National University
Of Computer & Emerging Sciences Karachi Campus

EE 117: Applied Physics

Motion in Two and Three Dimensions

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Chapter 4: Motion in Two and Three Dimension

4.1. What is Physics?

4.2. Motion

4.3. Position and Displacement

4.4. Average Velocity and Instantaneous Velocity

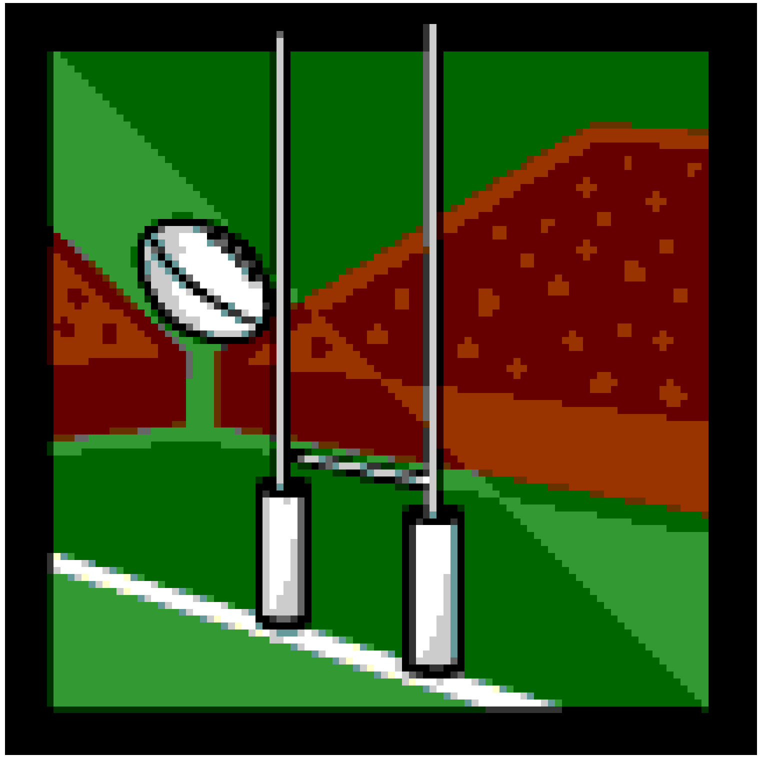
4.5. Average Acceleration and Instantaneous Acceleration

4.6. Projectile Motion

4.7. Uniform Circular Motion

Motion

- **Motion** – an object's change in position relative to a reference point



What is Mechanics?

- Mechanics is the science which describes and predicts the conditions of rest or motion of bodies under the action of forces.

- Categories of Mechanics:
  - Rigid bodies
    - *Statics*
    - Dynamics
  - Deformable bodies
  - Fluids

- Mechanics is an applied science.

- Mechanics is the foundation of most engineering sciences and is an indispensable prerequisite to their study.

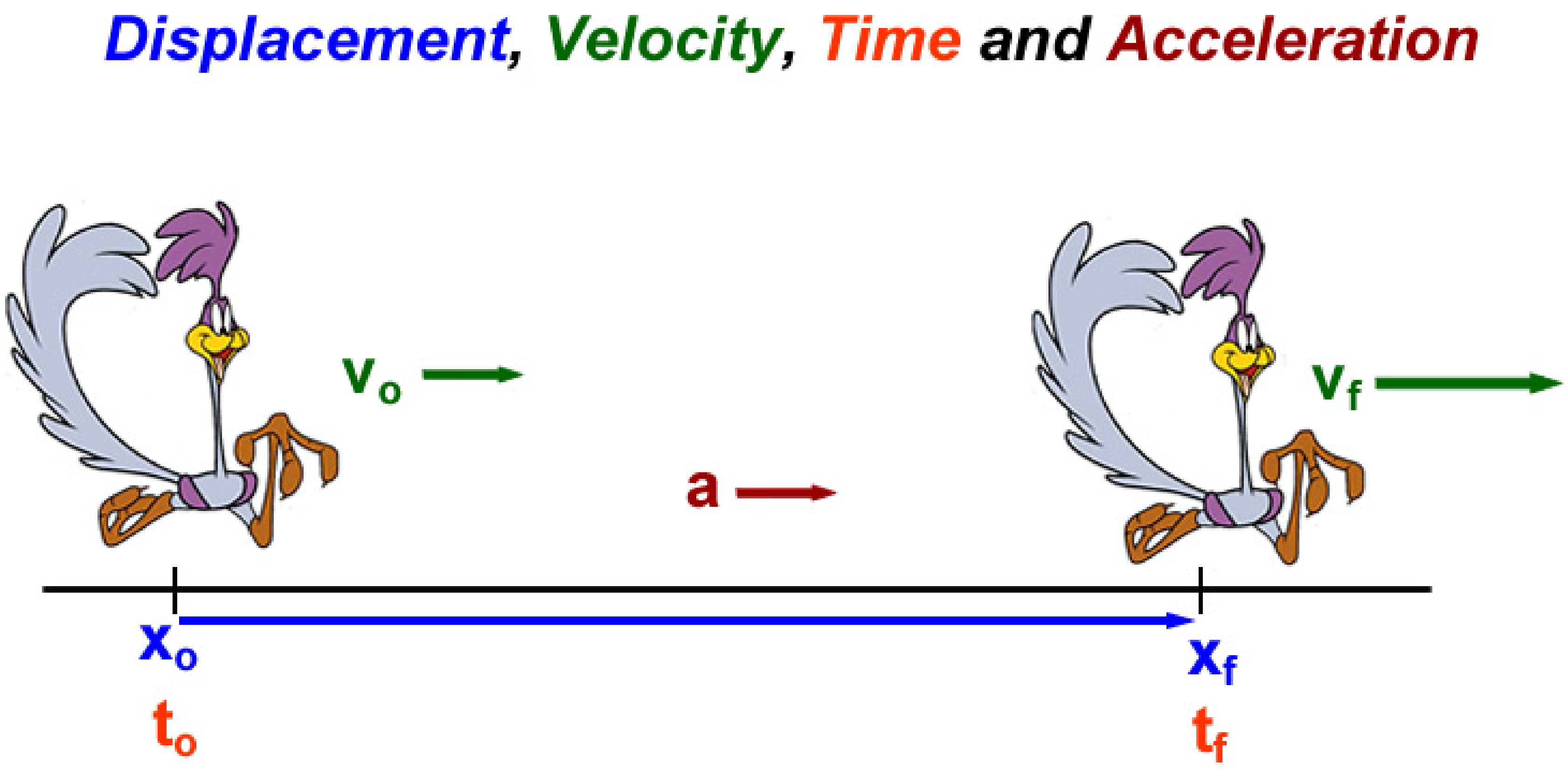
Kinematics – describing motion

Object is treated as a **particle** (a point-like concentration of matter that has no size, no shape and no internal structure).

Questions to ask:

- Where is the particle?
- How fast is it moving?
- How rapidly is it speeding up or slowing down?

Basic Quantities in Kinematics

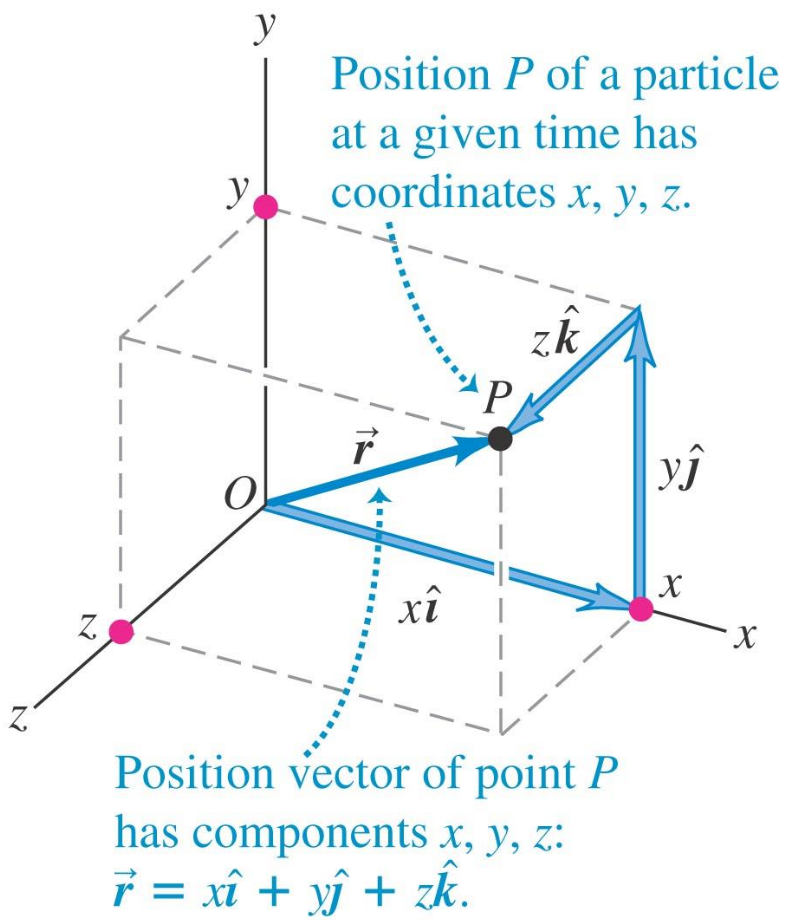


Introduction

- What determines where a batted baseball lands?
- If a cyclist is going around a curve at constant speed, is he accelerating?
- How is the motion of a particle described by different moving observers?
- We need to extend our description of motion to two and three dimensions.

