

$$A_1 = A: 2 \times 4 = P_0 \times P_1$$

$$A_2 = B: 4 \times 3 = P_1 \times P_2$$

$$A_3 = C: 3 \times 2 = P_2 \times P_3$$

$$A_4 = D: 2 \times 5 = P_3 \times P_4$$

$$A_5 = E: 5 \times 1 = P_4 \times P_5$$

$$P_0 = 2$$

$$P_1 = 4$$

$$P_2 = 3$$

$$P_3 = 2$$

$$P_4 = 5$$

$$P_5 = 1$$

m	A_1	A_2	A_3	A_4	A_5
A_1	0	24	36	56	36
A_2	—	0	24	64	28
A_3	—	—	0	30	16
A_4	—	—	—	0	10
A_5	—	—	—	—	0

K	A_2	A_3	A_4	A_5
A_1	1	2	3	1 ✓
A_2	—	2	3	2 ✓
A_3	—	—	3	3 ✓
A_4	—	—	—	4 ✓
A_5	—	—	—	—

回推表示式

$K[1,5] =$

$$(A_1)(A_2 A_3 A_4 A_5)$$

$K[2,5] =$

$$(A_2)(A_3 A_4 A_5)$$

$K[3,5] =$

$$(A_3)(A_4 A_5)$$

$K[4,5] =$

$$(A_4)(A_5)$$

$$\Rightarrow A_1(A_2(A_3(A_4 A_5)))$$

$$= A(B(C(DE)))$$

Assume that there are a few matrix need to multiple.

$$\begin{array}{cccccc} A_1 & A_2 & A_3 & A_4 & A_5 \\ 2 \times 4 & 4 \times 3 & 3 \times 2 & 2 \times 5 & 5 \times 1 \\ P_0 & P_1 & P_1 & P_2 & P_2 & P_3 & P_3 & P_4 & P_4 & P_5 \end{array} \quad , \{i \mid 1 \leq i \leq 5\} \Rightarrow A_i \in F^{P_{i-1} \times P_i} \Rightarrow A_i \text{ 的矩陣大小為 } P_{i-1} \times P_i$$

$m[i, j]$ 為第 i 個 matrix 到第 j 個 matrix 的最小乘法量

遞迴表示

$$m[i, j] = \begin{cases} 0 & , \text{ if } i = j \\ \min_{i \leq k < j} \{ m[i, k] + m[k+1, j] + P_{i-1} \cdot P_k \cdot P_j \} & , \text{ if } i < j \end{cases}$$

$$m[1,2] = \min_{1 \leq k < 2} = 24$$

$$k=1$$

$$m[1,1] + m[2,2] + p_0 p_1 p_2$$

$$= 0 + 0 + 2 \times 4 \times 3 = 24$$

$$m[2,3] = \min_{2 \leq k < 3} = 24$$

$$k=2$$

$$m[2,2] + m[3,3] + p_1 p_2 p_3$$

$$= 0 + 0 + 4 \times 3 \times 2 = 24$$

$$m[3,4] = \min_{3 \leq k < 4} = 30$$

$$k=3$$

$$m[3,3] + m[4,4] + p_2 p_3 p_4$$

$$= 0 + 0 + 3 \times 2 \times 5 = 30$$

$$m[4,5] = \min_{4 \leq k < 5} = 10$$

$$k=4$$

$$m[4,4] + m[5,5] + p_3 p_4 p_5$$

$$= 0 + 0 + 2 \times 5 \times 1 = 10$$

$$\forall k=3$$

$$m[2,3] + m[4,4] + p_1 p_3 p_4$$

$$= 24 + 0 + 4 \times 2 \times 5 = 64$$

$$m[1,3] = \min_{1 \leq k < 3} = 36$$

$$k=1$$

$$m[1,1] + m[2,3] + p_0 p_1 p_3$$

$$= 0 + 24 + 2 \times 4 \times 2 = 40$$

$$\forall k=2$$

$$m[1,2] + m[3,3] + p_0 p_2 p_3$$

$$= 24 + 0 + 2 \times 3 \times 2 = 36$$

$$m[3,5] = \min_{3 \leq k < 5} = 16$$

$$\forall k=3$$

$$m[3,3] + m[4,5] + p_2 p_3 p_5$$

$$= 0 + 10 + 3 \times 2 \times 1 = 16$$

$$k=4$$

$$m[3,4] + m[5,5] + p_2 p_4 p_5$$

$$= 30 + 0 + 3 \times 5 \times 1 = 45$$

$$m[2,4] = \min_{2 \leq k < 4} = 64$$

$$k=2$$

$$m[2,2] + m[3,4] + p_1 p_2 p_4$$

$$= 0 + 30 + 4 \times 3 \times 5 = 90$$

$$m[1,4] = \min_{1 \leq k < 4} = 56$$

$$k=1$$

$$m[1,1] + m[2,4] + p_0 p_1 p_4$$

$$= 0 + 64 + 2 \times 4 \times 5 = 104$$

$$k=2$$

$$m[1,2] + m[3,4] + p_0 p_2 p_4$$

$$= 24 + 30 + 2 \times 3 \times 5 = 84$$

$$\checkmark k=3$$

$$m[1,3] + m[4,4] + p_0 p_3 p_4$$

$$= 36 + 0 + 2 \times 2 \times 5 = 56$$

$$m[2,5] = \min_{2 \leq k < 5} = 28$$

$$\checkmark k=2$$

$$m[2,2] + m[3,5] + p_1 p_2 p_5$$

$$= 0 + 16 + 4 \times 3 \times 1 = 28$$

$$k=3$$

$$m[2,3] + m[4,5] + p_1 p_3 p_5$$

$$= 24 + 10 + 4 \times 2 \times 1$$

$$= 42$$

$$k=4$$

$$m[2,4] + m[5,5] + p_1 p_4 p_5$$

$$= 64 + 0 + 4 \times 5 \times 1 = 84$$

$$m[1,5] = \min_{1 \leq k < 5} = 36$$

$$\checkmark k=1$$

$$m[1,1] + m[2,5] + p_0 p_1 p_5$$

$$= 0 + 28 + 2 \times 4 \times 1$$

$$= 36$$

$$k=2$$

$$m[1,2] + m[3,5] + p_0 p_2 p_5$$

$$= 24 + 16 + 2 \times 3 \times 1 = 46$$

$$k=3$$

$$m[1,3] + m[4,5] + p_0 p_3 p_5$$

$$= 36 + 10 + 2 \times 2 \times 1$$

$$= 50$$

$$k=4$$

$$m[1,4] + m[5,5] + p_0 p_4 p_5$$

$$= 56 + 0 + 2 \times 5 \times 1$$

$$= 66$$