Explanation of the Contention Resolution Probability Formula

Introduction

In competitive allocation scenarios, multiple players may request the same item. To resolve such conflicts fairly, a probabilistic approach is used. The following formula defines the probability $r_{A,k}$ that a player k from the set of contenders A will receive the item:

$$r_{A,k} = \frac{1}{\sum_{i=1}^{n} p_i} \left(\sum_{i \in A \setminus \{k\}} \frac{p_i}{|A| - 1} + \sum_{i \notin A} \frac{p_i}{|A|} \right) \tag{1}$$

Definitions

- A: The set of players who have requested the item.
- $k \in A$: The specific player for whom we calculate the probability.
- p_i: The initial probability that player i requests the item.
- $|A|$: The number of players in the set A (i.e., number of contenders).
- n: The total number of players in the system.

Explanation of the Formula

Normalization Factor

$$\frac{1}{\sum_{i=1}^{n} p_i}$$

This ensures that all probabilities $r_{A,k}$ across contenders sum to 1, maintaining a valid probability distribution.

Competition Term

$$\sum_{i \in A \setminus \{k\}} \frac{p_i}{|A| - 1}$$

This part accounts for the competition within the set A, distributing the influence of other contenders' probabilities evenly across $|A| - 1$ players.

Non-Contender Adjustment

$$\sum_{i \notin A} \frac{p_i}{|A|}$$

This adjusts for players who did not request the item, distributing their total probability evenly among the contenders in A, ensuring fairness relative to the overall system.

Intuition

- The more players in A, or the higher their p_i, the lower the chance for any single player k.
- Players with higher initial probabilities p_i benefit from proportionally higher chances.
- This formula balances fairness and efficiency by considering both contenders and non-contenders.

Conclusion

The formula provides a fair method to resolve contention by probabilistically allocating items based on initial request probabilities, while ensuring that the total probability remains valid.