

K213186

Ch 15 OSCILLATIONS

(Q³) What is the maximum acceleration of a platform that oscillates at amplitude 2.20 cm and frequency 6.60 Hz?

Sol: $x_m = 2.20 \text{ cm} = \frac{2.20}{100} = 0.0220 \text{ m}$

$$a = \omega^2 x_m$$

$$a = (2\pi f)^2 x_m$$

$$a = [2\pi(6.60)]^2 (0.0220)$$

$$a = (41.474)^2 (0.0220)$$

$$a = (1720.09)(0.0220)$$

$$a = 37.842 \text{ m/s}^2$$

Ans.

(Q⁵) In an electrical shaver, the blade moves back and forth over a distance of 2.0 mm in 3 s, with frequency 120 Hz. Find a) the amplitude, b) the maximum blade speed, c) the magnitude of maximum blade acceleration.

Sol: D = 2.00 mm, f = 120 Hz.

a) For amplitude:

$$x_m = \frac{D}{2}$$

$$x_m = \frac{2.00}{2}$$

$$x_m = 1.00 \text{ mm}$$

$$x_m = 1 \times 10^{-3} \text{ m} \quad \text{Ans.}$$

For ω :

$$\omega = 2\pi f \quad \text{in radians/second}$$

$$\omega = 2 \times 3.142 \times 120$$

$$[\omega = 754.08 \text{ radians/second}]$$

Ans.

b) For V :

$$V = \omega x_m$$

$$V = (754.08) (1 \times 10^{-3})$$

$$V = 754 \times 754.08 \times 10^{-3}$$

$$[V = 0.754 \text{ m/s}] \quad \text{Ans.}$$

c) for a :

$$\text{As we know } a = \omega^2 x_m$$

$$a = (754.08)^2 (1 \times 10^{-3})$$

$$a = 568636.64 \times 10^{-3}$$

$$[a = 568.636 \text{ m/s}^2] \quad \text{Ans.}$$

13. An oscillator consists of a block of mass 0.500kg connected to a spring. When set into oscillation with amplitude 35.0cm, the oscillator repeats its motion every 0.500 s. Find the

- a) period, b) frequency, c) angular frequency,
- d) spring constant, e) maximum speed, and
- f) magnitude of the maximum force on the block from the spring.

Sol: $x_m = 35.0\text{cm}$ $x_m = 0.35\text{m}$

a) For Period:

$$T = 0.500\text{s}$$

Ans.

b) For Frequency:

$$f = 1/T$$

$$f = 1/0.500$$

$$f = 2.00\text{Hz}$$

Ans.

c) For Angular Frequency:

$$\omega = 2\pi f$$

$$\omega = 2 \times 3.142 \times 2.00$$

$$\omega = 12.6 \text{ radians/second}$$

Ans.

d) for spring constant

$$k = m\omega^2 \quad \text{where } \omega = 2\pi f$$

$$k = (0.500)(12.6)^2$$

$$k = 79.0 \text{ N/m} \quad \boxed{\text{Ans.}}$$

e) for maximum speed

$$V = \omega x_m$$

$$V = (12.6)(0.35)$$

$$V = 4.41 \text{ m/s} \quad \boxed{\text{Ans.}}$$

f) For max force

$$F_m = k x_m$$

$$F_m = (79)(0.35)$$

$$F_m = 27.65 \text{ N}$$

Ans.

23. A block is on a horizontal surface (a shake table) that is moving back and forth horizontally with simple harmonic motion of frequency 2.0 Hz. The coefficient of static friction between block and surface is 0.50. How great can the amplitude of the S.H.M be if the block is not to slip along the surface?

Sol:

$$f = 2.0 \text{ Hz}, \mu_s = 0.50, x_m = ?$$

If the block is not to slip along the surface then the force should be less than or equal to static force.

$$F_{\max} = f_s$$

$$m a_m = \mu_s F_N$$

$$m(\omega^2 x_m) = \mu_s m g$$

$$m[(2\pi f)^2 x_m] = \mu_s m g$$

For maximum displacement:

$$x_m = \frac{\mu_s m g}{m(2\pi f)^2}$$

$$x_m = (0.50)(9.8)$$

$$(2 \times 3.142 \times 2)^2$$

$$x_m = 0.031 \text{ m}$$

Aus.

Waves:

Q³) A wave has an angular frequency of 110 rad/s and a wave-length of 1.80 m. Calculate a) the angular wave number and b) the speed of the wave.

