Gauss’ Law

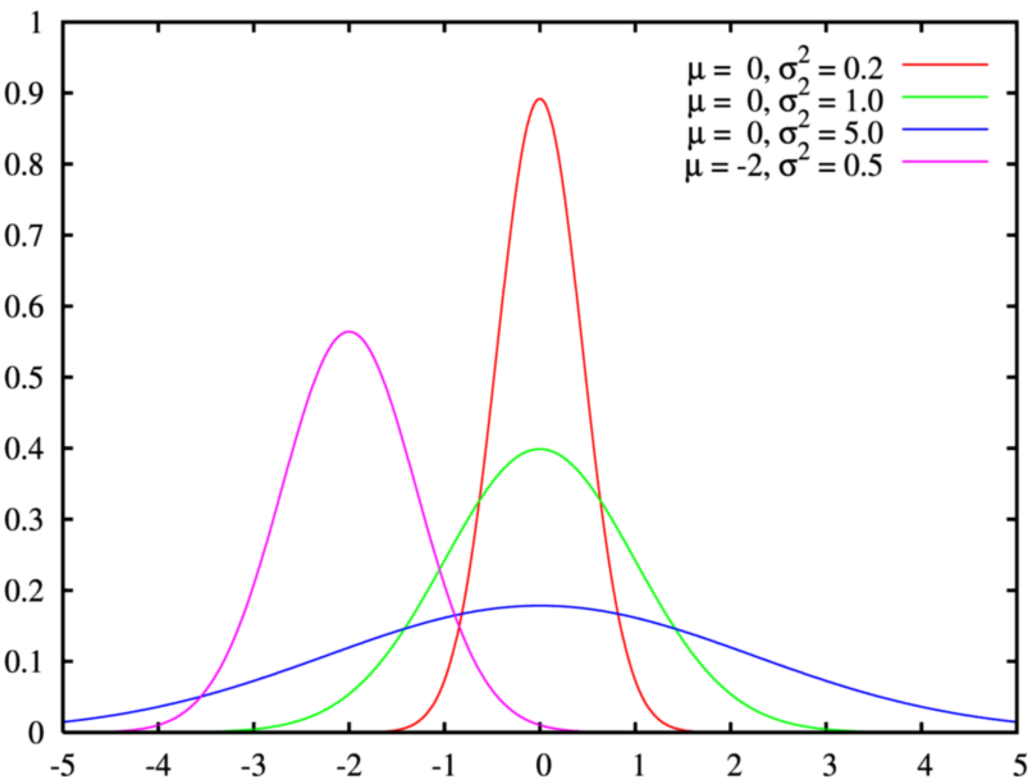
Chapter 23

Major Topics

- Flux
- Flux of an Electric Field
- Gauss’ Law
- Gauss’ Law and Coulomb’s Law
- A Charged Isolated Conductor
- Applying Gauss’ Law: Cylindrical Symmetry
- Applying Gauss’ Law: Planar Symmetry
- Applying Gauss’ Law: Spherical Symmetry

Karl Friedrich Gauss (1777 –1855)

- German mathematician (the Prince of Mathematicians)
- Contributed to number theory, astronomy, statistics, electrostatics, optics

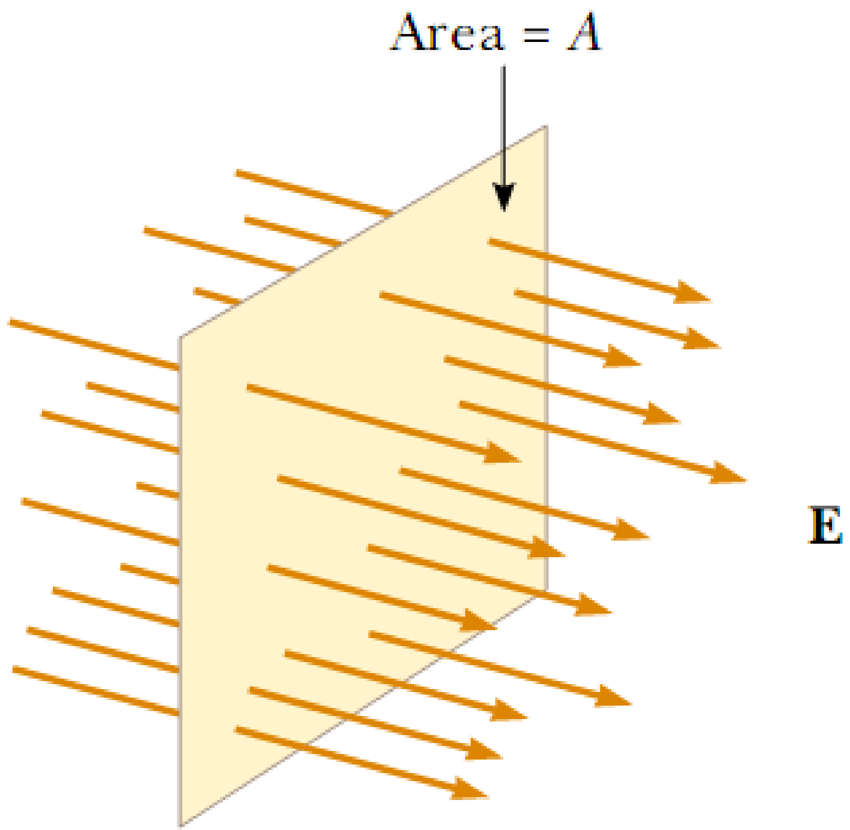


Gauss' Law Vs Coulomb's Law

- Gauss' law provides much simpler way to calculate electric fields in situations with a high degree of symmetry
- It can develop a system of equations for all electromagnetic phenomenon that illustrate more clearly the relationship between Electric and Magnetic fields
- Gauss' law is valid in case of fast moving charges, but Coulomb's law is valid only for the charges at rest or moving very slowly
- Gauss' Law is more general than Coulomb's Law.

Flux

- Flux is the measure of field lines intercepted by a body



Field Lines through a Plane Perpendicular to Field

The Flux of a Vector Field

- Flux is a Latin word means ; to flow
- It is the measure of the flow or penetration of the field vectors through an imaginary fixed elements of surface in the field
- "Flux" is the measure of the number of field lines passing through the loop
- Flux (ϕ) of the velocity field is defined as;
  $|\phi| = v. A$
  Where v is velocity field
  And A is the vector Area

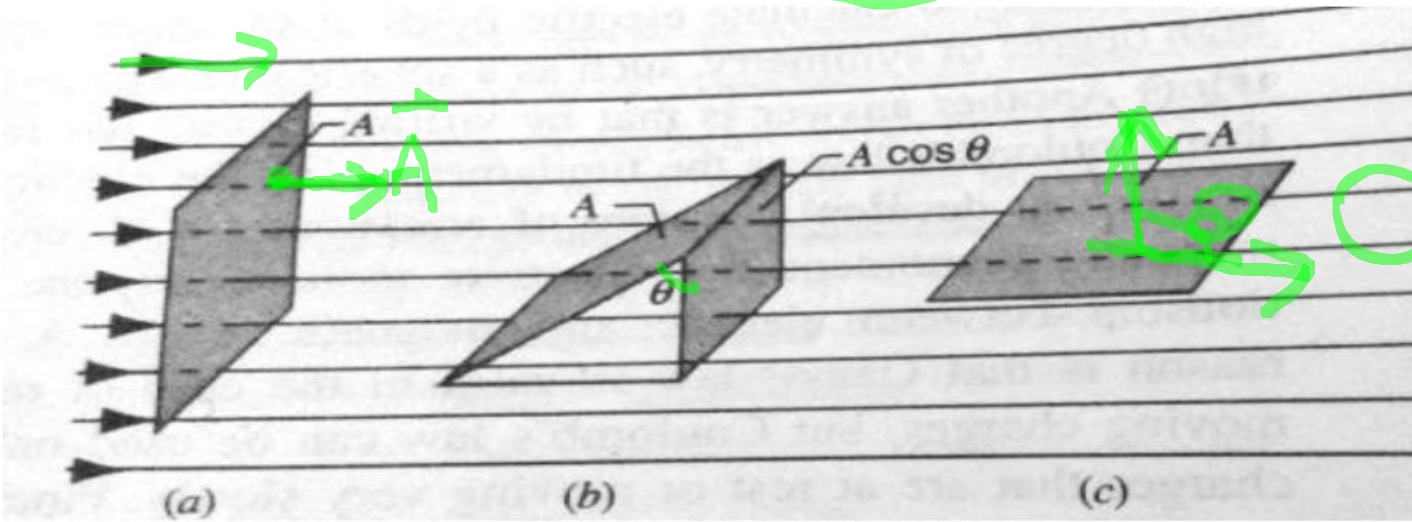
m/s · m²

m³/s

- The unit for "Flux" depends on the quantities being considered. Here its unit is m³/s

Continued

Let us consider an example of *velocity* field:



a. Plane is $\perp$ to the $\vec{v}$; $\vec{A}$ is $\parallel$ to the $\vec{v}$, and $\Theta = 0^o$

b. Plane is rotated at an angle Θ; $\vec{A}$ is $\cos\Theta$ with $\vec{v}$

c. Plane is $\parallel$ to the $\vec{v}$; $\vec{A}$ is $\perp$ to the $\vec{v}$; and $\Theta = 90^o$

