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Applied Physics
Assignment-1
Vectors

1. A displacement vector in the xy plane is 7.3m long and directed at angle of 30° in Fig 1. Determine the (a) x-component (b) y component of the vector.

Sol:-

$\vec{r} = 7.3m$

$r_x = ? \Rightarrow r_x = |\vec{r}|\cos\theta$

$r_y = ? \Rightarrow r_y = |\vec{r}|\sin\theta$

$r_x = \cos\theta\,|\vec{r}|$

$r_x = \cos 30^\circ\,|7.3m| \Rightarrow \boxed{r_x = 6.32m}$

$r_y = \sin\theta\,|\vec{r}|$

$r_y = \sin 30^\circ\,|7.3m| \Rightarrow \boxed{r_y = 3.65m}$

2. Find x and y components of their vector sum?

Data:

$\vec{a} = \vec{b} = 10\,m$

$\theta_1 = 30^\circ, \theta_2 = 105^\circ$

Sol:-

finding x and y components of $\vec{a}$ and $\vec{b}$.

$a_x = a\cos 30^\circ$

$a_x = \cos 30^\circ(10)$

$\boxed{a_x = 8.6602m}$

$a_y = \sin 30^\circ(10)$

$\boxed{a_y = 5m}$

$b_x = \cos 135^\circ\, b$

$b_x = \cos 135^\circ(10)$

$b_x = -7.0711m$

$b_y = (10)\sin 135^\circ$

$\boxed{b_y = 7.0711m}$

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x-component of vector sum:

$r_x = a_x + b_x \Rightarrow r_x = 8.6602 + (-7.0711)$

$\boxed{r_x = 1.5891}$

y-component of vector sum:

$r_y = a_y + b_y \Rightarrow 5 + 7.0711$

$\boxed{r_y = 12.0711 m}$

(b) the magnitude of r:

$(r) = \sqrt{(r_x)^2 + (r_y)^2}$

$\boxed{(r) = 12.1752 m}$

(c) the angle r makes with the positive x-axis

$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{12.0711}{1.5891}\right)$

$\boxed{\theta = 82.50^\circ}$

3. Data:

$\vec{a} = 4, \vec{B} = 3, \vec{c} = 5$

angle blw a and b $= \theta_1 = 90^\circ$

angle blw b and c $= \theta_2$

$\theta_2 = \tan^{-1}\left(\frac{3}{4}\right) \Rightarrow \theta_2 = 36.8699^\circ$

angle blw $\vec{a}$ and $\vec{c} = \theta_3$

$\theta_1 + \theta_2 + \theta_3 = 180^\circ$

$\theta_3 = 180^\circ - \theta_1 - \theta_2 = 180^\circ - 90^\circ - 36.8699^\circ$

$\theta_3 = 53.1301^\circ$

Magnitude and direction of

$a \times b = ?$ $a \times c = ?$ $b \times c = ?$

Sol:-

Magnitude of $a \times b$:

$|a \times b| = ab \sin\theta_1 = (4)(3)\sin(90^\circ)$

$\boxed{|a \times b| = 12 \text{ units}}$

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Magnitude of $\vec{a} \times \vec{c}$:

$|a \times c| = ac \sin\theta_3 = (4)(5)\sin(53.1301)^\circ$

$|a \times c| = 16$ units

Magnitude of $\vec{b} \times \vec{c}$:

$|b \times c| = bc \sin\theta_2 = (3)(5)\sin(36.8699)^\circ$

$\boxed{|b \times c| = 9 \text{ units}}$

direction of $\vec{a} \times \vec{b}$:

$\alpha = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{ab}\right) \Rightarrow \alpha = 90^\circ$ outward from paper

Direction of $\vec{b} \times \vec{c}$:

$\beta = \cos^{-1}\left(\frac{\vec{b} \cdot \vec{c}}{bc}\right) = \cos^{-1}(0.8)$

$\beta = 36.8699^\circ$ inward to the page.

