
بسم الله الرحمن الرحيم

Physics For Engineers EE (117)

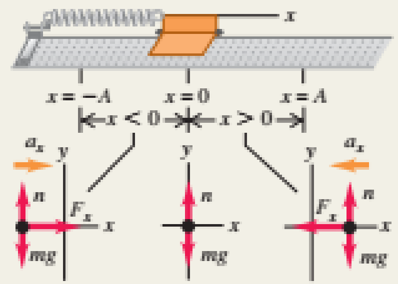
Week # 08

Date: 4th November, 2020

Periodic motion: Periodic motion is motion that repeats itself in a definite cycle. It occurs whenever a body has a stable equilibrium position and a restoring force that acts when it is displaced from equilibrium. Period T is the time for one cycle. Frequency f is the number of cycles per unit time. Angular frequency ω is 2π times the frequency. (See Example 14.1.)

$$f = \frac{1}{T} \qquad T = \frac{1}{f} \tag{14.1}$$

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{14.2}$$



Simple harmonic motion: If the restoring force F_x in periodic motion is directly proportional to the displacement x, the motion is called simple harmonic motion (SHM). In many cases this condition is satisfied if the displacement from equilibrium is small. The angular frequency, frequency, and period in SHM do not depend on the amplitude, but only on the mass m and force constant k. The displacement, velocity, and acceleration in SHM are sinusoidal functions of time; the amplitude A and phase angle ϕ of the oscillation are determined by the initial position and velocity of the body. (See Examples 14.2, 14.3, 14.6, and 14.7.)

$$F_x = -kx \tag{14.3}$$

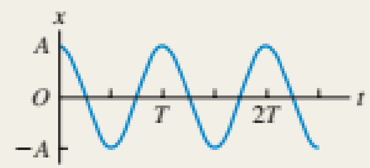
$$a_x = \frac{F_x}{m} = -\frac{k}{m}x \tag{14.4}$$

$$\omega = \sqrt{\frac{k}{m}} \tag{14.10}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{14.11}$$

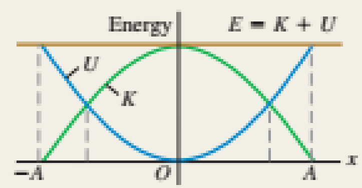
$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \tag{14.12}$$

$$x = A\cos(\omega t + \phi) \tag{14.13}$$



Energy in simple harmonic motion: Energy is conserved in SHM. The total energy can be expressed in terms of the force constant k and amplitude A. (See Examples 14.4 and 14.5.)

$$E = \tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \text{constant} \tag{14.21}$$



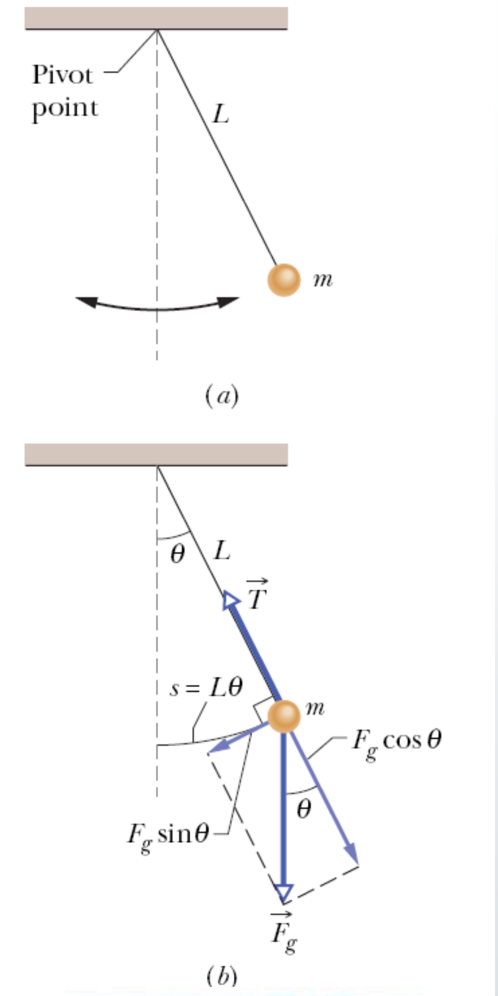
Angular simple harmonic motion: In angular SHM, the frequency and angular frequency are related to the moment of inertia I and the torsion constant κ.

$$\omega = \sqrt{\frac{\kappa}{I}} \quad \text{and} \quad f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}} \tag{14.24}$$

Pendulums

Pendulums

We turn now to a class of simple harmonic oscillators in which the springiness is associated with the gravitational force rather than with the elastic properties of a twisted wire or a compressed or stretched spring.



