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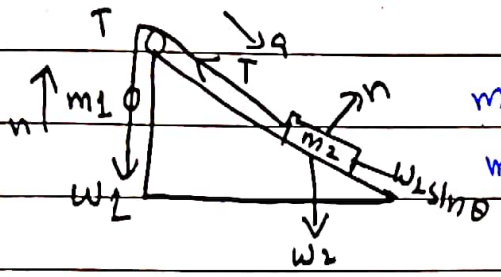
20K-1055 BDF-1A

Forces and Newton's law:

Question # 1:



$\Sigma F = 0$

$T - mg = m_1 a \rightarrow (1)$

$m_2 g - T = m_2 a$

$m_2 g \sin\theta - T = m_2 a \rightarrow (ii)$

for Tension:

$T = m_1 a + m_1 g$

$T = 5(2.54 + 9.81)$

$T = 61.75\ N$

for Acceleration:

Add both equations:

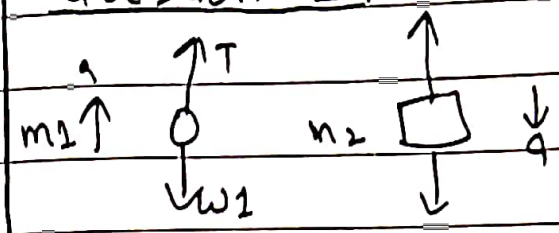
$m_2 g \sin\theta - m_1 g = m_1 a + m_2 a$

$a = \dfrac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}$

$a = \dfrac{(13 \times 9.81) \times \sin 48 - (5 \times 9.81)}{13 + 5}$

$a = 2.54\ m/s^2$

Question 2:



$\Sigma x = 0$

$T - m_1 g = m_1 a \rightarrow (i)$

$m_2 g - T = m_2 a \rightarrow (ii)$

for Tension:

$T = m_1 a + m_1 g$

$T = 4(3.997 + 9.81)$

$T = 55.22\ N$

for acceleration:

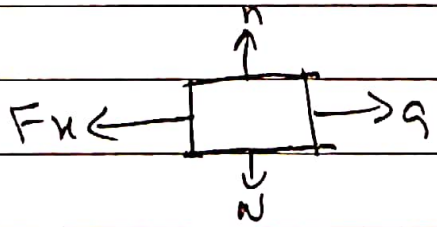
Add both equations

$m_2 g - m_1 g = m_1 a + m_2 a$

$a = \dfrac{m_2 g - m_1 g}{m_1 + m_2}$

$a = 3.997\ m/s^2$

Question 3:



$\Sigma F_y = w - n = 0 \rightarrow n = mg$

$\Sigma F_x = \mu n.$

$\mu = \dfrac{F_x}{n} \Rightarrow \mu = \dfrac{470}{79 \times 9.81}$

$\mu = 0.606$ Ans.

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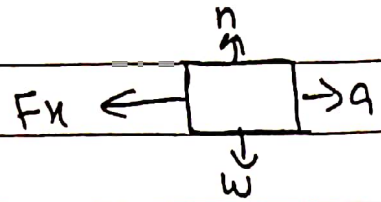
Question #4.

(a)

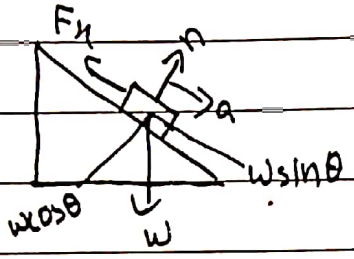


$\Sigma F_y = w - n = n = mg$

$F = \mu_2 n$

$F = 9120\ N$ Ans.