Direction of $\vec{a} \times \vec{c}$:

$\gamma = \left(\frac{\vec{c} \cdot \vec{a}}{ca}\right) = \boxed{\gamma = 53.1301^\circ}$

4. Data:

$a = b = 10$m

angle which $\vec{a}$ makes with x-axis $\theta_1 = 30^\circ$

angle which $\vec{b}$ makes with x-axis $\theta_2 = 135^\circ$

angle blw $\vec{a}$ and $\vec{b}$ = ?

Sol: angle blw $\vec{a}$ and $\vec{b} = \theta$

$\theta = 130^\circ + 30^\circ - 135^\circ$

$\boxed{\theta = 75^\circ}$

$\vec{a} \cdot \vec{b} = ab\cos\theta$

$= (10)(10)\cos 75^\circ$

$\boxed{\vec{a} \cdot \vec{b} = 25.8819\text{m}}$

$\vec{a} \times \vec{b} = ab\sin\theta$

$= (10)(10)\sin 75^\circ$

$\boxed{\vec{a} \times \vec{b} = 96.5926\text{m}}$

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5. Data:

$A_x = 25\,m$

$A_y = 40\,m$

mag. of A = ?

Sol:- $|A| = \sqrt{(25)^2 + (40)^2} \Rightarrow |A| = \sqrt{225}$

$\boxed{|A| = 47.1699}$ Ans.

(b) Sol:- $\theta = \tan^{-1}\left(\frac{40}{15}\right) \Rightarrow \boxed{\theta = 58^\circ}$

6. Data:

$F_x = 100\,km$

$F_y = 120\,km$

$F = ?$ $\theta = ?$

Sol:-

$F = \sqrt{(100)^2 + (120)^2} \Rightarrow \boxed{F = 156.2\,km}$

$\theta = \tan^{-1}\left(\frac{120}{100}\right) \Rightarrow \boxed{\theta = 50.1944^\circ}$

The ship has to travel 156.2050 km towards North-West at an angle of 50°.

7. Data:

$\vec{a} = \vec{b} = \vec{c} = 30\,m$

$\theta_1 = 30^\circ$, $\theta_2 = 195^\circ$

$\theta_3 = 315^\circ$

magnitude = ?, θ_f = ?

Sol:-

let vector a+b+c = F

$a_x = a\cos\theta_1$

$a_x = 30\cos(30^\circ)$

$\boxed{a_x = 25.98\,m}$

$a_y = a\sin\theta_1$

$a_y = 30\sin(30^\circ)$

$\boxed{a_y = 15\,m}$

$b_x = b\cos\theta_2$

$b_x = 30\cos(195^\circ)$

$\boxed{b_x = -28.9778\,m}$

$b_y = b\sin\theta_2$

$b_y = 30\sin(195^\circ)$

$\boxed{b_y = -7.7646\,m}$

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$c_x = 21.2132m$

$c_y = -21.2132m$

$F_x = a_x + b_x + c_x$ $\quad$ $F_y = a_y + b_y + c_y$

$\boxed{F_x = 18.2162m}$ $\quad$ $\boxed{F_y = -13.978m}$

$\boxed{F = 22.9610m} \rightarrow a+b+c$

$a_x = 43.3013m$ $\quad$ $b_x = -48.2963m$

$a_y = 25m$ $\quad$ $b_y = -12.9409m$

$\boxed{\begin{array}{l} c_x = 35.3553m \\ c_y = -35.3553m \end{array}}$

$F_x = 30.3603m$ $\quad$ $F_y = -23.2962m$

$F = 38.2683m \rightarrow a+b+c$

$\theta_F = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{23.2962}{30.3603}\right)$

$\boxed{\theta_F = 37.4999^\circ}$

(ii) the magnitude and angle of $a-b+c$?

SOL:-

let $a-b+c = G$

angle $= \theta_G$

$G_x = a_x + b_x + c_x \Rightarrow G_x = 43.3013 - (-48.2963) + 35.3553$

$\boxed{G_x = 126.9529m}$

$G_y = a_y - b_y + c_y$

$= 25 - (-12.9410) + (-35.3553)$

$\boxed{G_y = 2.5857}$

$\boxed{G = 126.9792m}$ $a-b+c$ $\quad$ $\boxed{\theta_G = 1.1668^\circ}$

(iii) mag and angle of fourth vector d such that $(a+b) - (c+d) = 0$?

