

1. Find the angle between the vector $A = 5i - 3j + 7k$ and the x, y, and z axes, respectively.

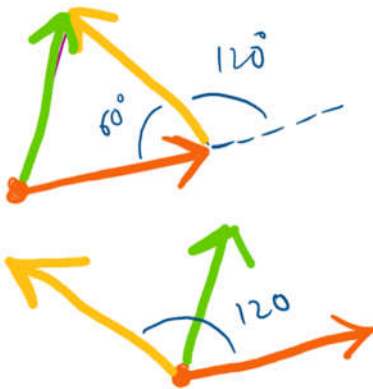
$A \cdot B = AB \cos \theta$
 $\theta_y = \cos^{-1} \left(\frac{A_y}{|A|} \right) = \cos^{-1} \left(\frac{-3}{9.11} \right)$
 $\theta_y = 109^\circ$
 $\theta_z = \cos^{-1} \left(\frac{A_z}{|A|} \right) = \cos^{-1} \left(\frac{7}{9.11} \right) \Rightarrow \theta_z = 37.90^\circ$
 $|A| = \sqrt{25 + 9 + 49} = \sqrt{83} = 9.11$

2. In the product $\vec{F} = q\vec{v} \times \vec{B}$, take $q = 2$, $\vec{v} = 2i + 4j + 6k$ and $\vec{F} = 4i - 20j + 12k$. What then is \vec{B} in unit-vector notation if $B_x = B_y$?

$\vec{F} = q\vec{v} \times \vec{B}$
 $|\vec{F}| = q|\vec{v} \times \vec{B}| = q|\vec{v}||\vec{B}|\sin \theta$
 $= q \begin{vmatrix} i & j & k \\ 2 & 4 & 6 \\ B_x & B_x & B_y \end{vmatrix}$
 $4i - 20j + 12k = 2i(4B_y - 6B_x) - 2j(2B_y - 6B_x) + 2k(2B_x - 4B_x)$
 solving these equation,
 $B_x = B_x = -3$
 $B_y = -4$
 Hence
 $\vec{B} = -3i - 3j - 4k$

2.

3. Two vectors (A and B) of equal magnitude are acting at a point. Find the angle between the vectors (A and B) when the magnitude of resultant is also equal to magnitude of either of these vectors.



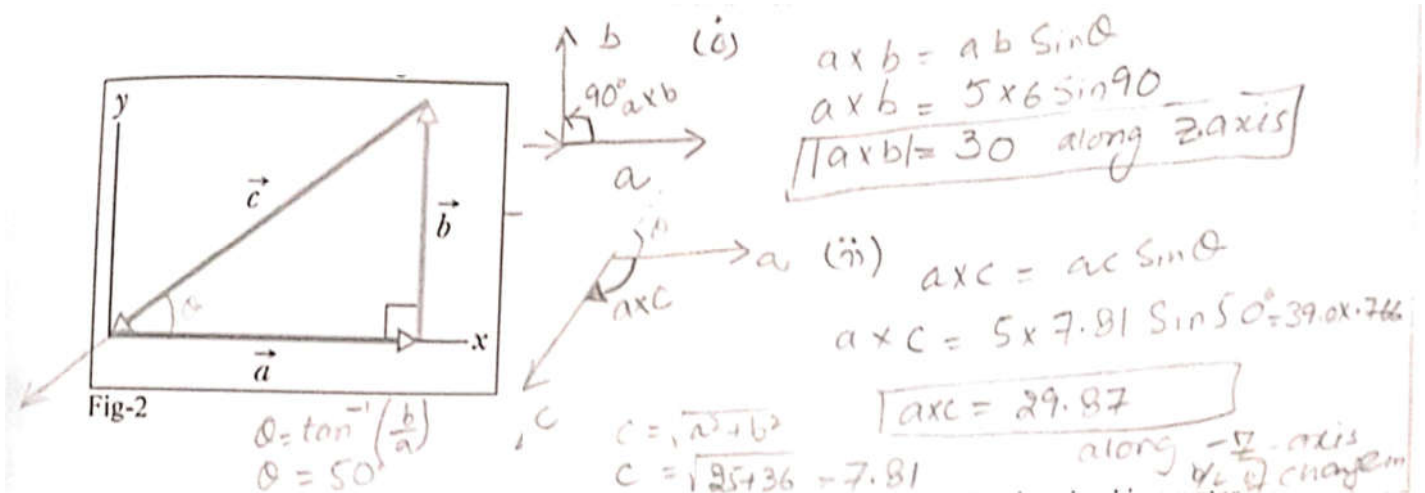
using Cosine law:

$R = \sqrt{A^2 + B^2 + 2AB \cos \theta_{AB}}$
 $a = \sqrt{a^2 + a^2 + 2a^2 \cos \theta_{AB}}$
 $-a^2 = 2a^2 \cos \theta_{AB}$
 $\theta_{AB} = \cos^{-1}(-1/2) = 120^\circ$

4. A sailboat sets out to sail to a point 215 km due north. An unexpected storm blows the ship to a point 195 km due east of its starting point. (i) How far and (ii) in what direction must it now sail to reach its original destination?

