National University
Of Computer & Emerging Sciences Karachi Campus

EE 117: Applied Physics

Introduction

Engr. Abdul Saboor Khan
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Office: 111-111-128 Ext:273

About the Instructor

- Engr.Abdul Saboor Khan
- Specialization
  - Machine Learning / Image Processing/Computer Vision/RFID Systems
- Education
  - Masters of Science (Electrical Engineering)(2013-2016)
    - Abasyn University Islamabad Campus
  - Bachelor of Engineering(Electronics Engineering)(2008-2012)
    - Usman Institute of Technology, Hamdard University

- Contact Detail
  - Email:abdul.saboor@nu.edu.pk
  - Phone: 111-111-128(273)
- Teaching Experiences
  - Lecturer (8-2017~Present) : FAST, NU Karachi
  - Lab Engineer(2-2013~7-2017): Abasyn University, Islamabad

Course Description

- Course Title: Applied Physics
- Course Code: EE117
- Credit Hours: 2(Theory)+1(Lab)=3
- Course & Lab Instructor: Engr. Abdul Saboor Khan
  - abdul.saboor@nu.edu.pk

Grading System

Grade	Grade Point	LL%	UL%
A+	4.00	≥90	–
A	4.00	≥86	<90
A−	3.67	≥82	<86
B+	3.33	≥78	<82
B	3.00	≥74	<78
B−	2.67	≥70	<74
C+	2.33	≥66	<70
C	2.00	≥62	<66
C−	1.67	≥58	<62
D+	1.33	≥54	<58
D	1.00	≥50	<54
F	0.00	–	<50

Course Grading Policy/Marks Distribution

- Quizzes & Assignments 10%
  - Late Assignments are not accepted
- Midterm Exam(I + II) 30%
  - As per University Schedule (Week 6 & 12)
- Lab Work /Lab Project 10%
  - As per semester schedule (Week 15)
- Final Exam 50%
  - As per University Schedule (Week 17/18)
- Total 100%

Homework

Homework will be assigned on regular basis. Work handed in must be original and not a duplicate.

Course Policies

- Come in time (First 15 minutes≈ allowable time).
- No disturbance during Lecture.
- Bring your textbook ,class notes,notebook,pen and calculator.
- DO NOT MISS YOUR QUIZ AND MID EXAMS.
- No cell phone calls , No SMS.
- Copying of assignments is strictly prohibited.
- Meet the deadlines of Assignments.
- Maintain your attendance (Atleast≥80%)
- Class Representative(CR) would collect the assignment and submit to me in my office or as directed.

Disclaimer

- Slides have been prepared using various online resources, ebooks, MIT OCW classroom.google.com/

Class Code

yeeldex	(BCS-1D)
a7ql7jd	(BSE-1A)
mhponcl	(BSE-1B)

COURSE OUTLINE
Week-wise Course Breakup

Credit Hours:	3
Pre-Requisites:	Nil (Physics taken at 12th year of Schooling)
Courses for which this can be pre-requisite:	Digital Logic Design, Computer Graphics, Modeling and Simulation

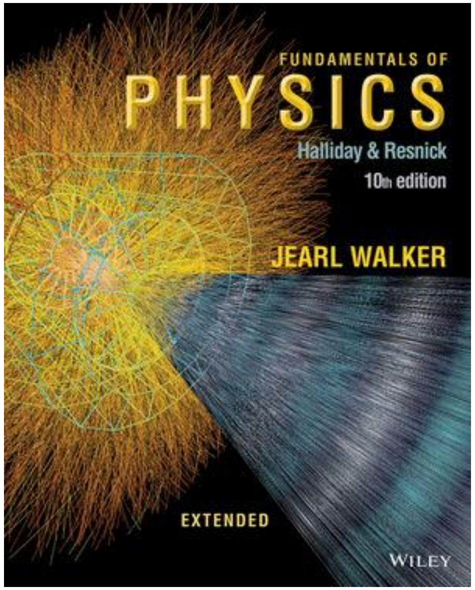
Week	Duration	Topics Covered	Tools
Week 1	3 hrs	Adding Vectors, Components of Vectors, Unit Vectors, Vector & Scalar Products, (1hr Lab Python for Applied Physics)	A1, M1, Q1, F
Week 2	3 hrs	Position & Displacement (2/3 dimensions) Average/Instantaneous Velocity/Acceleration, (1hr Lab Python for Applied Physics)	A1, M1, Q1, F
Week 3	3 hrs	Projectile Motion, Uniform Circular Motion horizontal/vertical motions, equation of the path and horizontal range, (1hr Lab Python for Applied Physics)	A2, M1, Q1, F
Week 4	3 hrs	Newton Laws of Motion, Forces (1D/2D): Gravitational, Friction, Tension, Weight, (1hr Lab Python for Applied Physics)	A2, M1, Q1, F,
Week 5	3 hrs	Simple Harmonic Motion, the Force Law for SHM, Angular SHM (1hr Lab Python for Applied Physics)	A2, M1, Q1, F
Week 6	3 hrs	**Mid Term –I**	

Week 7	3 hrs	Simple Pendulum, Damped SHM, Circular Motion & SHM, (1hr Lab Python for Applied Physics)	A3, Q2, M2, F
Week 8	3 hrs	Types of Waves, Sinusoidal Waves, Wavelength and Frequency (1hr Lab Python for Applied Physics)	A3, Q2, M2, F
Week 9	3 hrs	Electric Charge, Coulomb's Law, Electric Field, Electric Field Due to Point Charge and Dipole, (1hr Lab Python for Applied Physics)	A3, Q3, M2, F
Week 10	3 hrs	Gauss' Law, Flux, Flux of Electric Field, Gauss's Law, Equivalency of Gauss's Law and Coulombs' Law (1hr Lab Python for Applied Physics)	Q3, M2, F
Week 11	3 hrs	Capacitance, Parallel Plate, Cylindrical & Spherical Capacitors, Capacitors in Parallel and In Series.	A4, Q4, M2, F
		(1hr Lab Python for Applied Physics)	
Week 12	3 hrs	**Mid Term –II**	

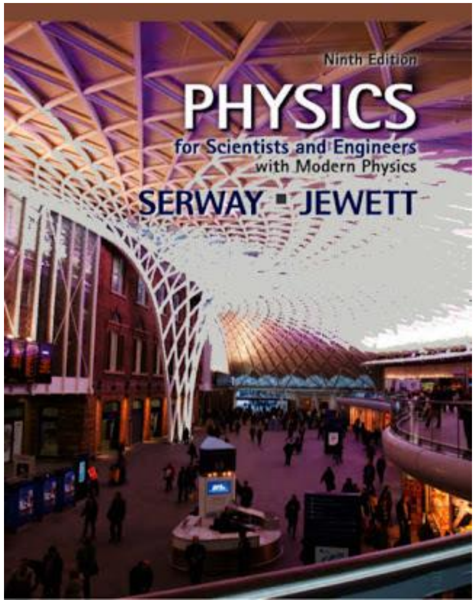
Week 13	3 hrs	Electric Current, Current Density and Drift Speed, Resistance & Resistivity, Ohm's Law, (1hr Lab Python for Applied Physics)	A4, Q4, F
Week 14	3 hrs	Magnetic Fields and Field Lines, Crossed Fields: Hall Effect, Circulating Charge Particles, Magnetic Force on Current Carrying Wire. (1hr Lab Python for Applied Physics)	A4, Q4, F
Week 15	3 hrs	Magnetic Field Due to Current, Ampere's Law, Magnetic Field Inside/Outside Wire, Solenoids & Toroid's & Between two Parallel Wires (1hr Lab Python for Applied Physics)	Q4, A4, P, F
Week 16	3 hrs	Revision	

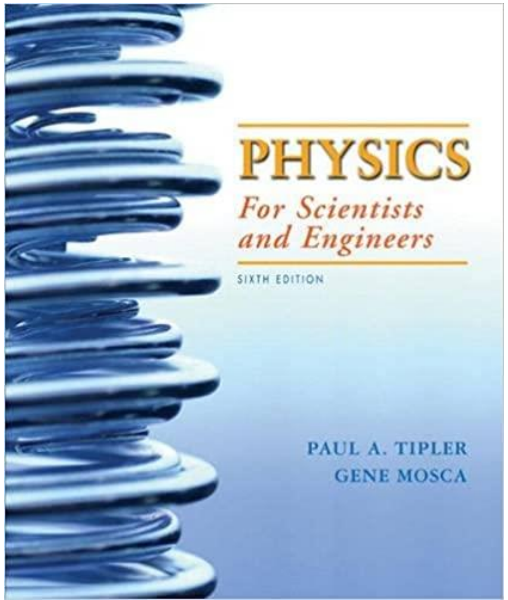
Text and Reference Books

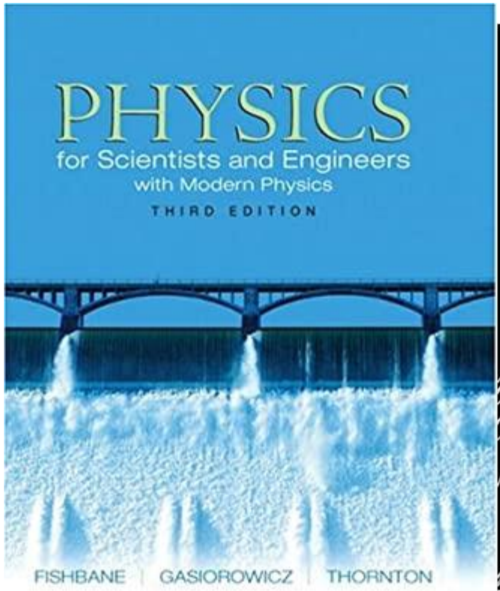
Textbooks

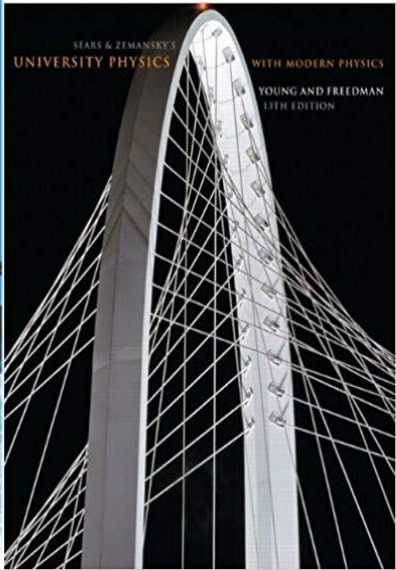


Reference Books









Reference Material

- Young & Freedman, *University Physics With Modern Physics*, 13th Edition, Pearson, 2012.

