# Systems of Linear Equations in Two Variables

# What You Should Learn

- Use the method of elimination to solve systems of linear equations in two variables.
- Graphically interpret the number of solutions of a system of linear equations in two variables.
- Use systems of linear equations in two variables to model and solve real-life problems.
- Please read all slides, but focus on 11-20 for the number of solutions and word problems!!!

# The Method of Elimination

# The Method of Elimination

Now we will study the **method of elimination** to solve a system of linear equations in two variables. The key step in this method is to obtain, for one of the variables, coefficients that differ only in sign so that *adding* the equations eliminates the variable.

$$
\begin{array}{rl}
3x + 5y = 7 & \quad \text{Equation 1} \\
-3x - 2y = -1 & \quad \text{Equation 2} \\
\hline
3y = 6 & \quad \text{Add equations.}
\end{array}
$$

# The Method of Elimination

Note that by adding the two equations, you eliminate the $x$-terms and obtain a single equation in $y$. Solving this equation for $y$ produces

$$y = 2$$

which you can then back-substitute into one of the original equations to solve for $x$.

# Example 1 – *Solving a System by Elimination*

Solve the system of linear equations.

$$\begin{cases} 3x + 2y = 4 & \text{Equation 1} \\ 5x - 2y = 8 & \text{Equation 2} \end{cases}$$

Solution:

Because the coefficients of *y* differ only in sign, you can eliminate the *y*-terms by adding the two equations.

$$\begin{array}{rcll} 3x + 2y &=& 4 & \text{Write Equation 1.} \\ 5x - 2y &=& 8 & \text{Write Equation 2.} \\ \hline 8x &=& 12 & \text{Add equations} \\ x &=& \frac{3}{2} & \text{Solve for } x \end{array}$$

# Example 1 – *Solution*

cont'd

So, $x = \frac{3}{2}$. By back-substituting into Equation 1, you can solve for *y*.

$$3x + 2y = 4 \qquad \text{Write Equation 1}$$

$$3\left(\frac{3}{2}\right) + 2y = 4 \qquad \text{Substitute } \frac{3}{2} \text{ for } x$$

$$y = -\frac{1}{4} \qquad \text{Solve for y}$$

The solution is

$$\left(\frac{3}{2}, -\frac{1}{4}\right).$$

You can check the solution *algebraically* by substituting into the original system.

# Example 1 – *Solution*

cont'd

**Check**

$$3\left(\tfrac{3}{2}\right) + 2\left(-\tfrac{1}{4}\right) \stackrel{?}{=} 4$$

Substitute into Equation 1

$$\tfrac{9}{2} - \tfrac{1}{2} = 4$$

Equation 1 checks

$$5\left(\tfrac{3}{2}\right) - 2\left(-\tfrac{1}{4}\right) \stackrel{?}{=} 8$$

Substitute into Equation 2

$$\tfrac{15}{2} + \tfrac{1}{2} = 8$$

Equation 2 checks

# The Method of Elimination

## The Method of Elimination

To use the **method of elimination** to solve a system of two linear equations in $x$ and $y$, perform the following steps.

1. Obtain coefficients for $x$ (or $y$) that differ only in sign by multiplying all terms of one or both equations by suitably chosen constants.
2. Add the equations to eliminate one variable; solve the resulting equation.
3. Back-substitute the value obtained in Step 2 into either of the original equations and solve for the other variable.
4. Check your solution in both of the original equations.

# Graphical Interpretation of Two-Variable Systems

# Graphical Interpretation of Two-Variable Systems

It is possible for any system of equations to have exactly one solution, two or more solutions, or no solution.

If a system of *linear* equations has two different solutions, then it must have an *infinite* number of solutions.

To see why this is true, consider the following graphical interpretations of a system of two linear equations in two variables.

# Graphical Interpretation of Two-Variable Systems

**Graphical Interpretations of Solutions**

For a system of two linear equations in two variables, the number of solutions is one of the following.