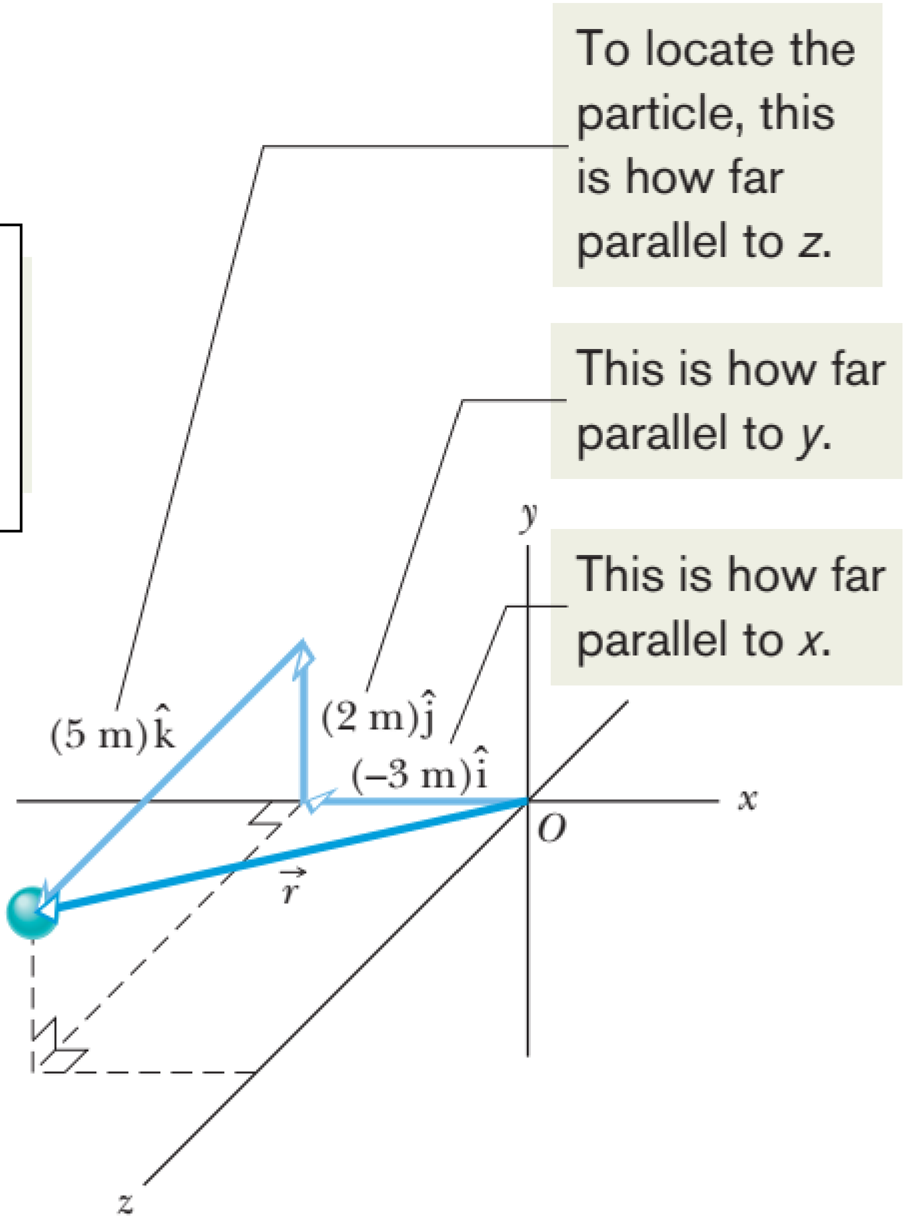
Position vector

- The position vector from the origin to point *P* has components *x*, *y*, and *z*.