In a *simple pendulum*, a particle of mass m is suspended from one end of an unstretchable massless string of length L that is fixed at the other end.

The restoring torque acting on the mass when its angular displacement is θ, is:

$$\tau = -L(F_g \sin\theta) = I\alpha$$

α is the angular acceleration of the mass. Finally,

$$\alpha = -\frac{mgL}{I}\theta, and$$

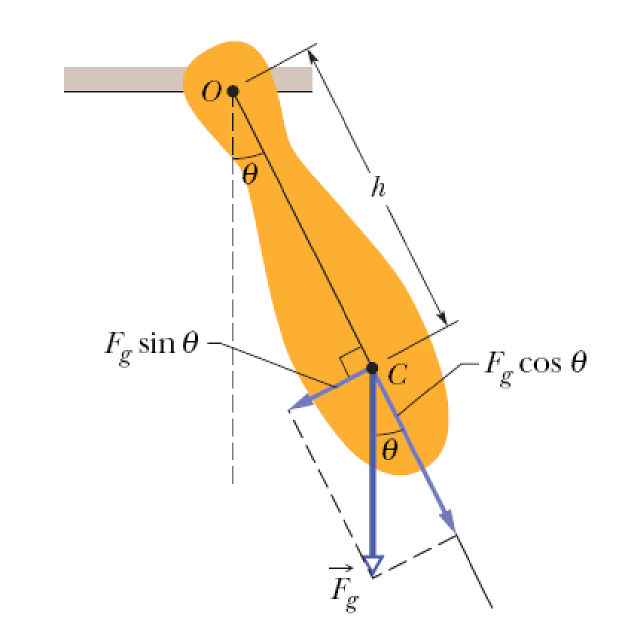
$$T = 2\pi\sqrt{\frac{L}{g}}$$

This is true for *small angular displacements*, θ.

Find the period and frequency of a simple pendulum 1.000 m long at a location where $g = 9.800\ \mathrm{m/s^2}$.

Pendulums

A physical pendulum can have a complicated distribution of mass. If the center of mass, C, is at a distance of h from the pivot point (figure), then for *small angular amplitudes*, the motion is simple harmonic.



The period, T, is:

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

Here, I is the rotational inertia of the pendulum about O.

In the **small-angle approximation** we can assume that $\theta << 1$ and use the approximation $\sin\theta \cong \theta$. Let us investigate up to what angle θ is the approximation reasonably accurate?

θ **(degrees)**	θ **(radians)**	**sin** θ
5	0.087	0.087
10	0.174	0.174
15	0.262	0.259 (1% off)
20	0.349	0.342 (2% off)

Conclusion: If we keep $\theta < 10°$ we make less than 1 % error.

SHM and uniform circular motion

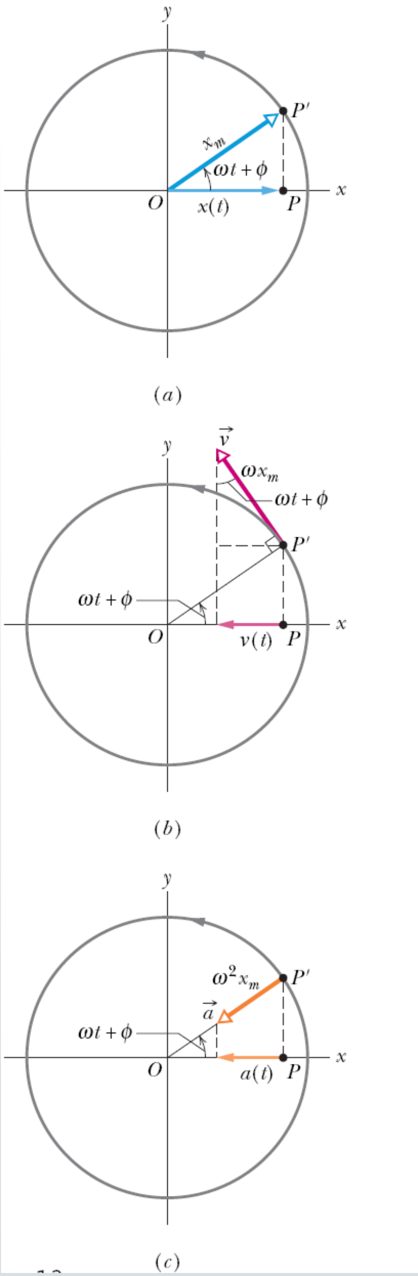
Consider a reference particle P’ moving in uniform circular motion with constant angular speed (w).

