Now let's analyze this **new function**:

$$f(x) = (x^3 - 3x + 1)^7$$

We are given **critical values** at:

- x = -1
- x = 1

♦ Step 1: First Derivative

We apply the chain rule:

Let $u = x^3 - 3x + 1$, so:

$$f(x) = u^7 \Rightarrow f'(x) = 7u^6 \cdot u'$$

Now compute u':

$$u' = \frac{d}{dx}(x^3 - 3x + 1) = 3x^2 - 3$$

So the derivative becomes:

$$f'(x) = 7(x^3 - 3x + 1)^6 \cdot (3x^2 - 3)$$

♦ Step 2: Find critical points

 $\operatorname{Set} f'(x) = 0:$

$$7(x^3 - 3x + 1)^6(3x^2 - 3) = 0$$

This gives us two cases:

1.
$$(x^3 - 3x + 1)^6 = 0 \Rightarrow x^3 - 3x + 1 = 0 \Rightarrow x = \pm 1$$
 (given)

2.
$$3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

So again, **critical points at** $x = \pm 1$.

♦ Step 3: Use Second Derivative Test

We'll try, but keep in mind: because of the high power, it might be inconclusive.

Let's write the first derivative again:

$$f'(x) = 7(x^3 - 3x + 1)^6(3x^2 - 3)$$

We will **not fully differentiate this complex expression** by hand right now—rather, use a **simpler approach**:

♦ Step 4: Use First Derivative Test (sign change)

Let's test values around x = -1 and x = 1 to check sign changes of f'(x):

At x = -1:

- Pick points:
 - Left of -1: x = -1.5

•
$$u = (-1.5)^3 - 3(-1.5) + 1 = -3.375 + 4.5 + 1 = 2.125 \Rightarrow u^6 > 0$$

•
$$3x^2 - 3 = 3(2.25) - 3 = 6.75 - 3 = 3.75 > 0$$

• So
$$f'(x) > 0$$

• Right of -1: x = -0.5

•
$$u = (-0.5)^3 - 3(-0.5) + 1 = -0.125 + 1.5 + 1 = 2.375 \Rightarrow u^6 > 0$$

•
$$3x^2 - 3 = 3(0.25) - 3 = 0.75 - 3 = -2.25 < 0$$

• So
$$f'(x) < 0$$

 \checkmark Derivative changes from **positive to negative** \rightarrow **Relative Maximum at** x = -1

At x = 1:

• Left of 1: x = 0.5

•
$$u = (0.5)^3 - 3(0.5) + 1 = 0.125 - 1.5 + 1 = -0.375 \Rightarrow u^6 > 0$$

•
$$3x^2 - 3 = 0.75 - 3 = -2.25 < 0$$

$$\bullet \quad f'(x) < 0$$

• Right of 1: x = 1.5

•
$$u = (1.5)^3 - 3(1.5) + 1 = 3.375 - 4.5 + 1 = -0.125 \Rightarrow u^6 > 0$$

•
$$3x^2 - 3 = 6.75 - 3 = 3.75 > 0$$

•
$$f'(x) > 0$$

 \checkmark Derivative changes from **negative to positive** \rightarrow **Relative Minimum at** x=1



Critical Point

Туре

$$x = -1$$

Relative Maximum

$$x = 1$$

Relative Minimum

Let me know if you'd like the graph or need help with a similar function.



$$f(x) = (x^4-4x+2)^5$$
 at $x=1$

Let's analyze the function:

$$f(x) = (x^4 - 4x + 2)^5$$

You're given a **critical point at** x=1. We'll determine whether it's a **relative** maximum, relative minimum, or neither.

♦ Step 1: First Derivative

Let:

$$u = x^4 - 4x + 2 \Rightarrow f(x) = u^5$$

Using the chain rule:

$$f'(x) = 5u^4 \cdot u'$$

Now compute u':

$$u' = \frac{d}{dx}(x^4 - 4x + 2) = 4x^3 - 4$$

So:

$$f'(x) = 5(x^4 - 4x + 2)^4(4x^3 - 4)$$

♦ Step 2: Find critical points

Set f'(x) = 0:

$$5(x^4 - 4x + 2)^4(4x^3 - 4) = 0$$

There are two factors:

1.
$$(x^4 - 4x + 2)^4 = 0 \implies x^4 - 4x + 2 = 0$$

2.
$$4x^3 - 4 = 0 \Rightarrow x^3 = 1 \Rightarrow x = 1$$

So x = 1 is a **critical point**, as given.

