古典伴隨矩陣回推矩陣.行列式

96 交大資訊

Give the $adj(A)$, find $det(A)$, A, $det(3A^{-1}A^{T})$, where

$$adj(A) = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ -2 & -1 & 2 \end{bmatrix}.$$

Ans.

$$A \cdot adj(A) = det(A) \cdot I$$

$$\Rightarrow det(A \cdot adj(A)) = det\begin{bmatrix} det(A) & & \\ & det(A) & \\ & & det(A) \end{bmatrix} = det(A)^3$$

$$\Rightarrow det(A) \cdot det(adj(A)) = det(A)^3$$

$$\Rightarrow det(adj(A)) = det(A)^2 = det\begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ -2 & -1 & 2 \end{bmatrix}$$

$$\begin{matrix} + & + & + & & - & - & - \end{matrix}$$

$$= \left|\begin{array}{ccc|cc} 2 & 1 & 0 & 2 & 1 \\ 4 & 3 & 2 & 4 & 3 \\ -2 & -1 & 2 & -2 & -1 \end{array}\right| = 2\times3\times2 + 1\times2\times(-2) + 0 - 0 - (-1)\times2\times2 - 1\times4\times2$$

$$= 12 - 4 + 4 - 8 = 4$$

$$\Rightarrow \boxed{det(A) = \pm 2}$$

$$A = det(A) \cdot adj(A)^{-1} = \pm 2\begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ -2 & -1 & 2 \end{bmatrix}^{-1}$$

用 Gauss-Jordan 求 $adj(A)^{-1}$

$$[adj(A) \mid I] = [I \mid adj(A)^{-1}]$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & & \\ 4 & 3 & 2 & & 1 & \\ -2 & -1 & 2 & & & 1 \end{array}\right] \xrightarrow{r_1^{(\frac{1}{2})}} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & \frac{1}{2} & & \\ 4 & 3 & 2 & 0 & 1 & \\ -2 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_{12}^{(-4)},\ r_{13}^{(2)}} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & \frac{1}{2} & & \\ 0 & 1 & 2 & -2 & 1 & \\ 0 & 0 & 2 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{r_{32}^{(-1)}} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & \frac{1}{2} & & \\ 0 & 1 & 0 & -3 & 1 & -1 \\ 0 & 0 & 2 & 1 & 0 & 1 \end{array}\right] \xrightarrow{r_{21}^{(-\frac{1}{2})},\ r_3^{\frac{1}{2}}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -3 & 1 & -1 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} \end{array}\right]$$

$$\Rightarrow \boxed{A = det(A) \cdot I \cdot adj(A)^{-1} = \pm 2 \begin{bmatrix} 2 & -\frac{1}{2} & 0 \\ -3 & 1 & -1 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} = \pm \begin{bmatrix} 4 & -1 & 0 \\ -6 & 2 & -2 \\ 1 & 0 & 1 \end{bmatrix}}$$

$$\boxed{det(3A^{-1}A^{T}) = det(3I_3) \cdot \frac{1}{\cancel{det(A)}} \cdot \cancel{det(A)} = 3^3 = 27.}$$