The projection of the particle on the x-axis is a point P, describing motion given by:

$$x(t) = x_m \cos(\omega t + \phi).$$

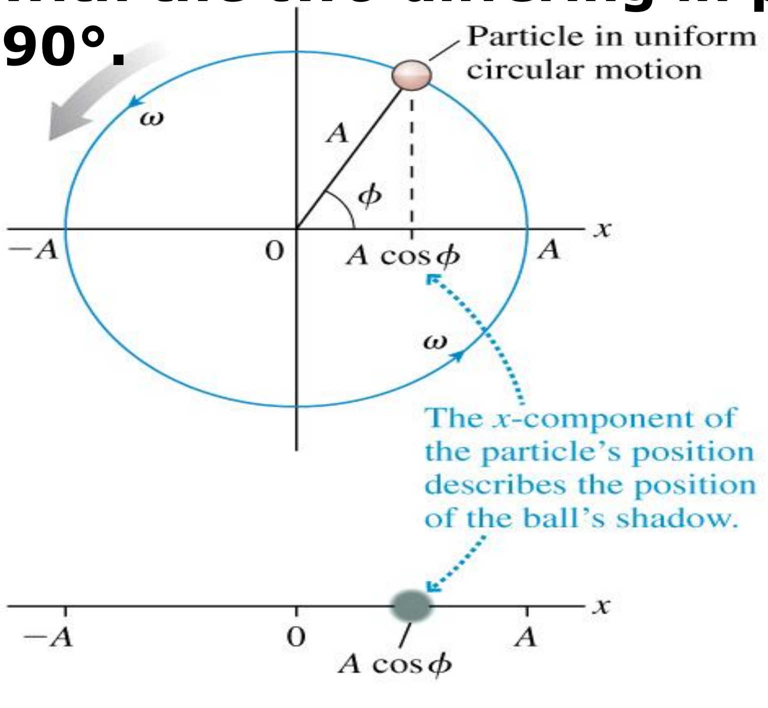
This is the displacement equation of SHM.

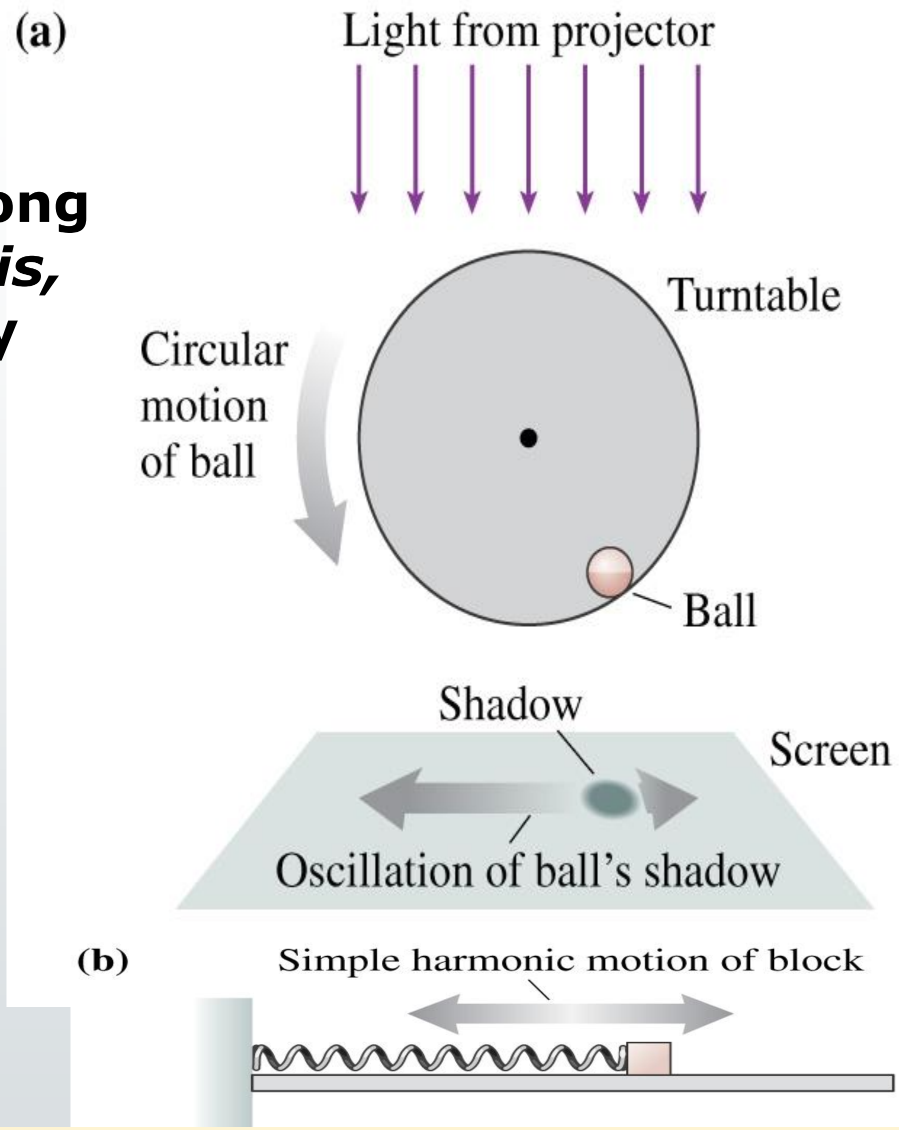
SHM, therefore, is the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.