(b)



$\Sigma F_y = w\cos\theta - n = 0 \Rightarrow n = w\cos\theta$

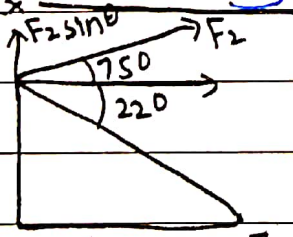
$\Sigma F_x = ma = w\sin\theta - F_x$

for max balancing force a

$F = \mu n$

$F = 9020\ N$ Ans

Q: No. 5



$\Sigma F_x = F_2\cos\theta + F_1\cos\theta$

$\Sigma F_y = F_2\sin\theta - F_1\sin\theta$

$\Sigma F_y = 7.5\sin(75) - 3.5\sin(22)$

$\Sigma F_x = 7.5\cos(75) + 3.5\cos(75)$

for acceleration:

$a_x = \Sigma F_x / m = a_x = 5.17/0.5 =$

$a_x = 10.35\ m/s^2$.

$a_y = \Sigma F_y / m \Rightarrow a_y = 5.93/0.5$

$a_y = 11.86\ m/s^2$

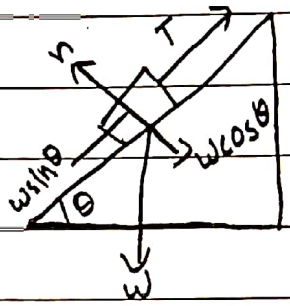
$a = \sqrt{a_x^2 + a_y^2} \Rightarrow a = 18.7\ m/s^2$

for direction:

$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{4.86}{10.34}\right)$

$\theta = 48.92^\circ$ from +ve x-axis

Question No. 6



$\Sigma F_x = T - w\sin\theta = 0$

$\Sigma F_y = n - w\cos\theta$

- Tension = $w\sin\theta$

  $T = mg\sin\theta$

  $T = 41.7\ N$

- normal force = $w\cos\theta$

  $n = mg\cos\theta$

  $n = 72.2\ N$

- when cord breaks down tension cease to exist so object tranch downwards

  $F = ma$

  $a = F/m$

  $a = 491\ m/s^2$

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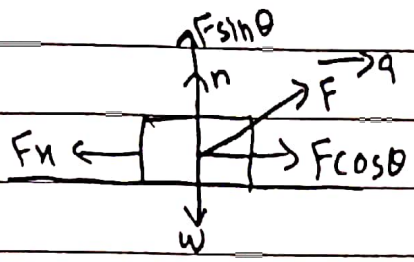
Question No #7:



$\Sigma F_y = F\sin\theta + n = w$

$\Sigma F_x = F\cos\theta - F_x = ma$

$F\cos\theta - (u_k \cdot n) = ma$

$n = \frac{F\cos\theta - ma}{u_k}$

$n = mg - F\sin\theta$

Equate both equations

$mg - F\sin\theta = \frac{F\cos\theta - ma}{u_k}$

$u_k(mg - F\sin\theta) = F\cos\theta - ma$

$a = F\cos\theta - \frac{[u_k(mg - F\sin\theta)]}{m}$

$a = \frac{F\cos\theta}{m} - u_k\left(g - \frac{F\sin\theta}{m}\right)$

$\frac{da}{d\theta} = \frac{F\sin\theta}{m} + u_k\frac{F\cos\theta}{m}$

$\frac{da}{d\theta} = \frac{-F\sin\theta}{m} + u_k\frac{F\cos\theta}{m}$

$\frac{-F\sin\theta}{m} + u_k\frac{F\cos\theta}{m} = 0$

$u_k = \tan\theta \rightarrow \theta = \tan^{-1}(0.4)$

$\theta = 21.80^\circ$

Replace θ with its value

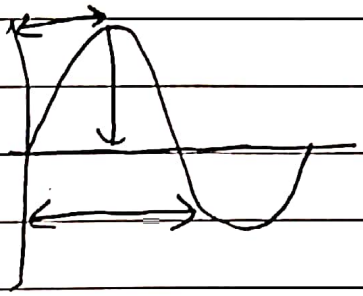
$a = \frac{F\cos\theta}{m} - u_k\left(g - \frac{F\sin\theta}{m}\right)$

$a = \frac{12\cos 21.80}{3} - 0.4\left[9.81 - \frac{12\sin 21.80}{3}\right]$

$a = 0.384\ m/s^2$ Ans.

