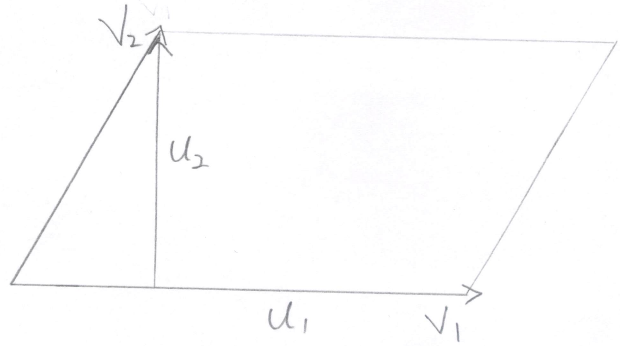
Gram-Schmidt process 計算平行四邊形 or 三角形面積。

補充:
六面體體積:
$\|u_1\|\|u_2\|\|u_3\|$



對 v_1, v_2 正交化後，得 u_1，u_2，

則平行四邊形面積為 $\|u_1\|\|u_2\|$。

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ex. Find the area of the triangle having vertices

$P(1,-2,3)$, $Q(3,7,6)$, and $R(6,-7,-3)$.

Ans.

$\overrightarrow{PQ} = (2,9,3)$

$\overrightarrow{PR} = (5,-5,-6)$

$u_1 = \overrightarrow{PQ} = (2,9,3)$, $\langle u_1, u_1 \rangle = 94$

$$u_2 = \overrightarrow{PR} - \frac{\langle \overrightarrow{PR}, u_1 \rangle}{\langle u_1, u_1 \rangle} u_1 = \frac{1}{94}(576, 7, -405)$$

$$\langle u_2, u_2 \rangle = \frac{1}{94^2}(576^2 + 49 + (-405)^2)$$

$$\Rightarrow \frac{1}{2}\|u_1\|\|u_2\| = \frac{1}{2} \cdot \sqrt{94} \cdot \left(\sqrt{\frac{576^2 + 49 + (-405)^2}{94^2}}\right)$$ #

(入射端)

ex. Find the volume of the parallelepiped whose incident sides extend from the first point to each of the other three.

$P_1 = (-1, 2, 3)$, $P_2 = (2, 5, 4)$, $P_3 = (1, 6, -3)$, and $P_4 = (6, -4, 7)$

Ans.

$\overrightarrow{P_1P_2} = (3, 3, 1)$

$\overrightarrow{P_1P_3} = (2, 4, -6)$

$\overrightarrow{P_1P_4} = (7, -6, 4)$

let, $V_1 = \overrightarrow{P_1P_2}$, $V_2 = \overrightarrow{P_1P_3}$, $V_3 = \overrightarrow{P_1P_4}$,

對 V_1, V_2, V_3 正交化：

$U_1 = V_1 = (3, 3, 1)$, $\langle U_1, U_1 \rangle = 19$

$U_2 = V_2 - \frac{\langle V_2, U_1 \rangle}{\langle U_1, U_1 \rangle} U_1 = (2, 4, -6) - \frac{6+12-6}{19}(3, 3, 1) = \frac{1}{19}(2, 40, -126)$

$\langle U_2, U_2 \rangle = \frac{920}{19}$

$U_3 = V_3 - \frac{\langle V_3, U_2 \rangle}{\langle U_2, U_2 \rangle} U_2 - \frac{\langle V_3, U_1 \rangle}{\langle U_1, U_1 \rangle} U_1 = \frac{1}{46}(275, -250, -175)$,

$\langle U_3, U_3 \rangle = \frac{3125}{46}$

$\Rightarrow \|U_1\| \|U_2\| \|U_3\| = \sqrt{\cancel{19}} \cdot \sqrt{\frac{\cancel{920}^{20}}{\cancel{19}}} \cdot \sqrt{\frac{3125}{\cancel{46}}} = 250$ #