

$$\sum_{k=1}^n k^5 = \Theta(n^a), \text{ 求 } a$$

~~积分法~~ 积分法

$$\Rightarrow \int_{\frac{n}{2}}^n k^5 dk \leq \sum_{k=1}^n k^5 \leq \int_1^n k^5 dk$$

$$\text{左: } \int_{\frac{n}{2}}^n k^5 dk = \left. \frac{1}{6} k^6 \right|_{k=\frac{n}{2}} - \left. \frac{1}{6} k^6 \right|_{k=\frac{n}{2}}$$

$$= \frac{1}{6} n^6 - \frac{1}{6} \left(\frac{n}{2}\right)^6$$

$$\text{右: } \int_1^n k^5 dk = \left. \frac{1}{6} k^6 \right|_{k=1} - \left. \frac{1}{6} k^6 \right|_{k=1}$$

$$= \frac{1}{6} n^6 - \frac{1}{6}$$

$$\therefore \sum_{k=1}^n k^5 = \Theta(n^6), \text{ } a=6$$