Define a position first

$$\vec{r} = \boxed{} + \boxed{} + \boxed{}$$

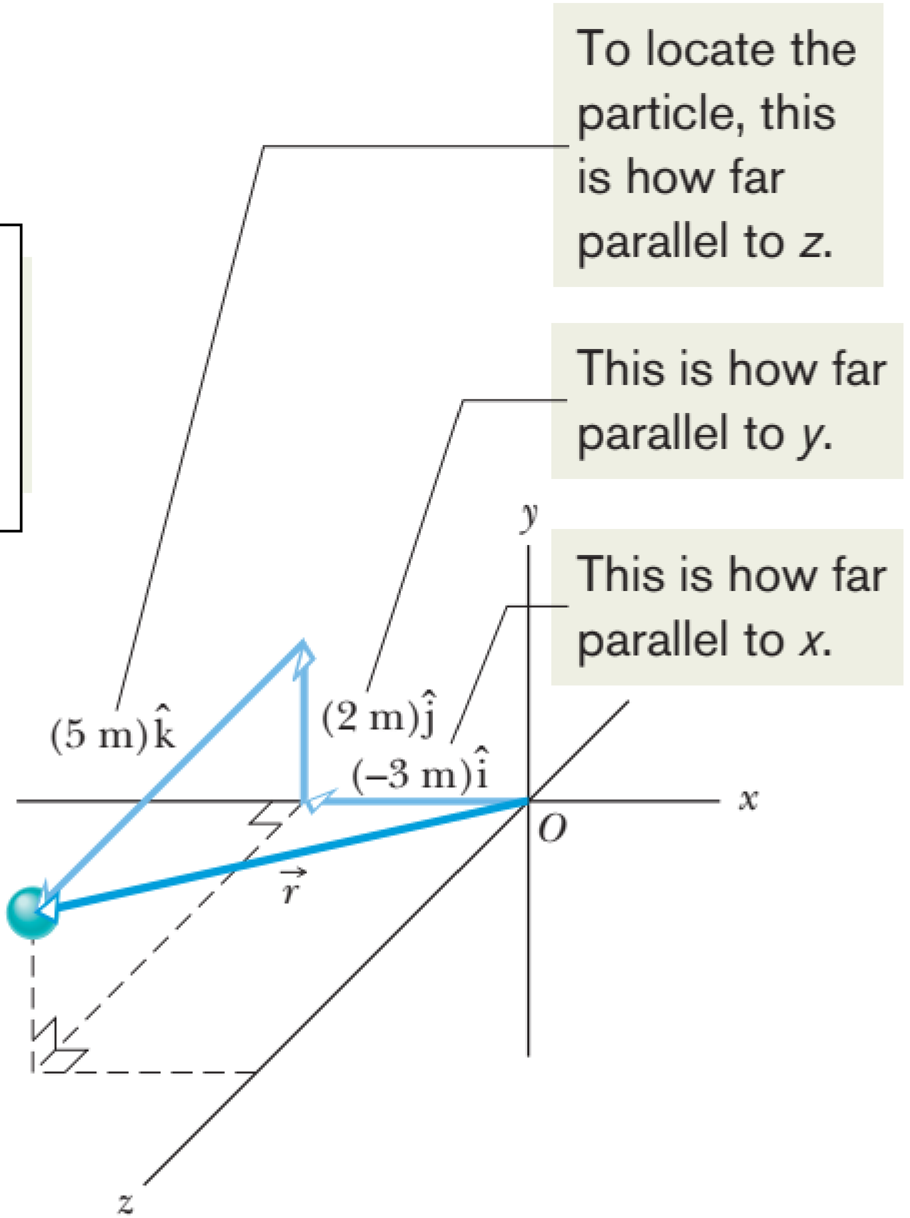


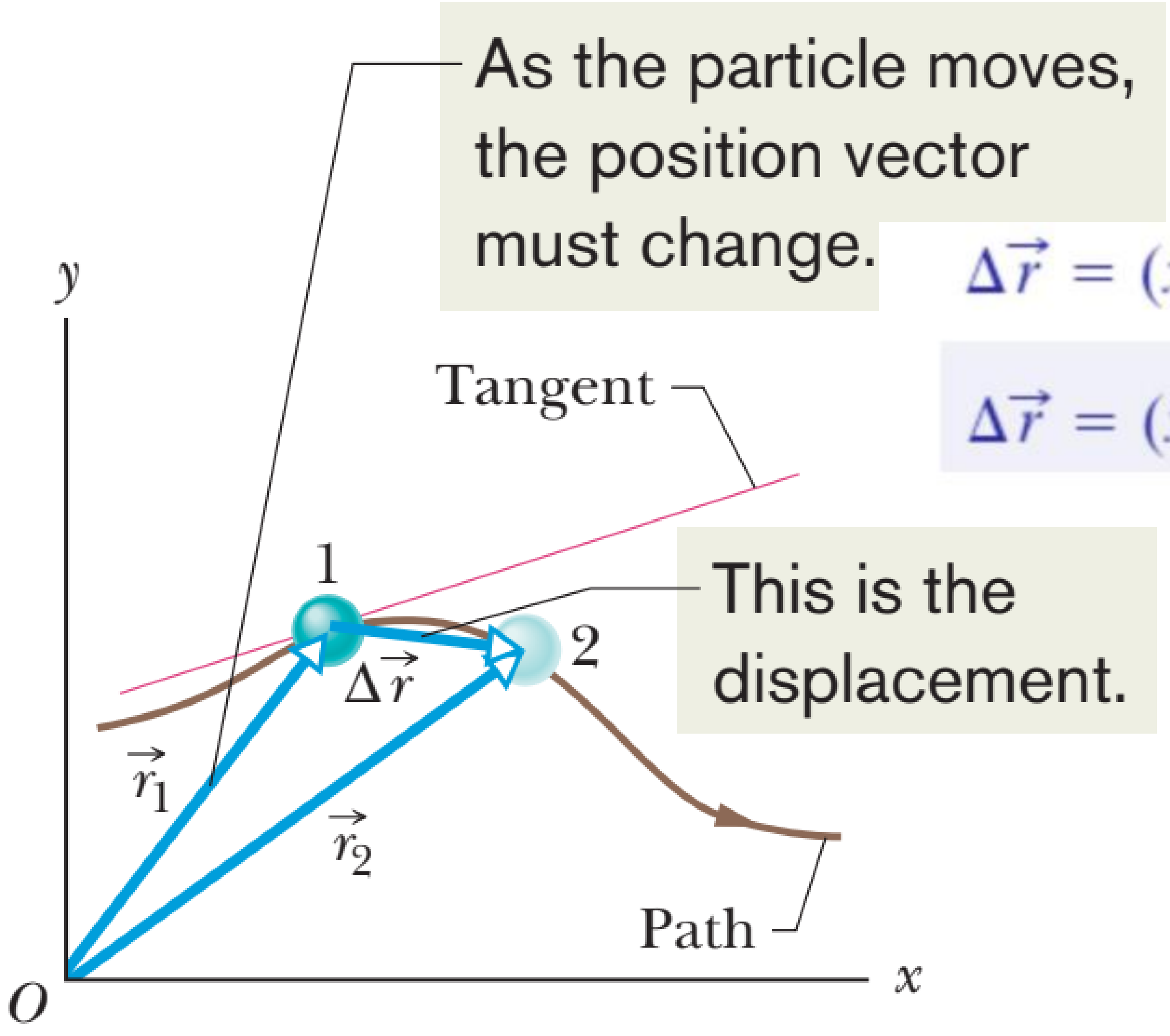
Define a position first

$$\vec{r} = \boxed{} + \boxed{} + \boxed{}$$

General position vector in 3D

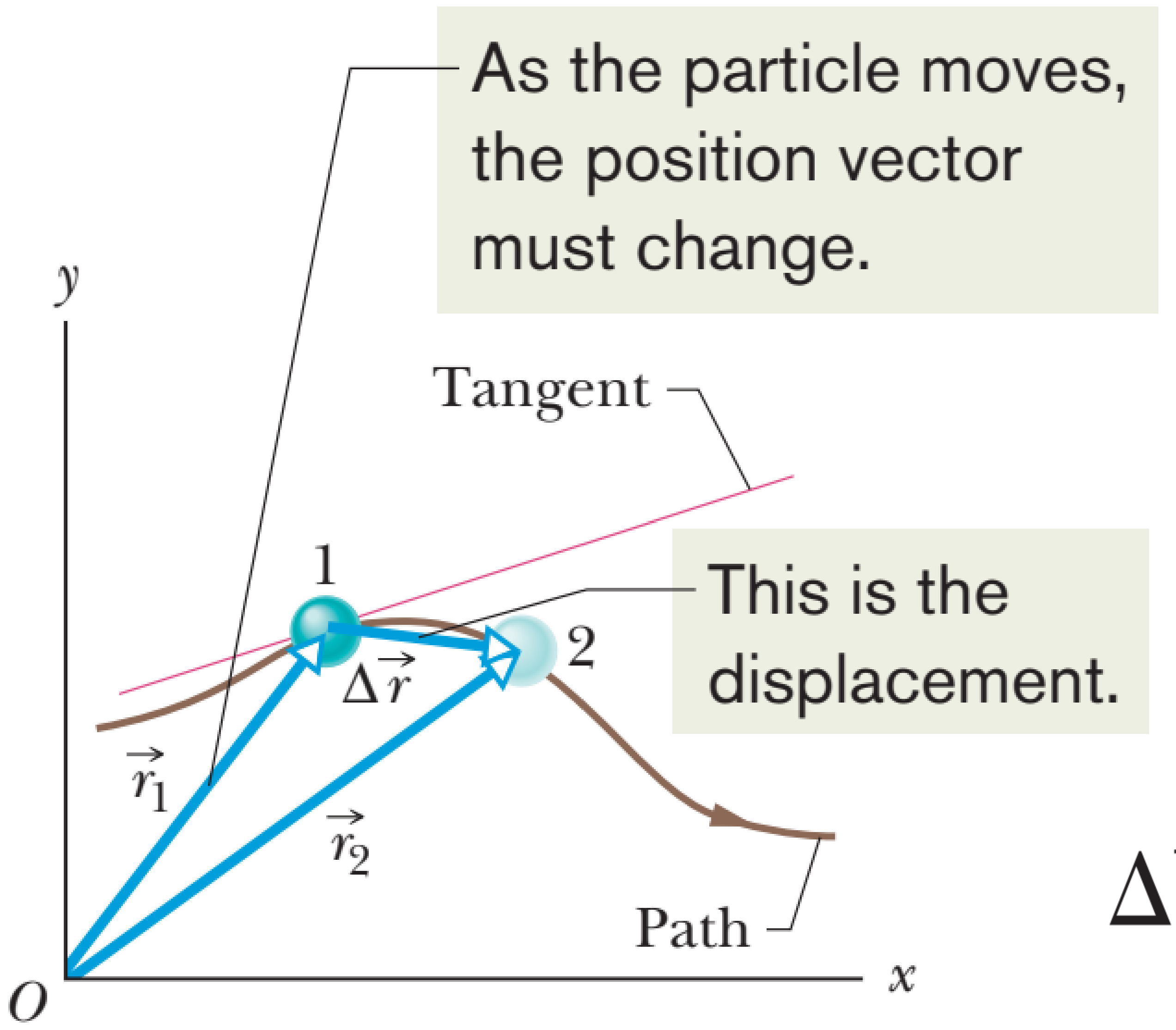
$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$$





$$\Delta\vec{r} = (x_2\hat{\mathrm{i}} + y_2\hat{\mathrm{j}} + z_2\hat{\mathrm{k}}) - (x_1\hat{\mathrm{i}} + y_1\hat{\mathrm{j}} + z_1\hat{\mathrm{k}})$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{\mathrm{i}} + (y_2 - y_1)\hat{\mathrm{j}} + (z_2 - z_1)\hat{\mathrm{k}},$$



$$\Delta\vec{r} = \Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}$$

Problem **two-dimensional motion**

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates of the rabbit's position as functions of time *t* (second) are given by