$(a_x + b_x) - (c_x + d_x) \Rightarrow (43.3013 - 48.2963) - (35.3553 + d_x) = 0$

$(-4.995) - (35.3553) + d_x = 0$

$-40.3503 - d_x = 0 \Rightarrow -d_x = 40.3503$

$\boxed{d_x = -40.3503}$

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$(4y+by)-(cy+dy)=0$

$(25-12.9410)-(-35.3553+dy)=0$

$47.4143-dy=0$ $\quad dy=-47.4143m$

$d=62.2596m$

$\theta=49.6017^\circ$

8.

Data: $\vec{A}=2i-3j+5k$

$\theta x=?,\ \theta y=?,\ \theta z=?$

Sol:- for θx

$$\cos\theta x=\frac{\vec{A}\cdot\vec{x}}{|A||x|}\Rightarrow\cos\theta x=\frac{2}{6.1644}$$

$$\theta x=\cos^{-1}\left(\frac{2}{6.1644}\right)\Rightarrow\theta x=35.7956^\circ$$

for θy:

$$\cos\theta y=\frac{\vec{A}\cdot\vec{y}}{|A||y|}\Rightarrow\cos\theta y=\frac{-3}{6.1644}$$

$$\theta y=\cos^{-1}\left(\frac{-3}{6.1644}\right)\qquad\theta y=119.1216^\circ$$

$\theta z=?$

$$\cos\theta z=\frac{\vec{A}.\vec{z}}{|A||z|}\Rightarrow\cos\theta z=\frac{5}{6.1644}$$

$$\theta z=\cos^{-1}\frac{5}{6.1644}\Rightarrow\theta z=35.7956^\circ$$

9.

Data:

$\vec{a}=5i+4j-6k$, $\vec{b}=-2i+2j+3k$, $\vec{c}=4i+3j+2k$

$r=a+b+c=?$

angle blw 'a' and 'b' $=\theta=?$

" " $=$ 'r' " +ve x-axis $=\theta x=?$

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Sol:-

$|a| = \sqrt{(5)^2+(4)^2+(6)^2} \Rightarrow |a| = 8.775$

$|b| = \sqrt{(-2)^2+(2)^2+(3)^2} \Rightarrow |b| = 4.1231$

$\vec{a}.\vec{b} = (5i+4j-6k).(-2i+2j+3k)$

$= -10+8-18$

$\boxed{\vec{a}.\vec{b} = -20}$

angle blw a and b

$\cos\theta = \frac{\vec{a}.\vec{b}}{ab} = \frac{-2}{(8.775)(4.1231)}$

$\cos\theta = -0.5528 \Rightarrow \boxed{\theta = 123.5593^\circ}$

$r = a+b+c = (5-2+4)i+(4+2+3)j+(-6+3+2)k$

$r = 7i+9j-k$

$|r| = \sqrt{(7)^2+(9)^2+(1)^2} \Rightarrow |r| = \sqrt{131}$

$\boxed{|r| = 11.4455}$

angle with z-axis

$\cos\theta = \frac{\vec{r}.\vec{z}}{rz} \Rightarrow \cos\theta = \frac{-1}{11.4455}$

$\boxed{\theta = 95^\circ}$

10. Data:

$\vec{A}$ = 6 units, $\vec{B}$ = 7 units, $\vec{A}.\vec{B} = 14$

Sol:-

$\cos\theta = \frac{\vec{A}.\vec{B}}{AB} \Rightarrow \cos\theta = \frac{14}{42}$

$\theta = \cos^{-1}(0.3333) \Rightarrow \boxed{\theta = 70.5288^\circ}$

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Motion in 1D

1. The position of a particle moving in an XY direction is given by
$r = \{(2)t^3 - 5t\}i + \{6 - 7t^4)\}j$; Calculate (a) r (b) V (c) a when $t = 2$ sec.