- 8.01SC Physics I: Classical Mechanics, Fall 2010
  - Peter Dourmashkin,
  - MIT OCW
  - http://ocw.mit.edu/courses/physics/8-01sc-physics-i-classical-mechanics-fall-2010/

- 8.02SC Physics II: Electricity and Magnetism,Fall 2010
  - Walter Lewin, John Belcher, and Peter Dourmashkin
  - MIT OCW
  - http://ocw.mit.edu/courses/physics/8-02sc-physics-ii-electricity-and-magnetism-fall-2010/index.htm

- Giancoli, D. C. *Physics for Scientists & Engineers*. Vol. 2. Prentice Hall.

Applied Physics

- Applied physics as a subject is rooted in the
  - fundamental truths and
  - basic concepts of the physical sciences
- but is concerned with the utilization of these scientific principles in practical devices and systems

- Reference: Wikipedia

Major Branches to be Studied

- Newtonian Mechanics
- Electricity & Magnetism

Newtonian Mechanics

- Concerned with the set of physical laws describing the motion of bodies under the action of a system of forces

Course Objective & Organization

- Objectives:
  - Learning the concepts associated with mechanics, electric field and magnetic field
  - Understanding of physical phenomena based on this knowledge
- Organization:
  - Mechanics
  - Electricity &Magnetism (E&M)

- **Definition of Physics**

“Physics is the study of physical phenomena of the universe. It is experimental science hence depends heavily on the objective observation and measurements.”

OR

- *“It is the branch of physical science that deals with interaction of matter and energy.”*

EE 117 Applied Physics

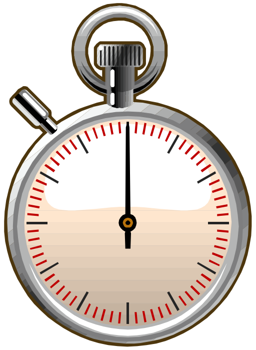
Vectors

Scalars

- A **scalar quantity** is a quantity that has magnitude only and has no direction in space

Examples of Scalar Quantities:

- Length
- Area
- Volume
- Time
- Mass



Scalar Quantities

- Quantities that can be completely described by **magnitude** (size).
- Scalars can be added algebraically.
- They are expressed as positive or negative numbers and a unit

Characteristics of a Scalar Quantity

- Only has magnitude
- Requires 2 things:
  1. A value
  2. Appropriate units

Example:

Mass: 5kg

Time = 20.0 s

Temperature = 20^{o}C

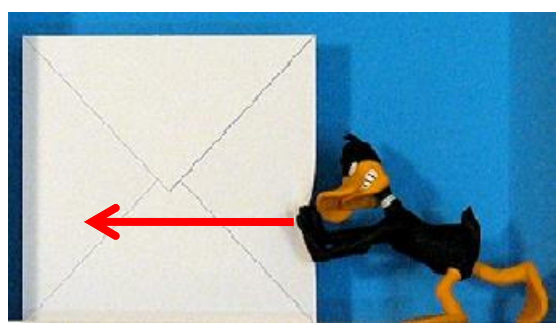
Speed = 20 m/s

Vectors

- A **vector quantity** is a quantity that has both magnitude and a direction in space

Examples of Vector Quantities:

- Displacement
- Velocity
- Acceleration
- Force



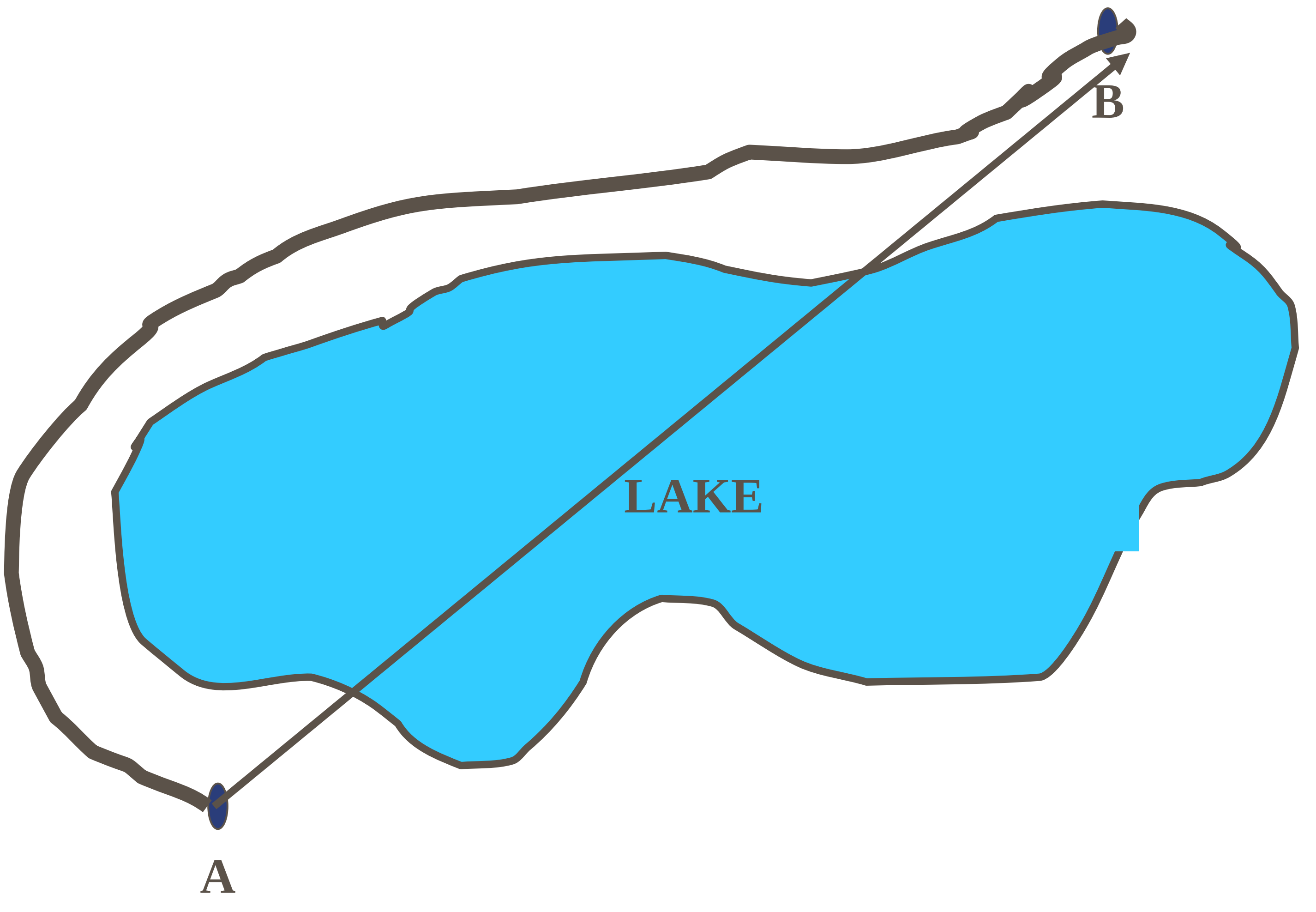
COMPRASION B/W SCALAR & VECTOR VALUES

- **Speed (a scalar) versus Velocity (a vector)**

- Speed is a *scalar*, (magnitude no direction) – such as 5 feet per second.
- Speed does not tell the direction the object is moving. All that we know from the speed is the magnitude of the movement.
- Velocity, is a *vector* (both magnitude and direction) – such as 5 ft/s Eastward. It tells you the magnitude of the movement, 5 ft/s, as well as the direction which is Eastward.

- Distance (a scalar) versus Displacement (a vector)

- We want to get from point A to point B. If we follow the road around the lake our direction is always changing. There is no specific direction. The distance traveled on the road is a scalar quantity.
- A straight line between A and B is the displacement. It has a specific direction and is therefore a vector.


B
LAKE
A

Vector Quantities

- Quantities that need both a **magnitude** and a **direction** to describe them (also a point of application)
- When expressing vectors as a symbol, you need to adopt a recognized notation
- They need to be added, subtracted and multiplied in a special way

Characteristics of a Vector Quantity

- Has magnitude & direction
- Requires 3 things:

    1. A value
    2. Appropriate units
    3. <u>A direction!</u>

  Example:

    Acceleration: 9.8 m/s^2 down

    Velocity: 25 mph West

Vectors

- A **vector** has magnitude as well as direction, and vectors follow **certain (vector) rules of combination**

- Some physical quantities that are vector quantities are displacement, velocity, and acceleration

- Not all physical quantities involve a direction

- Temperature, pressure, energy, mass, and time, for example, do not “point” in the spatial sense

- These are **scalars** quantities, and we deal with them by the rules of **ordinary algebra**

Representation of vectors

- Vectors can be represented in two form:
  - Graphical Representation (Polar):

    Polar form indicates a magnitude value and a directional value. the direction value may be in degrees, radians or geographic terms.

    Examples: 14.1 meters @ 315°, 14.1 meters @ (7/4)π radians, 14.1 feet at 45° south of east

  - Mathematical Representation (Rectangular):

Rectangular form identifies the x-y coordinates of the vector. the vector itself extends from origin to the x-y point.