COMPARING SIMPLE HARMONIC MOTION WITH UNIFORM CIRCULAR MOTION

Uniform circular motion can be considered a combination of two simple harmonic motions, one along the *x axis and one along the y axis, with the two* differing in phase by 90°.



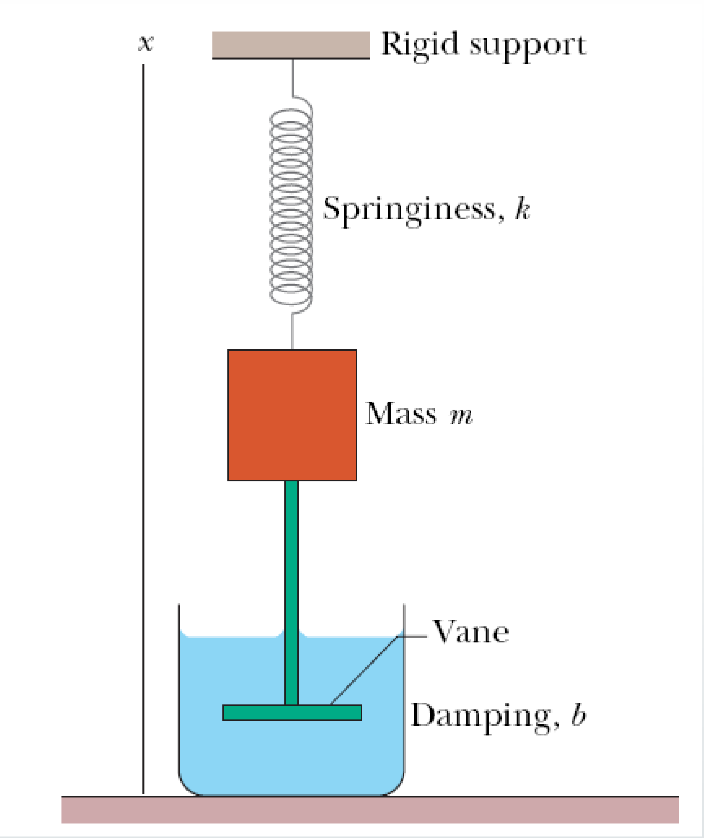


simple harmonic motion along a straight line can be represented by the projection of uniform circular motion along a diameter of a reference circle.

Damped Oscillations

In a damped oscillation, the motion of the oscillator is reduced by an external force.

Example: A block of mass *m* oscillates vertically on a spring with spring constant *k*.
From the block a rod extends to a vane which is submerged in a liquid.
The liquid provides the external damping force, F_d.