$a = 215 \text{ km}$
 $b = 195 \text{ km}$
 (i) $|\vec{r}| = \sqrt{(215)^2 + (195)^2} = \sqrt{46225 + 38025} = \sqrt{84250}$
 $|\vec{r}| = 290.25 \text{ km}$
 (ii) $\theta = \tan^{-1}\left(\frac{195}{215}\right) = \tan^{-1}(0.906) = 42.17^\circ$ (Wes of North)

5. For the vectors in Fig. 1, with $a = 5$, $b = 6$, what are (i) the magnitude and the direction of $\vec{a} \times \vec{b}$, (ii) the magnitude and the direction of $\vec{a} \times \vec{c}$.



6. Determine the value of "a" so that $\vec{A} = 2\hat{i} + a\hat{j} + \hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular.

$\vec{A} = 2\hat{i} + a\hat{j} + \hat{k}$
 $\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$
 $\vec{A} \cdot \vec{B} = 0$
 $8 - 2a - 2 = 0$
 $a = 3 \text{ units}$

7. Two forces of equal magnitude are acting at a point. Find the angle between the forces when the magnitude of resultant is also equal to magnitude of either of these forces.

[same as problem#3]

8. If $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$, Calculate: (i) Area of parallelogram (ii) Area of triangle.

$\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$
 $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$
 Area of parallelogram $|\vec{A} \times \vec{B}| = |\vec{C}|$
 $\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$
 $= 15\hat{i} - 10\hat{j} + 30\hat{k}$
 $|\vec{C}| = 35 \text{ units}$
 Area of Triangle $= \frac{1}{2} |\vec{C}|$
 $= 17.5 \text{ unit}^2$

9. Two vectors A and B have magnitudes $A = 3.00$ and $B = 3.00$. Their vector product is $\vec{A} \times \vec{B} = -5.00\hat{k} + 2.00\hat{i}$. What is the angle between A and B?

$$|A \times B| = |A||B|\sin\alpha$$

$$\sqrt{(2^2 + 5^2)} = 3 \cdot 3 \cdot \sin\alpha$$

$$\alpha = 36.75^\circ$$

10. If \vec{u} is added to $\vec{w} = 3.0\mathbf{i} + 4.0\mathbf{j}$, the result is a vector ($\vec{u} + \vec{w} = \vec{v}$) in the positive direction of the y-axis, with a magnitude equal to that of \vec{w} . What is the magnitude of \vec{u} ?

$\vec{u} + \vec{w} = \vec{x}$
 $\vec{u} + 3\mathbf{i} + 4\mathbf{j} = x\mathbf{j}$
 $|\vec{w}| = |\vec{x}| = \sqrt{3^2 + 4^2} = 5$
 Now, $\vec{u} + 3\mathbf{i} + 4\mathbf{j} = 5\mathbf{j}$
 $\vec{u} = -3\mathbf{i} + \mathbf{j}$
 $|\vec{u}| = \sqrt{3^2 + 1^2} = 3.2$

Isosceles triangle

$\tan \theta = (3/4) \rightarrow \theta = 36.9^\circ$
 $\sin\left(\frac{\theta}{2}\right) = \frac{u/2}{5} \rightarrow u = 2 \cdot 5 \cdot \sin\left(\frac{\theta}{2}\right) = 3.2$

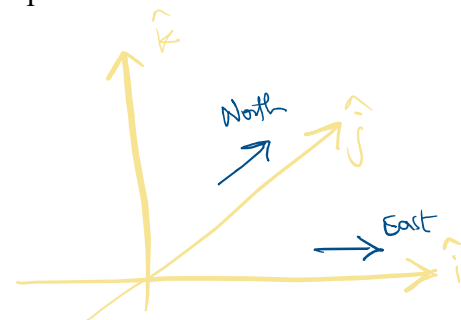
11. The scalar product of vectors \vec{A} and \vec{B} is +48.0 m. Vector \vec{A} has magnitude 9 m and direction -28° from -y axis. If vector \vec{B} has direction -39° from +x axis, what is the magnitude of vector \vec{B} ?

$\vec{A} \cdot \vec{B} = 48 = |\vec{A}||\vec{B}| \cos \theta_{AB}$
 $\frac{48}{9 \cdot \cos(79^\circ)} = |\vec{B}| = 27.9 \text{ units}$

12. Let \hat{i} be directed to the east, \hat{j} be directed to the north, and \hat{k} be directed to upward direction. What are the (i) values of $\hat{i} \cdot \hat{k}$ and $\hat{j} \cdot (-\hat{j})$, (ii) directions of $\hat{k} \times \hat{j}$ and $(-\hat{i}) \times (-\hat{j})$?

(i) $\hat{i} \cdot \hat{k} = 0$
 $\hat{j} \cdot (-\hat{j}) = -1$

(ii) $\hat{k} \times \hat{j} = -\hat{i}$ (west)
 $-\hat{i} \times -\hat{j} = +\hat{k}$ (upwards)



Advice:

Do not forget to read Problem Solving Tactics on Page#45 in the reference book.

Try to simplify the problem first, then add necessary details and complete the problem from every possible direction asked.

Drawing helps a lot. You don't have to be good at it but looking at the problem makes things clear.

Recheck your answer by running the problem in reverse and make sure everything (including vector directions, problem statement, units etc.) makes sense.