$\Phi =$

$\Phi = V.A\cos\theta \Rightarrow \Phi =$

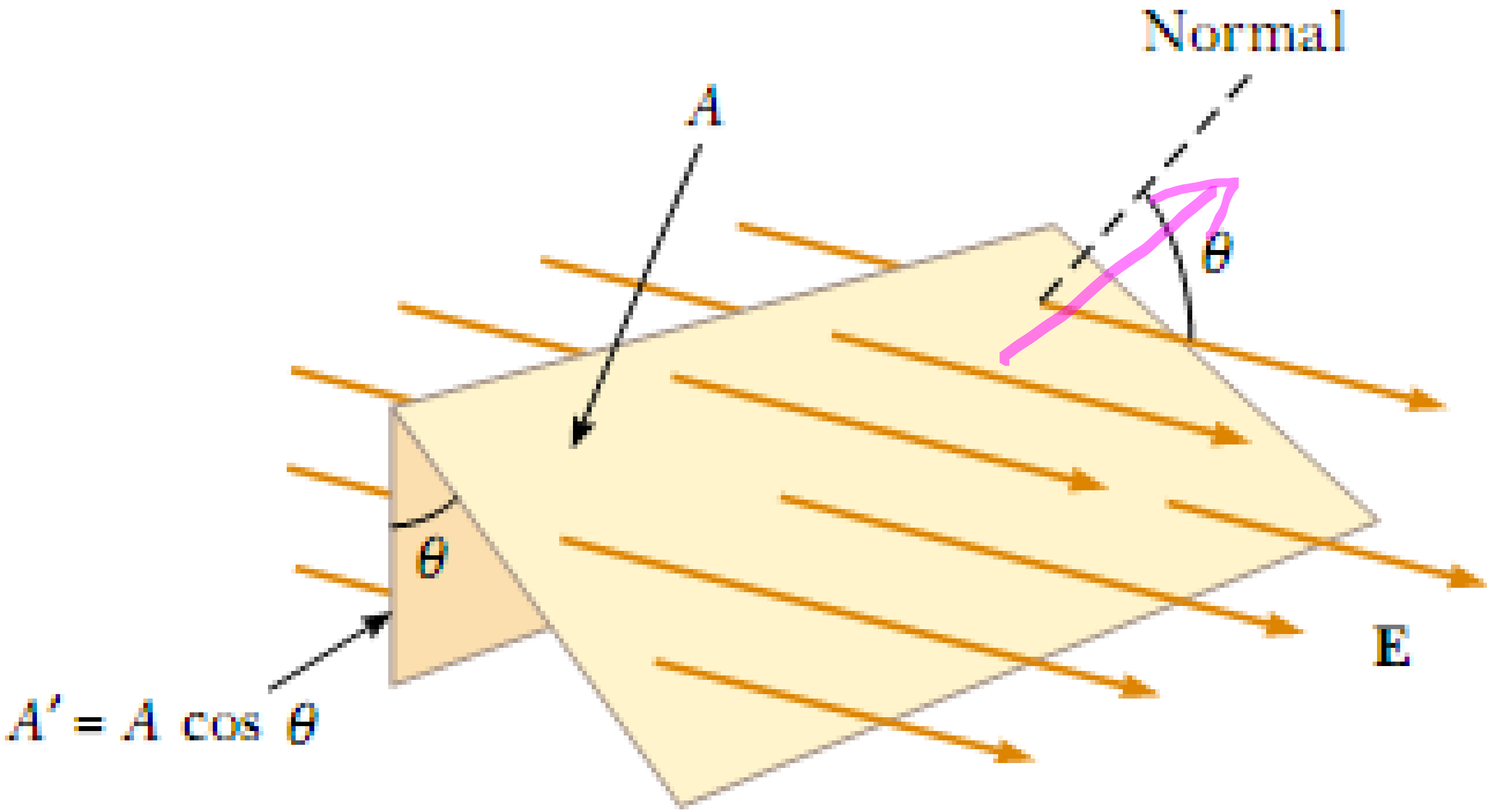
Continued

- Flux leaving the volume enclosed by the surface is taken +ive
- Flux entering the volume enclosed by the surface is considered -ive

$$\boxed{\phi = \sum \vec{v}.\vec{A}}$$

- Thus "Flux" is a scalar quantity as it is defined as the dot product of two vectors

Field Lines through an Inclined Plane



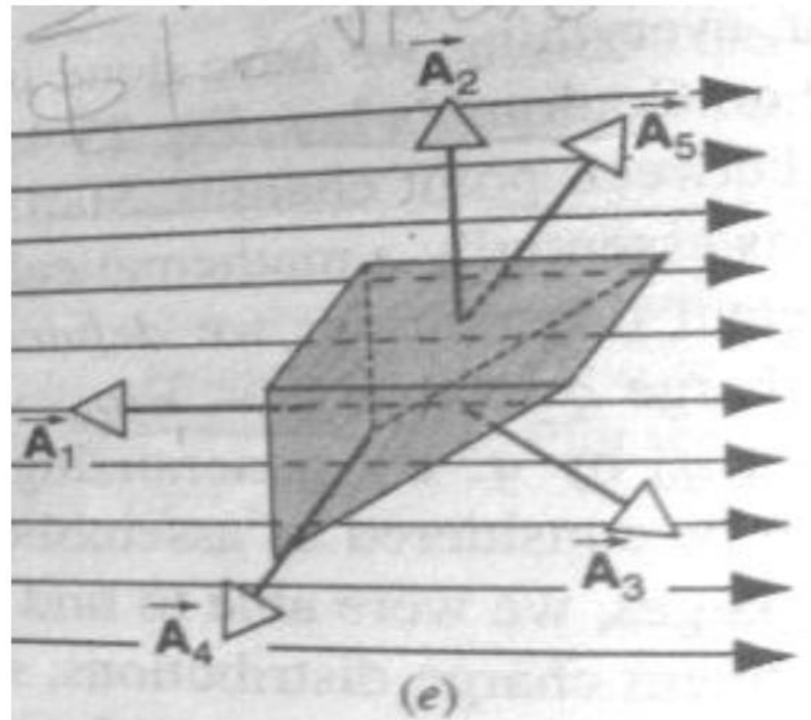
$$\Phi = (v \cos \theta)A$$

Do it ! (H.W)

Find the Total flux for the prism; a 5 faced object.

Using:

$$\phi = \sum \vec{v}.\vec{A}$$



Continued

For infinitesimal elements of area $d\vec{A}$;

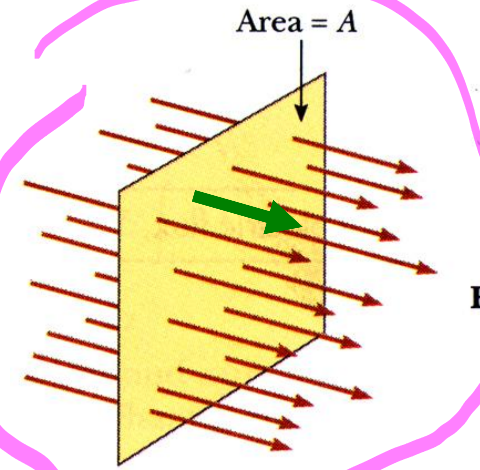
$$\phi = \int \vec{v} . \overrightarrow{dA}$$

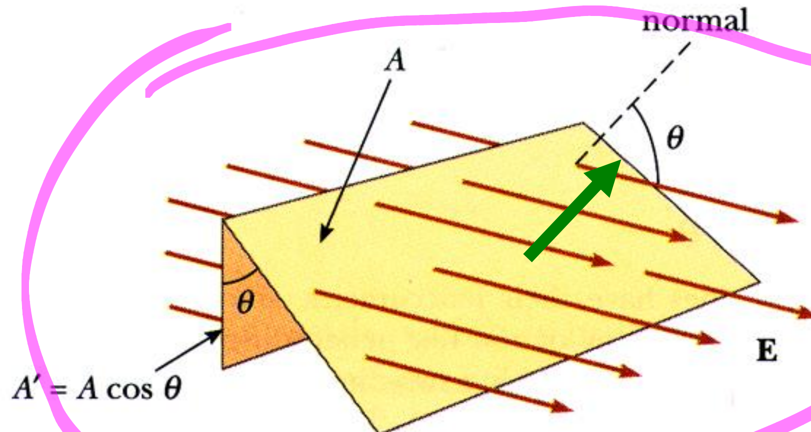
1. *Flux* is *zero*; if there is *no* sources *no* sinks
2. *Flux* is *positive* and equal; if there are *only* sources
3. *Flux* is *negative* and equal; if there are *only* sinks
4. *Flux* can be *positive*, can be *negative*, can be *zero*; if there exists *both* sources and sinks.

Definition of Electric Flux

- The amount of field, material or other physical entity passing through a surface.
- Surface area can be represented as vector defined normal to the surface it is describing
- Defined by the equation

$$\Phi_E = \int_{\text{surface}} \vec{E}\cdot d\vec{A}$$





The Flux of the Electric Field

- For Electric Flux;

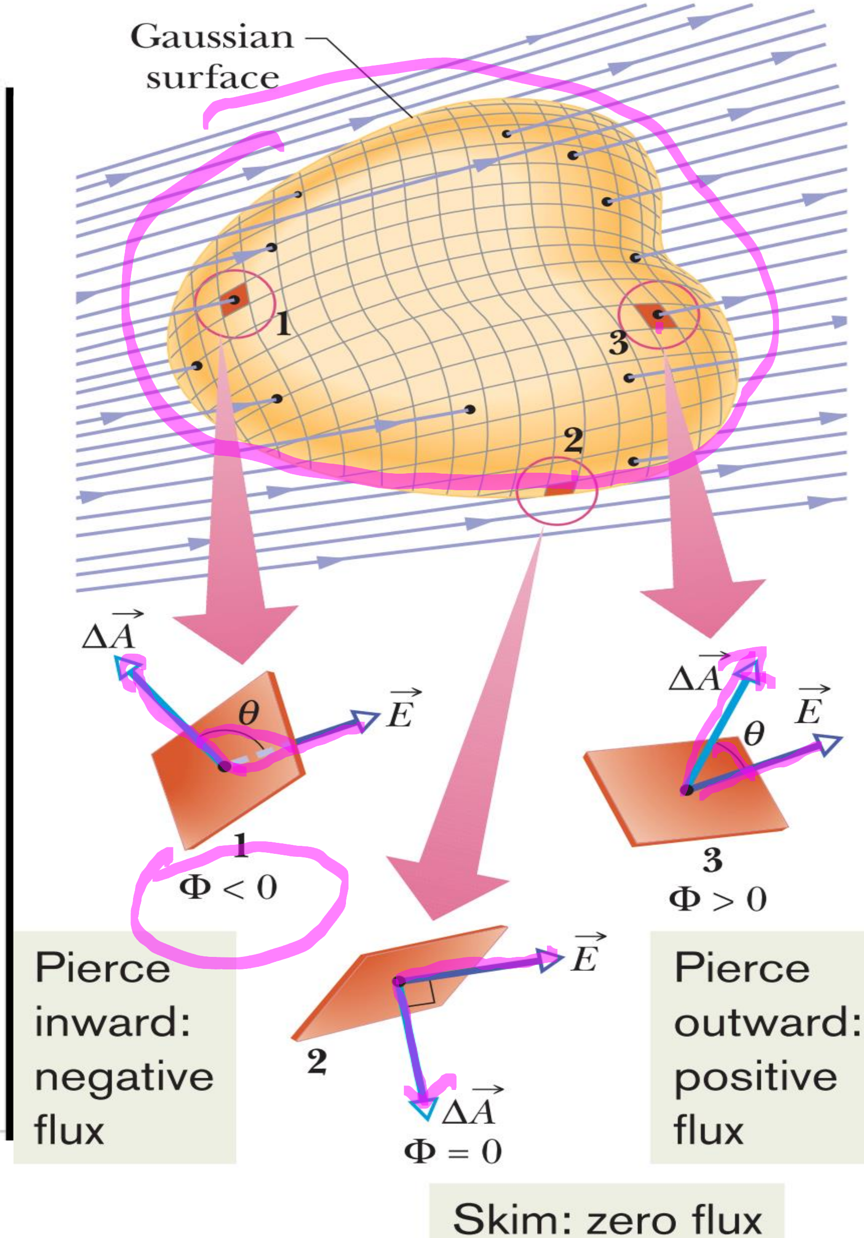
$$\phi_E = \sum \vec{E}.\vec{A}$$

- Electric Flux ϕ_E is scalar and its unit is Nm^2/C

For an irregular shaped object; like;

$N/C \cdot m^2$

Nm^2/C



CASES:

(a) For $\Theta > 90°$, $\underline{\vec{E}}$ is everywhere *inward*. Each $\underline{\vec{E}\ .\vec{\Delta A}}$ is *Negative* and ϕ_E for the surface is *Negative*.