Damped SHM

Often the damping force, F_d, is proportional to the 1st power of the velocity v. That is,

$$F_d = -bv$$

From Newton's 2nd law, the following DE results:

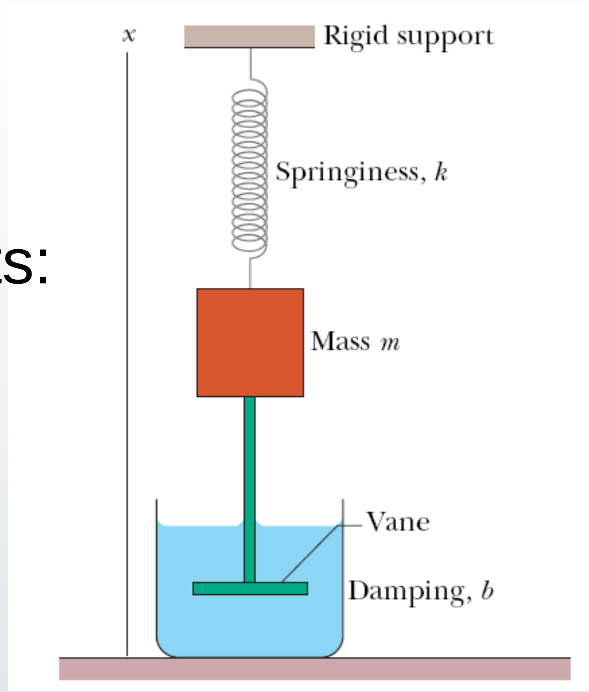
$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

The solution is:

$$x(t) = x_m e^{\frac{-bt}{2m}} \cos(\omega' t + \phi)$$

Here ω' is the angular frequency, and is given by:

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

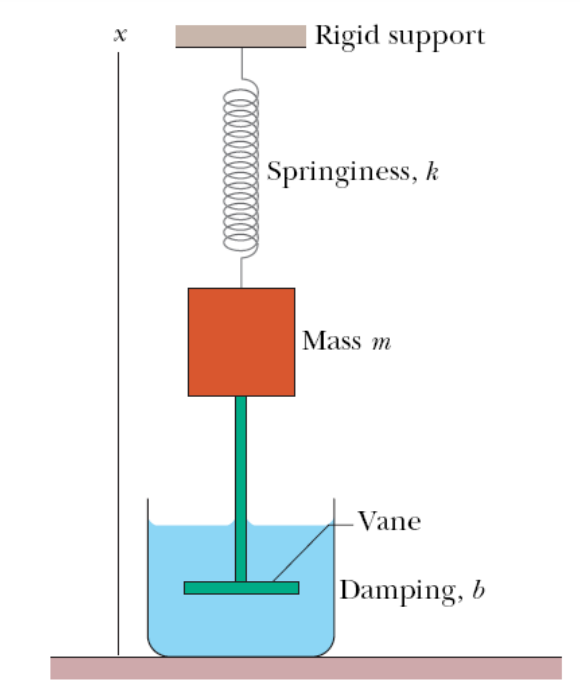


Damped Oscillations

Often the damping force, F_d, is proportional to the 1st power of the velocity v. That is,

$$F_d = -\gamma v$$

$$m\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + kx = 0$$



The solution is:

$$x(t) = x_0 e^{\frac{-\gamma t}{2m}} \cos(\omega' t + \varphi)$$

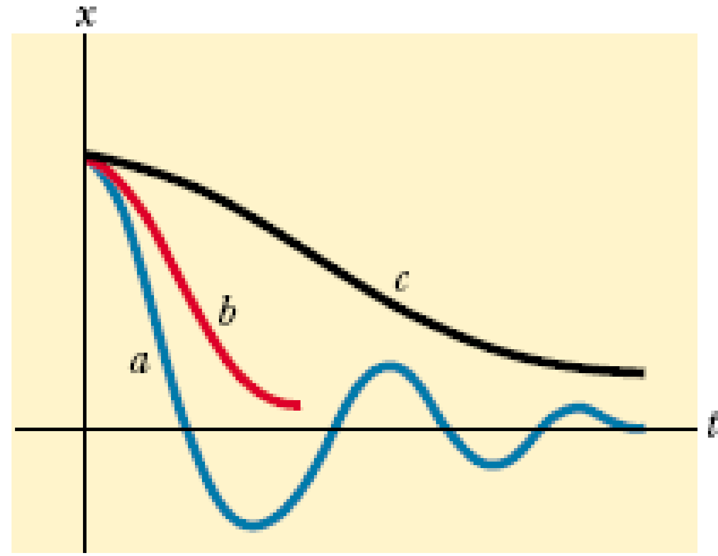
$$\omega' = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2m}\right)^2}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