$$x = -0.31t^2 + 7.2t + 28$$

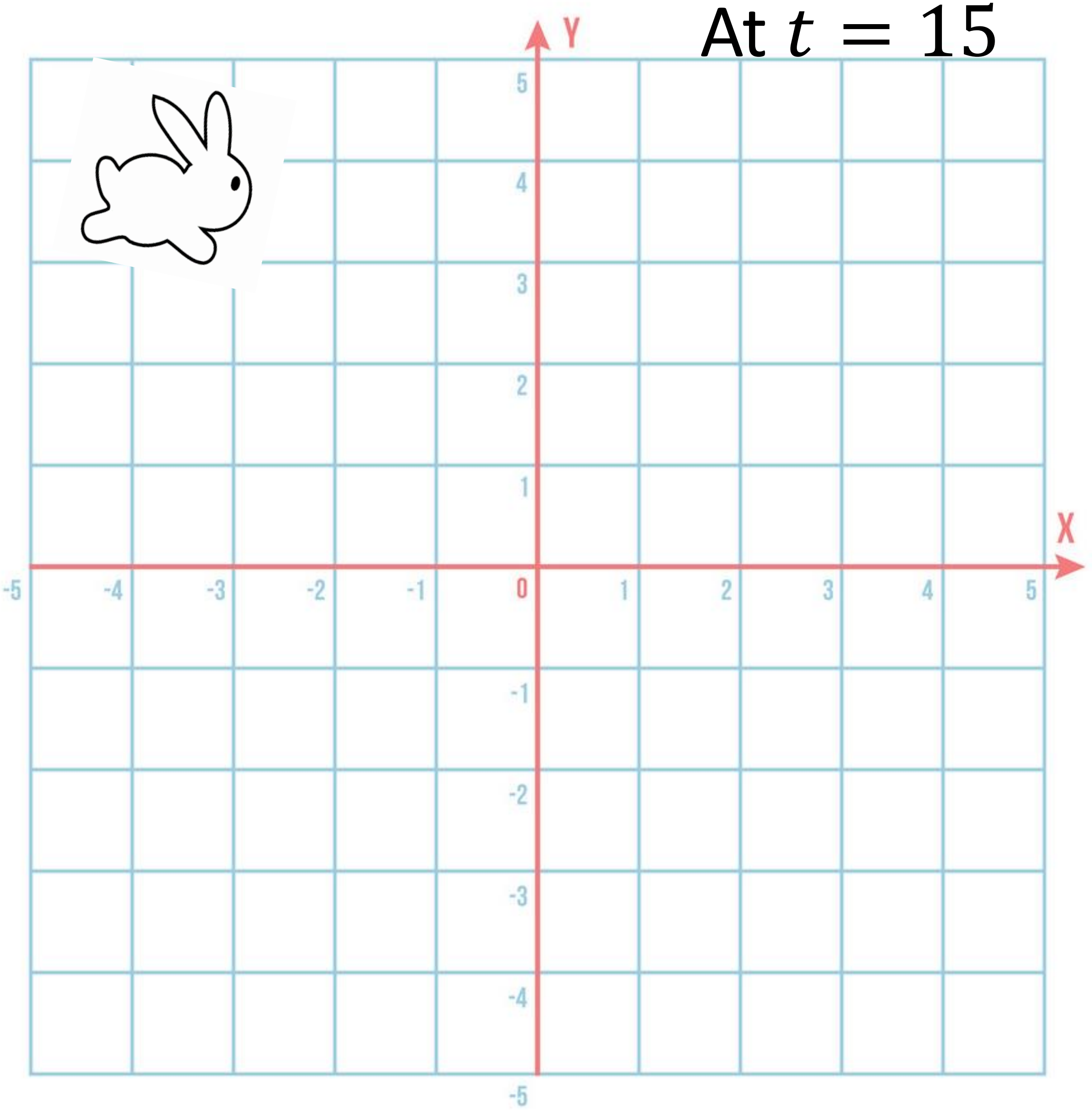
$$y = 0.22t^2 - 9.1t + 30,$$

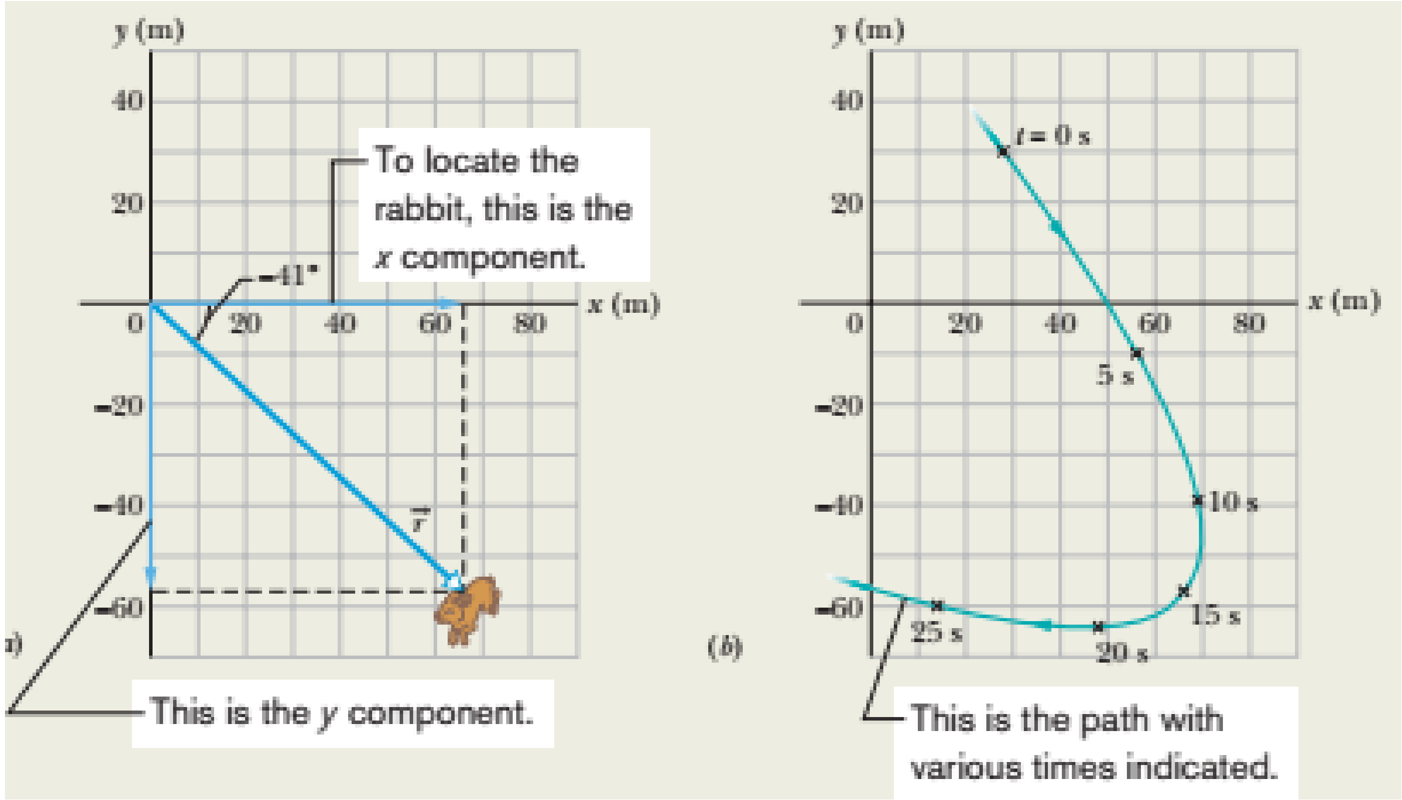
At t=15 s, what is the rabbit's position vector in unit-vector notation and in magnitude-angle notation?

$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

$$\vec{r}(t) = x(t)\hat{\mathrm{i}} + y(t)\hat{\mathrm{j}}$$

At $t = 15$



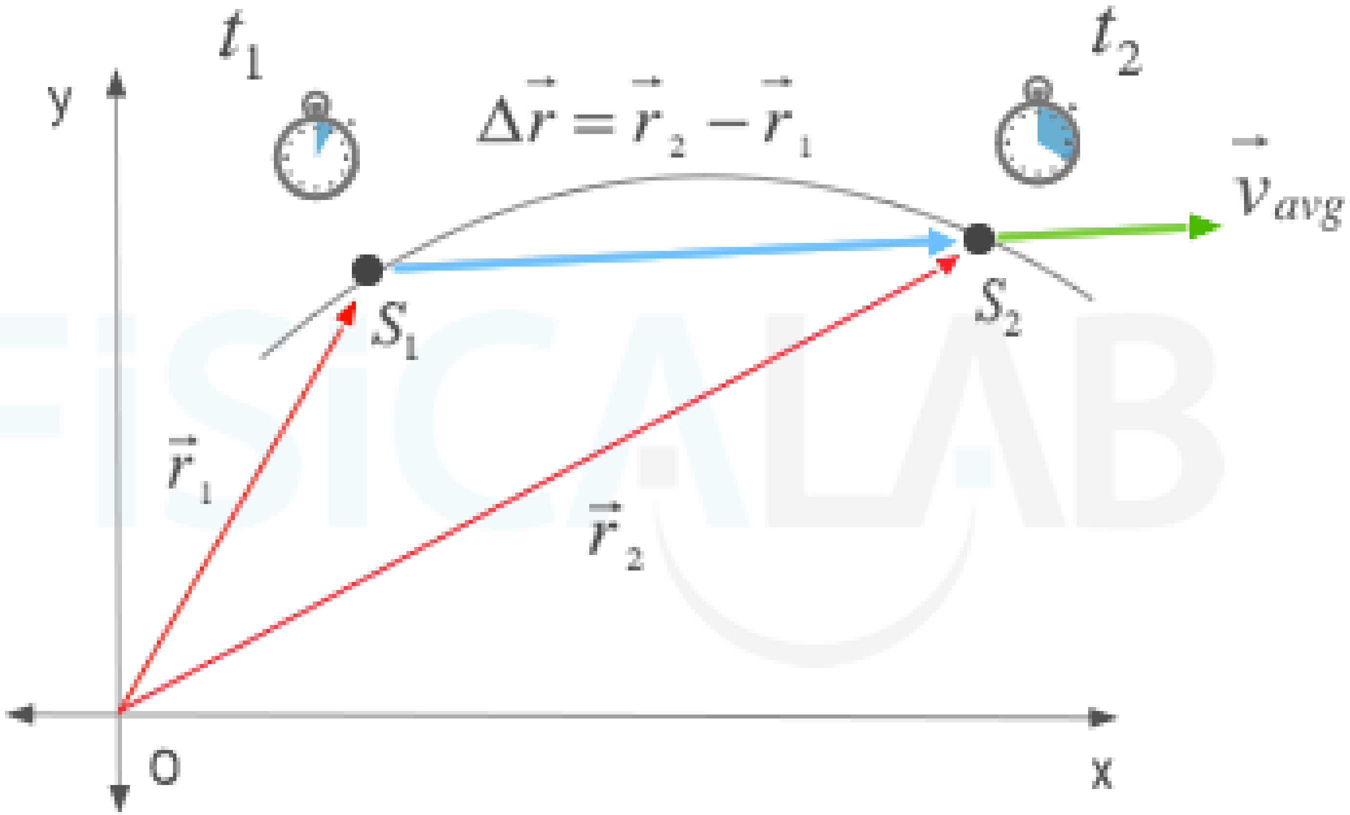


y (m)
40
20
0
−20
−40
−60
20
40
60
80
x (m)
−41°
To locate the rabbit, this is the x component.
$\vec{r}$
(a)
This is the y component.
(b)
y (m)
x (m)
t = 0 s
5 s
10 s
15 s
20 s
25 s
This is the path with various times indicated.

