AP ASSIGNMENT (VECTORS)

Q1) Find the angle between the vector $A = 5\hat{i} - 3\hat{j} + 7\hat{k}$ and the x, y and z axes, respectively.

Data:

$A = 5\hat{i} - 3\hat{j} + 7\hat{k}$

θ along x-axis $= \theta_1 = ?$

θ along y-axis $= \theta_2 = ?$

θ along z-axis $= \theta_3 = ?$

Solution:

~~UV~~

For θ_1:

unit vector along x-axis $= \hat{i}$

using,

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\vec{A}\cdot\hat{i} = Ai\cos\theta \longrightarrow (i)$$

$$|A| = \sqrt{(5)^2 + (-3)^2 + (7)^2}$$

$$\boxed{|A| = \sqrt{83}} \quad , \; i = 1$$

~~|A| =~~

$(i) \Rightarrow (5\hat{i} - 3\hat{j} + 7\hat{k})\cdot\hat{i} = \sqrt{83}\cos\theta.$

$5i^2 - 0 + 0 = \sqrt{83}\cos\theta,$

$5 = \sqrt{83}\cos\theta,$

$\hat{i}\cdot\hat{j} = 0$

$\hat{i}\cdot\hat{k} = 0$

$i^2 = 1$

$$\cos\theta_1 = 5/\sqrt{83}$$

$$\theta_1 = \cos^{-1}\left(\frac{5}{\sqrt{83}}\right)$$

$$\boxed{\theta_1 = 56.71^\circ}$$

For θ_2:

unit vector along y-axis $= \hat{j}$

$j = 1$

$$\vec{A}\cdot\hat{j} = Aj\cos\theta_2$$

$$(5\hat{i} - 3\hat{j} + 7\hat{k})\cdot\hat{j} = \sqrt{83}\cos\theta_2 \qquad \begin{bmatrix} \hat{i}\cdot\hat{j} = 0 \\ \hat{j}\cdot\hat{k} = 0 \\ j^2 = 1 \end{bmatrix}$$

$$0 - 3\hat{j}^2 + 0 = \sqrt{83}\cos\theta_2$$

$$-3 = \sqrt{83}\cos\theta_2$$

$$\cos\theta_2 = \frac{-3}{\sqrt{83}}$$

$$\theta_2 = \cos^{-1}\left(\frac{-3}{\sqrt{83}}\right)$$

$$\boxed{\theta_2 = 109.225^\circ}$$

For θ_3:

unit vector along z-axis $= \hat{k}$

$k = 1$

$$\vec{A}\cdot\hat{k} = Ak\cos\theta_3$$

$$(5\hat{i} - 3\hat{j} + 7\hat{k})\cdot\hat{k} = \sqrt{83}\cos\theta_3 \qquad \begin{bmatrix} \hat{i}\cdot\hat{k} = 0 \\ \hat{j}\cdot\hat{k} = 0 \\ k^2 = 1 \end{bmatrix}$$

$$0 - 0 + 7\hat{k}^2 = \sqrt{83}\cos\theta_3$$

$$\cos\theta_3 = 7/\sqrt{83}$$

$$\theta_3 = \cos^{-1}\left(\frac{7}{\sqrt{83}}\right)$$

$$\boxed{\theta_3 = 39.79^\circ}$$

Q2) In the product $\vec{F} = q\vec{v} \times \vec{B}$, take $q = 2$, $\vec{v} = 2\hat{i} + 4\hat{j} + 6\hat{k}$ and $\vec{F} = 4\hat{i} - 20\hat{j} + 12\hat{k}$. What is $\vec{B}$ in unit vector notation if $B_x = B_y$.

Data:

$q = 2$

$\vec{v} = 2\hat{i} + 4\hat{j} + 6\hat{k}$

$\vec{F} = 4\hat{i} - 20\hat{j} + 12\hat{k}$

$B_x = B_y$

$\vec{B} = ?$

Solution:

$\cancel{q\vec{v} = 2(2\hat{i} + 4\hat{j} + 6\hat{k})}$

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ B_x & B_y & B_z \end{vmatrix}$$

$B_x = B_y$

$$\vec{F} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ B_x & B_x & B_z \end{vmatrix}$$

$$\vec{F} = 2\left[\hat{i}(4B_z - 6B_x) - \hat{j}(2B_z - 6B_x) + \hat{k}(2B_x - 4B_x)\right]$$

$$\vec{F} = 2\left[\hat{i}(4B_z - 6B_x) - \hat{j}(2B_z - 6B_x) - 2B_x\hat{k}\right]$$

$$(4\hat{i} - 20\hat{j} + 12\hat{k}) = 2(4B_z - 6B_x)\hat{i} - 2(2B_z - 6B_x)\hat{j} - 4B_x\hat{k}$$