DAMPED OSCILLATIONS

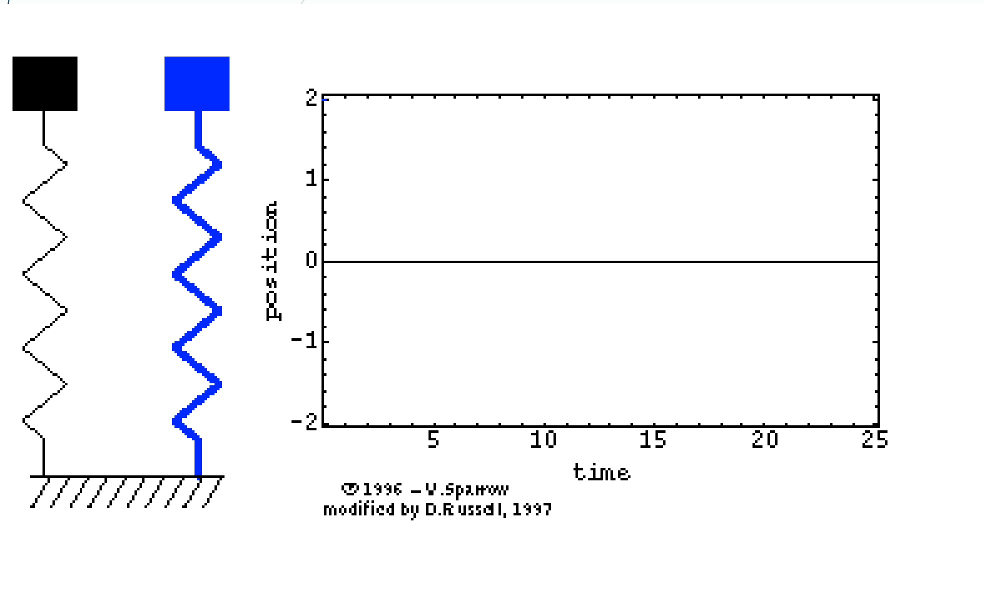
$$\omega' = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2m}\right)^2}$$

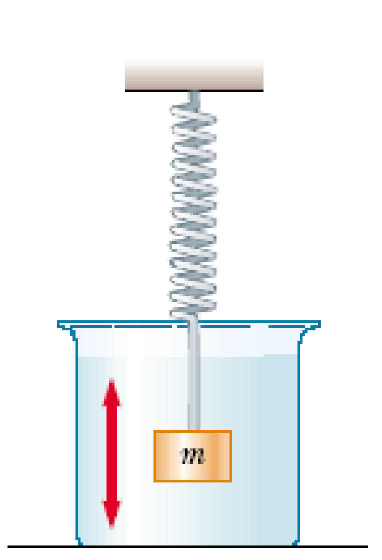
where $\omega_0 = \sqrt{k/m}$ represents the angular frequency in the absence of a retarding force (the undamped oscillator) and is called the **natural frequency**



Graphs of displacement versus time for (a) an underdamped oscillator, (b) a critically damped oscillator, and (c) an overdamped oscillator.