(b) For $\Theta = 90°$, $\vec{E}$ is everywhere *parallel*. Each $\underline{\vec{E}\ .\vec{\Delta A}}$ is *Zero* and ϕ_E for the surface is *Zero*.

(c) For $\Theta < 90°$, $\underline{\vec{E}}$ is everywhere *outward*. Each $\underline{\vec{E}\ .\vec{\Delta A}}$ is *Positive* and ϕ_E for the surface is *Positive*.

Do it !

1. Figure shows a hypothetical closed cylinder of radius R immersed in a uniform electric field, the cylinder axis being parallel to the field. What is Φ_E for this closed surface?

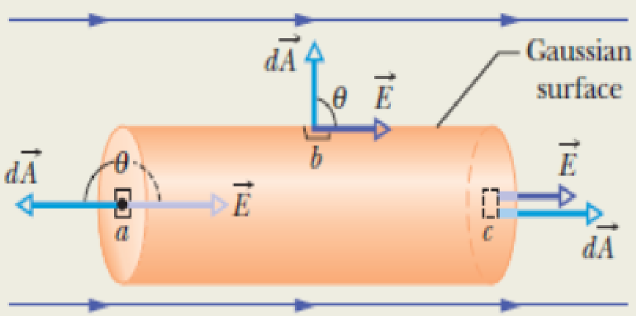
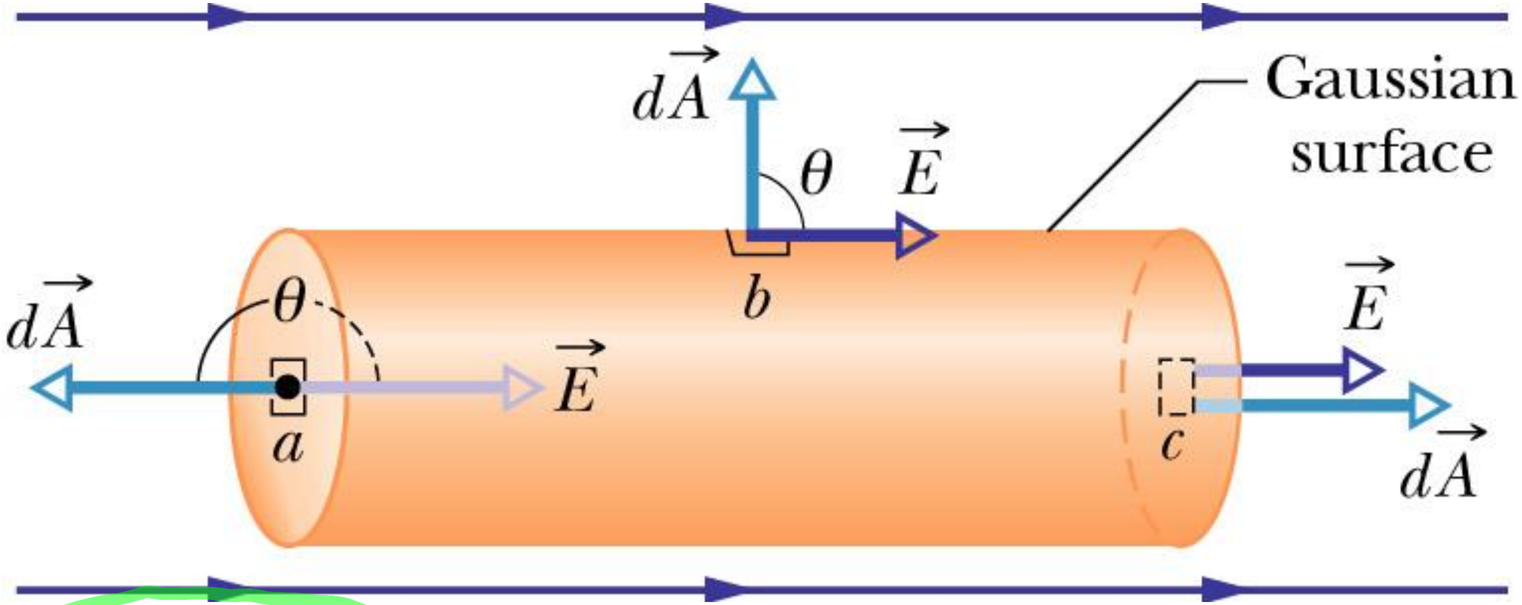


Figure 23-6 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

Answer ???

Find the electric flux through a cylindrical surface in a uniform electric field **E**



$$\Phi = \oint \vec{E} \bullet d\vec{A} = \oint E\cos\theta dA$$

$$d\vec{A} = \hat{n}dA$$

$(\because A = \pi r^2)$

a. $\Phi = \int E\cos 180 dA = -\int EdA = -E\pi R^2$

b. $\Phi = \int E\cos 90 dA = 0$

c. $\Phi = \int E\cos 0 dA = \int EdA = E\pi R^2$

A

Flux from a. + b. + c. = 0

What is the flux if the cylinder were vertical ?

Suppose it were any shape?

1. Figure shows a hypothetical closed cylinder of radius R immersed in a uniform electric field, the cylinder axis being parallel to the field. What is Φ_E for this closed surface?

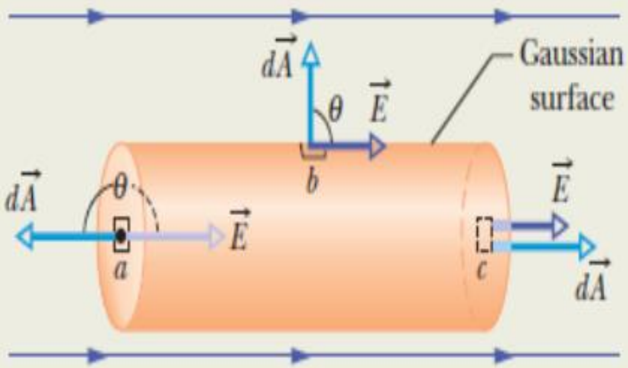
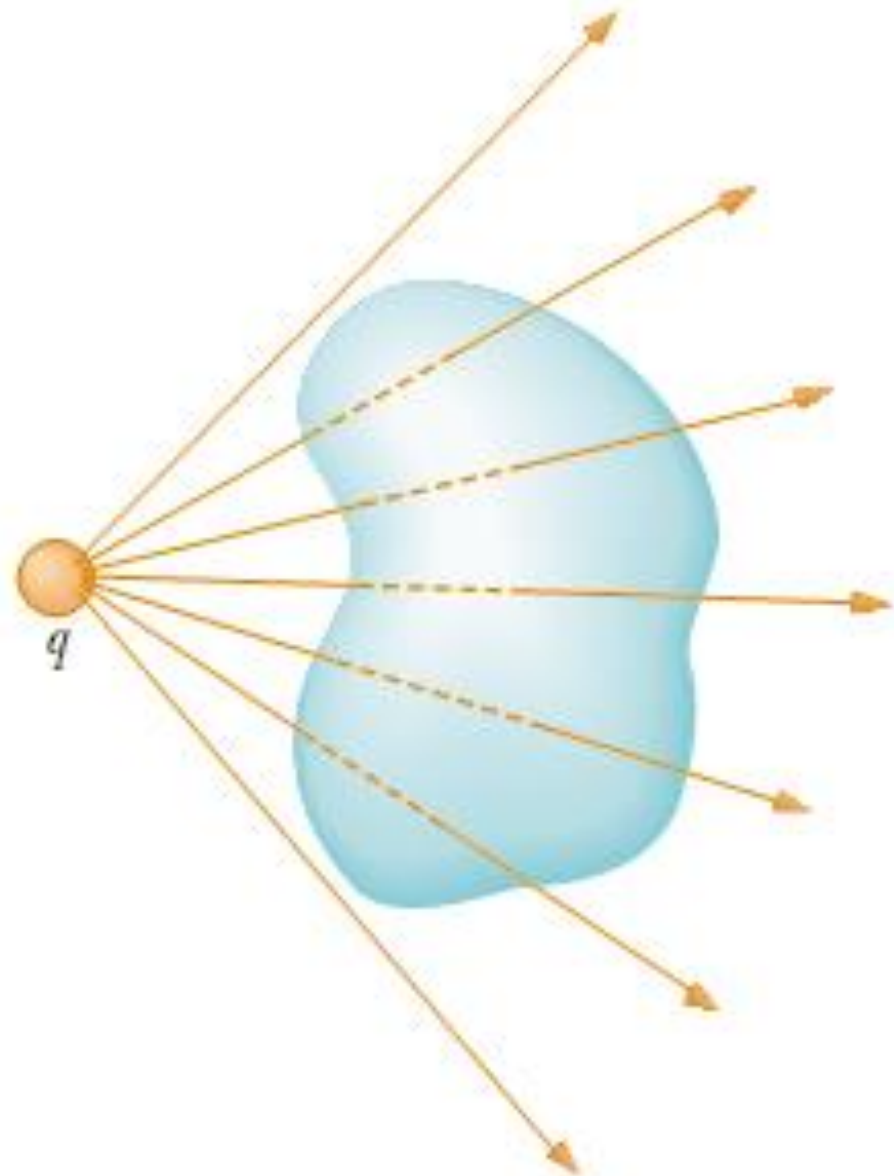


Figure 23-6 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

Answer: Φ_E = Zero

A Point Charge Located Outside a Closed Surface



Electric Flux for Arbitrary Closed Surface

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

- The loop on the integral sign indicates that the integration is to be taken over the entire (closed) surface
- The electric flux ϕ_E through a Gaussian surface is proportional to the net number of Electric field lines passing through that surface

