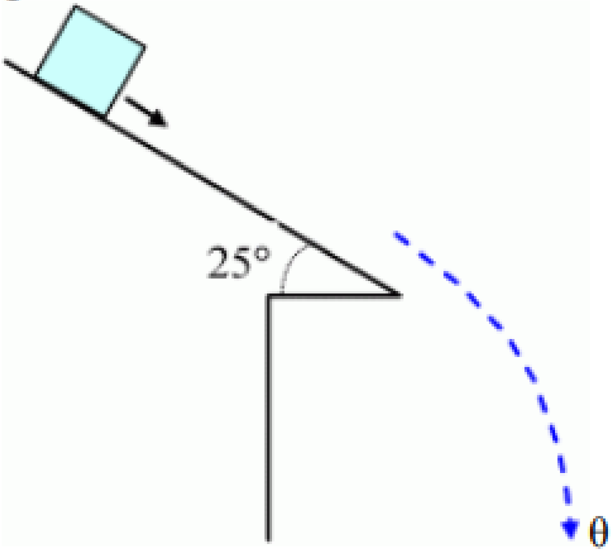
(a) Determine the speed of the tile just as it leaves the roof.

(b) Determine the vertical component of the velocity just before it leaves the roof.

(c) Determine the horizontal component of the velocity just before it leaves the roof.

(d) Determine how long it takes to hit the ground after leaving the roof.

(e) Determine how far from the edge of the roof that the tile lands.



Answers

(a) 3.9686 m/s

(b) -1.6772 m/s

(c) 3.5968 m/s

(d) t=1.149s

(e) 4.13 m

When the tile slides down the roof, it travels in a straight line. That is a 1D kinematics problem. When it leaves the roof, it becomes a projectile problem.