Sol.

$$\omega = 110 \text{ rad/s}, \lambda = 1.80 \text{ m}$$

a) For angular wave number:

$$k = \frac{2\pi}{\lambda}$$

$$k = \frac{2 \times 3.142}{1.80}$$

$$k = 3.49 \text{ m}^{-1} \text{ Ans.}$$

b) For speed of the wave:

$$V = f\lambda$$

$$V = \left(\frac{\omega}{2\pi} \right) \lambda$$

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

$$V = \frac{110}{2 \times 3.142} \times 1.80$$

$$V = 31.50 \text{ m/s}$$

Ans.

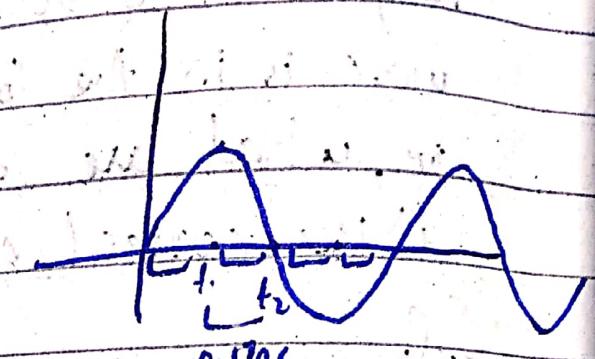
(Q5) A sinusoidal wave travels along a string. The time for a particular point to move from maximum displacement to zero is 0.170 s. What are: a) period, b) frequency? c) the wavelength is 1.40 m; what is the wave speed?

$$\lambda = 1.40 \text{ m}$$

a) For period:

$$T = 4(0.170)$$

$$T = 0.680 \text{ s} \quad \boxed{\text{Ans.}}$$



b) For frequency:

$$f = \frac{1}{T}$$

$$f = \frac{1}{0.680 \text{ s}}$$

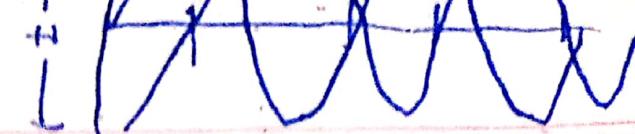
$$f = 1.47 \text{ Hz} \quad \boxed{\text{Ans.}}$$

c) for wave Speed:

$$V = f\lambda$$

$$V = (1.47)(1.40)$$

$$V = 2.058 \text{ m/s} \quad \boxed{\text{Ans.}}$$



 Q9) A sinusoidal wave moving along a string is shown twice in figure, as crest A travels in the positive direction of an x-axis by distance $d = 6.0\text{cm}$ in 4.0ms . The tick marks along the axis are separated by 10cm ; height $H = 6.00\text{mm}$. The equation for the wave is in the form $y(u, t) = y_m \sin(kx \pm \omega t)$, so, what are a) y_m b) k ; c) ω and d) the correct choice of sign in front of ω ?

~~Sol:~~

$$H = 6.00\text{mm} \quad d = 6.0\text{cm}$$

$$d = \frac{6}{100} = 0.060\text{m}$$

$$t = 4\text{ ms}$$

$$\lambda = 40\text{cm} = \frac{40}{100} = 0.4\text{m}$$

$$t = 4 \times 10^{-3}\text{s}$$

$$100$$

a) For maximum amplitude:

$$y_m = \frac{H}{2}$$

$$y_m = \frac{6}{2}$$

$$y_m = 3\text{ mm}$$

Ans

b) For k :

$$k = \frac{2\pi}{\lambda}$$

$$k = \frac{2 \times 3.142}{0.40}$$

$$k = 15.71 \text{ rad/m}$$

c) For ω :

$$\omega = kv \rightarrow \text{(i)}$$

$$v = \frac{d}{t}$$

$$v = \frac{0.060}{4 \times 10^{-3}}$$

$$v = 15 \text{ m/s}$$

$$\omega = 15.71 \times 15$$

$$\omega = 235.65 \text{ rad/s}$$

Ans

d) For choice of sign:

Wave is moving towards right, so
correct choice of sign will be -ve (minus)

(Q13) A sinusoidal wave of frequency 500Hz has a speed of 350m/s. a) How far apart are two points that differ in phase by $\pi/3$ rad? b) What is the phase difference between two displacements at a certain point at times 1.00ms apart?

