

The *matrix-chain multiplication problem* can be stated as follows: given a chain $\langle A_1, A_2, \dots, A_n \rangle$ of matrices, where for $i = 1, 2, \dots, n$, matrix A_i has dimension $p_{i-1} * p_i$, fully parenthesize the product A_1, A_2, \dots, A_n in a way that minimizes the number of scalar multiplications. Suppose you have 6 matrices: A_1 has dimension 30×35 , A_2 has dimension 35×18 , A_3 has dimension 18×5 , A_4 has dimension 5×10 , A_5 has dimension 10×25 , A_6 has dimension 25×30 . Please calculate the minimum number of scalar multiplications.

【105 年成大資工所】

Ans.

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14
18 26=59

$$P_0 = 30$$

$$P_1 = 35$$

$$P_2 = 18$$

$$P_3 = 5$$

$$P_4 = 10$$

$$P_5 = 25$$

$$P_6 = 30$$

m	1	2	3	4	5	6
1	0	18900	8400	9900	13400	18300 117900
2		0	3150	4900	4475 8775	13400
3			0	900	3500	7700
4				0	1250	5000
5					0	17500
6						0

$$m[1.2] = 30 \times 35 \times 18 = 18900$$

$$\begin{array}{r} 105 \\ \times 18 \\ \hline 840 \\ 105 \\ \hline 1890 \end{array}$$

$$m[2.3] = 35 \times 18 \times 5 = 3150$$

$$\begin{array}{r} 35 \\ \times 90 \\ \hline 3150 \end{array}$$

$$m[3.4] = 18 \times 5 \times 10 = 900$$

$$m[4.5] = 5 \times 10 \times 25 = 1250$$

$$m[5.6] = 10 \times 25 \times 30 = 7500$$

$$m[1.3] = 8400$$

$$k=1 \quad \begin{array}{cc} 3150 & 5250 \end{array}$$

$$m[1.1] + m[2.3] + 30 \times 5 \times 35 = 8400$$

$$\begin{array}{r} 150 \\ \times 35 \\ \hline 5250 \end{array}$$

$$k=2 \quad \begin{array}{cc} 18900 & 0 \end{array}$$

$$m[1.2] + m[3.3] + 30 \times 5 \times 18 =$$

$$m[2.4] = 4900$$

$$k=2 \quad \begin{array}{cc} 900 & 6300 \end{array}$$

$$m[2.2] + m[3.4] + 35 \times 10 \times 18 =$$

$$\begin{array}{r} 350 \\ \times 18 \\ \hline 6300 \end{array}$$

$$k=3 \quad \begin{array}{cc} 3150 & 1750 \end{array}$$

$$m[2.3] + m[4.4] + 35 \times 10 \times 5 = 4900$$

$$\begin{array}{r} 35 \\ \times 50 \\ \hline 1750 \end{array}$$

$$m[3.5] = 3500$$

$$k=3 \quad \begin{array}{cc} 1250 & 2250 \end{array}$$

$$\checkmark m[3.3] + m[4.5] + 18 \times 25 \times 5 = 3500$$

$$k=4 \quad \begin{array}{cc} 900 & 0 \end{array}$$

$$m[3.4] + m[5.5] + 18 \times 25 \times 10 =$$

$$m[4.6] = 5000$$

$$k=4 \quad \begin{array}{cc} 1750 & 0 \end{array}$$

$$m[4.4] + m[5.6] + 5 \times 30 \times 10 =$$

$$k=5 \quad \begin{array}{cc} 1250 & 0 \end{array}$$

$$\checkmark m[4.5] + m[6.6] + 5 \times 30 \times 25 = 5000$$

$$\begin{array}{r} 1750 \\ \times 5 \\ \hline 8750 \\ + 1250 \\ \hline 10000 \end{array}$$

$$m[1.4] = 9900$$

$$k=1 \quad \begin{array}{cc} 4900 & 0 \end{array}$$

$$m[1.1] + m[2.4] + 30 \times 10 \times 35 =$$

$$k=2 \quad \begin{array}{cc} 18900 & 0 \end{array}$$

$$m[1.2] + m[3.4] + 30 \times 10 \times 18 =$$

$$k=3 \quad \begin{array}{cc} 8400 & 1500 \end{array}$$

$$\checkmark m[1.3] + m[4.4] + 30 \times 10 \times 5 =$$

$$m[2.5] = \cancel{9775} \quad 8775$$

$$k=2$$

$$m[2.2] + m[3.5] + 35 \times 25 \times 18$$

$$k=3$$

$$m[2.3] + m[4.5] + 35 \times 25 \times 5 = 9775$$

$$k=4$$

$$m[2.4] + m[5.5] + 35 \times 25 \times 10$$

$$m[3.6] = 7700$$

$$k=3$$

$$m[3.3] + m[4.6] + 18 \times 30 \times 5 = 7700$$

$$k=4$$

$$m[3.4] + m[5.6] + 18 \times 30 \times 10$$

$$k=5$$

$$m[3.5] + m[6.6] + 18 \times 30 \times 25$$

$$m[1.5] = 13400$$

$$m[1.1] + m[2.5] + 30 \times 25 \times 35$$

$$m[1.2] + m[3.5] + 30 \times 25 \times 18$$

$$m[1.3] + m[4.5] + 30 \times 25 \times 5 = 13400$$

$$m[1.4] + m[5.5] + 30 \times 25 \times 10$$

$$\begin{array}{r} 25 \\ 150 \\ \hline 1250 \\ 25 \\ \hline 3750 \\ + 9650 \\ \hline 13400 \end{array}$$

計算錯誤

$$\begin{array}{r} 125 \\ \times 35 \\ \hline 625 \\ 375 \\ \hline 4375 \\ + 5400 \\ \hline 9775 \end{array}$$

$$m[2.6] = 13400$$

$$m[2.2] + m[3.6] + 35 \times 30 \times 18$$

$$m[2.3] + m[4.6] + 35 \times 30 \times 5$$

$$m[2.4] + m[5.6] + 35 \times 30 \times 10$$

$$m[2.5] + m[6.6] + 35 \times 30 \times 25$$

$$\begin{array}{r} 150 \\ \times 35 \\ \hline 750 \\ 450 \\ \hline 5250 \\ + 8150 \\ \hline 13400 \end{array}$$

$$17900$$

$$m[1.6] = \cancel{18300}$$

$$m[1.1] + m[2.6] + 35 \times 30 \times 30$$

$$m[1.2] + m[3.6] + 18$$

$$m[1.3] + m[4.6] + 5$$

$$m[1.4] + m[5.6] + 10$$

$$m[1.5] + m[6.6] + 25$$

$$\begin{array}{r} 4500 \\ 9900 \\ + 8400 \\ \hline 18300 \end{array}$$

計算錯誤

$$\begin{array}{r} 9500 \\ + 8400 \\ \hline 17900 \end{array}$$