Chapter 15 Oscillations

P1 :-

Data

$W = 14.N$

$\Theta = 40.0^\circ$

$l = 0.450m$

$k = 120$

$Req = ?$

(a) $x = ?$

(b) $T = ?$

Solution :-

$$x = \frac{W_y \sin\Theta}{k}$$

$$x = \frac{14 \sin(14)}{120}$$

$$\boxed{x = 0.074m}$$

$$T = 2\pi\sqrt{m/k}$$

$$T = 2(3.142)\sqrt{\frac{1.429}{120}}$$

$$\boxed{T = 0.685 \text{ sec}}$$

P.2

DATA

$k = 400\ N/m$

$x = 0.100\ m$

$v = 13.6\ m/s$

$a = -123\ m/s^2$

Solution:-

(a) $f = ?$

(b) $m = ?$

(c) $x_m = ?$

- $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$f = \frac{1}{2(3.14)}\sqrt{\frac{400}{m}}$

$\therefore m = \frac{-kx}{-a}$

(b) $\boxed{m = 0.325 kg}$

So put m

$f = \frac{1}{2(3.14)}\sqrt{\frac{400}{0.325}}$

(a) $\boxed{f = 5.578\ Hz}$

(c) $x_m = \sqrt{(m/k)(v)^2 + x^2}$

$x_m = \sqrt{(0.325/400)(13.6)^2 + (0.100}$

$\boxed{x_m = 1.15 \times 10^{-3}\ m}$

P3

$y_i = 10\ cm$

$g = 9.8$

$x_i = 10\ cm$

(a) $f = ?$

(b) $V = ?$

(c) $m_{s1} = ?$

(d) $y = ?$

$$f = \frac{1}{2\pi}\sqrt{g/L}$$

$$f = \frac{1}{2\pi}\sqrt{9.8/0.05}$$

(a) $\boxed{f = 2.22\ Hz}$

(b) $$V = \sqrt{2gy - \frac{k}{m}y^2}$$

$$V = \sqrt{2(9.81)(0.08) - \left(\frac{9.81}{0.05}\right)(0.08)^2}$$

$$\boxed{V = 0.56\ m/s}$$

(c) $m + \Delta m = 4m$

$\Delta m = 4m - m$

$\Delta m = 3m$

$m = \Delta m/3$

$$\boxed{m = 0.100\ kg}$$

(d) $kx = mg$

$y = (m + \Delta m)g$

$$y = \frac{(m + \Delta m)g}{k}$$

$$y = \frac{(0.100 + 0.300)(9.80)}{(0.100)(2\pi \times 2.23)^2}$$

$y = 0.200\ m$

P4

DATA

$m = 5.00\,kg$

$k = 1000$

$x = 50.0\,cm = 0.5\,m$

$v = 10.0\,m/s$

(a) $f = \frac{1}{2\pi}\sqrt{k/m}$

$f = 2.250\,Hz$

(b) $P.E = \frac{1}{2}kx^2$

$P.E = 125\,joule$

(c) $K.E = \frac{1}{2}mv^2$

$K.E = 250$

(d) $E = \frac{1}{2}kxm^2$

$xm = \sqrt{\frac{2E}{k}}$

$xm = 0.866\,m$

P.5)

① $\theta_m = \pi$ rad

$T = 0.5$ s

$\theta(t) = \theta_m \cos(\omega t)$

$\theta(t) = \theta_m \cos\left(\frac{2\pi t}{T}\right)$

(a) for angular velocity

$$V = \frac{-2\pi}{T}\,\theta_m \sin\left(\frac{2\pi t}{T}\right)$$

The angular velocity (V_m) max when cosine = 1

$$V_m = \frac{2\pi\theta_m}{T}$$

$$V_m = \frac{2\pi(\pi)}{0.5}$$

$$V_m = 39.5 \text{ rad/s}$$

(b) $\theta = \frac{\pi}{2}$

$$\frac{\theta}{\theta_m} = \frac{\pi}{\pi/2}$$

$$\frac{\theta}{\theta_m} = \frac{1}{2} \quad -(i)$$

① gives $\cos\left(\frac{2\pi t}{T}\right) = \frac{1}{2}$

$\therefore \sin^2 x + \cos^2 x = 1$

$$\sin\left(\frac{2\pi t}{T}\right) = \sqrt{1 - \cos^2\left(\frac{2\pi t}{T}\right)}$$

$$= \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{2}}$$

$$\Omega = \frac{-2\pi}{T}\,\theta_m \sin\left(\frac{2\pi t}{T}\right)$$

$\Omega = -34.2$ rad/s.