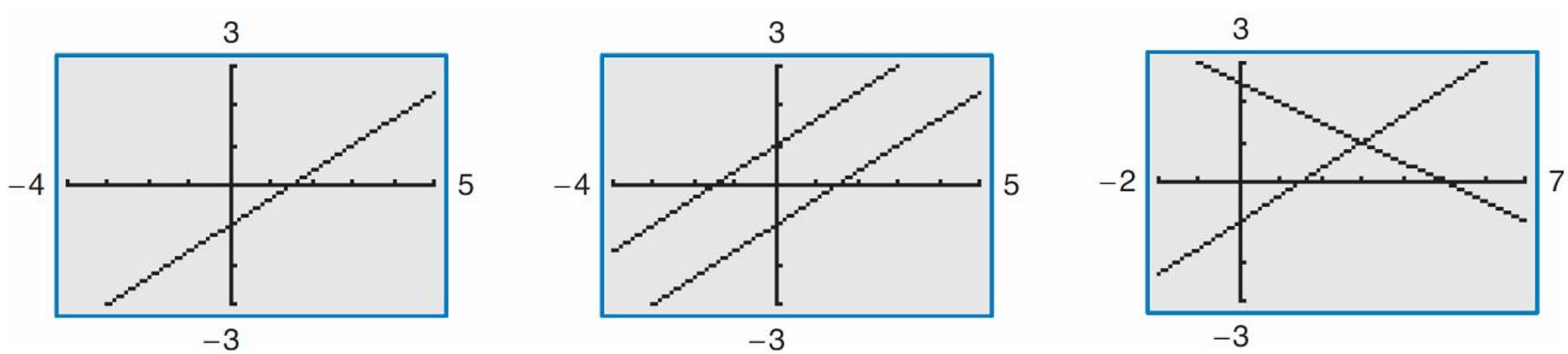
| *Number of Solutions* | *Graphical Interpretation* |
|---|---|
| **1.** Exactly one solution | The two lines intersect at one point. |
| **2.** Infinitely many solutions | The two lines are coincident (identical). |
| **3.** No solution | The two lines are parallel. |

A system of linear equations is **consistent** when it has at least one solution. It is **inconsistent** when it has no solution.

# Example 2 – *Recognizing Graphs of Linear Systems*

Match each system of linear equations (a, b, c) with its graph (i, ii, iii) in Figure 7.10. Describe the number of solutions. Then state whether the system is consistent or inconsistent.

**a.** $\begin{cases} 2x - 3y = 3 \\ -4x + 6y = 6 \end{cases}$ **b.** $\begin{cases} 2x - 3y = 3 \\ x + 2y = 5 \end{cases}$ **c.** $\begin{cases} 2x - 3y = 3 \\ -4x + 6y = -6 \end{cases}$




**i.** **ii.** **iii.**

**Figure 7.10**

# Example 2 – *Solution*

Begin by rewriting each system of equations in slope-intercept form.

**a.** The graph of system (a) is a pair of parallel lines (ii). The lines have no point of intersection, so the system has no solution. The system is inconsistent.

$$\text{System (a): } \begin{cases} y = \frac{2}{3}x - 1 \\ y = \frac{2}{3}x + 1 \end{cases}$$

**b.** The graph of system (b) is a pair of intersecting lines (iii). The lines have one point of intersection, so the system has exactly one solution.

# Example 2 – *Solution*

cont'd

The system is consistent.

$$\text{System (b):}\quad \begin{cases} y = \frac{2}{3}x - 1 \\ y = -\frac{1}{2}x + \frac{5}{2} \end{cases}$$

**c.** The graph of system (c) is a pair of lines that coincide (i). The lines have infinitely many points of intersection, so the system has infinitely many solutions. The system is consistent.

$$\text{System (c):}\quad \begin{cases} y = \frac{2}{3}x - 1 \\ y = \frac{2}{3}x - 1 \end{cases}$$

Application

# Application

At this point, we may be asking the question "How can I tell which application problems can be solved using a system of linear equations?" The answer comes from the following considerations.