Oscillations:

Q.1



Time period = 2(t) = 0.5 sec

Frequency = 1/0.5 = 2 Hz

Amplitude = $\frac{\pi}{4} = \frac{2\times 36}{4} = 18$cm

Q.2

(a) $F = ma$ and $a = \omega^2 x$

$F = m(-\omega^2 x)$

$F = -0.12\left(\left(\frac{2\pi}{0.2}\right)^2 \times \frac{8.5}{100}\right)$

$F = -10.07N$

(-) sign will be neglected

(b) $\omega = \sqrt{K/m}$

$\omega^2 * m = K$

$K = \left(\frac{2\pi}{0.2}\right)^2 \times 0.12$

$K = 118\ N/m$

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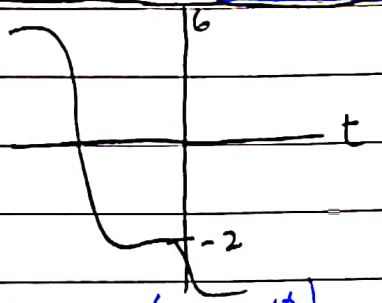
Q.3

(a) $\omega = \frac{2\pi}{T} \Rightarrow \omega = \frac{2\pi}{1}$ $\boxed{\omega = 6.28 \text{ rad/s}}$

(b) Max-speed = ωx max. displacement

$x = 1 \times 10^{-3} \Rightarrow \frac{x}{6.28} \Rightarrow \boxed{x = 159 \text{ m}}$ Ans

Q4



$x = x_m \cos(\omega t + \phi)$

at $t=0$, $x=-2$

$-2 = 6\cos(\omega(0) + \phi)$

$\phi = \cos^{-1}(-2/6)$

$\boxed{\phi = 1.91}$

Q.5

Total Energy $= \frac{1}{2} k (x_m)^2$

$= \frac{1}{2} k (0.12)^2 \rightarrow \boxed{k = 833 \text{ N/m}}$

Q.6

(a) $\frac{x}{m} e^{-\frac{bt}{2m}} \cdot \frac{1}{3} \frac{x}{m}$

$= -\frac{bt}{2m} \ln e^1 = \ln\left(\frac{1}{3}\right)$

$t = \frac{-2m}{b} \ln(1/3) \rightarrow t = \frac{2m}{b} \ln(3)$

$t = \frac{2 \times 1.5 \times \ln(3)}{0.23} \Rightarrow \boxed{t = 14.3 \text{ s}}$

(b) $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

$= \omega' = \sqrt{\frac{8}{105} - \frac{(0.23)^2}{4(105)^2}}$

$= \omega' = 2.31 \text{ rad/s}$

$T = \frac{2\pi}{\omega'} \Rightarrow T' = \frac{2\pi}{2.31}$

$T' = 2.72 \text{ sec}$

No. of oscillation $= \frac{14.3}{2.72} = 5.27$.

Q.7

(a) $x = 6\cos(3\pi x t + \pi/3)$

at $t=2 \rightarrow x = 6\cos(3\pi \times 2 + \pi/3)$

$x = 6\cos(6\pi + \pi/3) \Rightarrow \boxed{x = 3 \text{ m}}$

(b) $v = \frac{dx}{dt} \rightarrow v = -6(3\pi)\sin(3\pi x t + \frac{\pi}{3})$

at $t=2 \rightarrow$

$v = -6(3\pi)\sin(3\pi \times 2 + \pi/3)$

$v = -18\pi \sin(6\pi + \pi/3)$

$\boxed{v = -49 \text{ m/s}}$

(c) $a = \frac{dv}{dt} \rightarrow a = -6(3\pi)^2 \cos(3\pi x t + \pi/3)$

at $t=2 \rightarrow a = -6(3\pi)^2 \cos(3\pi \times 2 + \pi/3)$.