Sol:-

$$r = \{2t^3 - 5t\}i + \{6 - 7t^4\}j$$

given $t = 2$ sec.

$$r = \{2(2)^3 - 5(2)\}i + \{6 - 7(2)^4\}j$$

$$r = \{16 - 10\}i + \{6 - 112\}j$$

$$\boxed{r = \{6\}i - \{106\}j \quad \text{Ans}}$$

(b) V

$$\frac{dr}{dt} = V \Rightarrow \frac{dr}{dt} = \frac{d}{t}\{2t^3 - 5t\}i + \{6 - 7t^4\}j$$

$$\frac{dr}{dt} = (6t^2 - 5)i + (-28t^3)j$$

given $t = 2$sec

$$V = (6(2)^2 - 5)i + (-28(2)^3)j$$

$$V = (24 - 5)i + (-224)j$$

$$\boxed{V = 19i - 224j \text{ Ans.}}$$

(c) a

$$a = \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = \frac{d}{dt}\{6t^2 - 5\}i + \{-28t^3\}j$$

$$a = (12t)i + (-84t^2)j$$

given $t = 2$ sec

$$a = 12(2)i + (-84(2)^2)j$$

$$\boxed{a = 24i - 336j}$$

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2. A particle had a velocity of 18m/s in the +x direction and 2.4 sec later its velocity was 30m/s in the opposite direction. What was the average acceleration of the particle during this 2.4 sec interval?

Data:

$v_i = 18\ m/s$

$v_f = -30\ m/s$

$t = 2.4\ s$

$a_{avg} = ?$

Sol:-

$$a_{avg} = \frac{v_f - v_i}{t} \Rightarrow a_{avg} = \frac{-30\ m/s - 18\ m/s}{2.4\ s}$$

$$a_{avg} = \frac{-48\ m/s}{2.4\ s} \Rightarrow \boxed{a_{avg} = -20\ m/s} \text{ Ans.}$$

3. A rocket ship in free space moves with constant acceleration equal to $9.8\ m/s^2$ (a) if it starts from rest, how long will it take to acquire a speed one-tenth that of light? (b) How far will it travel in so doing? (The speed of light is $3 \times 10^8\ m/s$).

Data:

$a = 9.8\ m/s^2$

$v_i = 0$

$v_f = 3 \times 10^7\ m/s$

(a) $t = ?$

(b) $s = ?$

$t = ? \Rightarrow v_f = v_i + at \Rightarrow t = v_f - v_i / a$

$$t = \frac{3 \times 10^7\ m/s - 0}{9.8\ m/s^2} \Rightarrow \boxed{t = 3.06 \times 10^6\ s}$$ Ans.

$s = ? \quad s = v_i t + \frac{1}{2} a t^2$

$$s = (0)t + \frac{1}{2} \times 9.8 \times (3.06 \times 10^6)^2 \Rightarrow \boxed{s = 4.7 \times 10^{13}\ m}$$ Ans.

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4. At a construction site a pipe wrench strikes the ground with a speed of 24 m/s (a) From what height was it inadvertently dropped? (b) For how long was it falling?

Data:

$V_f = 24\ m/s$

$V_i = 0$

$a = g = 9.8\ m/s^2$

(a) s = height $= h = ?$ $\quad V_f^2 - V_i^2 = 2gh \Rightarrow h = V_f^2 - V_i^2 / 2g$

(b) t = time $= ?$ $\quad t = V_f - V_i / g$

(a) $h = ?$ $\quad h = \dfrac{V_f^2 - V_i^2}{2g} \Rightarrow h = \dfrac{(24)^2 - (0)^2}{2(9.8)}$

$\boxed{h = 29.38\ m}$ Ans

(b) $t = ?$ $\quad t = \dfrac{V_f - V_i}{g} \Rightarrow t = \dfrac{24 - 0}{9.8}$

$\boxed{t = 2.448\ s}$ Ans

5. Data:

$t = 2.25\ s$

$h = 36.8\ m$

(a) $V_i = ?$ $\quad s = V_i t + 1/2\ gt^2 \Rightarrow h = Vt + \frac{1}{2} gt^2$

(b) V_f (at $h = 36.8\ m$) $= ?$

(c) total $h = ?$

(a) $V_i = ?$ $\quad 36.8\ m = V_i (2.25\ s) + \dfrac{1}{2} \times 9.8 \dfrac{m}{s^2} \times (2.25)^2$

$36.8\ m = V_i (2.25\ s) + 24.806$

$V_i = \dfrac{36.8 - 24.806}{2.25\ s} = \dfrac{25}{25.775}$

$\boxed{V_i = 27.775\ m/s}$

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(b) v_f (at $h = 36.8\,m$) = ?