DAMPED OSCILLATIONS

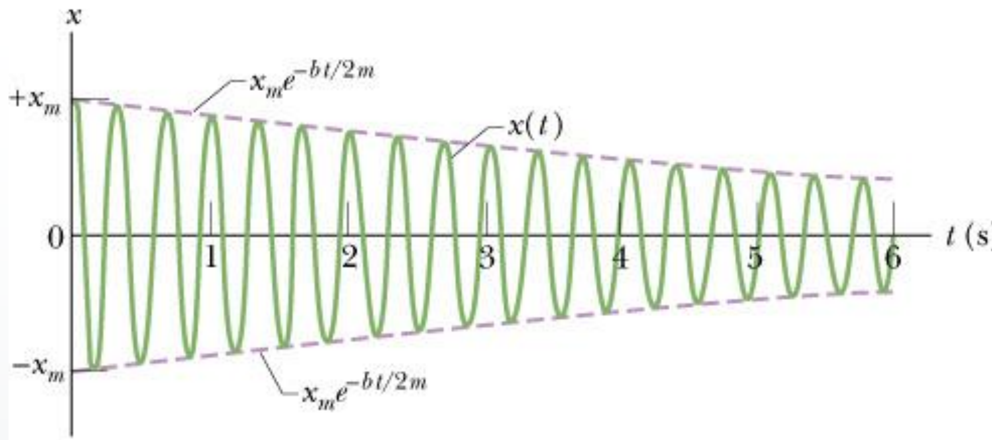




http://www.lon-capa.org/~mmp/applist/damped/d.htm

15.5 Damped Oscillations

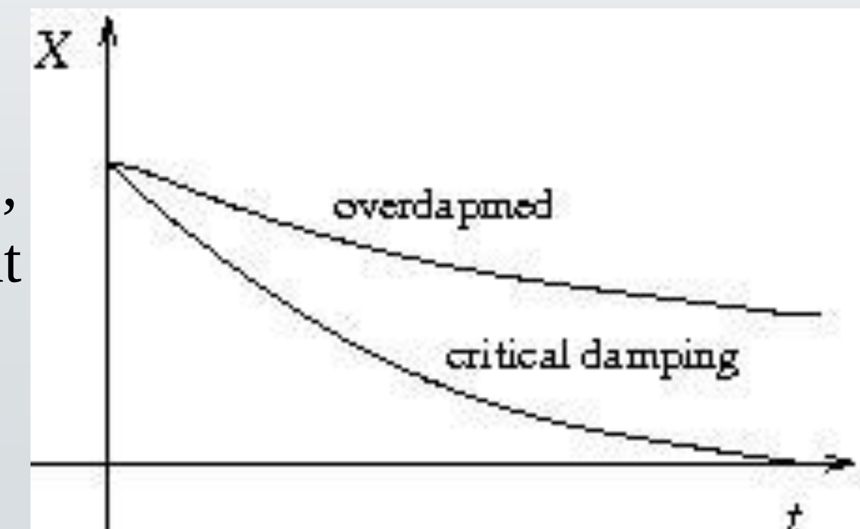
$$x(t) = x_0 e^{\frac{-\gamma t}{2m}} \cos(\omega' t + \varphi)$$



The above figure shows the displacement function *x(t)* for the damped oscillator described before.
The amplitude decreases as x_0 exp ($-\gamma t$ / $2m$) with time.

The above is for $\gamma < 2m\omega_0$ (underdapmed).

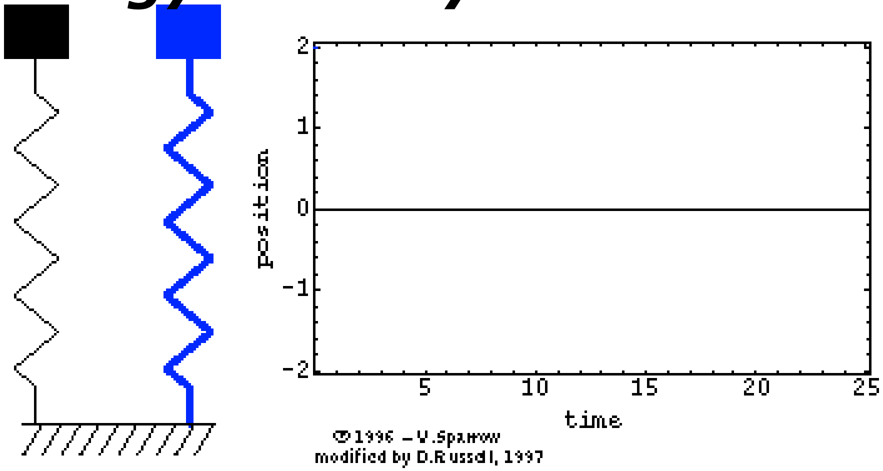
For $\gamma > 2m\omega_0$ (overdapmed) and $\gamma = 2m\omega_0$ (critical damping), the oscillation goes like the right figure.



$$E(t) \approx \tfrac{1}{2}kx_m^2 e^{-bt/m},$$

DAMPED OSCILLATIONS

In many real systems, dissipative forces, such as friction, retard the motion.
Consequently, the mechanical energy of the system diminishes in time, and the motion is said to be Damped.
Retarding force $\mathbf{R} = -b\mathbf{v}$