Gauss' Law

- Gauss' law relates the net flux ϕ of an electric field through a closed surface (a Gaussian surface) to the net charge q_{enc} that is enclosed by that surface

$$\varepsilon_0 \Phi = q_{enc} \qquad \text{(Gauss' law)}$$

or

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \qquad \text{(Gauss' law)}$$

$$\phi = E \cdot A = \frac{q_{enc}}{\varepsilon_0}$$

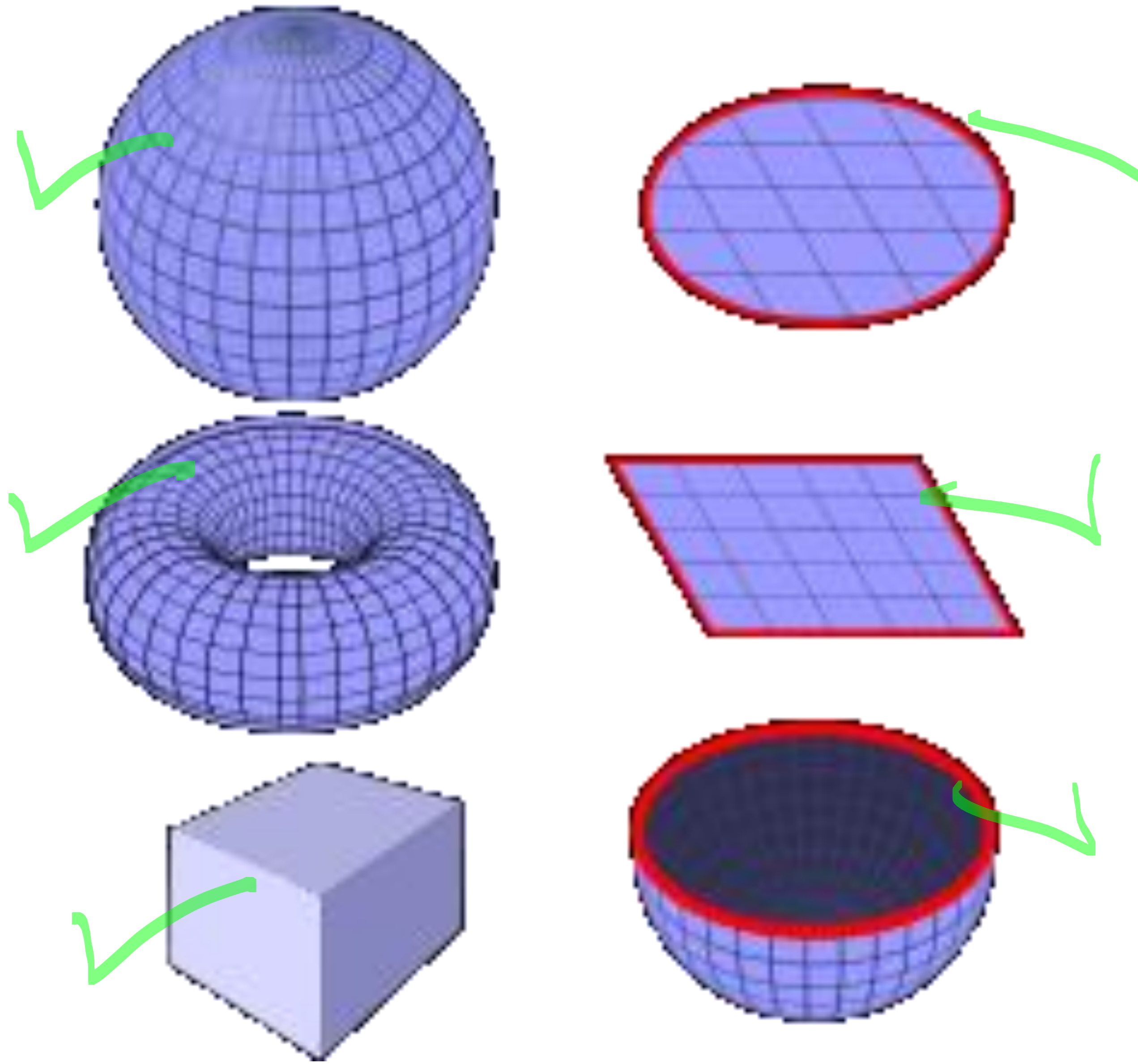
- Quite useful for certain charge distributions involving symmetry

Gauss’ Law & Gaussian Surface

- Instead of considering the fields of charge elements in a given charge distribution, Gauss’ law considers a hypothetical (imaginary) closed surface enclosing the charge distribution

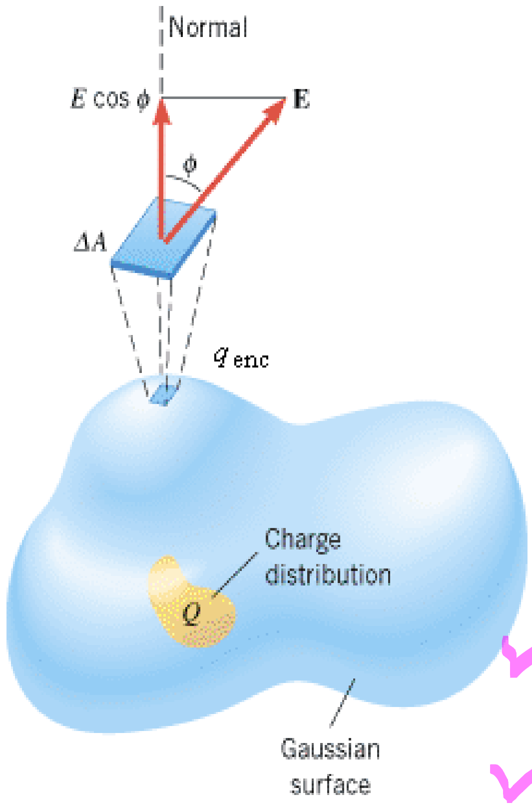
- This Gaussian surface, as it is called, can have any shape, but the shape that minimizes our calculations of the electric field is one that mimics the symmetry of the charge distribution

Gaussian vs Non-Gaussian Shapes



Gauss' Law

For charge distribution Q:



The electric flux through a Gaussian surface times by ε_0 (the permittivity of free space) is equal to the net charge Q enclosed :

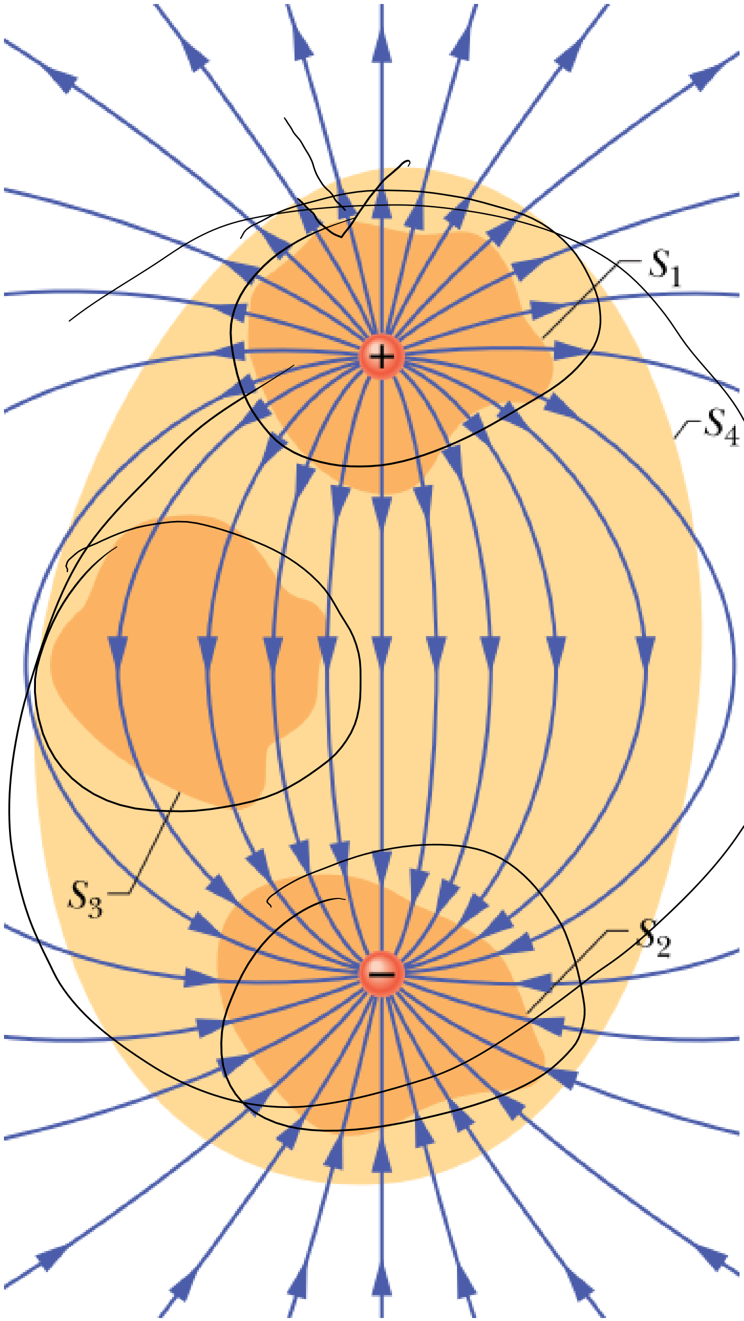
$$\varepsilon_0 \Phi = q_{enc} \qquad \text{(Gauss' law)},$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \qquad \text{(Gauss' law)}.$$

- The net charge q_{enc} is the algebraic sum of all the *enclosed* charges.
- Charge outside the surface, no matter how large or how close it may be, is not included in the term q_{enc}.

Determine the ϕE for each surface enclosed ??

Figure 23-8 Two charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field. Four Gaussian surfaces are shown in cross section. Surface S_1 encloses the positive charge. Surface S_2 encloses the negative charge. Surface S_3 encloses no charge. Surface S_4 encloses both charges and thus no net charge.



ϕ_E

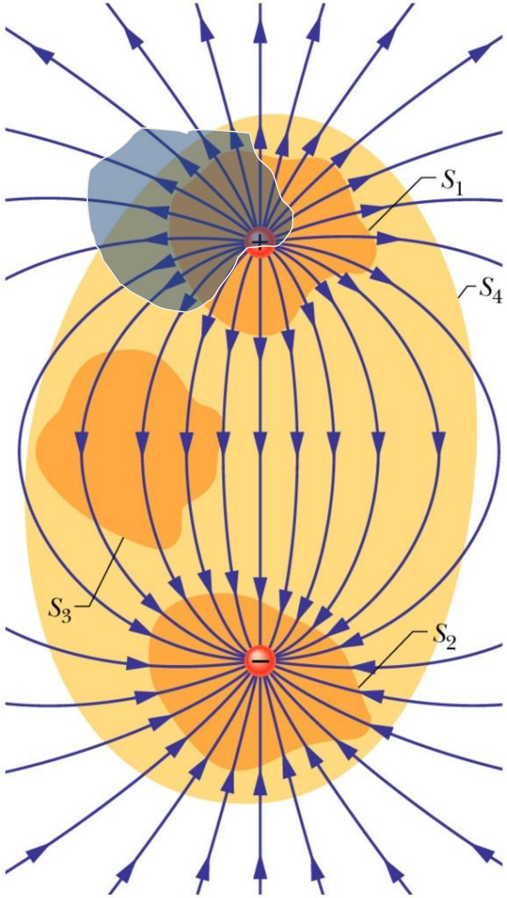
S_1

S_2

S_3

Example of Gauss' Law

- Consider a dipole with equal positive and negative charges.
- Imagine four surfaces S_1, S_2, S_3, S_4, as shown.
- S_1 encloses the positive charge. Note that the field is everywhere outward, so the flux is positive.
- S_2 encloses the negative charge. Note that the field is everywhere inward, so the flux through the surface is negative.
- S_3 encloses no charge. The flux through the surface is negative at the upper part, and positive at the lower part, but these cancel, and there is no net flux through the surface.
- S_4 encloses both charges. Again there is no net charge enclosed, so there is equal flux going out and coming in—no net flux through the surface.



DO- IT !!

5. A point charge of 1.84 μC is at the center of a cubical Gaussian surface 55 cm on edge. Fine Φ_E through the surface

ANSWER: ???

$$\Phi_E = \frac{q}{\varepsilon_0}$$

$$= \frac{1.84 \times 1}{8.85 \times}$$

DO- IT !!