**1.** Does the problem involve more than one unknown quantity?

**2.** Are there two (or more) equations or conditions to be satisfied?

When one or both of these conditions are met, the appropriate mathematical model for the problem may be a system of linear equations.

# Example 3 – *Aviation*

An airplane flying into a headwind travels the 2000-mile flying distance between Cleveland, Ohio and Fresno, California in 4 hours and 24 minutes. On the return flight, the same distance is traveled in 4 hours. Find the airspeed of the plane and the speed of the wind, assuming that both remain constant.

Solution:

The two unknown quantities are the speeds of the wind and the plane. If $r_1$ is the speed of the plane and $r_2$ is the speed of the wind, then

$r_1 - r_2$ = speed of the plane *against* the wind

# Example 3 – *Solution*

cont'd

$r_1 + r_2$ = speed of the plane *with* the wind

as shown in Figure 7.13.

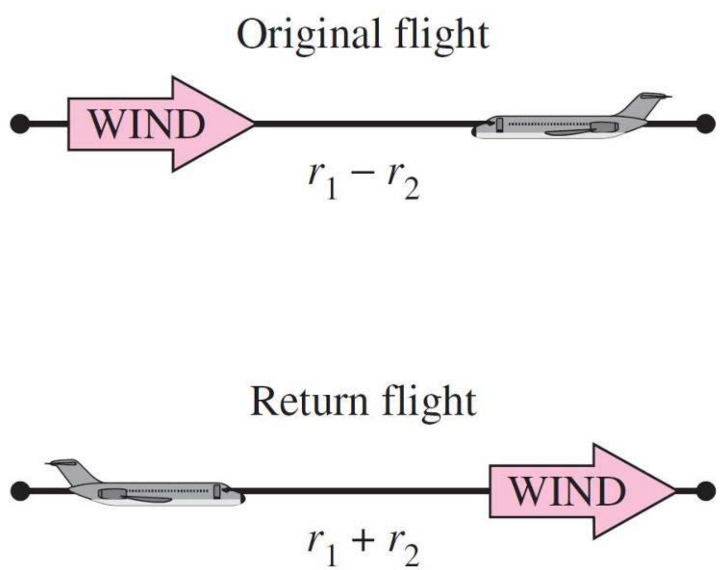



**Figure 7.13**

# Example 3 – *Solution*

cont'd

Using the formula

distance = (rate) (time)

for these two speeds, you obtain the following equations.

$$2000 = (r_1 - r_2)\left(4 + \frac{24}{60}\right)$$

$$2000 = (r_1 + r_2)(4)$$

These two equations simplify as follows.

$$\begin{cases} 5000 = 11r_1 - 11r_2 & \text{Equation 1} \\ 500 = r_1 + r_2 & \text{Equation 2} \end{cases}$$

# Example 3 – *Solution*

cont'd

To solve this system by elimination, multiply Equation 2 by 11.

$$\begin{array}{rcl} 5000 = 11r_1 - 11r_2 & \Rightarrow & 5000 = 11r_1 - 11r_2 \\ \underline{500 = \phantom{11}r_1 + \phantom{11}r_2} & \Rightarrow & \underline{5500 = 11r_1 + 11r_2} \\ & & 10{,}500 = 22r_1 \end{array}$$

Write Equation 1

Multiply Equation 2 by 11

Add Equations

So,

$$r_1 = \frac{10{,}500}{22} = \frac{5250}{11} \approx 477.27 \text{ miles per hour}$$

Speed of plane

and

$$r_2 = 500 - \frac{5250}{11} = \frac{250}{11} \approx 22.73 \text{ miles per hour.}$$

Speed of wind