Velocity

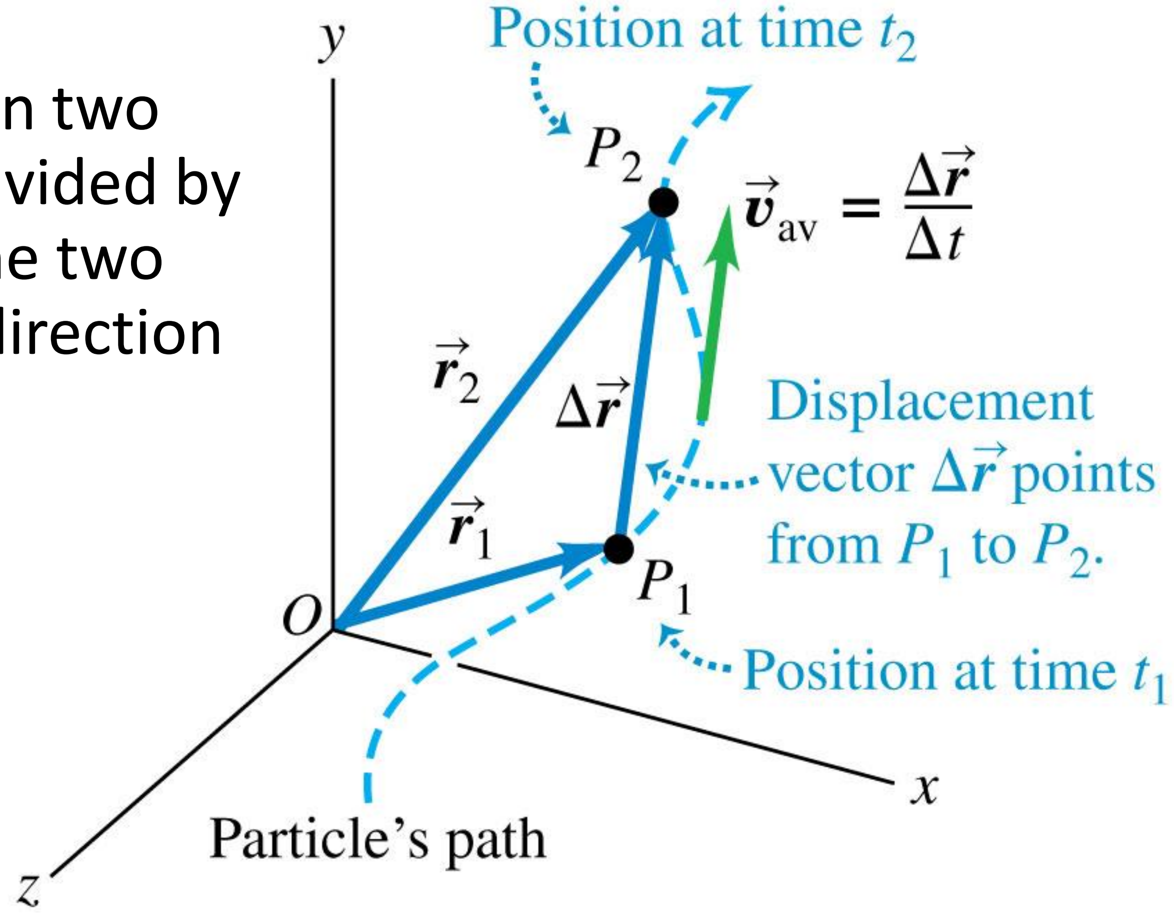
Average Velocity

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}.$$



Average velocity—Figure 3.2

- The average velocity between two points is the displacement divided by the time interval between the two points, and it has the same direction as the displacement.



Average and Instantaneous Velocity

$$\text{average velocity} \equiv \frac{\text{displacement}}{\text{time interval}},$$

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t}.$$

$$\vec{v}_{\text{avg}} = \frac{\Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\mathrm{i}} + \frac{\Delta y}{\Delta t}\hat{\mathrm{j}} + \frac{\Delta z}{\Delta t}\hat{\mathrm{k}}.$$

Instantaneous velocity is:

$$\vec{v} = \frac{d\vec{r}}{dt}.$$

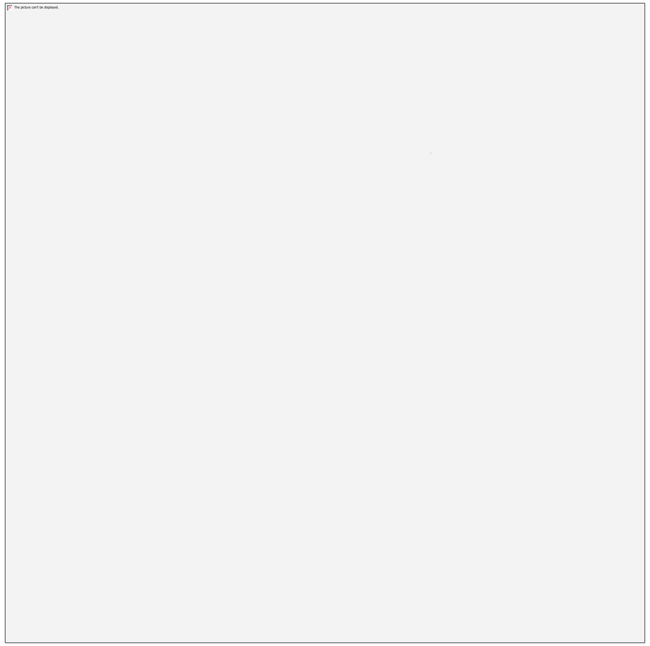
$$\vec{v} = \frac{d}{dt}\left(x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}\right) = \frac{dx}{dt}\hat{\mathrm{i}} + \frac{dy}{dt}\hat{\mathrm{j}} + \frac{dz}{dt}\hat{\mathrm{k}}.$$

$$v = \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$$

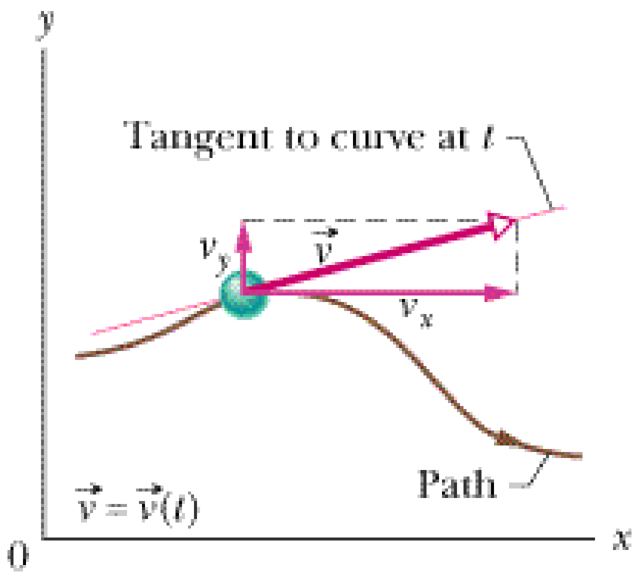
$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}},$$

Particle’s Path vs Velocity

Displacement:



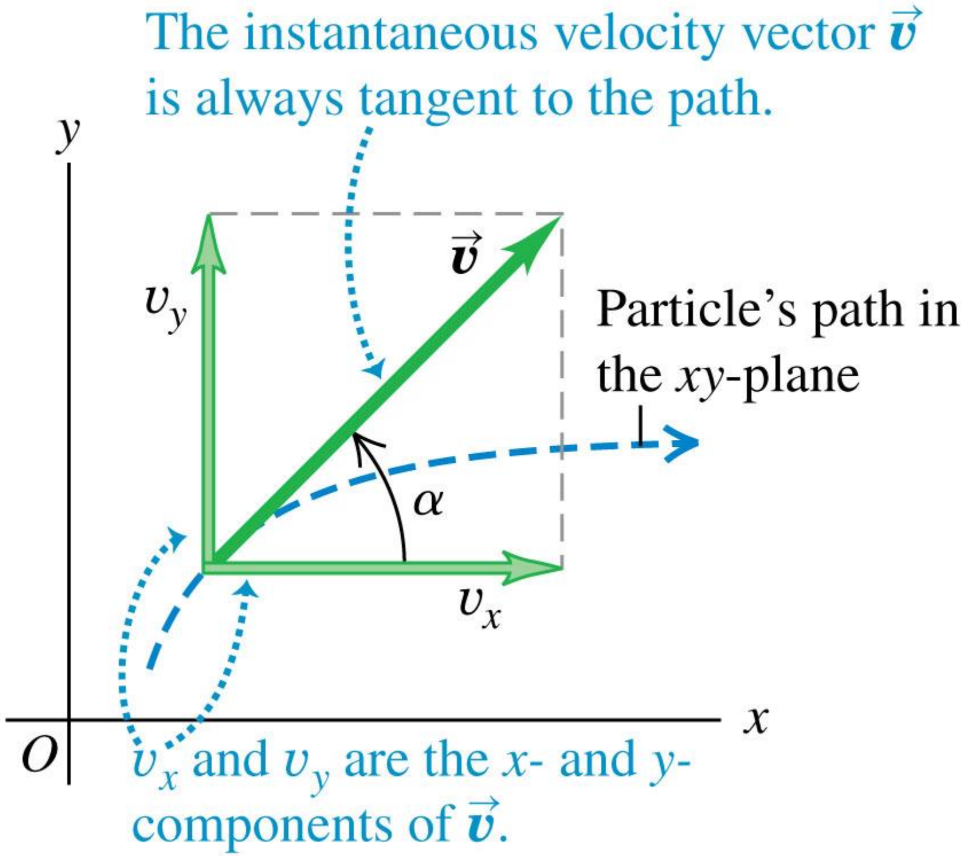
The velocity vector



The direction of the instantaneous velocity of a particle is always tangent to the particle’s path at the particle’s position.

Instantaneous velocity

- The *instantaneous velocity* is the instantaneous rate of change of position vector with respect to time.
- The components of the instantaneous velocity are $v_x = dx/dt$, $v_y = dy/dt$, and $v_z = dz/dt$.
- The instantaneous velocity of a particle is always tangent to its path.