Examples: 10, -10 (x = +10, y = -10) the magnitude of the vector can be found using the Pythagorean theorem $(10^2 + (-10^2))^{1/2} = 14.1$

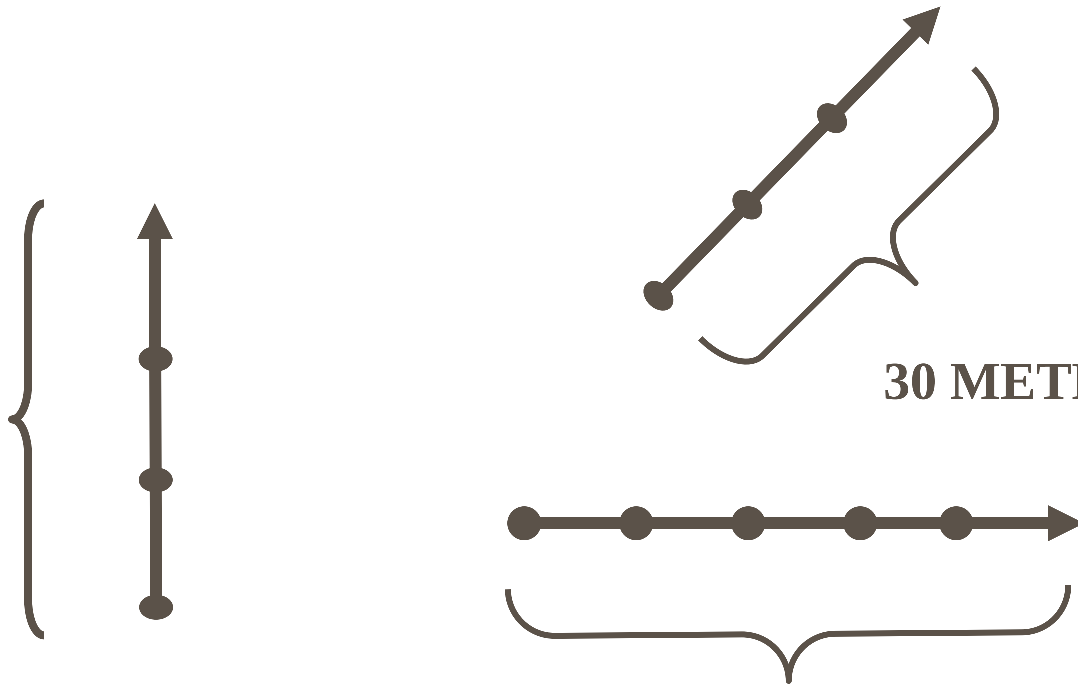
the direction can be found using an inverse tangent function $\tan^{-1}(10/10) = \tan^{-1}(1.0) = 45^o$ since x is positive and y is negative the angle is -45^o and is in quadrant iv or 315^o

Graphical Representation of a Vector

The goal is to draw a *mini version* of the vectors to give you an accurate picture of the magnitude and direction. To do so, you must:

1. Pick a scale to represent the vectors. Make it simple yet appropriate.
2. Draw the tip of the vector as an arrow pointing in the appropriate direction.
3. Use a ruler & protractor to draw arrows for accuracy. The angle is always measured from the horizontal or vertical.

Graphical Representation of a Vector



30 METERS @ 90^{O}

30 METERS @ 45^{O}

50 METERS @ 0^{O}

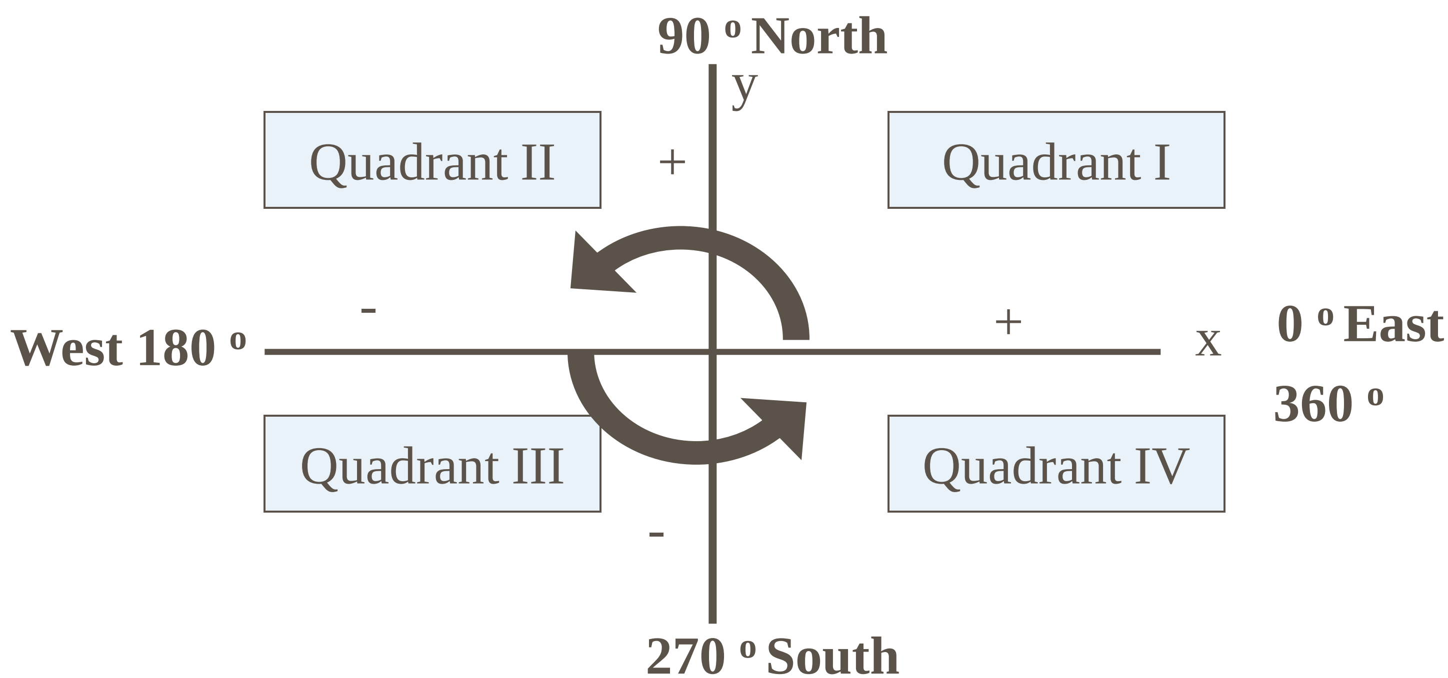
= 10 METERS

SCALE

VECTOR ARROWS MAY BE DRAWN ANYWHERE ON THE PAGE AS LONG AS THE PROPER LENGTH AND DIRECTION ARE MAINTAINED

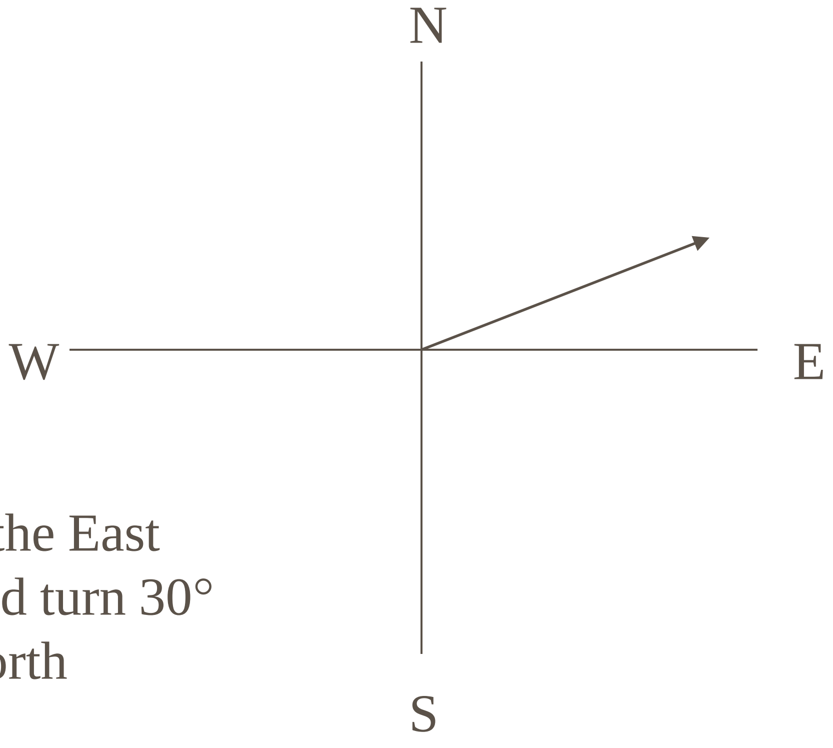
Up = + **Down = -** **Right = +** **Left = -**

Rectangular Coordinates



Understanding Vector Directions

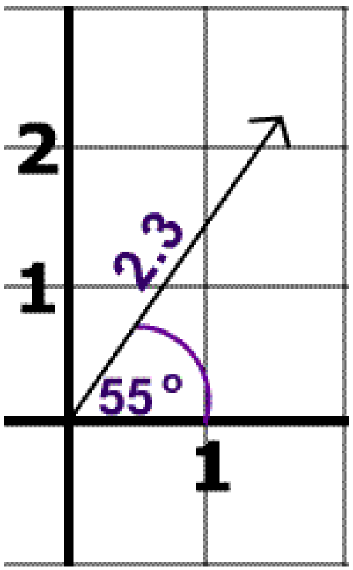
To accurately draw a given vector, start at the **second** direction and move the given degrees to the **first** direction.



30° N of E

Start on the East origin and turn 30° to the North

Example



•The direction of the vector is 55° North of East

•The magnitude of the vector is 2.3.

Addition of Vectors

- Vectors can be added or subtracted however not in the usual arithmetic manner. The directional components as well as the magnitude components must each be considered.

- The addition and subtraction of vectors can be accomplished used graphic methods (drawing) or component methods (mathematical).

- Graphical addition and subtraction requires that each vector be represented as an arrow with a length proportional to the magnitude value and pointed in the proper direction assigned to the vector.

Graphical Method

Graphical Addition of Vectors

<u>Head-To-Tail Method</u>

1. Pick appropriate scale, write it down.
2. Use a ruler & protractor, draw 1^{st} vector to scale in appropriate direction, label.
3. Start at tip of 1^{st} vector, draw 2^{nd} vector to scale, label.
4. Connect the vectors starting at the tail end of the 1^{st} and ending with the tip of the last vector.
5. This = sum of the original vectors, its called the resultant vector.

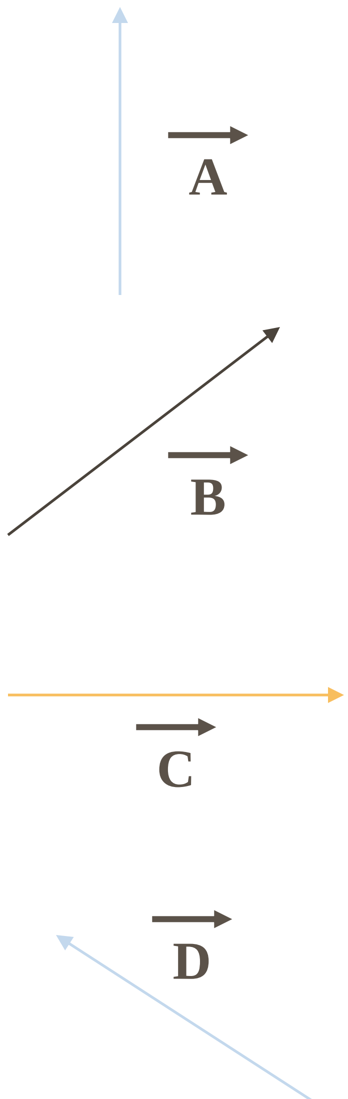
Graphical Addition of Vectors (cont.)