Equating $\hat{i}$, $\hat{j}$ and $\hat{k}$

$$4 = 2(4B_z - 6B_x) \longrightarrow (i)$$
$$-20 = -2(2B_z - 6B_x) \longrightarrow (ii)$$
$$12 = -4B_x \longrightarrow (iii)$$

(iii) $\Rightarrow$ $B_x = -\frac{12}{4}$

$$\boxed{B_x = -3}$$

putting it in (i)

$$\frac{4}{2} = \{4B_z - 6(-3)\}$$

$$2 - 18 = 4B_z$$

$$\boxed{B_z = -4}$$

Hence

$$\boxed{\vec{B} = -3\hat{i} - 3\hat{j} - 4\hat{k}}$$

(Q3) Two vectors ($\vec{A}$ and $\vec{B}$) of equal magnitude are acting at a point. Find the angle between the vectors, when magnitude of resultant is also equal to magnitude of either of these vectors.

Data:

$$|\vec{A}| = |\vec{B}|$$

and

$$|\vec{A}| = |\vec{B}| = |\vec{A} + \vec{B}|$$

$$\therefore |\vec{B}| = B \quad , \quad |\vec{A}| = A$$

$$B = \sqrt{A^2 + B^2 + 2AB\cos\theta} \qquad \because A = B$$

$$B = \sqrt{B^2 + B^2 + 2B^2\cos\theta}$$

squaring on both sides

$$B^2 = 2B^2 + 2B^2\cos\theta$$

$$B^2 = 2B^2(1 + \cos\theta)$$

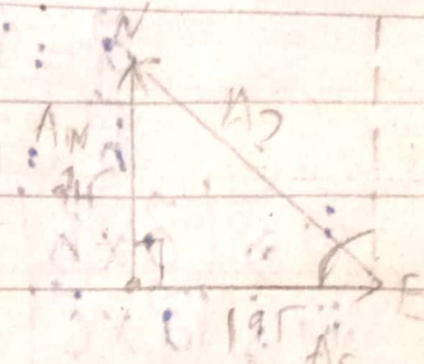
$$\frac{1}{2} = 1 + \cos\theta$$

$$\cos\theta = -1/2$$

$$\theta = \cos^{-1}(-1/2)$$

$$\boxed{\theta = 120^\circ}$$

Q4) A sailboat sets out to sail to a point 215 km due north. An unexpected ships blows the ship to a point 195 km due east of its starting point. (i) How far and (ii) in what direction must it know sail to reach its original destination?



<u>Data:</u> let

$A_N = 215\,km$

$A_E = 195\,km$

i) far, $A = ?$

ii) direction $= ?$

<u>Solution:</u> Using

$H^2 = P^2 + B^2$

$(A_N)^2 = (A)^2 + (A_E)^2$

$(215)^2 = A^2 + (195)^2$

$A = \sqrt{(215)^2 - (195)^2}$

$A =$

$H^2 = P^2 + B^2$

$(A)^2 = (215)^2 + (195)^2$

$A^2 = (215)^2 + (195)^2$

$A = \sqrt{(215)^2 + (195)^2}$

$\boxed{A = 290.25\,km}$

Direction:

Ship must sail in northwest direction to reach its original destination.

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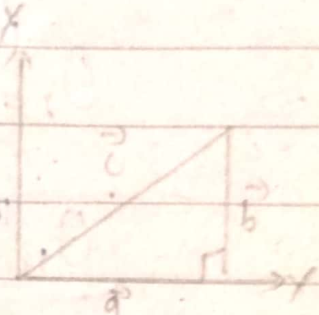
Angle:

$$\theta = \tan^{-1}\left(\frac{195}{215}\right)$$

$$\boxed{\theta = 42.20^\circ}$$

Q5) For vectors in Fig 1, with $a = 5$, $b = 6$, what are i) magnitude and direction of $\vec{a} \times \vec{b}$ ii) magnitude and direction of $\vec{a} \times \vec{c}$?



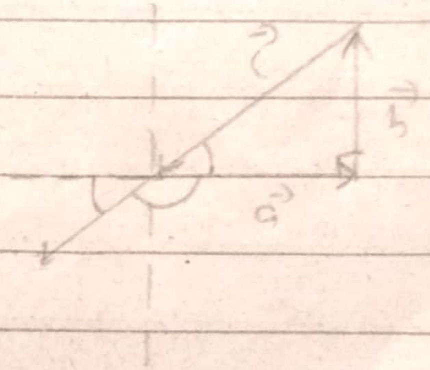
Data:

$\vec{a} = 5$

$\vec{b} = 6$

(i) $|\vec{a} \times \vec{b}| = ?$ $\theta_1 = ?$

(ii) $|\vec{a} \times \vec{c}| = ?$ $\theta_2 = ?$



i) For $|\vec{a} \times \vec{b}|$:

