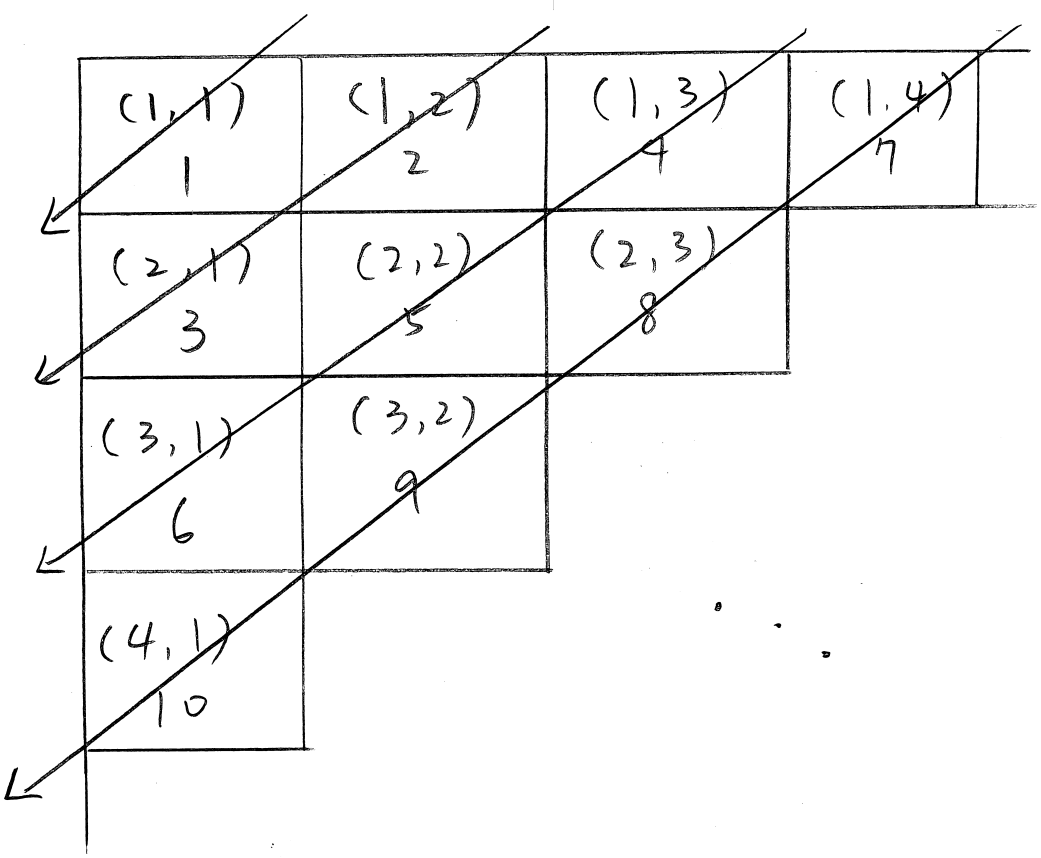
編碼題目：

We can show that the set $\{(x,y) \mid x,y \text{ are positive integers}\}$ is countable by finding a 1-1 mapping from (x,y) to an integer. Given $(1,1)\to 1$, $(1,2)\to 2$, $(1,3)\to 4$, $(1,4)\to 7$, $(2,1)\to 3$, $(2,2)\to 5$, $(2,3)\to 8$, $(3,1)\to 6$, $(3,2)\to 9$, $(4,1)\to 10$, please find which (x,y) maps to 465?

Ans.

其編碼方式如下



以斜線算格子數到 465 的作法：

第一條斜線有一格，第二條有 2 格，....

$\Rightarrow 1+2+\cdots+n \geq 465$

可能在某條中間

$\Rightarrow \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \geq 465$

$\Rightarrow n^2+n \geq 930 \Rightarrow n^2+n-930 \geq 0$

$n = \frac{-1 \pm \sqrt{1+3720}}{2} = \frac{-1 \pm \sqrt{3721}}{2} = \frac{-1 \pm 61}{2} = -31, 30$

$\because n \in Z^+, \therefore n = 30$

$\Rightarrow \frac{n(n+1)}{2}$ 代 30 $\Rightarrow \frac{30 \times 31}{2} = 465$

所以是在第 30 條斜線的最後一格，為 (30, 1) ※