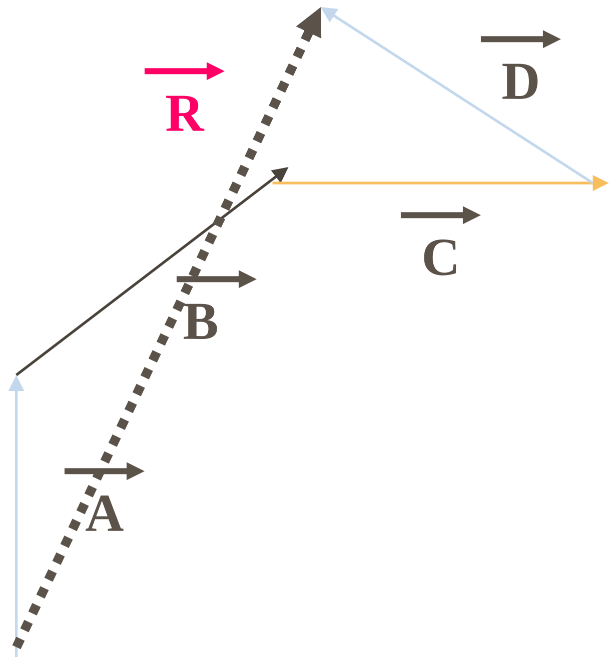
<u>Head-To-Tail Method</u>

5. Measure the magnitude of R.V. with a ruler. Use your scale and convert this length to its actual amt. and record with units.
6. Measure the direction of R.V. with a protractor and add this value along with the direction after the magnitude.

Addition of Vectors

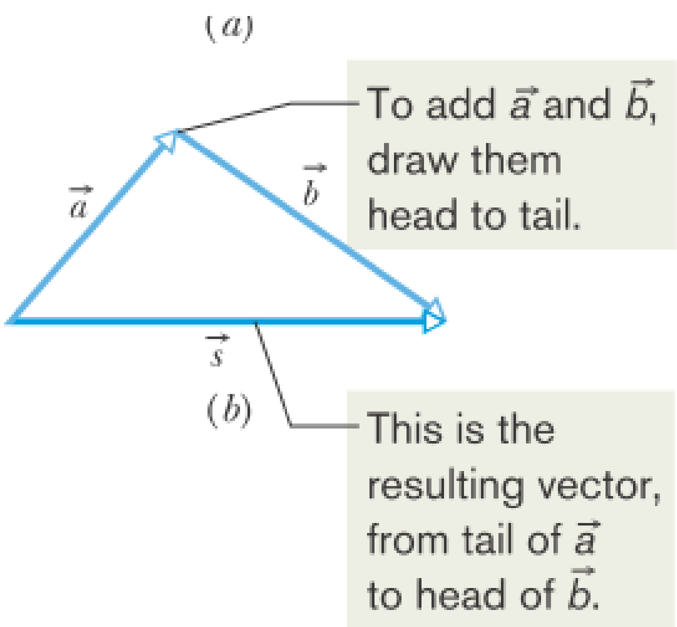
ALL VECTORS MUST BE DRAWN TO SCALE & POINTED IN THE PROPER DIRECTION

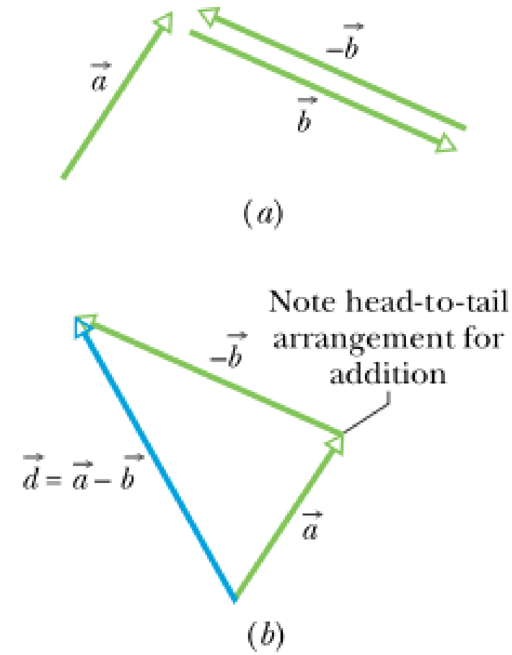




$$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{R}$$

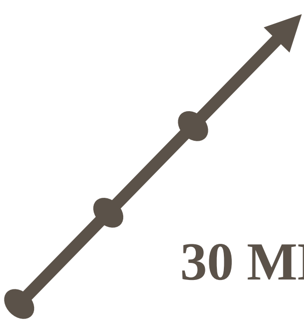
Adding Vectors Geometrically Two vectors $\vec{a}$ and $\vec{b}$ may be added geometrically by drawing them to a common scale and placing them head to tail. The vector connecting the tail of the first to the head of the second is the vector sum $\vec{s}$. To subtract $\vec{b}$ from $\vec{a}$, reverse the direction of $\vec{b}$ to get $-\vec{b}$; then add $-\vec{b}$ to $\vec{a}$. Vector addition is commutative and obeys the associative law.



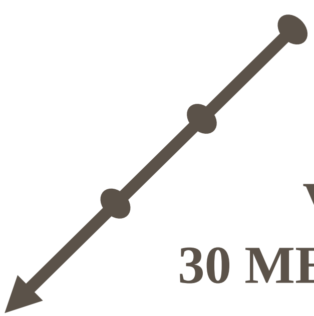


GRAPHIC SUBTRACTION OF VECTORS

- In algebra, a – b = a + (–b) or in other words, adding a negative value is actually subtraction. This is also true in vector subtraction. If we add a negative vector b to vector a this is really subtracting vector b from vector a.
- Vector values can be made negative by reversing the vector's direction by 180 degrees. If vector a is 30 meters directed at 45 degrees (quadrant i), negative vector a is 30 meters at 225 degrees (quadrant iii).

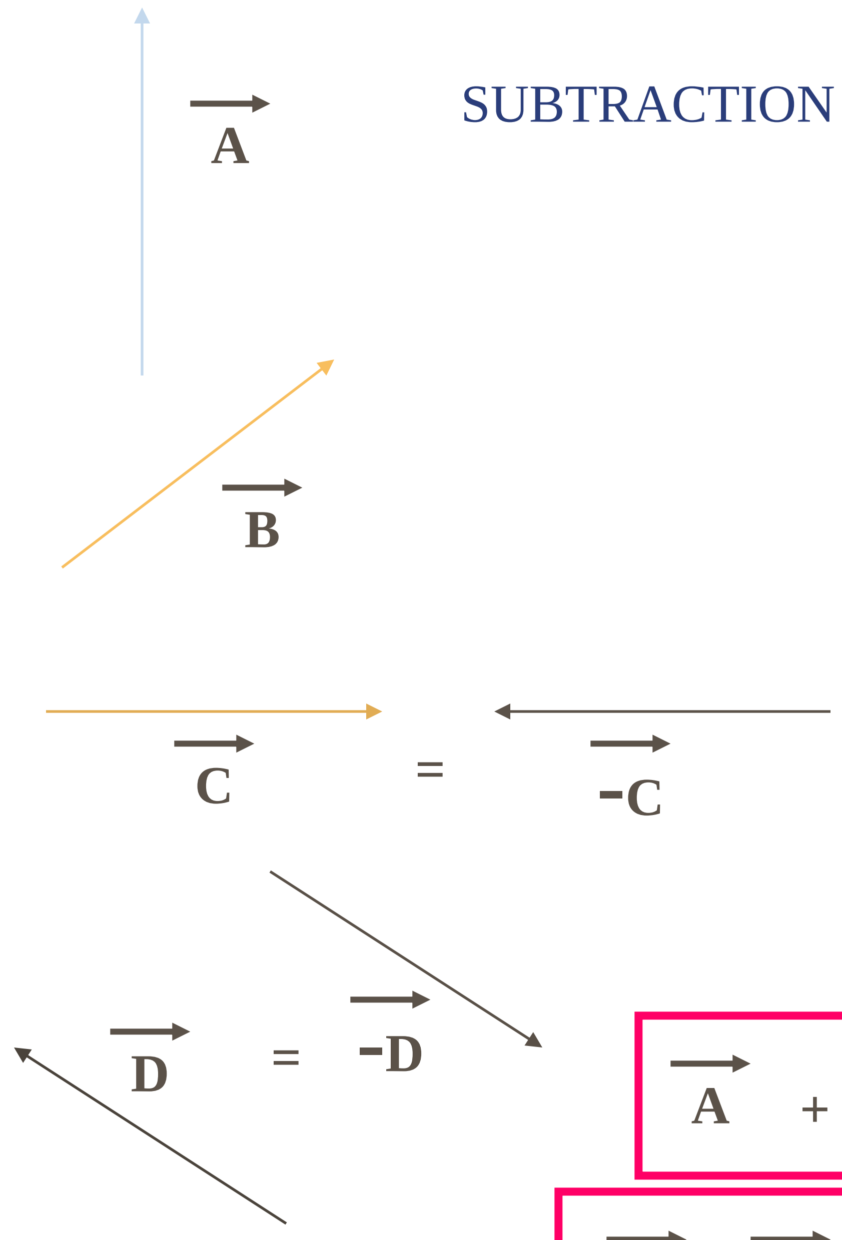


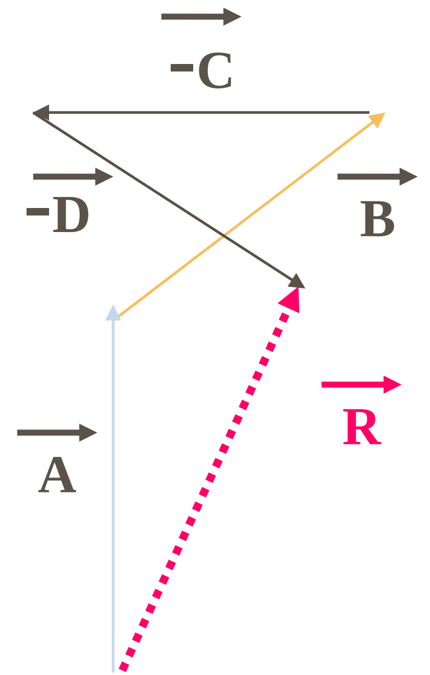
Vector A
30 METERS @ 45^{O}



Vector - A
30 METERS @ 225^{O}

SUBTRACTION OF VECTORS





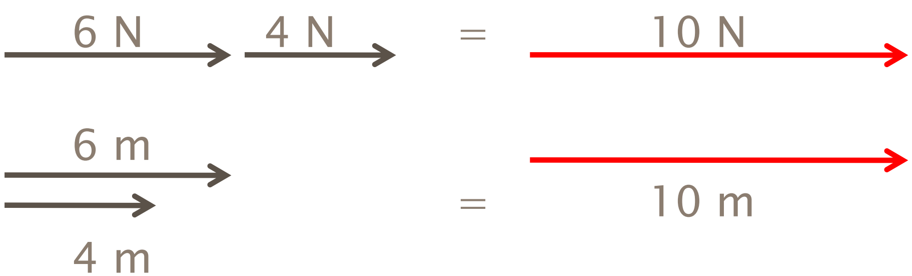
$$\boxed{\vec{A} + \vec{B} - \vec{C} - \vec{D} = \vec{R}}$$