(c) $a = \frac{d^2\theta}{dt^2} = \frac{-2\pi^2}{T}\theta_m \cos\left(\frac{2\pi t}{T}\right)$

$\therefore \theta = \theta_m \cos\left(\frac{2\pi t}{T}\right)$

$a = -(2\pi t/T)$

angular acc when d is $\theta = \pi/4$

$a = -(2\pi/0.5)^2(\pi/4)$

$$\boxed{a = -124 \text{ rad/s}^2}$$

P6

Data

mass = 0.06 kg

θ = 0.08 rad $\quad \cos[(4.43)t + \theta]$

ω = 4.43 rad

a) $\omega = \sqrt{g/L}$ $\quad L = 0.499\,m$

$L = \frac{9.8}{(4.43)^2}$

$V_m = \omega x_m \quad (4.43)(0.499)(0.08)$

$V_m \quad 0.176\,m/s$

b) $K.E = \frac{1}{2} m v_m^2$

$K.E = 9.40 \times 10^{-4}\,J$

P 7) (B)

$R = 2.35\ cm$

$T = ?$

$I = I_{cm} + mh^2$

$I_{cm} = ?$

$d = 1.75\ cm$

$I_{cm} = mR^2/2$

$I = m(R^2/2) + mh^2$

for t

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$T = 2\pi\sqrt{\frac{mR^2/2 + mh^2}{mgd}}$$

$$T = 2\pi\sqrt{\frac{m(R^2/2 + d^2)}{mgd}}$$

$T = 0.366\ s$

P 8

DATA

$L = 2.20\,m$

$m = 22.1\,g = 0.0221\,kg$

(a) $T = ?$

Sol

$T = 2\pi\sqrt{I/mgh}$

$I_{cm} = \frac{1}{2}mL^2$

$I = \frac{1}{2}mL^2 + mx^2$

Substitute in eq (a)

$T = 2\pi\sqrt{\frac{mL^2}{2} + mx^2}$

Substitute in eq (a)

$T = 2\pi\sqrt{\frac{L^2 + 2x^2}{2gx}}$

For finding x we set der

$\frac{dT}{dx} = 0$

$= \frac{1}{2}\sqrt{\frac{2gx}{L^2 2x^2}}$

$288gx^2 - 12gL^2 - 144gx^2 = 0$

$12gL^2 = 144gx^2$

$L^2 = 12x^2$

$x = L/\sqrt{12}$

$= 0.635$

Substitute in T

$T = 2.262\,m/s$

(b) if L gets increased time period also increased as

$T \propto \sqrt{L}$

(c) change in mass will not effect T at all because T is independent of mass. Ans

P9

DATA

$k = 85\ N/m$

$m = 250\ g = 0.25\ kg$

$b = 70\ g/s = 0.07\ kg/s$

$t = 20\ T$

SOL:-

$T = 2\pi\sqrt{0.25/85}$

$T = 0.34\ sec$

$t = 20(0.34$

$e^{-bt/2m} = e^{-(0.070)(20)(0.34)/2(0.25)}$

$e^{-bt/2m} = 39.$

P10

DATA:-

$m = 500\,kg$, $x = 10cm = 0.1m$

$e^{-bt/2m} = 0.5$

(a) $k = ?$ (b) $b = ?$

Sol:-

(a)

$F = kx$

$mg = kx$

$k = mg/x$

$k = 4.9 \times 10^3\ N/m$

(b) $e^{-b\pi/2m} = 0.5 = T = 2\pi/\omega = 2\pi\sqrt{m/k} = \omega = \sqrt{\frac{k}{m}}$

$e^{-b\pi\sqrt{1/mk}} = 0.5$

$\ln e^{-b\pi\sqrt{1/mk}} = \ln(0.5)$

$\pi b\sqrt{1/mk} = 0.693$

$b = \frac{0.693}{\pi\sqrt{1/mk}}$

$b = 1080\ kg/s$

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P11:-

DATA:-

$v = 50\ m/s$

$L = 5\ m$

$m = 0.06\ kg$

$T = ?$

$\therefore \quad v = \sqrt{T/M}$

$M = m/L$

SoL:-

$v = \sqrt{T/m/L}$

$v^2 = T/m/L$

$v^2 = TL/m$

$\frac{v^2 m}{L} = T$

$T = \frac{(50)^2 (0.06)}{5}$

$T = 30N$ Ans.