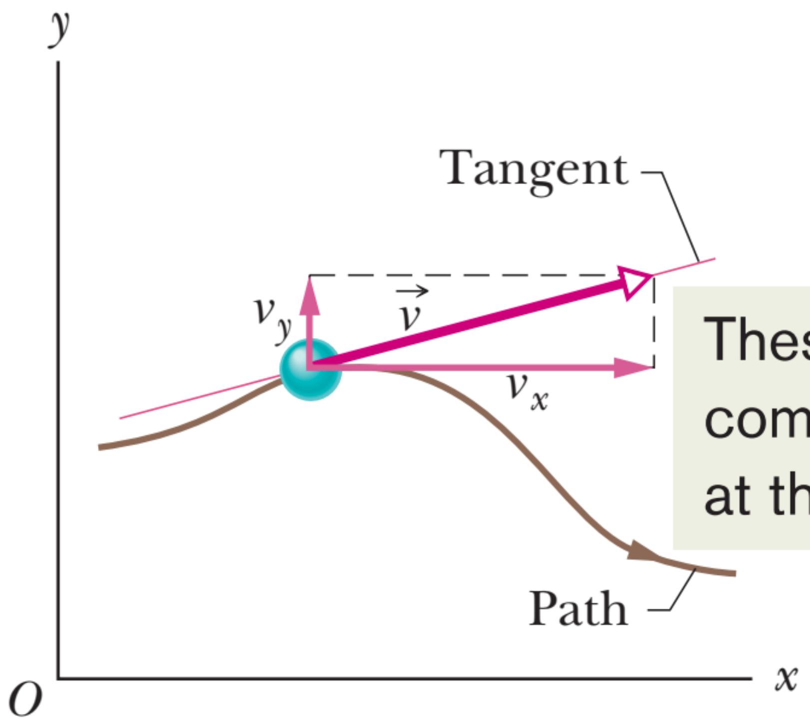
Velocity

The velocity vector is always tangent to the path.

Instantaneous Velocity

$$\Delta t \rightarrow 0$$

$$\vec{v} = \frac{d\vec{r}}{dt}.$$



These are the *x* and *y* components of the vector at this instant.

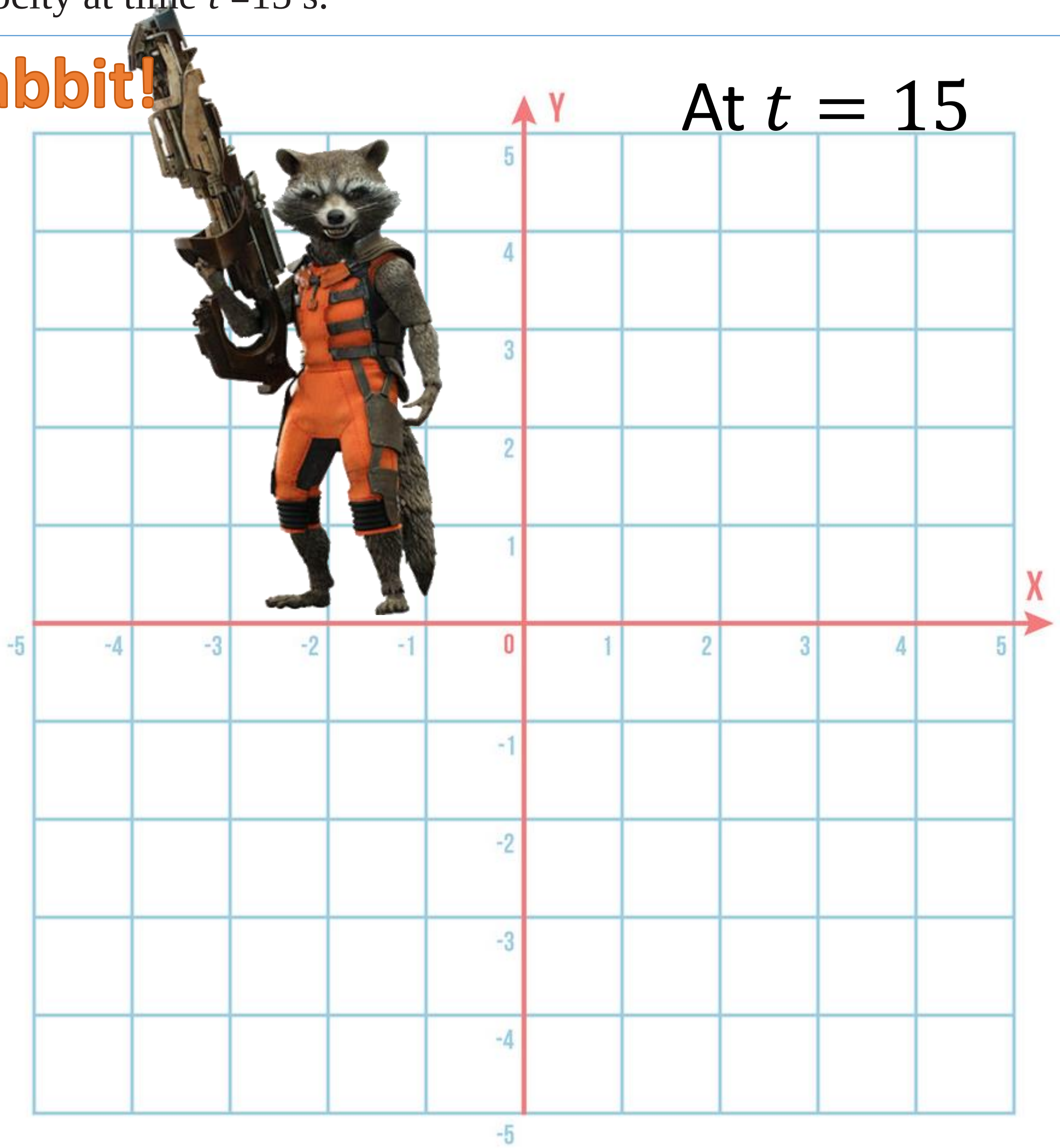
For the rabbit in the preceding sample problem, find the velocity at time t =15 s.

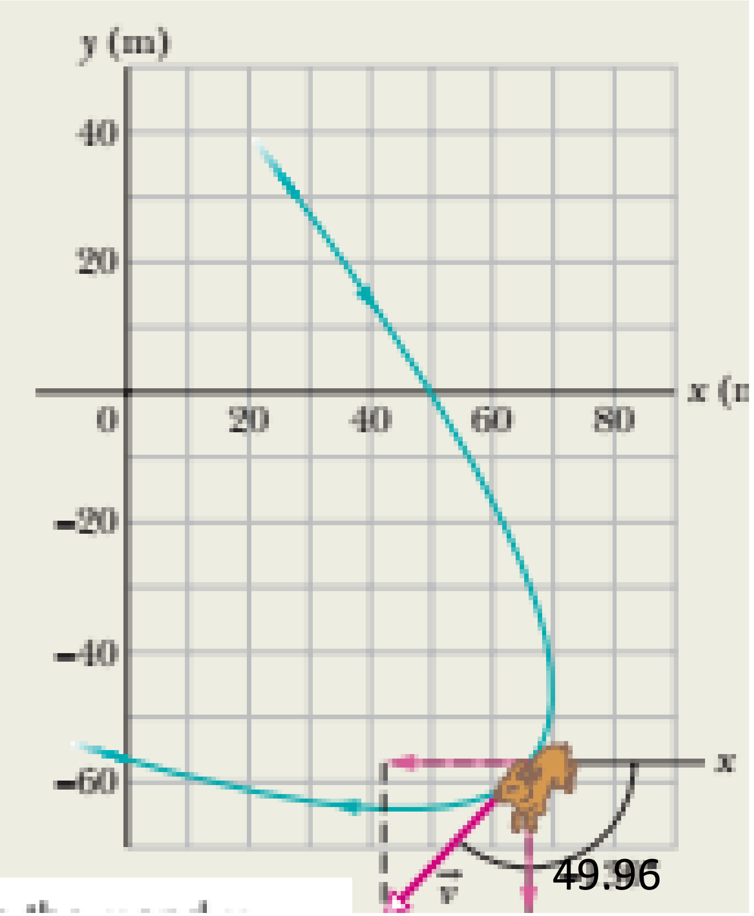
$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

Bring back the rabbit!