$$\boxed{\vec{A} + \vec{B} + (-\vec{C}) + (-\vec{D}) = \vec{R}}$$

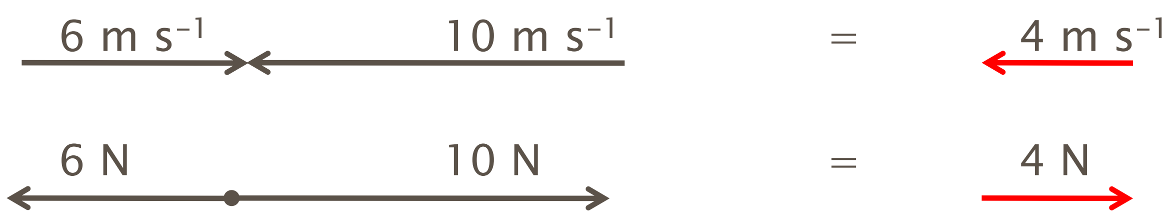
Resultant of Two Vectors

- The **resultant** is the sum or the **combined effect** of two vector quantities

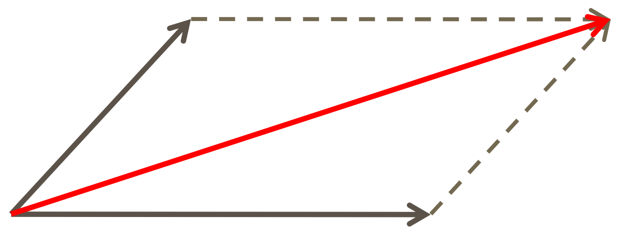
Vectors in the same direction:



Vectors in opposite directions:

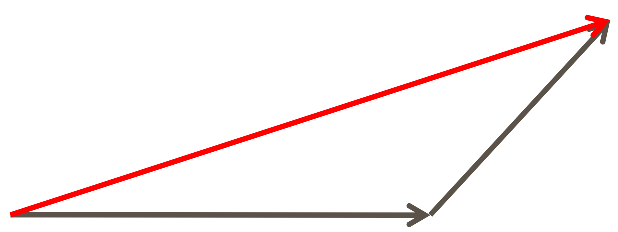


The Parallelogram Law



- When two vectors are joined tail to tail
- Complete the parallelogram
- The resultant is found by drawing the diagonal

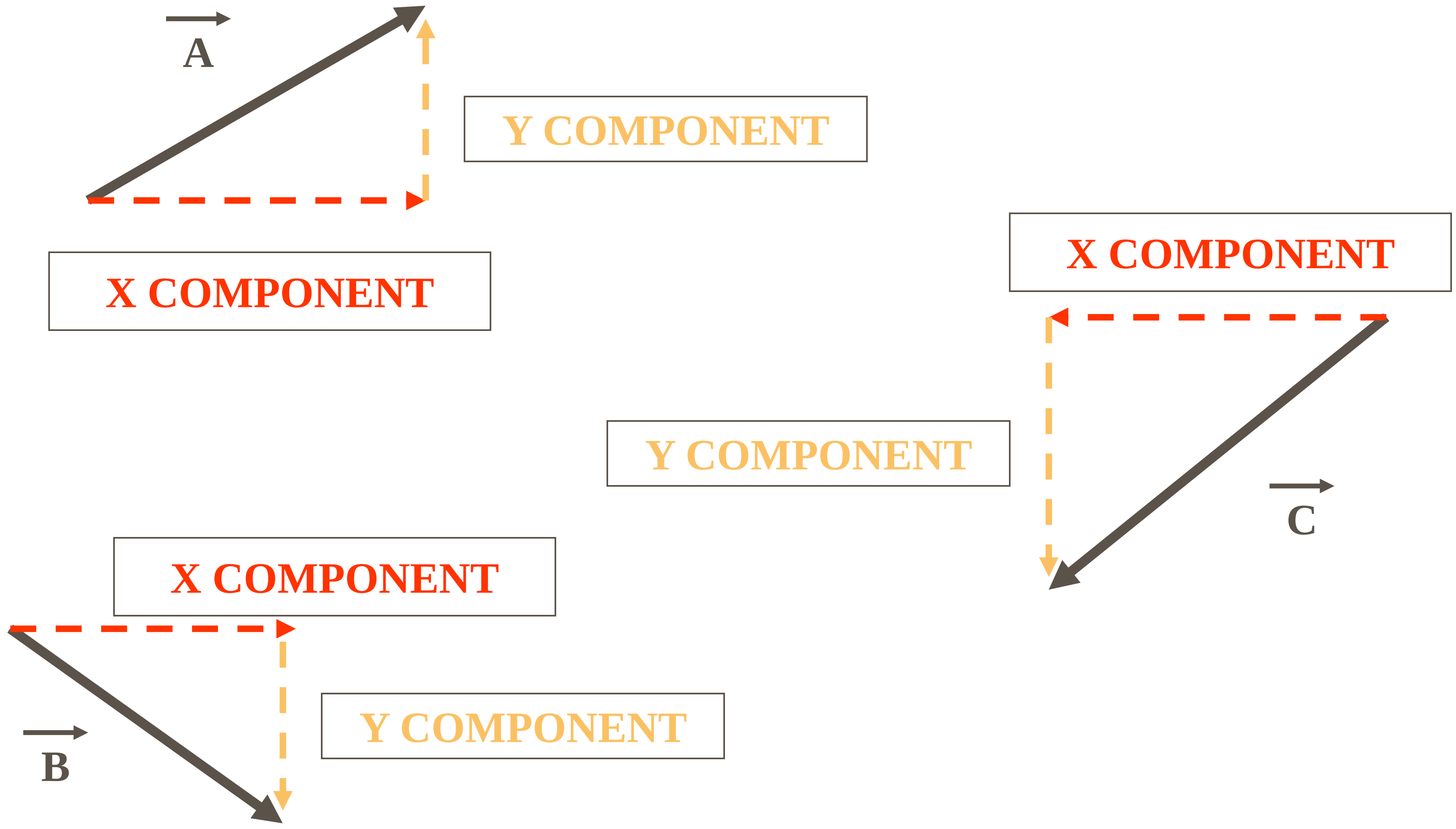
The Triangle Law



- When two vectors are joined head to tail
- Draw the resultant vector by completing the triangle

Component Method

VECTOR COMPONENTS



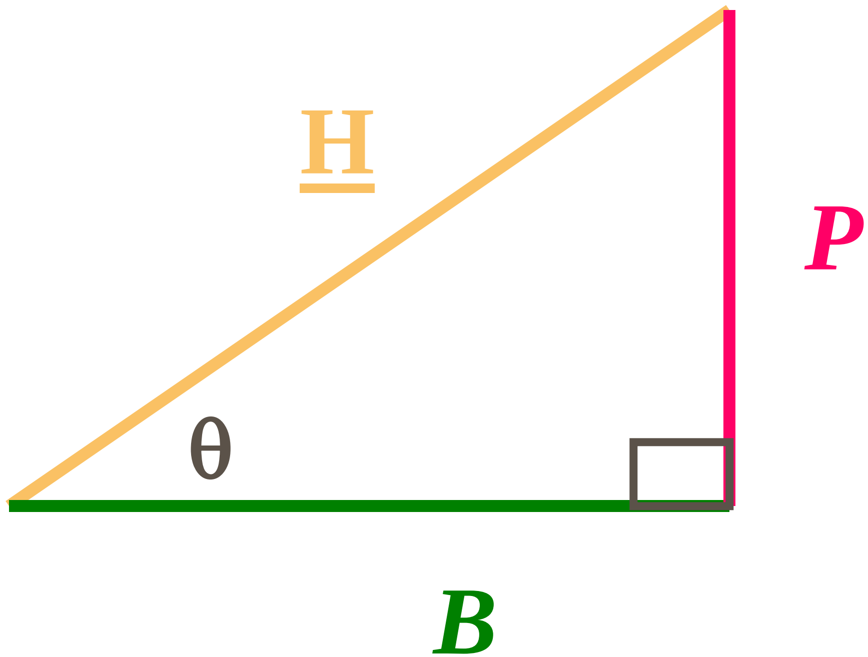
VECTOR COMPONENTS

- As we have seen two or more vectors can be added together to give a new vector. Therefore, any vector can considered to be the sum of two or more other vectors.
- When a vector is resolved (made) into components two component vectors are considered, one lying in the x axis plane and the other lying in the y axis plane. The component vectors are thus at right angles to each other.
- The x-y axis components are chosen so that right triangle trigonometry and the Pythagorean theorem can be used in their calculation.

VECTOR COMPONENTS

- Vector components can be found mathematically using sine and cosine functions. Recall sine of an angle for a right triangle is the side opposite the angle divided by the hypotenuse of the triangle and the cosine is the side adjacent to the angle divided by the hypotenuse.
- Using these facts, the x component of the vector is calculated by multiplying the cosine of the angle by the vector value and the y component is calculated by multiplying the sine of the angle by the vector value. Angular values are measured from 0 degrees (due east or positive x) on the Cartesian coordinate system.