$v_f = v_i + gt$

$v_f = 27.775 - 9.8(2.25)$

~~$v_f = 49.825$~~ $\quad \boxed{v_f = 5.725\,m/s}$

(c) total h = ?

$v_f = 0\,m/s$

$v_i = 27.385\,m/s$

$a = g = -9.8\,m/s^2$

h = ?

Sol:-

$2as = v_f^2 - v_i^2$

$2gh = v_f^2 - v_i^2 \Rightarrow 2(-9.8)h = 0 - (27.7)^2$

$-19.6h = -749.6$

$\boxed{h = 38.2\,m}$ Ans.

6. Data:

$v_i = 40\,km/h$

$v_f = 60\,km/h$

Sol:- $v_{f_{avg}} = \dfrac{2s}{t_1 + t_2} \Rightarrow v_{f_{avg}} = \dfrac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} \Rightarrow v_{f_{avg}} = \dfrac{2s}{\frac{sv_2 + sv_1}{v_1 v_2}}$

$v_{f_{avg}} = \dfrac{2s \times v_1 v_2}{s(v_1 + v_2)}$ $\qquad v_{avg} = \dfrac{2s(v_1 \cdot v_2)}{(v_1 + v_2)}$ $\quad \therefore v_1 = v_i$, $v_2 = v_f$

$v_{avg} = \dfrac{2(40 \times 60)}{40 + 6} = \dfrac{4800}{100} = \boxed{48\,m/s}$ Ans.

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Q.7

Data:

$v_i = 12.4 m/s$

$s = h = 81.3 m$

$a = g = 9.8 m/s^2$

$v_f = ?$

$t = ?$

(a) $2gh = v_f^2 - v_i^2$

$2(9.8)(81.3) = v_f^2 - (12.4)^2$

$1593.48 = v_f^2 - 153.76$

$v_f^2 = 1747.24$

$\boxed{v_f = \pm 41.8 m/s}$ Ans

(b) $t = \frac{v_f - v_i}{g}$

$t = \frac{41.8 - 12.4}{9.8}$

$\boxed{t = 3 seconds}$

Q.8

Data:

$v_f = 360 km/h = \frac{360 \times 1000}{3600} = 100 m/s$

$v_i = 0$

$s = 1.8 km = 1800 m$

$a = ?$

Sol:-

$2as = v_f^2 - v_i^2$

$2a(1800) = (100)^2 - 0^2$

$3600a = 1000$

$\boxed{a = 2.7778 m/s^2}$

Q.9

Data:

$a = -4.92 m/s^2$

$v_i = 24.6 m/s$

$v_f = 0$

$t = ?$

$s = ?$

Sol:-

(a) $t = \frac{v_f - v_i}{a}$

$t = \frac{0 - 24.6}{-4.92}$

$\boxed{t = 5 sec}$

(b) $s = v_i t + \frac{1}{2} a t^2$

$s = (24.6)(5) + \frac{1}{2}(-4.92)(5)^2$

$s = 123 - 61.5$

$\boxed{s = 61.5 m}$

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Q. 10

Data:

$x = 50t + 10t^2$

$t_o = 0\,s$

$t_f = 3\,s$

$v_{avg} = ?$

$v_{ins} = ?$

$a_{ins} = ?$

Sol:-

(a) Distance at $t=0$

$x_o = 50(0) + 10(0)^2 \quad x_o = 0$

Distance at $t = 3$ sec

$x_f = 50(3) + 10(3)^2$

$x_f = 150 + 10(9) \Rightarrow \boxed{x_f = 240}$ Ans.

(b) velocity $\frac{dx}{dt} = \frac{d}{dt}(50t + 10t^2)$

$v = (50 + 20t)$

$t = 3$ sec

$v = 50 + 20(3) \Rightarrow \boxed{v = 110\ m/s}$

(c) acceleration $\frac{dv}{dt} = \frac{d}{dt}(50 + 20t)$

$a = 20 \Rightarrow \boxed{a = 20\ m/s^2}$ Ans.

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