, we can write Newton's second law as

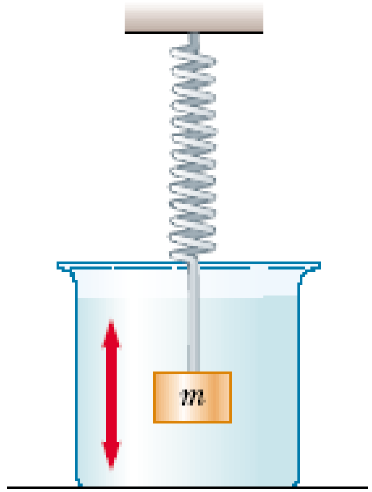
$$\sum F_x = -kx - bv = ma_x$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

The solution of this equation $x = Ae^{-\frac{b}{2m}t}\cos(\omega t + \phi)$

where the angular frequency of oscillation is

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$



http://www.lon-capa.org/~mmp/applist/damped/d.htm

Example

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(a) What are the angular frequency, the frequency, and the period of the resulting motion?

KEY IDEA The block–spring system forms a linear simple harmonic oscillator, with the block undergoing SHM.

Calculations: The angular frequency is given by Eq. 15-12:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65\ \text{N/m}}{0.68\ \text{kg}}} = 9.78\ \text{rad/s}$$

$$\approx 9.8\ \text{rad/s.} \qquad \text{(Answer)}$$

The frequency follows from Eq. 15-5, which yields

$$f = \frac{\omega}{2\pi} = \frac{9.78\ \text{rad/s}}{2\pi\ \text{rad}} = 1.56\ \text{Hz} \approx 1.6\ \text{Hz.} \qquad \text{(Answer)}$$

The period follows from Eq. 15-2, which yields

$$T = \frac{1}{f} = \frac{1}{1.56\ \text{Hz}} = 0.64\ \text{s} = 640\ \text{ms.} \qquad \text{(Answer)}$$

Example

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(b) What is the amplitude of the oscillation?

KEY IDEA With no friction involved, the mechanical energy of the spring–block system is conserved.

Reasoning: The block is released from rest 11 cm from its equilibrium position, with zero kinetic energy and the elastic potential energy of the system at a maximum. Thus, the block will have zero kinetic energy whenever it is again 11 cm from its equilibrium position, which means it will never be farther than 11 cm from that position. Its maximum displacement is 11 cm:

$$x_m = 11 \text{ cm.} \qquad \text{(Answer)}$$

Example

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(c) What is the maximum speed v_m of the oscillating block, and where is the block when it has this speed?

KEY IDEA The maximum speed v_m is the velocity amplitude ωx_m in Eq. 15-6.

Calculation: Thus, we have

$$v_m = \omega x_m = (9.78 \text{ rad/s})(0.11 \text{ m})$$
$$= 1.1 \text{ m/s.} \qquad \text{(Answer)}$$

This maximum speed occurs when the oscillating block is rushing through the origin; compare Figs. 15-4*a* and 15-4*b*, where you can see that the speed is a maximum whenever $x = 0$.

Example

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(d) What is the magnitude a_m of the maximum acceleration of the block?

KEY IDEA The magnitude a_m of the maximum acceleration is the acceleration amplitude $\omega^2 x_m$ in Eq. 15-7.

Calculation: So, we have

$$a_m = \omega^2 x_m = (9.78 \text{ rad/s})^2(0.11 \text{ m})$$
$$= 11 \text{ m/s}^2. \qquad \text{(Answer)}$$

This maximum acceleration occurs when the block is at the ends of its path. At those points, the force acting on the block has its maximum magnitude; compare Figs. 15-4*a* and 15-4*c*, where you can see that the magnitudes of the displacement and acceleration are maximum at the same times.

Example

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(e) What is the phase constant ϕ for the motion?

Calculations: Equation 15-3 gives the displacement of the block as a function of time. We know that at time $t = 0$, the block is located at $x = x_m$. Substituting these *initial conditions*, as they are called, into Eq. 15-3 and canceling x_m give us

$$1 = \cos\phi. \tag{15-14}$$

Taking the inverse cosine then yields

$$\phi = 0 \text{ rad.} \qquad \text{(Answer)}$$

(Any angle that is an integer multiple of 2π rad also satisfies Eq. 15-14; we chose the smallest angle.)

Homework

- Do all the Sample Problems of the Chapter other than the slides !