$\theta_1 = 90°$

$|\vec{a} \times \vec{b}| = ab\sin\theta$

$|\vec{a} \times \vec{b}| = (6)(5)\sin 90$

$|\vec{a} \times \vec{b}| = 30(1)$

$\boxed{|\vec{a} \times \vec{b}| = 30}$

For c:

$H^2 = P^2 + B^2$

$c^2 = (6)^2 + (5)^2$

$c^2 = 36 + 25$

$\boxed{c = 7.810}$

$$\theta_2 = 180 - \tan^{-1}\left(\frac{b}{a}\right)$$

$$\theta_2 = 180 - \tan^{-1}\left(\frac{6}{5}\right)$$

$$\theta_2 = 180 - 50.19$$

$$\boxed{\theta_2 = 129.81^\circ}$$

For $|\vec{a} \times \vec{c}|$:

$$|\vec{a} \times \vec{c}| = ac\sin\theta$$

$$|\vec{a} \times \vec{c}| = (5)(7.810)\sin(129.81)$$

$$\boxed{|\vec{a} \times \vec{c}| = 30}$$

Q6) Determine the value of 'a' so that $\vec{A} = 2\hat{i} + a\hat{j} + \hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular.

Data:

$\vec{A} = 2\hat{i} + a\hat{j} + \hat{k}$

$\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$

$a = ?$

Solution:

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\because \theta = 90^\circ$

$\vec{A}\cdot\vec{B} = AB\cos 90$

$\because \cos 90 = 0$

$$\boxed{\vec{A}\cdot\vec{B} = 0}$$

$\vec{A}\cdot\vec{B} = (2\hat{i} + a\hat{j} + \hat{k})\cdot(4\hat{i} - 2\hat{j} + 2\hat{k})$

$\vec{A}\cdot\vec{B} = 8\hat{i}^2 - 2a\hat{j}^2 - 2\hat{k}^2$ $\left[\begin{array}{l} \hat{i}^2 = 1 \\ \hat{j}^2 = 1 \\ \hat{k}^2 = 1 \end{array}\right]$

$0 = 8 - 2a - 2$

$0 = 6 - 2a$

$2a = 6$

$$\boxed{a = 3}$$

Q7) Two forces of equal magnitude are acting at a point. Find the angle between the vectors when magnitude of resultant is also equal to magnitude of either of these forces.

let ~~force~~ one force be $\vec{A}$ and other be $\vec{B}$

This ~~question~~ solution is similar to Q3 ~~and~~
~~[illegible]~~

Q8) If $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$.
Calculate (i) Area of parallelogram
(ii) Area of triangle.

Data:

$\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$

$\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$

Area of parallelogram $= |\vec{A} \times \vec{B}| = ?$

Area of triangle $= \dfrac{|\vec{A} \times \vec{B}|}{2} = ?$

Solution:

For $\vec{A} \times \vec{B}$:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i}(6+9) - \hat{j}(-2+12) + \hat{k}(6+24)$$

$$\boxed{\vec{A} \times \vec{B} = 15\hat{i} - 10\hat{j} + 30\hat{k}}$$

For $|\vec{A} \times \vec{B}|$ or (Area Of Parallelogram):

$$|\vec{A} \times \vec{B}| = \sqrt{(15)^2 + (10)^2 + (30)^2}$$

$$|\vec{A} \times \vec{B}| = \sqrt{1225}$$

$$\boxed{|\vec{A} \times \vec{B}| = \cancel{35 \text{ units}}\ 35 \text{ sq.units}}$$

Area of parallelogram is 35 sq.units

For Area Of Triangle:

$$\text{Area of triangle} = \frac{1}{2}|\vec{A} \times \vec{B}|$$

$$\text{Area of triangle} = \frac{35}{2}$$

$$\boxed{\text{Area of triangle} = 17.5 \text{ sq. units}}$$

Q9) Two vectors $\vec{A}$ and $\vec{B}$ have magnitudes $A = 3.00$ and $B = 3.00$. Their vector product is $A \times B = -5.00\hat{k} + 2.00\hat{i}$. What is angle b/w them

Data:

$A = 3$

$B = 3$

$A \times B = 2\hat{i} - 5\hat{k}$

$\theta = ?$

Solution:

For $|\vec{A} \times \vec{B}|$:

$$|\vec{A} \times \vec{B}| = \sqrt{(2)^2 + (-5)^2}$$

$$|\vec{A} \times \vec{B}| = \sqrt{4 + 25}$$

$$\boxed{|\vec{A} \times \vec{B}| = \sqrt{29}}$$

For θ. using,

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

$$\sqrt{29} = (3)(3)\sin\theta$$

$$\sin\theta = \frac{\sqrt{29}}{9}$$

$$\boxed{\theta = \sin^{-1}\left(\frac{\sqrt{29}}{9}\right)}$$

or

$$\boxed{\theta = 36.75^\circ}$$

Q10) If $\vec{u}$ is added to $\vec{w} = 3.0\hat{i} + 4.0\hat{j}$, the result is a vector ($\vec{u} + \vec{w} = \vec{v}$) in the positive direction of the y-axis with magnitude equal to that of $\vec{w}$. What is magnitude of $\vec{u}$?