P12:-

Data:-

$v_1 = 20\ m/s$

$T = 6\ N$

$v_2 = 30\ m/s$

$T_2 = ??$

SoL:-

$M = T_1/v^2$

$M = 6/(20)^2$

$M = 0.015$

$T_2 = v_2^2 \times M$

$T_2 = 13.5N$

P13

DATA:-

$A = 8cm = 0.08m$

$\lambda = 80.0cm = 0.8m$

$f = 3Hz$

(a) $y(0,t) = 0$ at $t = 0$

(b) $y(x,0) = 0$ at $x = 0.1m$

Sol:-

$$k = \frac{2\pi}{\lambda}$$

$$k = \frac{2\pi}{0.8}$$

$$k = 7.85 \text{ rad/m}$$

$$\omega = 2\pi f$$

$$\omega = 2(3.14)(3)$$

$$\omega = 18.84 \text{ rad/s}$$

(a) $y(x,t) = 0.08 \sin(7.853x + 6\pi t + \phi)$

$y(0,t) = 0.08 \sin(7.853(0) + 6\pi(0) + \phi) = 0$

$y(0,t) = 0.08 \sin(0 + \phi) = 0$

$y(0,t) = \sin + \phi = 0$

$y(0,t) = \phi = \sin^{-1}(0) = 0.$

Hence $y = 0.08 \sin(7.853x + 6\pi t)$

(b) $y(0, 0.1)$ to get ϕ

$y(0, 0.1) = 0.08 \sin(7.853(0.1) + 6\pi(0) + \phi) = 0$

$y(0, 0.1) = \sin(7.853(0.1) + \phi) = 0$

~~Sin~~

$(7.853)(0.1) + \phi = 0$

$\phi = -0.785 \text{ rad}$

Hence

$y = 0.08 \sin(7.85x + 6\pi t - 0.785)$

P14.

DATA :-

$y = (0.25\ m) \sin(0.30x - 40t)$.

(a) A=? (b) w=? (c) k=? (d) λ=? (e) v=? (f) Direction of motion=?.

Sol:

$y = (0.25\ m) \sin(0.30x - 40t)$.

(a) $A = 0.25$ (b) $w = 40$ rad/s (c) $k = 0.30$

(d) $\lambda = 2\pi/k = \frac{2\pi}{0.30} = 20.94\ m$

(e) $v = w/k = 133.3$

(f) Direction of motion is +ve.

Ans.

P15.

DATA :-

$k = 3.10$ rad/cm $= 310$ rad/m

$w = 9.30$ rad

$\Delta t = 10$ sec

$y = (0.51\ cm) \sin(kx - wt)$

~~$k = 3.10$ rad~~

$\Delta x = ??$

$\Delta x = vt$

$v = w/k$

$v = 0.03\ m/s$

$\Delta x = vt$

$\Delta x = (0.03)(10)$

$\Delta x = 0.3\ m$ (+ve direction)

P16 :-

DATA :-

$y = 0.02 \sin(kx - \omega t)$

$y = A \sin(kx - \omega t)$

$k = 2.11$ rad/m

$\omega = 3.62$ rad/s

(a) $A = ?$ (b) $\lambda = ?$ (c) $f = ?$ (d) $v = ?$

Solution :-

(a) $A = 0.02$

(b) $v = \omega / k$

$v = 3.62 / 2.11$

$v = 1.71$ m/s

(c) $\omega = 2\pi f$

$f = \omega / 2\pi$

$f = 0.58$ Hz

(b) $\lambda = 2\pi / k$

$\lambda = 2.97$ m

P.17

DATA:

$y = (0.15\,m)\sin(0.80x - 50t)$.

(a) $V = ?$ (b) $f = ?$ (c) $\lambda = ?$ (d) $P = ?$

Solution:

(a) $v = \omega / k$

$v = 50 / 0.80$

$v = 62.5\ m/s$

(b) $f = \omega / 2\pi$

$f = 50/2\,(3.14)$ ~~Hz~~

~~$f = 78.55$ Hz~~

$f = 7.96$ Hz