5. A point charge of 1.84 μC is at the center of a cubical Gaussian surface 55 cm on edge. Fine Φ_E through the surface

ANSWER: $\Phi_E = 2.08 \times 10^5$ Nm²/C

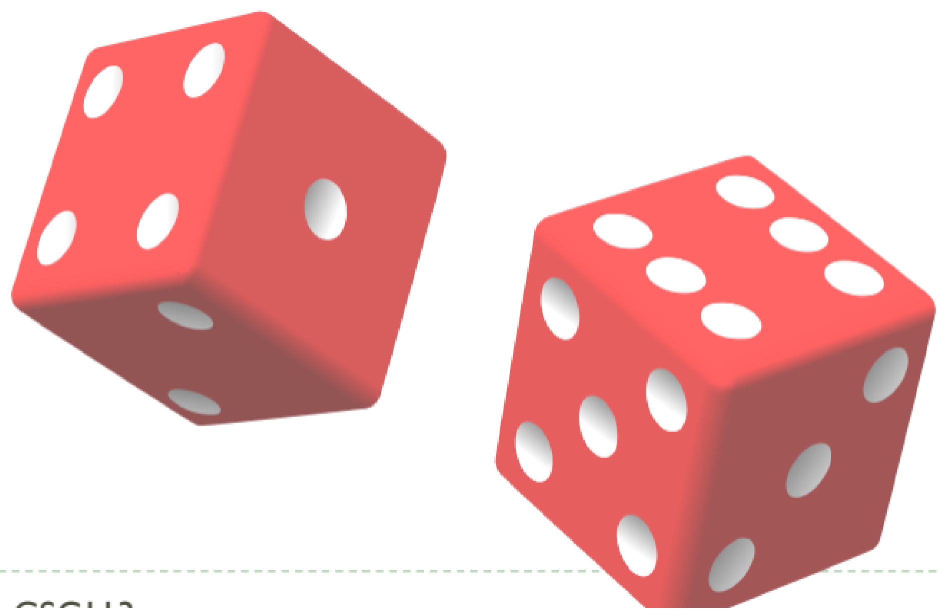
$\boxed{\Phi_E = 2.08 \times 10^5 \text{ N}}$

Please verify in

DO – IT !!

6. The net electric flux through each face of a die (one member of a pair of dice) has magnitude in units of 10^3 N.m^2/C equal to the number N of spots on the face (1 through 6). The flux is inward for N odd and outward for N even. What is the net charge inside the die?

- ANSWER: ???



DO – IT !! (Homework)

6. The net electric flux through each face of a die (one member of a pair of dice) has magnitude in units of 10^3 N.m^2/C equal to the number N of spots on the face (1 through 6). The flux is inward for N odd and outward for N even. What is the net charge inside the die?

ANSWER: q = 2.66 exp -8 C

There will be Solu

Gauss' Law and Coulomb's Law

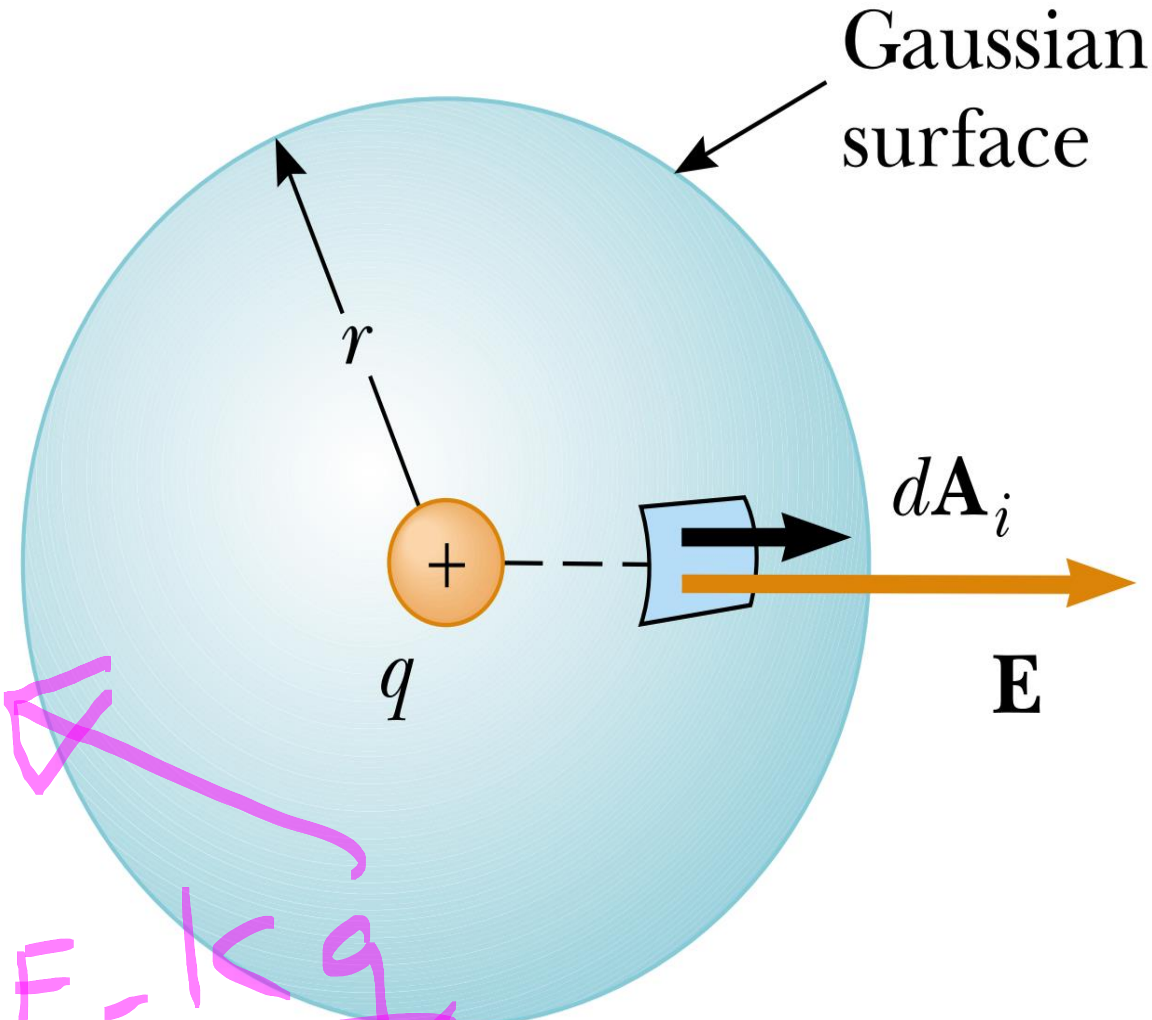
- Consider a point charge
- Enclose it in a Gaussian
- We have;

$$\boxed{\epsilon_o \oint \vec{E}.\overrightarrow{dA} = q}$$

- Ultimately we get:

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}$$

$E = k\frac{q}{r^2}$



Spherical Gaussian Surface for Point Charge

Electric lines of flux and Derivation of Gauss' Law using Coulombs law

- Consider a sphere drawn around a positive point charge. Evaluate the net flux through the closed surface.

Net Flux = $\Phi = \oint \vec{E} \bullet d\vec{A} = \oint E\cos\theta dA = \oint E dA$