♦ Step 3: Use First Derivative Test

Test sign of f'(x) on either side of x = 1:

- Let's pick:
 - Left of 1: x = 0.5

•
$$u = (0.5)^4 - 4(0.5) + 2 = 0.0625 - 2 + 2 = 0.0625 \Rightarrow u^4 > 0$$

•
$$4x^3 - 4 = 0.5 - 4 = -3.5$$

- So f'(x) < 0
- Right of 1: x = 1.5

•
$$u = (1.5)^4 - 4(1.5) + 2 = 5.06 - 6 + 2 \approx 1.06 \Rightarrow u^4 > 0$$

•
$$4x^3 - 4 = 13.5 - 4 = 9.5$$

• So
$$f'(x) > 0$$

 \checkmark Since derivative changes from **negative to positive**, there is a **relative minimum** at x=1

✓ Final Answer:

Critical Point

Type

x = 1

Relative Minimum

Let me know if you'd like help with the graph or more examples.

$$f(x) = (x^2-4)^4(x^2-1)^3$$
 at $x=1$, $x=2$

Let's analyze the function:

$$f(x) = (x^2 - 4)^4 (x^2 - 1)^3$$

We are given **critical points** at:

•
$$\chi = 1$$

• x = 2

We'll determine if these are **relative maxima**, **minima**, or **neither**, using the **First Derivative Test** (since second derivative would be messy here).

Critical Point Analysis

♦ Step 1: Understand the structure

Note that:

•
$$x^2 - 4 = (x - 2)(x + 2)$$

•
$$x^2 - 1 = (x - 1)(x + 1)$$

So we could write:

$$f(x) = [(x-2)(x+2)]^4 \cdot [(x-1)(x+1)]^3$$

Critical points occur where f'(x) = 0 — we're told x = 1 and x = 2 are critical points.

♦ Step 2: First Derivative Test (Sign Analysis)

We'll test the sign of f'(x) around x = 1 and x = 2 using test points.

We don't need the full derivative. Since all factors are powers of polynomials, we can look at how the function changes sign near each point.

riangle Around x = 1:

Use test values slightly less than and slightly more than 1:

- At x = 0.9:
 - $(x^2 4)^4 = ((0.9)^2 4)^4 = (-3.19)^4 > 0$
 - $(x^2 1)^3 = ((0.9)^2 1)^3 = (-0.19)^3 < 0$
 - So f(x) < 0
- At x = 1.1:
 - $(x^2-4)^4 = ((1.1)^2-4)^4 = (-2.79)^4 > 0$
 - $(x^2-1)^3 = ((1.1)^2-1)^3 = (0.21)^3 > 0$
 - So f(x) > 0
- ✓ Function goes from negative to positive around x = 1 → Relative Minimum

- ightharpoonup Around x = 2:
- At x = 1.9:
 - $(x^2-4)^4 = ((1.9)^2-4)^4 = (-0.39)^4 > 0$
 - $(x^2 1)^3 = ((1.9)^2 1)^3 = (2.61)^3 > 0$
 - So f(x) > 0
- At x = 2.1:
 - $(x^2-4)^4 = ((2.1)^2-4)^4 = (0.41)^4 > 0$
 - $(x^2-1)^3 = ((2.1)^2-1)^3 = (3.41)^3 > 0$
 - $\operatorname{So} f(x) > 0$
- Function stays positive on both sides \rightarrow No sign change, so neither maximum nor minimum at x=2

✓ Final Answer:

Critical Point

Туре

x = 1

Relative Minimum

x = 2

Neither (Flat Point or Plateau)

Let me know if you want the full derivative or graph for confirmation.

 $f(x)= (x^3-48)^1/3$ at x=4 from i/3 i mean it is whole cube root

Thanks for the clarification! You're working with the function:

$$f(x) = \sqrt[3]{x^3 - 48} = (x^3 - 48)^{1/3}$$

And you're interested in analyzing the point x = 4: is it a **relative maximum**, **relative minimum**, or **neither**?

\diamond Step 1: Check if x = 4 is a critical point

A critical point occurs where:

•
$$f'(x) = 0$$
, or

• f'(x) does not exist

Let's find the derivative.

Let:

$$f(x) = (x^3 - 48)^{1/3}$$

Using the chain rule:

$$f'(x) = \frac{1}{3}(x^3 - 48)^{-2/3} \cdot (3x^2) = \frac{x^2}{(x^3 - 48)^{2/3}}$$

\diamond Step 2: Plug in x = 4

First compute:

$$x^3 - 48 = 64 - 48 = 16 \Rightarrow \text{Denominator is } 16^{2/3} > 0$$

 $x^2 = 16 \Rightarrow \text{Numerator is also } > 0$

So:

$$f'(4) = \frac{16}{16^{2/3}} > 0 \Rightarrow$$
 Derivative is positive at $x = 4$

So:

- f'(4) = 0
- f'(4) exists

✓ Conclusion:

x = 4 is **NOT** a critical point, because the derivative exists and is not zero.

✓ Final Answer:

Point Type

x = 4 Neither (not a max/min/critical point)

Let me know if you'd like to find actual extrema or graph this function!