Trigonometric Functions



A RIGHT TRIANGLE

$\sin\theta = P / H$

$\cos\theta = B / H$

$\tan\theta = P / B$

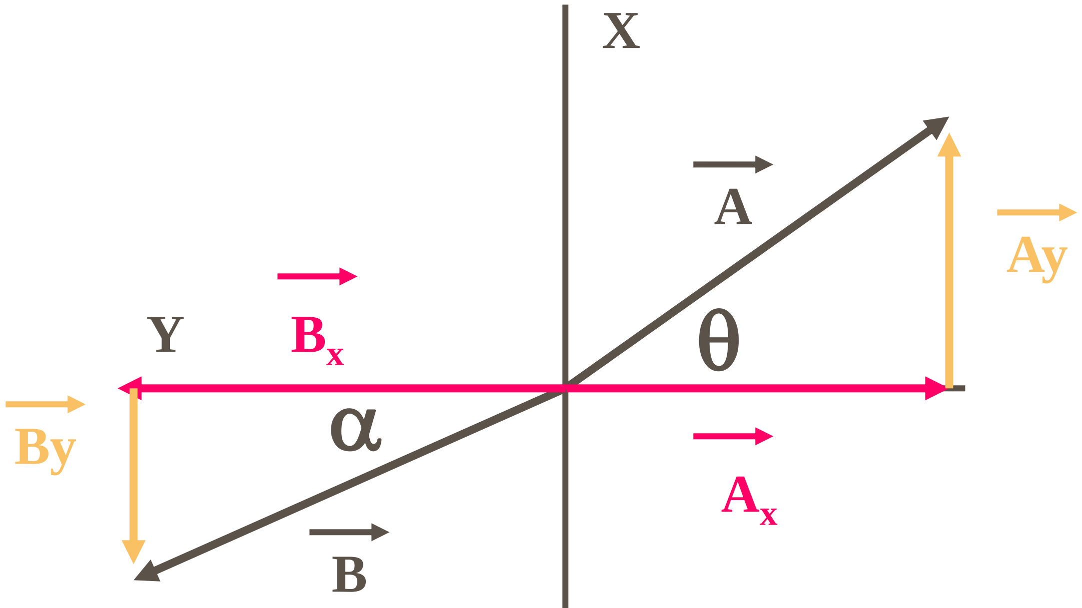
Components of Vectors

- A component of a vector is the projection of the vector on an axis
- The projection of a vector on an x axis is its x component, and similarly the projection on the y axis is the y component
- The process of finding the components of a vector is called resolving the vector

$$a_x = a \cos \theta \quad \text{and} \quad a_y = a \sin \theta,$$

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan \theta = \frac{a_y}{a_x}$$

VECTOR COMPONENTS



$$\vec{B}_x = \vec{B}\ \cos\ \alpha$$

$$\vec{B}_y = \vec{B}\ \sin\ \alpha$$

$$\vec{A}_x = \vec{A}\ \cos\ \theta$$

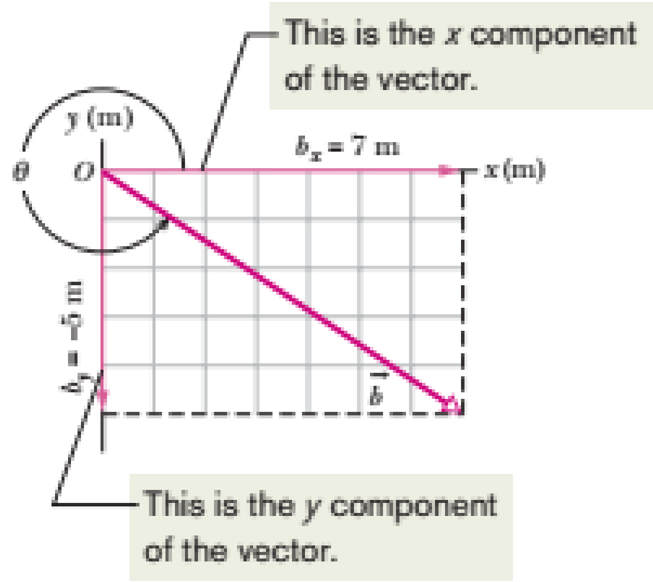
$$\vec{A}_y = \vec{A}\ \sin\ \theta$$

VECTOR COMPONENTS

- The signs of the x and y components depend on which quadrant the vector lies.
- Vectors in <u>quadrant i</u> (0 to 90 degrees) have positive x and positive y values
- Vectors in <u>quadrant ii</u> (90 to 180 degrees) have negative x values and positive y values.
- Vectors in <u>quadrant iii</u> (180 to 270 degrees) have negative x values and negative y values.
- Vectors in <u>quadrant iv</u> (270 to 360 degrees) have positive x values and negative y values.

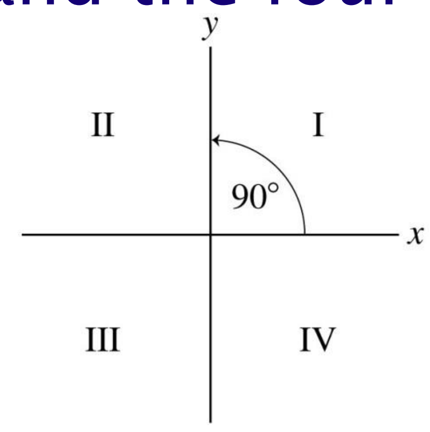
VECTOR COMPONENTS

If we know a vector in *component notation* (b_x and b_y) and want it in *magnitude-angle notation* (b and θ)



Coordinate Systems and Vector Components

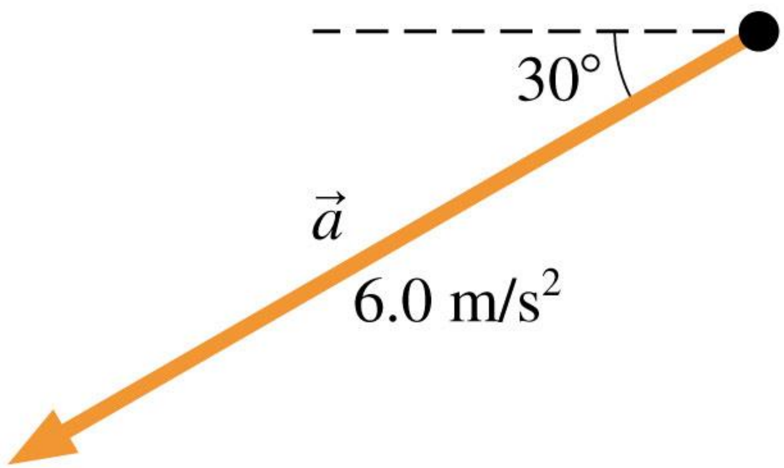
- A *coordinate system* is an artificially imposed grid that
you place on a problem.
- You are free to choose:
  - Where to place the origin, and
  - How to orient the axes.
- Below is a conventional xy–coordinate system and the four **quadrants** I through IV.



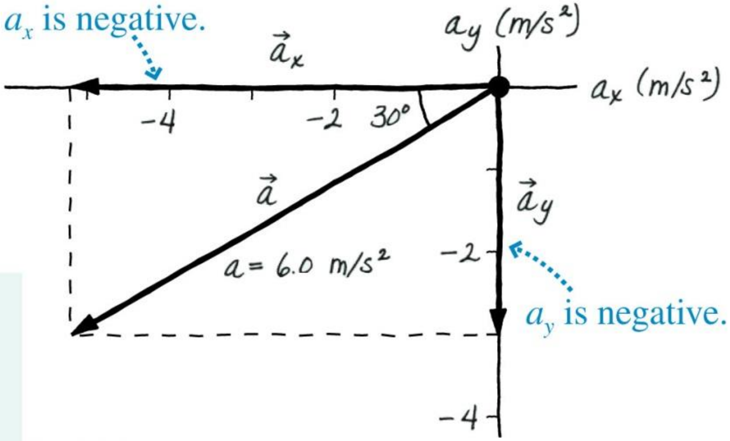
Example: Finding the Components of an Acceleration Vector

Find the x- and y-components of the acceleration vector a shown below.

→



Example: Finding the Components of an Acceleration Vector



EXAMPLE 3.3 Finding the components of an acceleration vector

VISUALIZE It's important to *draw* vectors. The figure on the right shows the original vector $\vec{a}$ decomposed into components parallel to the axes. Notice that the axes are "acceleration axes," not *xy*-axes, because we're measuring an acceleration vector.

SOLVE The acceleration vector $\vec{a} = (6.0\text{ m/s}^2, 30°$ below the negative x-axis) points to the left (negative x-direction) and down (negative y-direction), so the components a_x and a_y are both negative:

$$a_x = -a\cos 30° = -(6.0\text{ m/s}^2)\cos 30° = -5.2\text{ m/s}^2$$

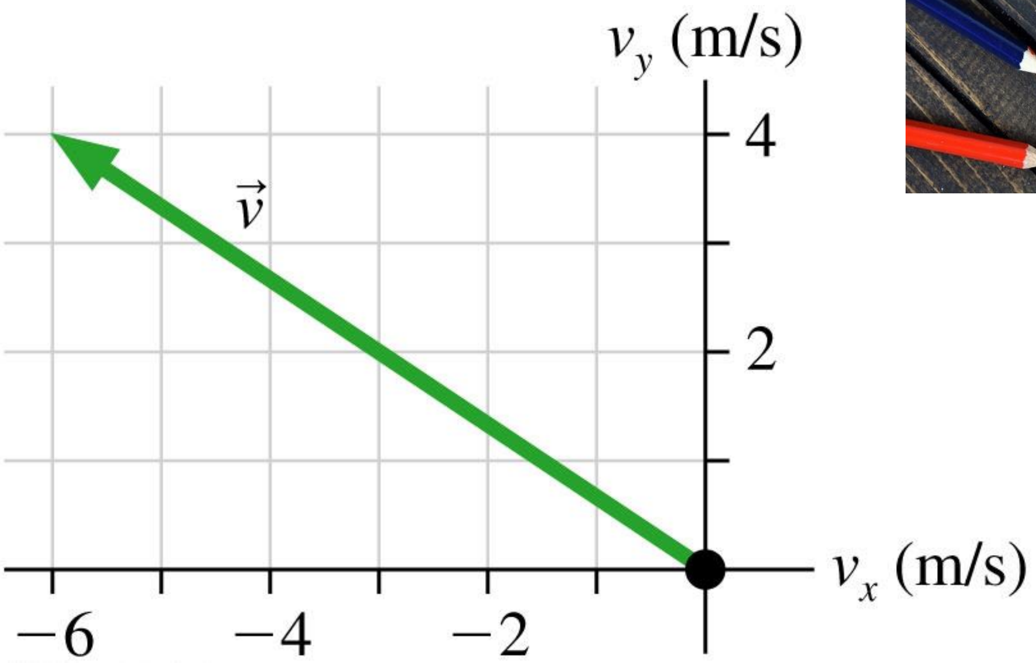
$$a_y = -a\sin 30° = -(6.0\text{ m/s}^2)\sin 30° = -3.0\text{ m/s}^2$$

ASSESS The units of a_x and a_y are the same as the units of vector $\vec{a}$. Notice that we had to insert the minus signs manually by observing that the vector points left and down.