$a = -54\pi^2 \cos(6\pi + \pi/3)$

$\boxed{a = -266 \text{ m/s}^2}$

(d) $x = x_m \cos(\omega t + \phi)$ } By comparing both eqs:

$x = x_m \cos(3\pi t + \phi)$ }

$\boxed{\omega = 3\pi}$

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(e) frequency $= 2\pi f = \omega$

$f = \frac{\omega}{2\pi} \Rightarrow f = \frac{3\pi}{2\pi} \Rightarrow \boxed{f = 1.5\,Hz}$

(f) Time period:

$T = 1/f \Rightarrow T = \frac{1}{1.5} \Rightarrow \boxed{T = 0.67\,s}$

Q. 8 (a) for max velocity $\Rightarrow v = \omega x_m$

max Ex = Total Energy $= \frac{1}{2} m (\omega x_m)^2$

$= \frac{1}{2}(3)\left(\frac{\pi}{3} \times 5\right)^2 = 41.12\,J$

$T.E = P.E + K.E$

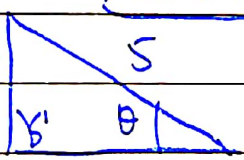
$K.E = \frac{T.E}{2}$

$\frac{1}{2} mv^2 = \frac{41.12}{2}$

$m\left(\omega\sqrt{x_m^2 - x^2}\right)^2 = 41.12$

$3\left(\frac{\pi}{3}\right)^2 (25 - x^2) = 41.12 \Rightarrow x^2 = 25 - 12.5$

$\boxed{x = 3.53\,m}$

(b) 

$\sin\theta = \frac{3.53}{5} \Rightarrow \theta = \sin^{-1}\left(\frac{3.53}{5}\right)$

$\boxed{\theta = 0.78\,rad}$

$\omega = \frac{\theta}{t} \Rightarrow t = \frac{\theta}{\omega} \Rightarrow t = \frac{0.78}{\pi/3}$

$\boxed{t = 0.74\,sec}$

Q. 9:

(a) $\omega = \sqrt{\frac{K}{m}} \Rightarrow 2\pi f = \sqrt{\frac{K}{m}}$

$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{1000}{5}}$

$\boxed{f = 2.25\,Hz}$

(b) $P.E = \frac{K x_m^2 m}{2} \Rightarrow P.E = \frac{1000(0.5)^2}{2}$

$\boxed{P.E = 125\,J}$

(c) $K.E = \frac{1}{2} mv^2 \Rightarrow K.E = \frac{1}{2}(5)(10)^2$

$\boxed{K.E = 250\,J}$

(d) $T.E = \frac{1}{2} K (x_m)^2$

$K.E + P.E = \frac{1}{2} K (x_m)^2$

$= \frac{2(K.E + P.E)}{1000} = x_m^2$

$(x_m)^2 = \frac{2(125 + 250)}{1000}$

$\boxed{x_m = 0.87\,m}$

Q. 10:

(a) $b = 4 \quad \sqrt{Km} = 82.46$

$\omega' = \sqrt{\frac{K}{m} - \frac{b^2}{4m}} \Rightarrow 2\pi = \sqrt{\frac{K}{m} - \frac{b^2}{4m}}$

$\frac{2\pi}{T} = \sqrt{\frac{85}{80} - \frac{(4)^2}{4 \times 80}} \Rightarrow 2\pi = 1.006$

$\boxed{T = 6.24\,s}$

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(b) $x(t) = x_m e^{-\frac{bt}{2m}}$ → for amplitude

$\frac{1}{2} x_m = x_m e^{-bt/2m}$

$= \ln\left(\frac{1}{2}\right) = \ln e^{-\frac{bt}{2m}}$

$\frac{(\ln 2) \times 2m}{b} = t \rightarrow t \Rightarrow \frac{\ln(2) \times 2(85)}{85}$

$\boxed{t = 1.305\,s}$

(c) $E(t) = E_0 e^{-\frac{bt}{2m}}$ → for energy

$\frac{1}{2} E_0 = E_0 e^{-\frac{bt}{2m}} \rightarrow \ln\left(\frac{1}{2}\right) =$

$\frac{\ln e^{-bt}}{m} \Rightarrow \frac{(\ln 2) \times m}{b}$

~~t = t~~ $t = \frac{(\ln 2) \times 80}{40}$

$\boxed{t = 63.86\,s}$

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