行列式題目：98 交大資訊

Assume that $A = [a_1 \ a_2 \ a_3]$ and $B = \begin{bmatrix} 2a_2^T \\ a_1^T + a_2^T + a_3^T \\ a_3^T \end{bmatrix}$.

If $\det(A) = 2$, find $\det(AB^{-1})$.

Ans.

$$A^T = \begin{bmatrix} a_1^T \\ a_2^T \\ a_3^T \end{bmatrix} \xrightarrow{r_{21}^{(1)}\ r_{31}^{(1)}} \begin{bmatrix} a_1^T + a_2^T + a_3^T \\ a_2^T \\ a_3^T \end{bmatrix} \xrightarrow{r_{12},\ r_1^{(2)}} \begin{bmatrix} 2a_2^T \\ a_1^T + a_2^T + a_3^T \\ a_3^T \end{bmatrix}$$

$$\Rightarrow R_1^{(2)} R_{12} R_{31}^{(1)} R_{21}^{(1)} A^T = B$$

$$\Rightarrow \det(R_1^{(2)} R_{12} R_{31}^{(1)} R_{21}^{(1)} A^T) = 2 \cdot (-1) \cdot 1 \cdot 1 \cdot 2 = -4 = \det(B)$$

$$\det(AB^{-1}) = \det(A) \cdot \frac{1}{\det(B)} = 2 \cdot \frac{1}{-4} = -\frac{1}{2} \quad \#$$