Example 3.4 Finding the Direction of Motion

EXAMPLE 3.4 **Finding the direction of motion**

The figure below shows a car's velocity vector $\vec{v}$. Determine the car's speed and direction of motion.



Ans : Magnitude = 7.2 m/s , Direction= 146°

Homework(Sample Problem 3.2)

Sample Problem

Finding components, airplane flight

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. How far east and north is the airplane from the airport when sighted?

KEY IDEA

We are given the magnitude (215 km) and the angle (22° east of due north) of a vector and need to find the components of the vector.

Calculations: We draw an xy coordinate system with the positive direction of x due east and that of y due north (Fig. 3-10). For convenience, the origin is placed at the airport. The airplane's displacement $\vec{d}$ points from the origin to where the airplane is sighted.

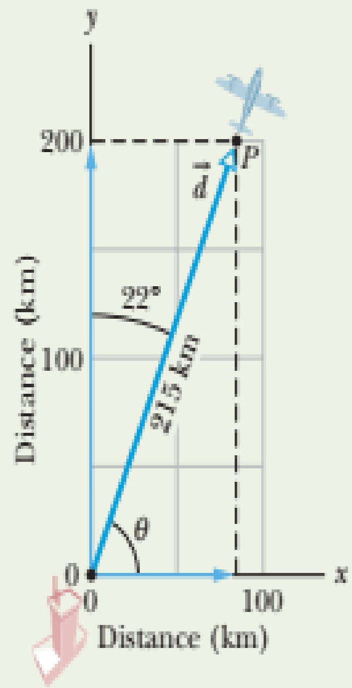


Fig. 3-10 A plane takes off from an airport at the origin and is later sighted at P.

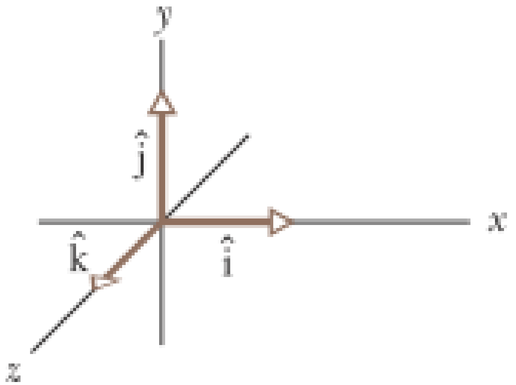
Unit Vectors

Unit-Vector Notation *Unit vectors* $\hat{\mathrm{i}}$, $\hat{\mathrm{j}}$, and $\hat{\mathrm{k}}$ have magnitudes of unity and are directed in the positive directions of the x, y, and z axes, respectively, in a right-handed coordinate system. We can write a vector $\vec{a}$ in terms of unit vectors as

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}, \tag{3-7}$$

in which $a_x\hat{\mathrm{i}}$, $a_y\hat{\mathrm{j}}$, and $a_z\hat{\mathrm{k}}$ are the **vector components** of $\vec{a}$ and a_x, a_y, and a_z are its **scalar components.**

The unit vectors point along axes.

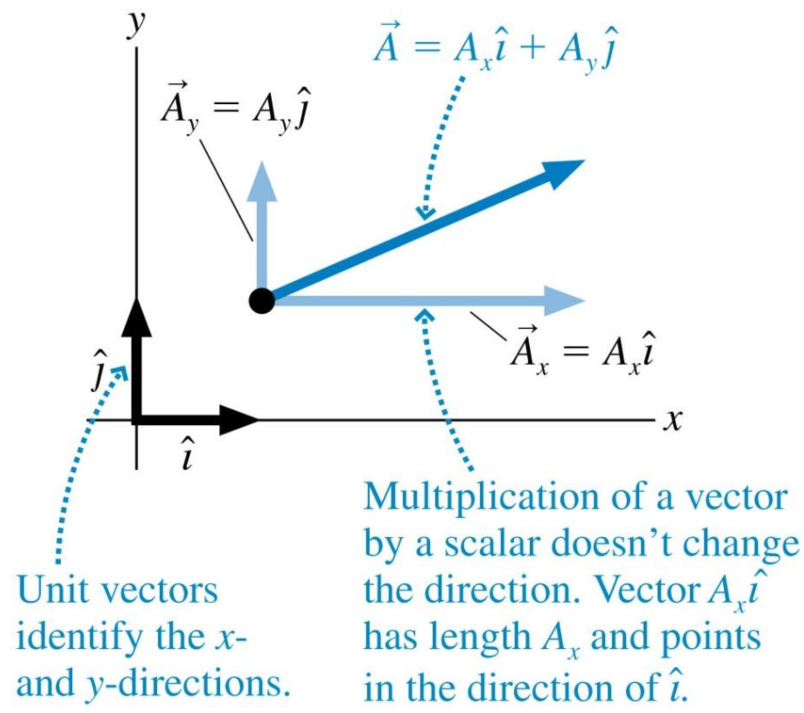


Vector Algebra

When decomposing a vector, unit vectors provide a useful way to write component vectors:

$$\vec{A}_x = A_x\,\hat{\imath}$$

$$\vec{A}_y = A_y\,\hat{\jmath}$$



The full decomposition of the vector $\vec{A}$ can then be written:

$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\,\hat{\imath} + A_y\,\hat{\jmath}$$

Sample Problem

Adding vectors, unit-vector components

Figure 3-15*a* shows the following three vectors:

$$\vec{a} = (4.2\text{ m})\hat{\text{i}} - (1.5\text{ m})\hat{\text{j}},$$
$$\vec{b} = (-1.6\text{ m})\hat{\text{i}} + (2.9\text{ m})\hat{\text{j}},$$

and
$$\vec{c} = (-3.7\text{ m})\hat{\text{j}}.$$

What is their vector sum $\vec{r}$ which is also shown?

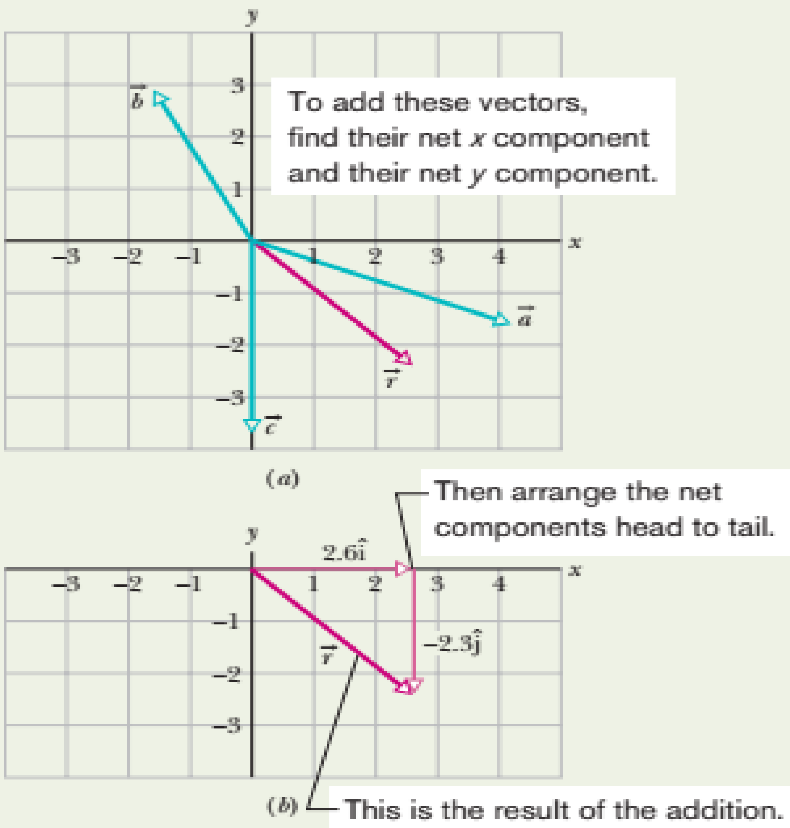


Fig. 3-15 Vector $\vec{r}$ is the vector sum of the other three vectors.

KEY IDEA

We can add the three vectors by components, axis by axis, and then combine the components to write the vector sum $\vec{r}$.

Calculations: For the x axis, we add the x components of $\vec{a}$, $\vec{b}$, and $\vec{c}$, to get the x component of the vector sum $\vec{r}$:

$$r_x = a_x + b_x + c_x$$
$$= 4.2\text{ m} - 1.6\text{ m} + 0 = 2.6\text{ m}.$$

Similarly, for the y axis,

$$r_y = a_y + b_y + c_y$$
$$= -1.5\text{ m} + 2.9\text{ m} - 3.7\text{ m} = -2.3\text{ m}.$$

We then combine these components of $\vec{r}$ to write the vector in unit-vector notation:

$$\vec{r} = (2.6\text{ m})\hat{\text{i}} - (2.3\text{ m})\hat{\text{j}}, \qquad \text{(Answer)}$$

where $(2.6\text{ m})\hat{\text{i}}$ is the vector component of $\vec{r}$ along the x axis and $-(2.3\text{ m})\hat{\text{j}}$ is that along the y axis. Figure 3-15*b* shows one way to arrange these vector components to form $\vec{r}$. (Can you sketch the other way?)

We can also answer the question by giving the magnitude and an angle for $\vec{r}$. From Eq. 3-6, the magnitude is

$$r = \sqrt{(2.6\text{ m})^2 + (-2.3\text{ m})^2} \approx 3.5\text{ m} \qquad \text{(Answer)}$$

and the angle (measured from the $+x$ direction) is

$$\theta = \tan^{-1}\left(\frac{-2.3\text{ m}}{2.6\text{ m}}\right) = -41°, \qquad \text{(Answer)}$$

where the minus sign means clockwise.