E || n
Cos 0 = 1

sphere

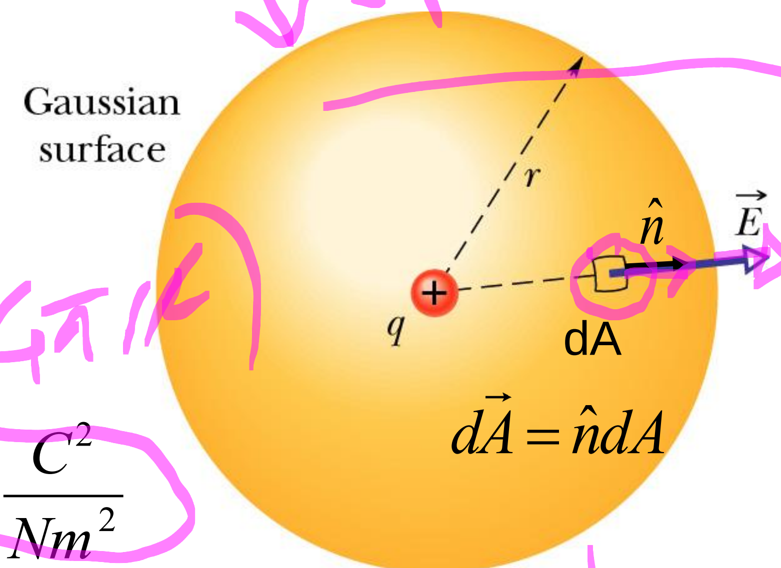
For a Point charge **E=kq/r²**

$$\Phi = \oint E dA = \oint kq/r^2 dA$$

$$\Phi = kq/r^2 \oint dA = kq/r^2 (4\pi r^2)$$

$= kq(4\pi r^2)$

$$\Phi = 4\pi kq$$

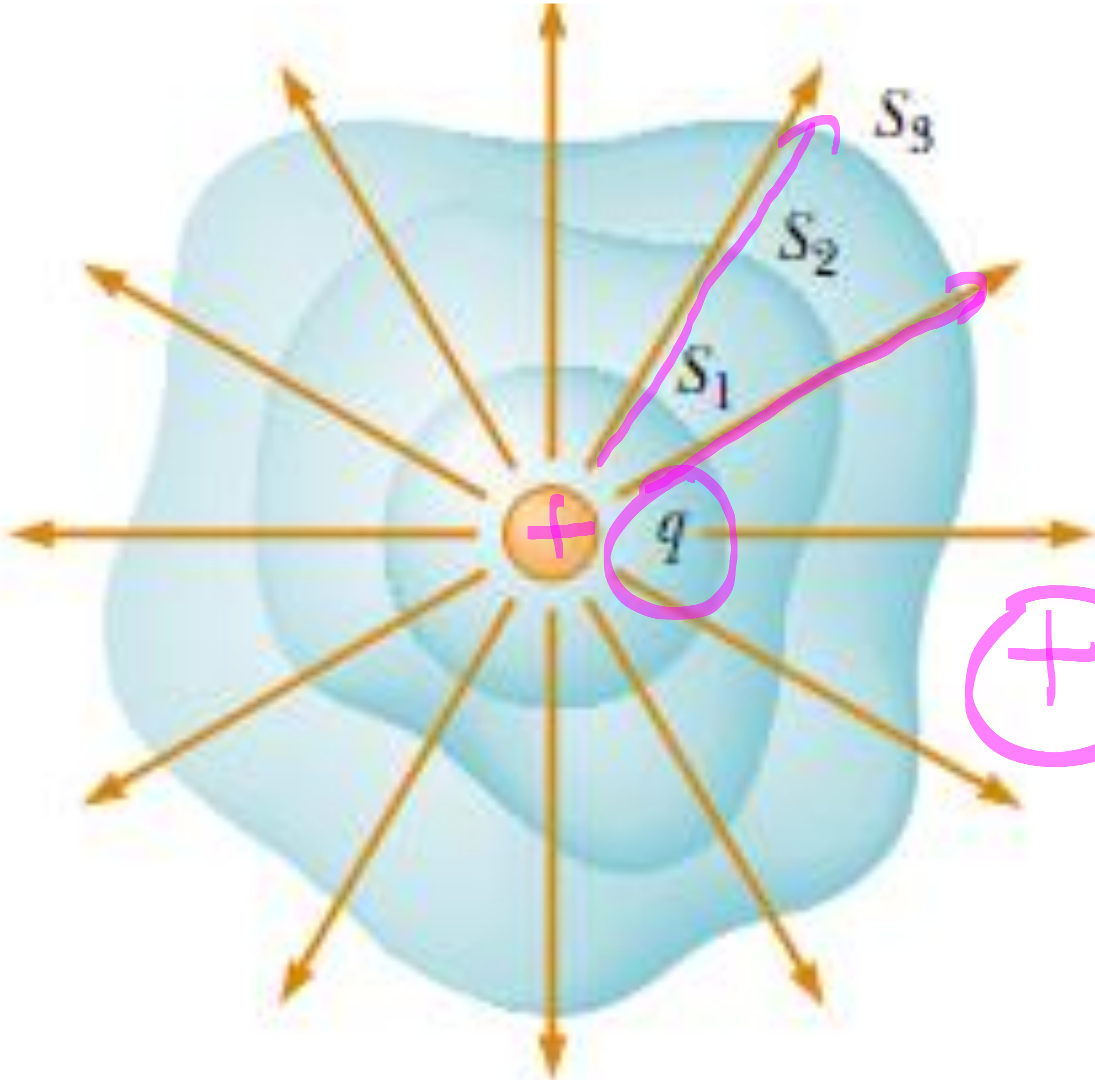


$$4\pi k = 1/\varepsilon_0 \text{ where } \varepsilon_0 = 8.85\text{x}10^{-12} \frac{C^2}{Nm^2}$$

$$\boxed{\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}}$$ Gauss' Law

using Coulombs

Closed Surfaces of Various Shapes Surrounding a Charge ‘q’



⊕ or ⊖

Conductors in Electrostatic Equilibrium

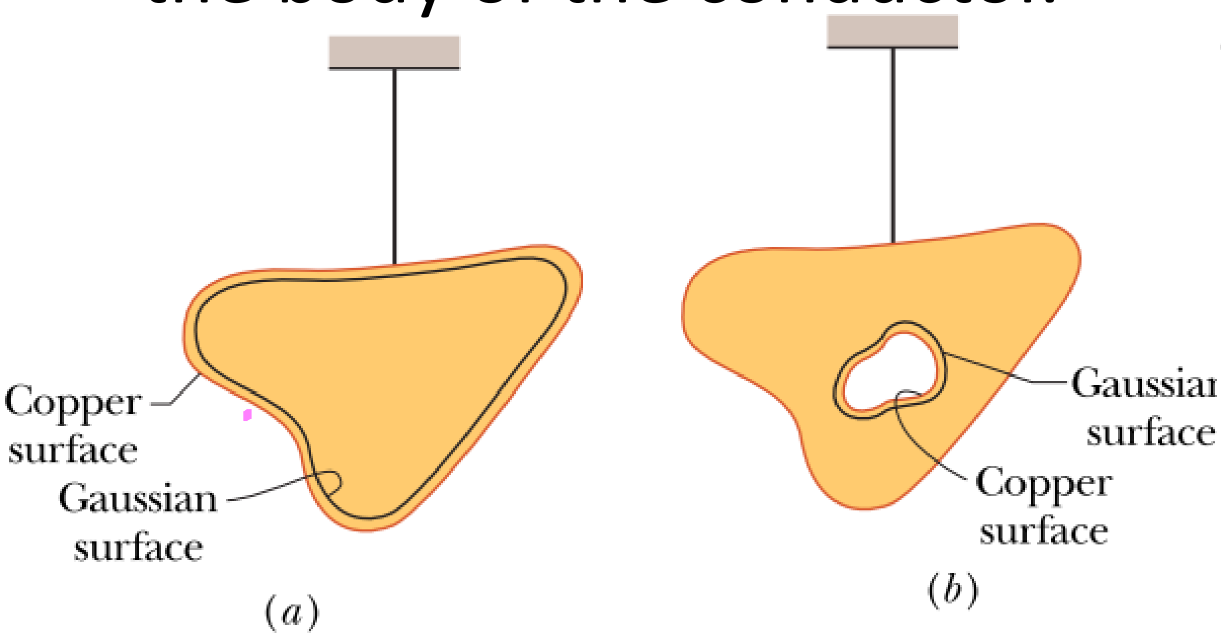
- A good electrical conductor contains charges (electrons) that are not bound to any atom and therefore are free to move about within the material
- When there is no net motion of charge within a conductor, the conductor is in electrostatic equilibrium

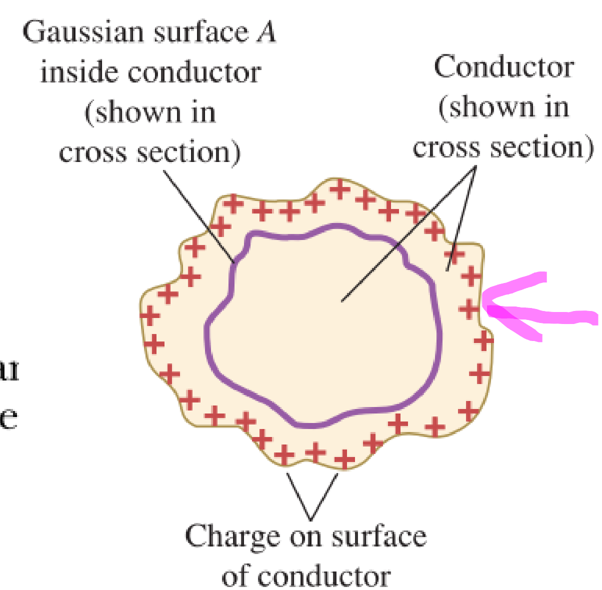
Conductors in Electrostatic Equilibrium - Properties

- The electric field is zero everywhere inside the conductor, whether the conductor is solid or hollow
- If the conductor is isolated and carries a charge, the charge resides on its surface
- The electric field at a point just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude σ/ϵ_0, where σ is the surface charge density at that point
- On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest

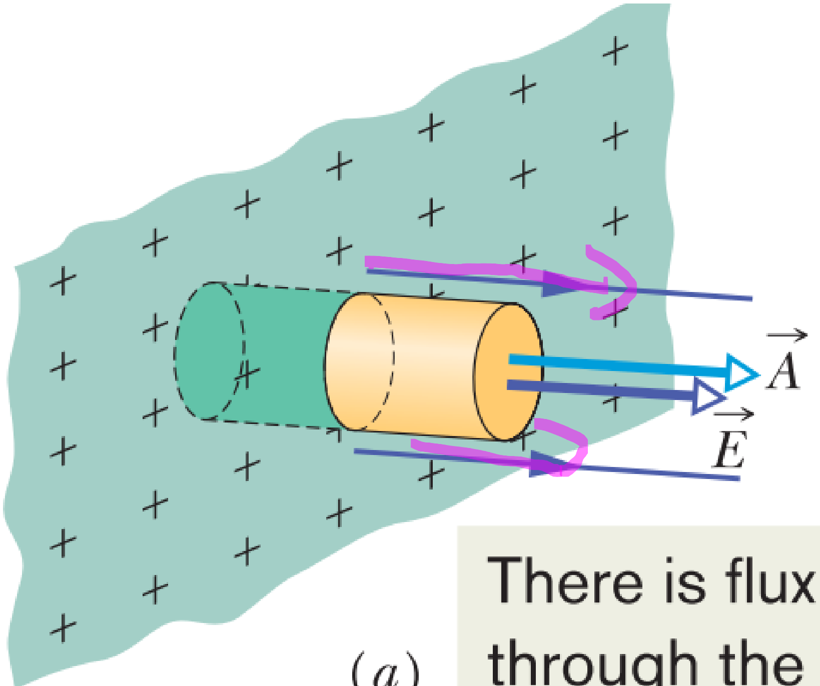
Inside a Charged Isolated Conductor

- If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor.
- None of the excess charge will be found within the body of the conductor.



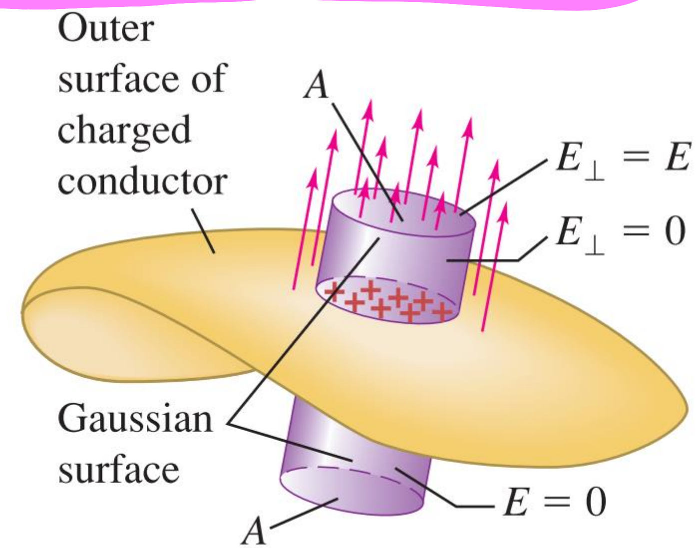


Outside a Charged Isolated Conductor



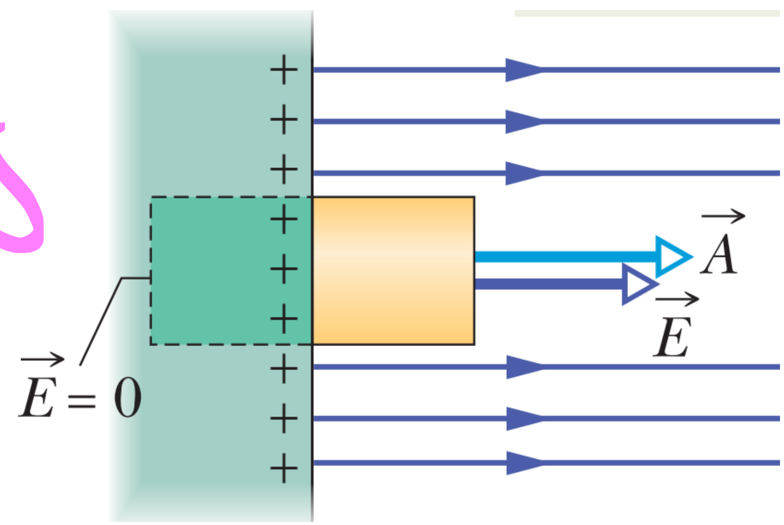
(a)

There is flux only through the *external* end face.