~~Sol:~~

$$f = 500 \text{ Hz} \quad v = 350 \text{ m/s}$$

~~For a):~~

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$\lambda = \frac{350}{500}$$

$$\boxed{\lambda = 0.7 \text{ m}} \quad \text{or} \quad \boxed{\lambda = 0.7 \times 1000 \text{ mm}}$$

We know that

$$\lambda = 2\pi$$

and

$$\chi = \frac{\pi}{3}$$

$$\lambda = 700 \text{ mm}$$

Ans

$$2\pi n = \frac{\pi}{3}$$

$$n = \frac{1}{6}$$

$$x = \frac{700}{6}$$

$$x = 116.66\text{m}$$

m

Ans.

b) For phase difference:

$$f = \frac{1}{T}$$

$$T = \frac{1}{f}$$

$$T = \frac{1}{500}$$

$$T = 2 \times 10^{-3} \text{ s}$$

$$T = 2\text{ms}$$

At 2ms $\lambda = 700\text{mm}$

At 1ms $\lambda = 350\text{ mm}$

At ~~700~~ $\lambda = 700\text{mm}$

$$700 = 2\pi$$

$$350 = x$$

$$700\lambda = 700\pi$$

$$x = \pi \text{ rad}$$

The phase difference between two displacements at a certain point at 100s apart is π rad.

Q 23

Sol.

a) $y_m = 5 \text{ cm}$

Ans.

$$\mu = 25 \text{ g/m}$$

$$= 25 \times 10^{-3} \text{ kg/m}$$

b) For wavelength:

$$\lambda = (45 \text{ cm} - 5 \text{ cm})$$

$$\lambda = 40 \text{ cm}$$

or

$$\lambda = 40 = 0.4 \text{ m}$$

100

$$\lambda = 0.4 \text{ m}$$

Ans

c) For wave speed:

$$V = \sqrt{\frac{T}{\mu}}$$

$$V = \frac{3.6}{\sqrt{25 \times 10^{-3}}}$$

$$V = 12 \text{ m/s}$$

Ans

d) For period:

$$T = \frac{1}{f} \rightarrow ii$$

$$v = f\lambda$$

$$f = \frac{\lambda}{v}$$

$$f = 0.4$$

12

$$f = 30 \text{ Hz}$$

$$ii) \Rightarrow T = \frac{1}{30}$$

$$T = 0.033s \quad \text{Ans.}$$

e) For maximum transverse Speed:

$$U_m = \omega y_m$$

$$U_m = 2\pi f y_m$$

$$U_m = 2 \times 3.142 \times 30 \times \left(\frac{5}{100} \right)$$

$$U_m = 9.424 \text{ m/s}$$

Ans.

f) For k :

$$k = \frac{2\pi}{\lambda}$$

$$k = \frac{2 \times 3.142}{0.4}$$

$$k = 15.70 \text{ m}^{-1}$$
 Ans.

g) For ω :

$$\omega = 2\pi f$$

$$\omega = 2 \times 3.142 \times 30$$

$$\omega = 188.49 \text{ rad/s}$$
 Ans.

h) for ϕ :

$$y'(x, t) = y_m \sin(kx - \omega t + \phi)$$

$$\text{At } x=0, t=0$$

$$y'(0, 0) = y_m \sin[k(0) - \omega(0) + \phi]$$

$$y'(0, 0) = y_m \sin \phi \rightarrow \text{iii}$$

$$y_m = 4 \text{ cm} \rightarrow \text{at } x=0, t=0$$

$$\therefore y = \frac{4}{100}$$

$$y = 0.04 \text{ m} \rightarrow \text{iii}$$

$$y_m \sin \phi = 0.04$$

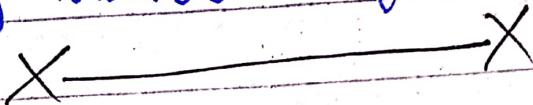
$$\therefore \sin \phi = \frac{0.04}{0.05}$$

$$\phi = \sin^{-1} \left(\frac{0.04}{0.05} \right)$$

$$\boxed{\phi = 0.92 \text{ rad}} \quad \text{Ans.}$$

(i) For signs in front of w :

+ve (positive) sign because wave is moving towards left (i.e negative x -direction)



Q 27 Sol:

$$\text{For Amplitude: } R_x = \frac{1}{2} uvw^2 y_m^2$$

$$\therefore R_x = 10 W$$

$$10 = \frac{1}{2} uvw^2 y_m^2 \quad \therefore \begin{cases} \lambda = 0.2 \\ f = 1/2 \times 10^{-3} \\ f = 500 Hz \end{cases}$$

$$\omega = (0.002)(f\lambda)(2\pi f)^2 y_m^2$$

$$y_m^2 = \frac{\omega}{0.002 \times f^3 \pi^2 \lambda \times k_1}$$

$$\sqrt{y_m^2} = \sqrt{\frac{\omega}{5}}$$

$$\sqrt{0.002 \times (500)^3 (3.142)^2 (0.2)}$$

$$\boxed{y_m = 0.00318 \text{ m}} \quad \text{Ans.}$$