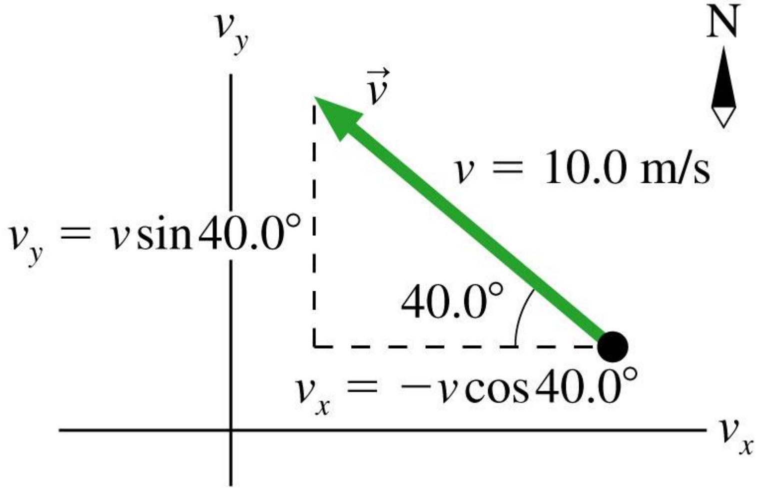
EXAMPLE 3.5 **Run rabbit run!**

A rabbit, escaping a fox, runs 40.0° north of west at 10.0 m/s. A coordinate system is established with the positive *x*-axis to the east and the positive *y*-axis to the north. Write the rabbit's velocity in terms of components and unit vectors.

ANS

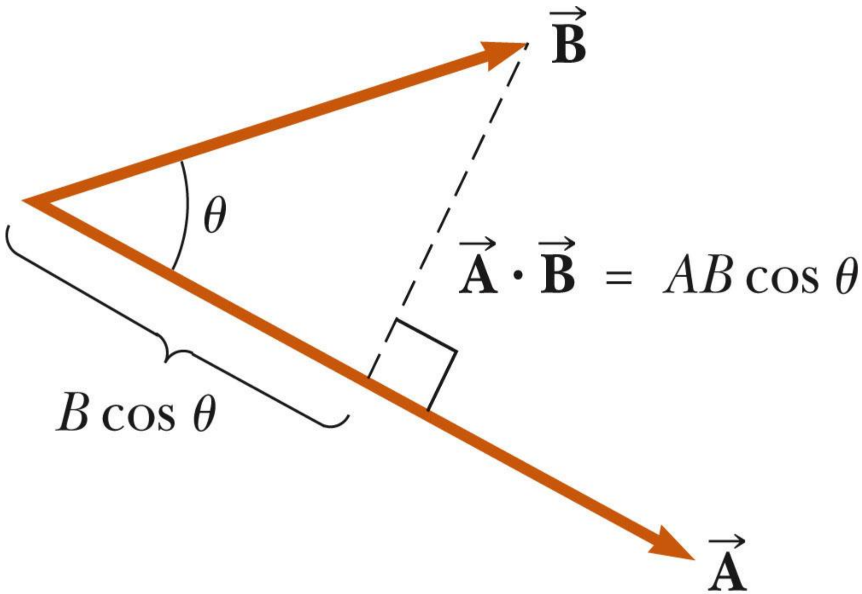
Vx= - 7.66 m/s, Vy=6.43 m/s

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (-7.66\hat{\imath} + 6.43\hat{\jmath})\text{ m/s}$$



Vector Multiplication (Vector . Vector) = Scalar

- The scalar product of two vectors is $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ written as
  - It is also called the dot product
- $$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv A\,B\cos\theta$$
  - θ is the angle *between A* and *B*
- Applied to work, this means



$$\hat{i}\cdot\hat{j}=0;\ \hat{i}\cdot\hat{k}=0;\ \hat{j}\cdot\hat{k}=0$$
$$\hat{i}\cdot\hat{i}=1;\ \hat{j}\cdot\hat{j}=1;\ \hat{k}\cdot\hat{k}=1$$

$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$

Multiplying Vectors(Scalar Product)

Sample Problem

Angle between two vectors using dot products

What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$? (*Caution:* Although many of the following steps can be bypassed with a vector-capable calculator, you will learn more about scalar products if, at least here, you use these steps.)

KEY IDEA

The angle between the directions of two vectors is included in the definition of their scalar product (Eq. 3-20):

$$\vec{a} \cdot \vec{b} = ab \cos \phi. \tag{3-24}$$

Calculations: In Eq. 3-24, a is the magnitude of $\vec{a}$, or

$$a = \sqrt{3.0^2 + (-4.0)^2} = 5.00, \tag{3-25}$$

and b is the magnitude of $\vec{b}$, or

$$b = \sqrt{(-2.0)^2 + 3.0^2} = 3.61. \tag{3-26}$$

We can separately evaluate the left side of Eq. 3-24 by writing the vectors in unit-vector notation and using the distributive law:

$$\begin{aligned}\vec{a} \cdot \vec{b} &= (3.0\hat{i} - 4.0\hat{j}) \cdot (-2.0\hat{i} + 3.0\hat{k}) \\ &= (3.0\hat{i}) \cdot (-2.0\hat{i}) + (3.0\hat{i}) \cdot (3.0\hat{k}) \\ &\quad + (-4.0\hat{j}) \cdot (-2.0\hat{i}) + (-4.0\hat{j}) \cdot (3.0\hat{k}).\end{aligned}$$

We next apply Eq. 3-20 to each term in this last expression. The angle between the unit vectors in the first term ($\hat{i}$ and $\hat{i}$) is 0°, and in the other terms it is 90°. We then have

$$\begin{aligned}\vec{a} \cdot \vec{b} &= -(6.0)(1) + (9.0)(0) + (8.0)(0) - (12)(0) \\ &= -6.0.\end{aligned}$$

Substituting this result and the results of Eqs. 3-25 and 3-26 into Eq. 3-24 yields

$$-6.0 = (5.00)(3.61) \cos \phi,$$

so $$\phi = \cos^{-1} \frac{-6.0}{(5.00)(3.61)} = 109° \approx 110°. \qquad \text{(Answer)}$$

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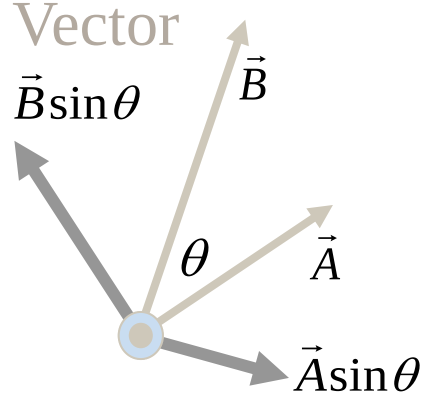
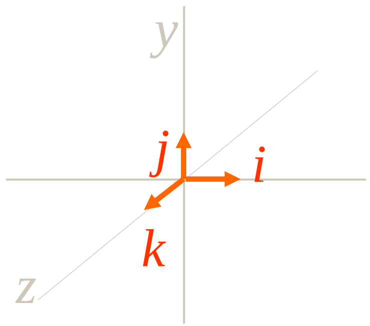
Cross Product (Vector x Vector) = Vector

$$\vec{C} = \vec{A} \times \vec{B}$$

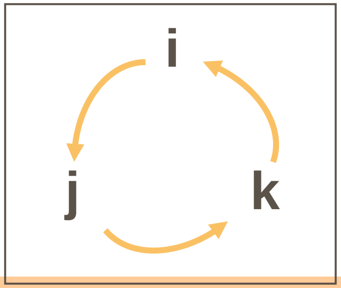


- The cross product of two vectors says something about how perpendicular they are.
- Magnitude:

$$\left|\vec{C}\right| = \left|\vec{A} \times \vec{B}\right| = AB\sin\theta$$



  - θ is smaller angle between the vectors
  - Cross product of any parallel vectors = zero
  - Cross product is maximum for perpendicular vectors
  - Cross products of Cartesian unit vectors:



$$\hat{i} \times \hat{j} = \hat{k};\ \hat{i} \times \hat{k} = -\hat{j};\ \hat{j} \times \hat{k} = \hat{i}$$

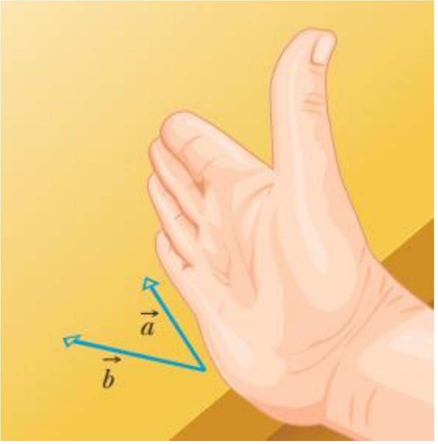
$$\hat{i} \times \hat{i} = 0;\ \hat{j} \times \hat{j} = 0;\ \hat{k} \times \hat{k} = 0$$

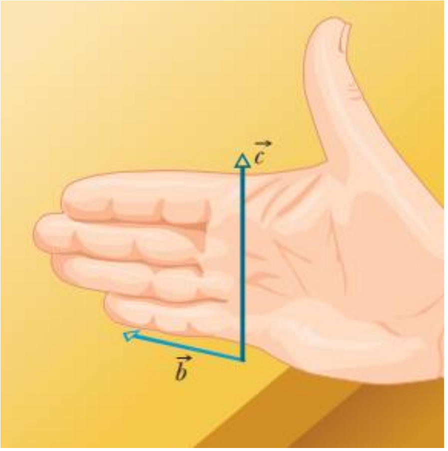
Vector Multiplication (Vector x Vector) = Vector

Cross Product

Rotational Information

The system must be in three dimensions or more.





Vector Product(Example)

Sample Problem

Cross product, unit-vector notation

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

KEY IDEA

When two vectors are in unit-vector notation, we can find their cross product by using the distributive law.

Calculations: Here we write

$$\begin{aligned}\vec{c} &= (3\hat{i} - 4\hat{j}) \times (-2\hat{i} + 3\hat{k}) \\ &= 3\hat{i} \times (-2\hat{i}) + 3\hat{i} \times 3\hat{k} + (-4\hat{j}) \times (-2\hat{i}) \\ &\quad + (-4\hat{j}) \times 3\hat{k}.\end{aligned}$$

We next evaluate each term with Eq. 3-27, finding the direction with the right-hand rule. For the first term here, the angle ϕ between the two vectors being crossed is 0. For the other terms, ϕ is 90°. We find

$$\begin{aligned}\vec{c} &= -6(0) + 9(-\hat{j}) + 8(-\hat{k}) - 12\hat{i} \\ &= -12\hat{i} - 9\hat{j} - 8\hat{k}. \qquad \text{(Answer)}\end{aligned}$$

This vector $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, a fact you can check by showing that $\vec{c} \cdot \vec{a} = 0$ and $\vec{c} \cdot \vec{b} = 0$; that is, there is no component of $\vec{c}$ along the direction of either $\vec{a}$ or $\vec{b}$.

Additional examples, video, and practice available at *WileyPLUS*

Thanks