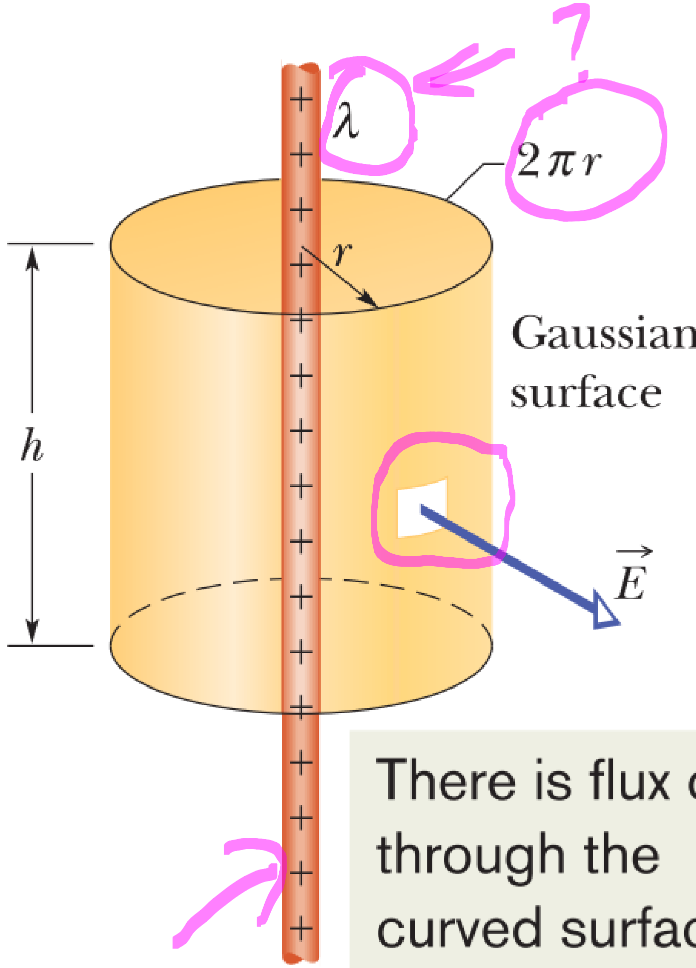


Surface charge density

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(conducting surface)}$$



A Cylindrically Symmetric Charge Distribution for Non-Conducting Rod

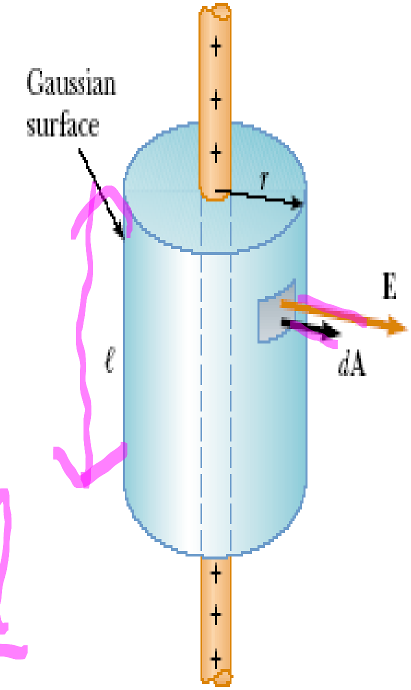


$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{(line of charge).}$$

There is flux only through the curved surface.

A Cylindrically Symmetric Charge Distribution

$$\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = E\oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda\ell}{\epsilon_0}$$

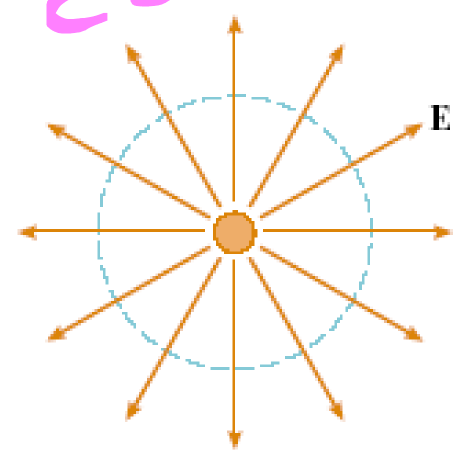


$$A = 2\pi r\ell$$

$$E \cdot A = \frac{\lambda\ell}{\epsilon_0}$$

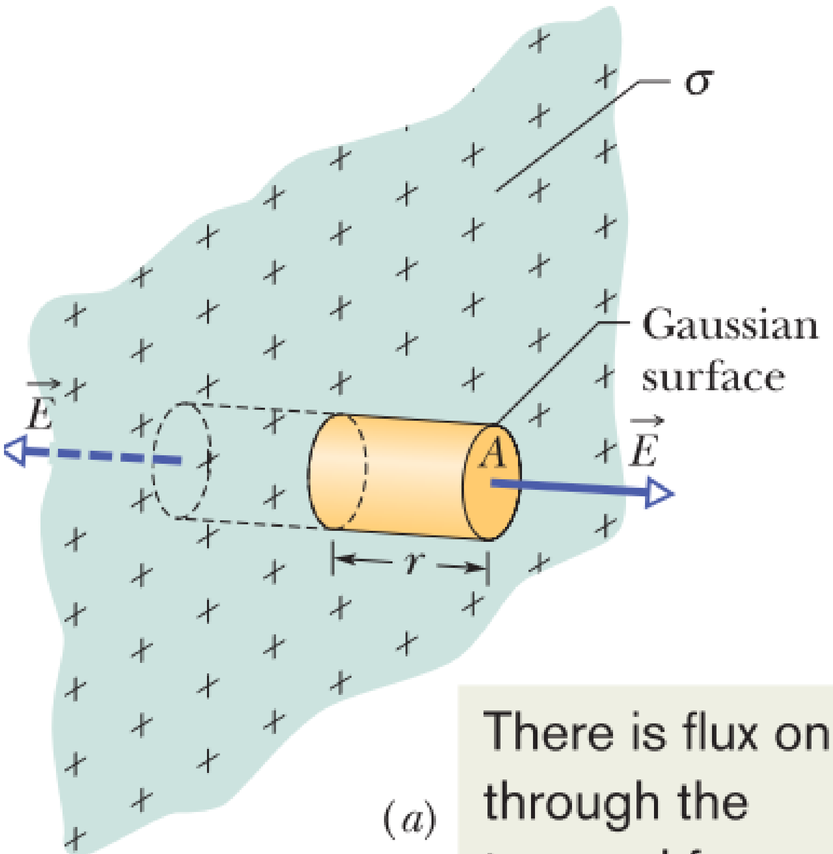
λ

$$E(2\pi r\ell) = \frac{\lambda\ell}{\epsilon_0}$$



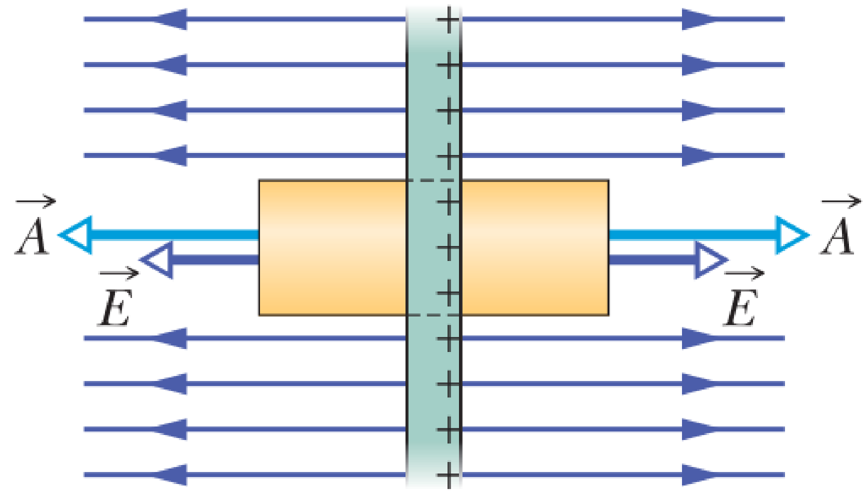
$$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2k_e\frac{\lambda}{r}$$

A Non-Conducting Plane of Charge



(a)

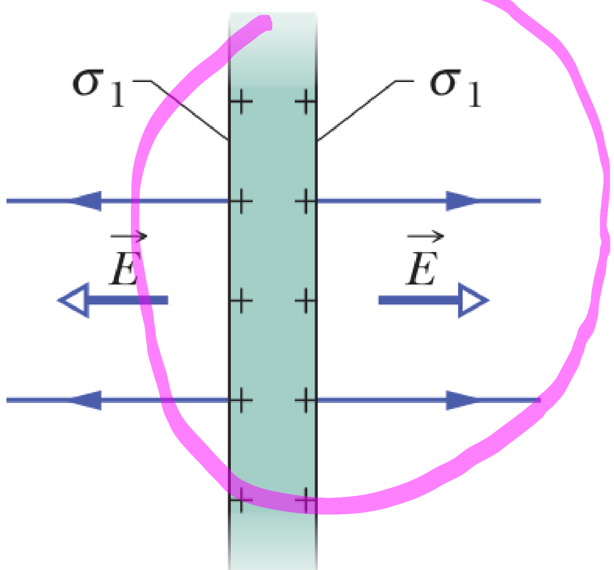
There is flux only through the two end faces.

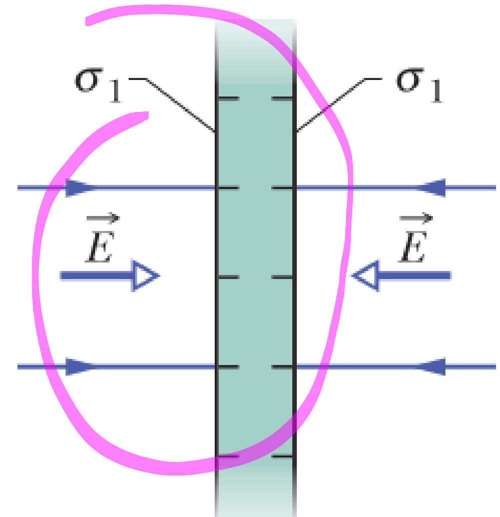


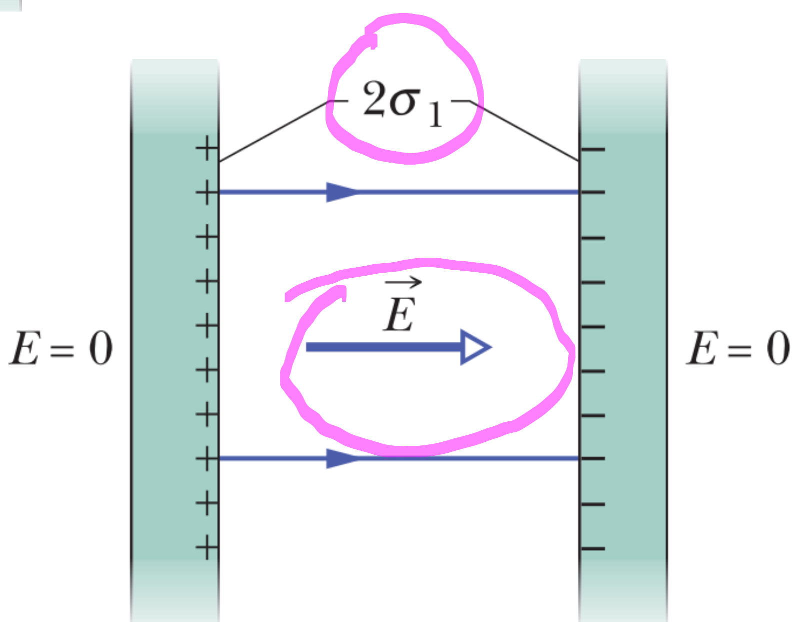
$$\Phi_E = 2EA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

