

$$\sum_{k=1}^n k^2 (\log k)^3 = \Theta(n^d (\log n)^e), \text{ for } d, e$$

for O :

$$\sum_{k=1}^n k^2 (\log k)^3 \leq \sum_{k=1}^n n^2 (\log n)^3 = n^3 (\log n)^3$$

$$\therefore \sum_{k=1}^n k^2 (\log k)^3 = O(n^3 (\log n)^3)$$

for Ω :

$$\sum_{k=1}^n k^2 (\log k)^3 \geq \sum_{k=\frac{n}{2}}^n \left(\frac{n}{2}\right)^2 \left(\log \frac{n}{2}\right)^3 = \left(\frac{n}{2}\right)^3 \left(\log \frac{n}{2}\right)^3$$

$$\therefore \sum_{k=1}^n k^2 (\log k)^3 = \Omega(n^3 (\log n)^3)$$

$$\Rightarrow \sum_{k=1}^n k^2 (\log k)^3 = \Theta(n^3 (\log n)^3), \text{ } d=3, e=3$$