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}$$

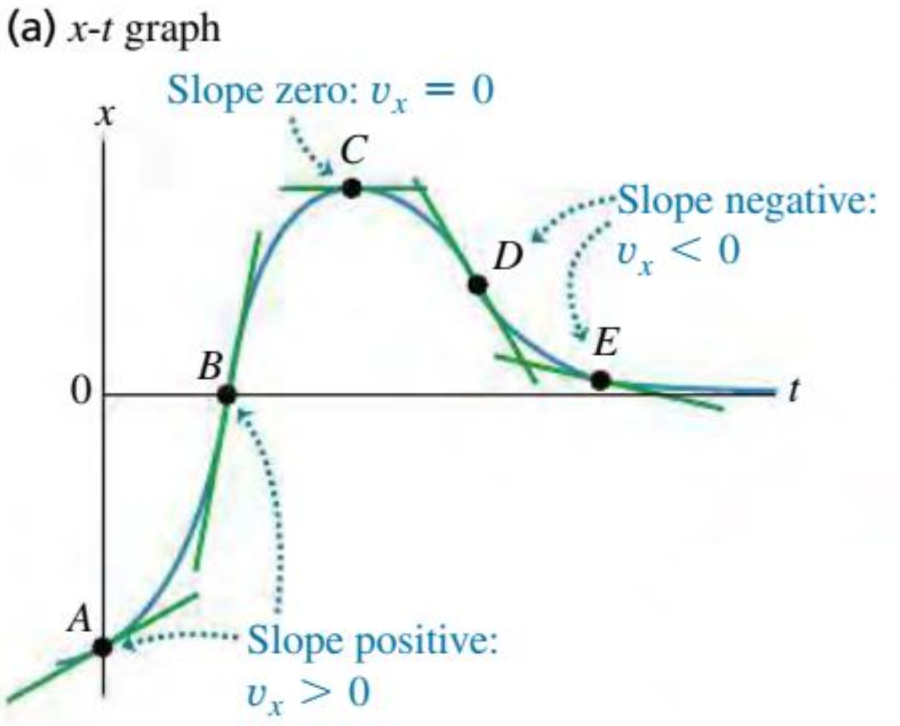
At $t = 15$

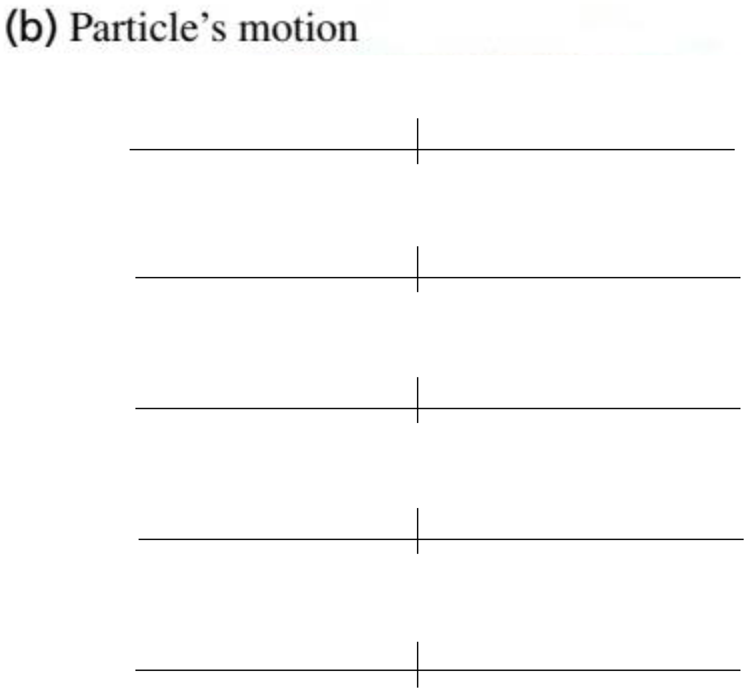




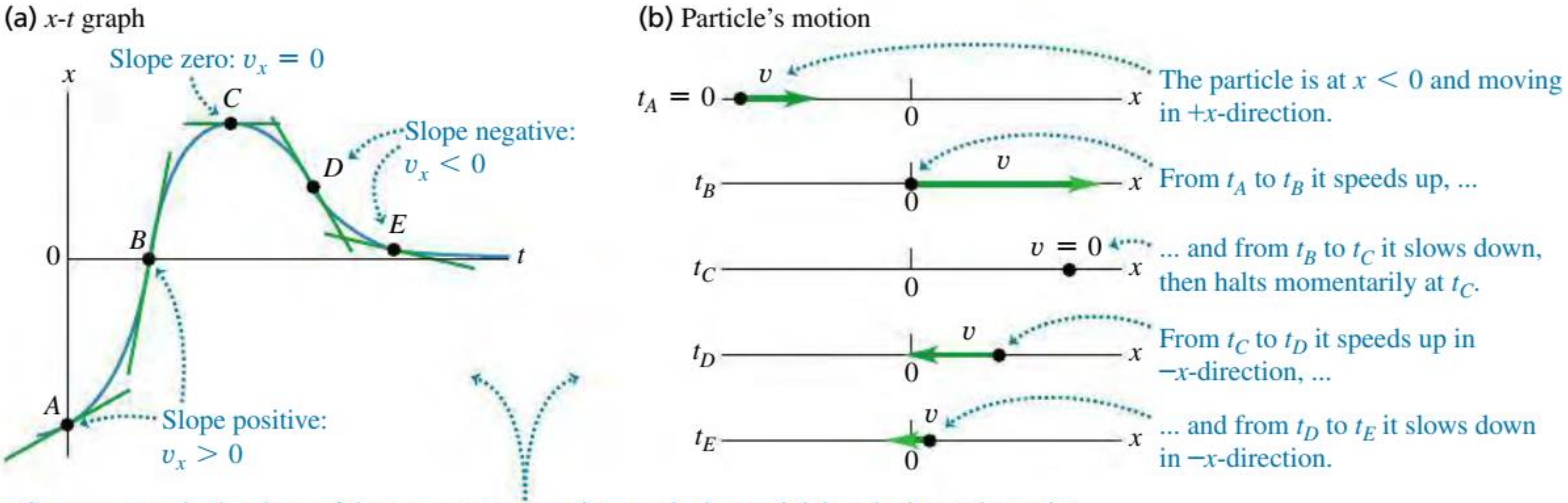
y (m)
40
20
0
20
40
60
80
−20
−40
−60
x
$\vec{v}$
49.96

Constant Velocities – graphical analysis





Constant Velocities – graphical analysis



- On an x-t graph, the slope of the tangent at any point equals the particle's velocity at that point.
- The steeper the slope (positive or negative), the greater the particle's speed in the positive or negative x-direction.

Average and Instantaneous Acceleration

Average acceleration is

$$\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} + \frac{\Delta v_z}{\Delta t}\vec{k} = \langle a_x \rangle \hat{i} + \langle a_y \rangle \hat{j} + \langle a_z \rangle \vec{k}$$

Instantaneous acceleration is

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

$$\begin{aligned} \vec{a} &= \frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right) \\ &= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}. \end{aligned}$$

Speed up or slow down

- If the velocity and acceleration components along a given axis have the *same sign* then they are in the same direction. In this case, the object will *speed up*.

- If the acceleration and velocity components have *opposite signs*, then they are in opposite directions. Under these conditions, the object will *slow down*.

Acceleration

$$\boxed{\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}}$$

$$\boxed{\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}.}$$

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt}.}$$

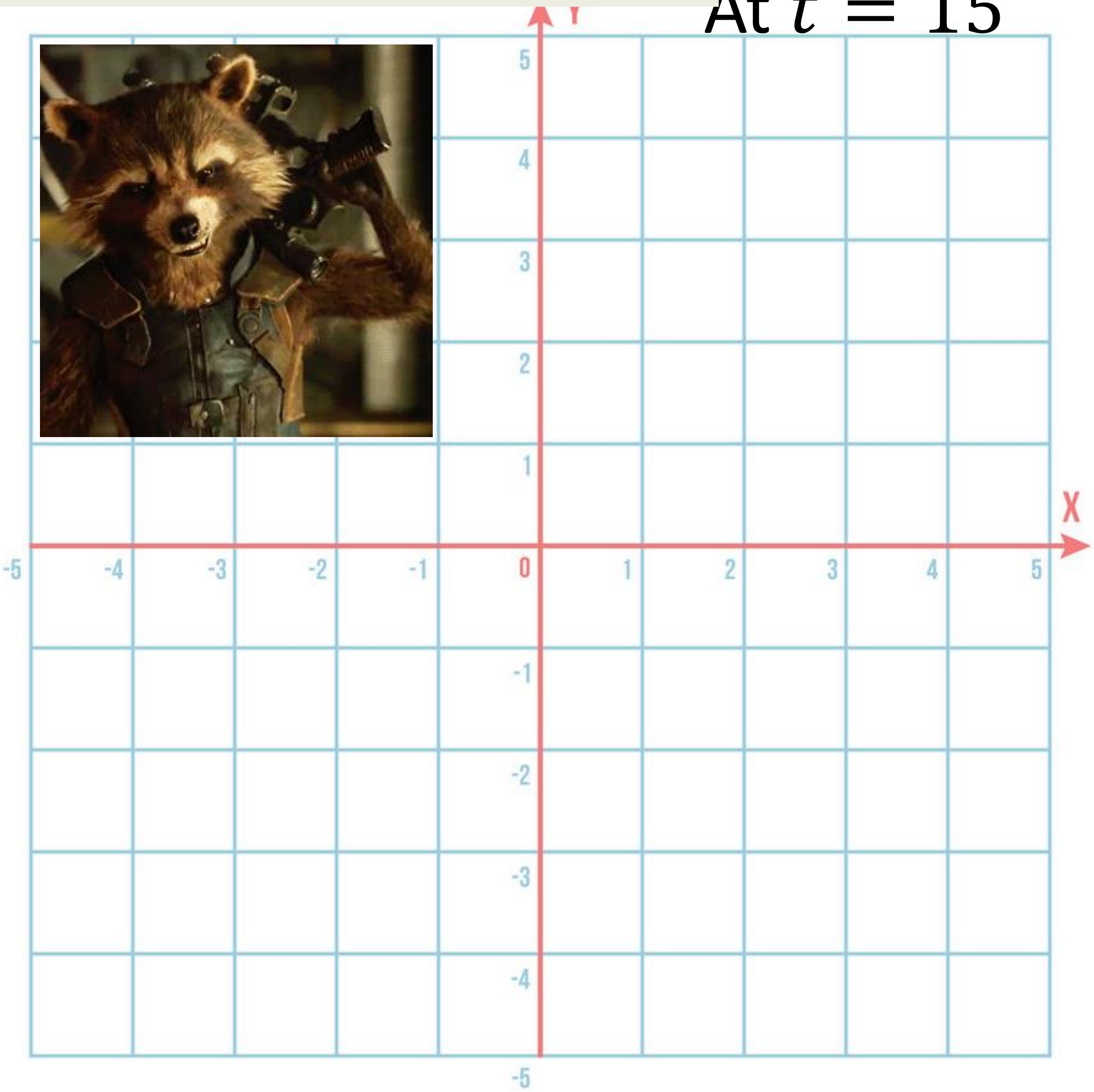
For the rabbit in the preceding two sample problems, find the acceleration $\vec{a}$ at time $t = 15$ s.