Two Conducting Plates



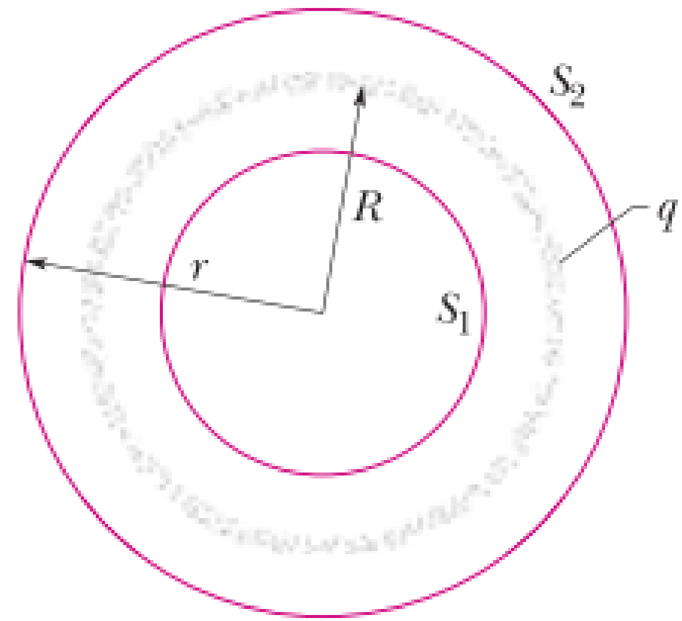




$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

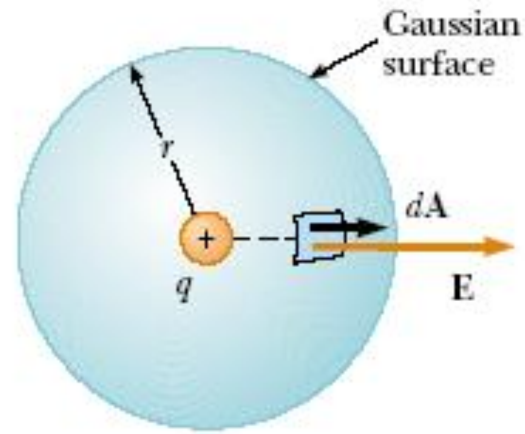
Shell Theorems

1. A shell of uniform charge attracts or repels a charged particle that is **outside the shell** as if **all the shell's charge** were **concentrated** at the **center of the shell**
2. If a charged particle is located **inside** a shell of uniform charge, there is **no** electrostatic force on the particle from the shell



The Electric Field Due to a Point Charge

Starting with Gauss's law, calculate the electric field due to an isolated point charge q.



$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E\, dA = \frac{q}{\epsilon_0}$$

$$\oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

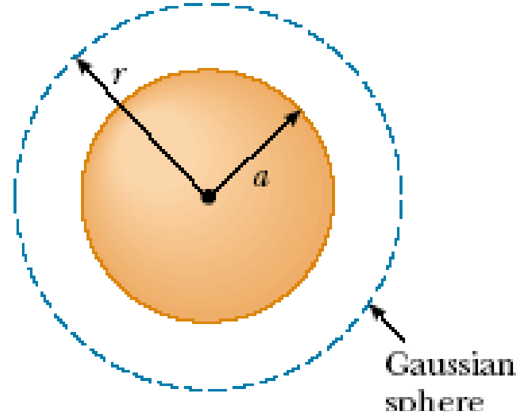
$$E = \frac{q}{4\pi\epsilon_0 r^2} = \boxed{k_e \frac{q}{r^2}}$$

$$\frac{Kq}{r^2}$$

A Spherically Symmetric Charge Distribution

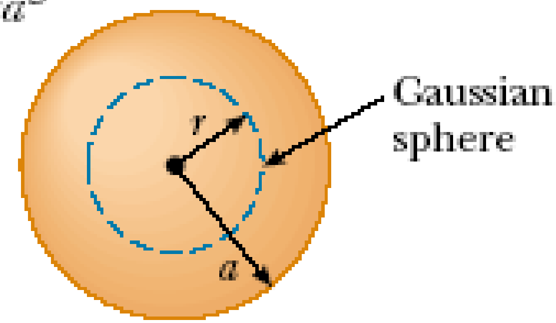
$$E = k_e \frac{Q}{r^2} \quad (\text{for } r > a)$$

$r < a$



$$q_{\text{in}} = \rho V' = \rho\left(\tfrac{4}{3}\pi r^3\right)$$

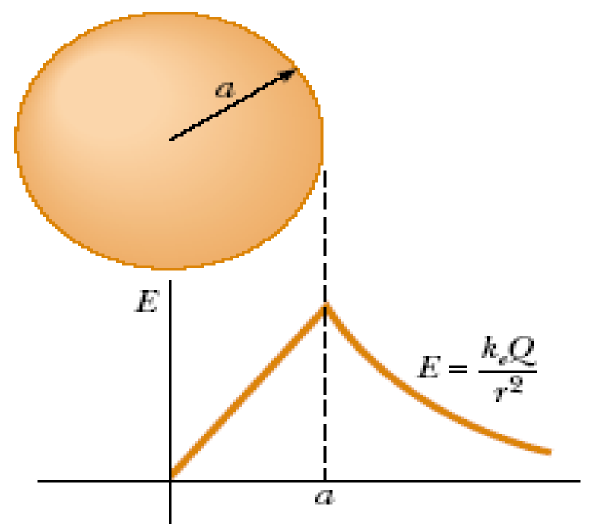
$$\rho = Q/\tfrac{4}{3}\pi a^3$$



$$\oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E = \frac{q_{\text{in}}}{4\pi\epsilon_0 r^2} = \frac{\rho\frac{4}{3}\pi r^3}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0} r$$

$$E = \frac{Qr}{4\pi\epsilon_0 a^3} = \frac{k_e Q}{a^3} r \quad (\text{for } r < a)$$

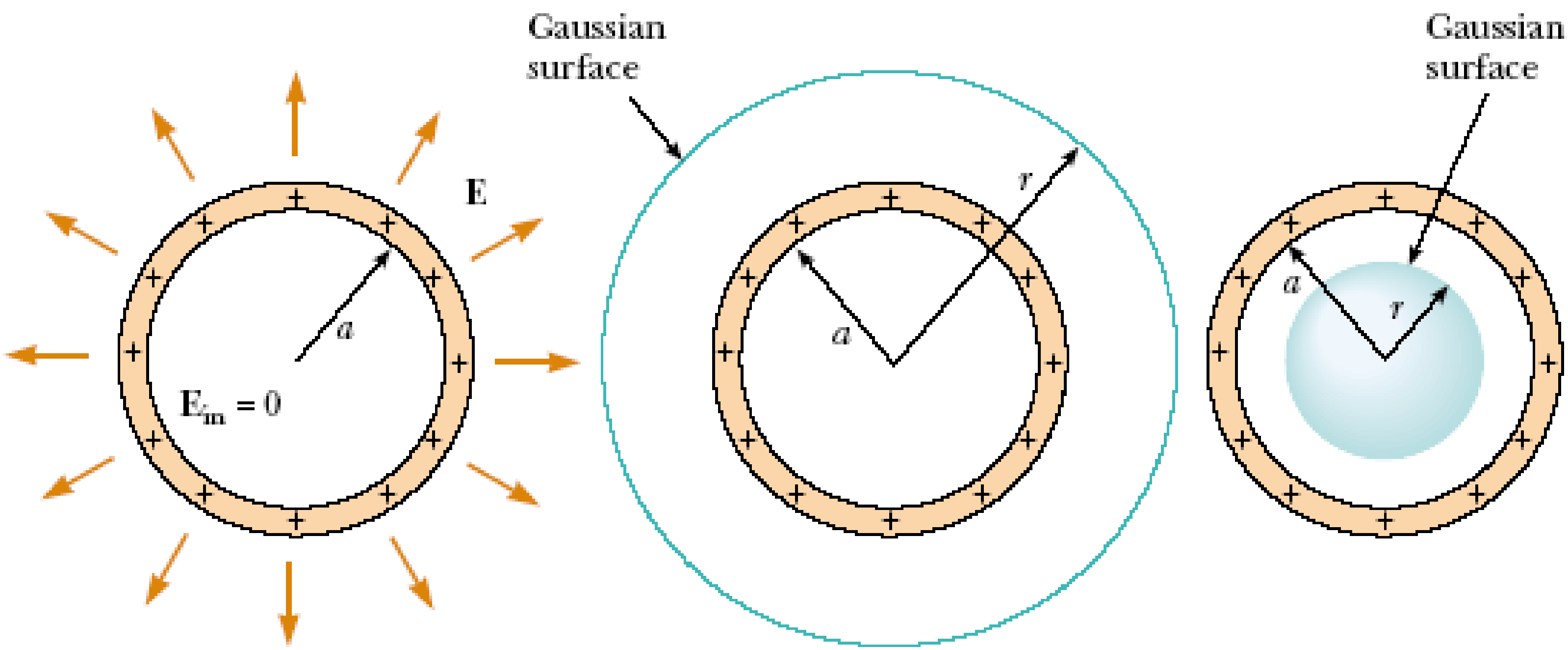


The Electric Field Due to a Thin Spherical Shell

- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.
- If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.

$$E = k_e \frac{Q}{r^2} \quad \text{(for } r > a\text{)}$$

$E = 0$ in the region $r < a$.



Electric field of various symmetric charge distributions: The following table lists electric fields caused by several symmetric charge distributions. In the table, q, Q, λ, and σ refer to the *magnitudes* of the quantities.

Charge Distribution	Point in Electric Field	Electric Field Magnitude
Single point charge q	Distance r from q	$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$
Charge q on surface of conducting sphere with radius R	Outside sphere, $r > R$	$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$
	Inside sphere, $r < R$	$E = 0$
Infinite wire, charge per unit length λ	Distance r from wire	$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$
Infinite conducting cylinder with radius R, charge per unit length λ	Outside cylinder, $r > R$	$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$
	Inside cylinder, $r < R$	$E = 0$
Solid insulating sphere with radius R, charge Q distributed uniformly throughout volume	Outside sphere, $r > R$	$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$
	Inside sphere, $r < R$	$E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}$
Infinite sheet of charge with uniform charge per unit area σ	Any point	$E = \frac{\sigma}{2\epsilon_0}$
Two oppositely charged conducting plates with surface charge densities $+\sigma$ and $-\sigma$	Any point between plates	$E = \frac{\sigma}{\epsilon_0}$
Charged conductor	Just outside the conductor	$E = \frac{\sigma}{\epsilon_0}$

What next ??

Chap 25 Capacitance