Data:

$\vec{w} = 3\hat{i} + 4\hat{j}$

$\vec{u} = ?$

Solution: for $|\vec{v}|$:

∴ magnitude of $\vec{v}$ is equal to $\vec{w}$, so.

$|\vec{w}| = \sqrt{(3)^2 + (4)^2}$

$|\vec{w}| = \sqrt{25}$

$\boxed{|\vec{w}| = 5}$, $\boxed{|\vec{v}| = |\vec{w}| = 5}$

For $\vec{v}$:

since $\vec{v}$ is in direction of y-axis, so

$$\boxed{\vec{v} = 5\hat{j}}$$

For $\vec{u}$:

According to given condition

$$\vec{u} + \vec{w} = \vec{v}$$

$$\vec{u} + (3\hat{i} + 4\hat{j}) = 5\hat{j}$$

$$\vec{u} = -3\hat{i} + 5\hat{j} - 4\hat{j}$$

$$\cancel{\boxed{\vec{u} = -3\hat{i} + \hat{j}}} \qquad \boxed{\vec{u} = -3\hat{i} + \hat{j}}$$

for $|\vec{u}|$:

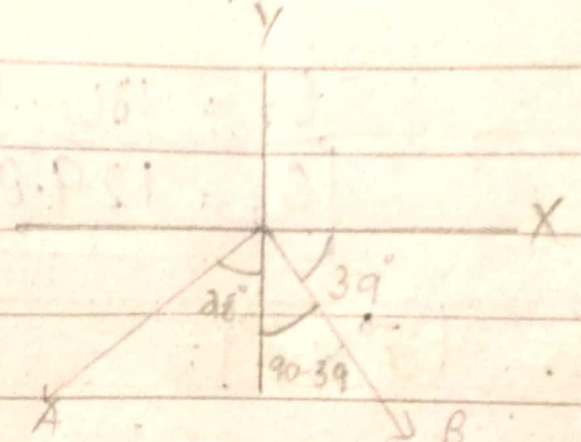
$$|\vec{u}| = \sqrt{(-3)^2 + (1)^2}$$

$$\boxed{|\vec{u}| = \sqrt{10}}$$

Question 11:

Q11) The scalar product of vectors $\vec{A}$ and $\vec{B}$ is +48.0m. Vector $\vec{A}$ has magnitude 9m and direction $-28°$ from $-y$-axis, if vector $\vec{B}$ has direction $-39°$ from $+x$-axis, what is magnitude of vector '$\vec{B}$'?

Data:



$\vec{A}\cdot\vec{B} = 48m$

$|\vec{A}| = 9m$

$|\vec{B}| = ?$

$\theta = (90-39)+28$

$\boxed{\theta = 79°}$

For B:

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$48 = 9(B)\cos 79°$

$\frac{48}{9} = B(0.1908)$

$B = \frac{5.333}{0.1908}$

$\boxed{B = 27.95}$

Q^{12}) Let $\hat{i}$ be directed to the east, $\hat{j}$ be directed to the north, and $\hat{k}$ be directed towards the uppear direction. what are the:

i) values of $\hat{i}\cdot\hat{k}$ and $\hat{j}\cdot(-\hat{j})$

ii) direction of $\hat{k}\times\hat{j}$ and $(-\hat{i})\times(-\hat{j})$?

Data:

$\hat{i}$ towards east

$\hat{j}$ towards north

$\hat{k}$ towards upward

i) $\Rightarrow \hat{i}\cdot\hat{k} = ?$

$\hat{j}\cdot(-\hat{j}) = ?$

ii) $\Rightarrow \hat{k}\times\hat{j}$ (direction) $= ?$

$(-\hat{i})\times(-\hat{j})$ (direction) $= ?$

Solution:

Part (i): $\hat{i}\cdot\hat{k} = 0$ [$\because$ vectors are perpendicu

$\hat{j}\cdot(-\hat{j}) = -1$ $\because [\hat{j}^2 = 1]$.

Part (ii): $\hat{k}\times\hat{j} = -\hat{i}$

$-\hat{i}$ is directed towards west because $\hat{i}$ is directed towards east.

$(-\hat{i})\times(-\hat{j}) = +\hat{k}$

It is directed towards upward direction.

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