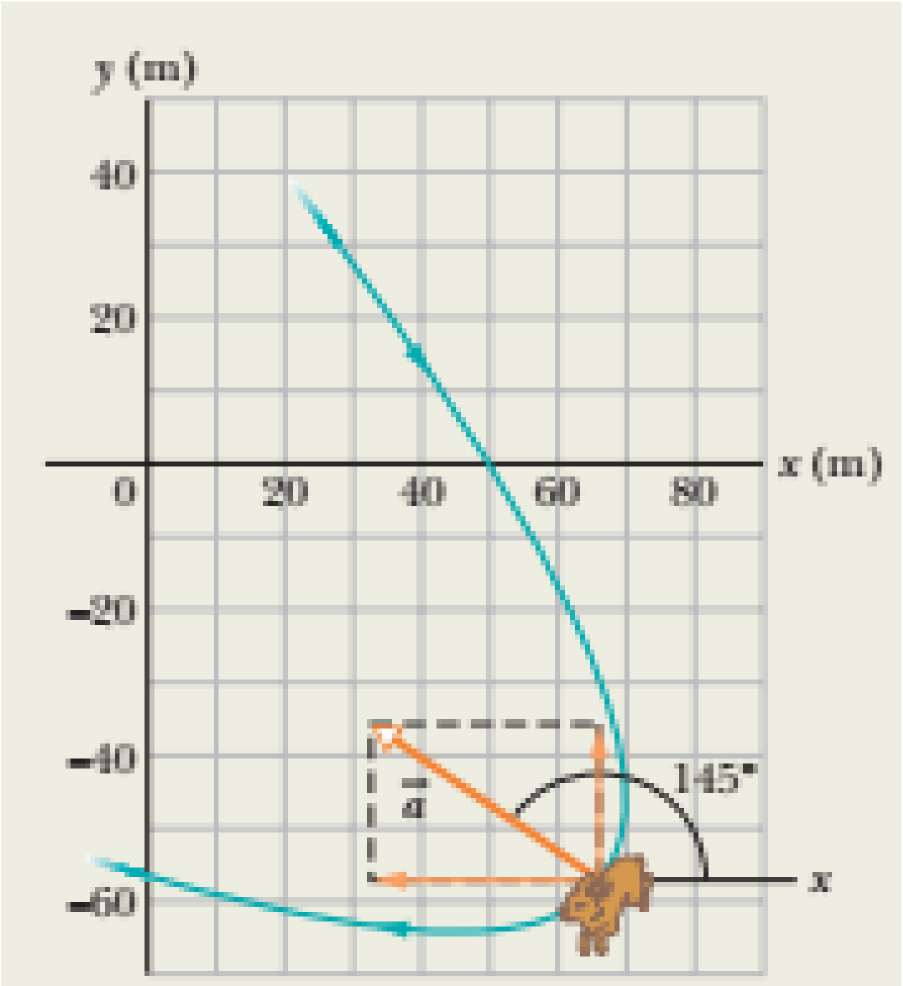
$$x = -0.31t^2 + 7.2t + 28$$

$$y = 0.22t^2 - 9.1t + 30.$$

$$a_x = \frac{dv_x}{dt} \qquad a_y = \frac{dv_y}{dt}$$

At $t = 15$

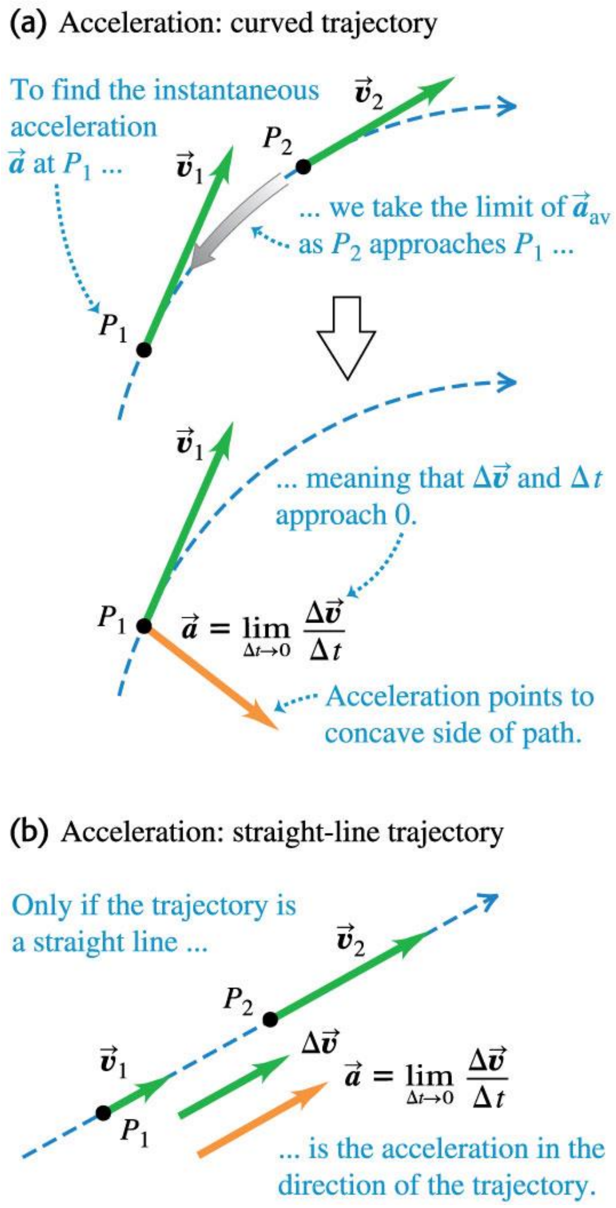




y (m)
40
20
0
20
40
60
80
x (m)
−20
−40
−60
145°
$\vec{a}$
x

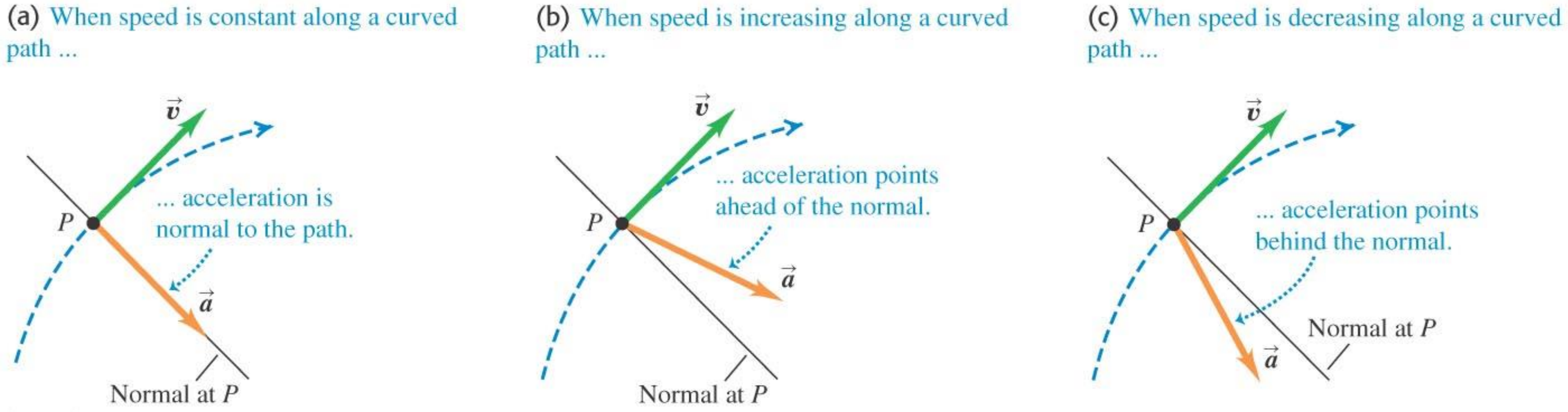
Instantaneous acceleration

- The *instantaneous acceleration* is the instantaneous rate of change of the velocity with respect to time.
- Any particle following a curved path is accelerating, even if it has constant speed.
- The components of the instantaneous acceleration are $a_x = dv_x/dt$, $a_y = dv_y/dt$, and $a_z = dv_z/dt$.



Direction of the acceleration vector

- The direction of the acceleration vector depends on whether the speed is constant, increasing, or decreasing, as shown in Figure 3.12.



Defining Motion in Two/Three Dimensions

One Dimension

- Position x
- Displacement Δx
- Velocity: displacement per unit time. Sign is equal to the sign of the displacement Δx
- Acceleration: change in velocity Δv per unit time. Sign is equal to the sign of the velocity difference Δv.

Two/Three Dimensions

- Position vector $\boldsymbol{r}$
- Displacement vector $\Delta\boldsymbol{r}$
- Velocity vector: change in the position vector per unit time. The direction is equal to the direction of the displacement vector $\Delta\boldsymbol{r}$.
- Acceleration vector: change in the velocity vector per unit time. The direction is equal to the direction of the velocity difference vector $\Delta\boldsymbol{v}$.

HomeWorks

- **PROBLEM** 1: The position vector for an electron is 1 = (5.0 m) l (3.0 m) J + (2.0 m) k. (a) Find the magnitude of 1. (b) Sketch the vector on a right-handed coordinate system.

- **PROBLEM 2**: An ion's position vector is initially 1 = 5.0I- 6.0J + 2.0k, and 10 s later it is 1 = -2.01 + 8.0J - 2.0k, all in meters. In unit vector notation, what is its vavg during the 10 s?

- Answer

$$\vec{v}_{avg} = (-0.70\hat{i} + 1.40\hat{j} - 0.40\hat{k})\mathrm{m/s}$$

Assignment 1 to be uploaded on Google Classroom(Assignments) Folder @ 30th September,2020

Thanks