The *matrix-chain multiplication problem* can be stated as follows: given a chain $\langle A_1, A_2, \ldots, A_n \rangle$ of matrices, where for $i = 1,2,\ldots,n$, matrix A_i has dimension $p_{i-1} * p_i$, fully parenthesize the product $A_1, A_2, \ldots, A_n$ in a way that minimizes the number of scalar multiplications. Suppose you have 6 matrices: A_1 has dimesion 30x35, A_2 has dimension 35x18, A_3 has dimension 18x5, A_4 has dimension 5x10, A_5 has dimension 10x25, A_6 has dimension 25x30. Please calculate the minimum number of scalar multiplications. 【105 年成大資工所】

Ans.

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$P_0 = 30$

$P_1 = 35$

$P_2 = 18$

$P_3 = 5$

$P_4 = 10$

$P_5 = 25$

$P_6 = 30$

m	1	2	3	4	5	6
1	0	18900	8400	9900	13400	~~18300~~ 17900
2		0	3150	4900	~~4775~~ 8775	13400
3			0	900	3500	7700
4				0	1250	5000
5					0	17500
6						0

$m[1,2] = 30 \times 35 \times 18 = 18900$

105 × 18 = 840 + 105 = 1890

$m[2,3] = 35 \times 18 \times 5 = 3150$

35 × 90 = 3150

$m[3,4] = 18 \times 5 \times 10 = 900$

$m[4,5] = 5 \times 10 \times 25 = 1250$

$m[5,6] = 10 \times 25 \times 30 = 17500$

$m[1,3] = 8400$

$k=1$

$m[1,1] + m[2,3] + 30 \times 5 \times 35 = 8400$ (0, 3150, 5250)

150 × 35 = 750 + 450 = 5250

$k=2$

$m[1,2] + m[3,3] + 30 \times 5 \times 18 =$ (18900, 0)

$m[2,4] = 4900$

$k=2$

$m[2,2] + m[3,4] + 35 \times 10 \times 18 =$ (0, 900, 6300)

350 × 18 = 2800 + 350 = 6300

$k=3$

$m[2,3] + m[4,4] + 35 \times 10 \times 5 = 4900$ (3150, 0, 1750)

35 × 50 = 1750

$m[3,5] = 3500$

$k=3$

✓ $m[3,3] + m[4,5] + 18 \times 25 \times 5 = 3500$ (0, 1250, 2250)

$k=4$

$m[3,4] + m[5,5] + 18 \times 25 \times 10 =$ (900, 0)

$m[4,6] = 5000$

$k=4$

$m[4,4] + m[5,6] + 5 \times 30 \times 10 =$ (0, 17500)

$k=5$

✓ $m[4,5] + m[6,6] + 5 \times 30 \times 25 = 5000$ (1250, 0)

1750 × 5 = 3750 + 1250 = 5000

$m[1,4] = 9900$

$k=1$

$m[1,1] + m[2,4] + 30 \times 10 \times 35$ (0, 4900)

$k=2$

$m[1,2] + m[3,4] + 30 \times 10 \times 18$ (18900, 900)

$k=3$

✓ $m[1,3] + m[4,4] + 30 \times 10 \times 5$ (8400, 0, 1500)

$m[2,5] = $ ~~9775~~ 8775

計算錯誤

$k=2$

$m[2,2] + m[3,5] + 35\times25\times18$

$k=3$

$m[2,3] + m[4,5] + 35\times25\times5 = 9775$ ✓

$k=4$

$m[2,4] + m[5,5] + 35\times25\times10$

$m[3,6] = 7700$

$k=3$

$m[3,3] + m[4,6] + 18\times30\times5 = 7700$ ✓

$k=4$

$m[3,4] + m[5,6] + 18\times30\times10$ ..

$k=5$

$m[3,5] + m[6,6] + 18\times30\times25$ ..

$m[1,5] = 13400$

$m[1,1] + m[2,5] + 30\times25\times35$

$m[1,2] + m[3,5] + 30\times25\times18$

$m[1,3] + m[4,5] + 30\times25\times5 = 13400$ ✓

$m[1,4] + m[5,5] + 30\times25\times10$

$m[2,6] = 13400$

$m[2,2] + m[3,6] + 35\times30\times18$

$m[2,3] + m[4,6] + 35\times30\times5 =$

$m[2,4] + m[5,6] + 35\times30\times10$

$m[2,5] + m[6,6] + 35\times30\times25$

$m[1,6] = $ ~~18500~~

$m[1,1] + m[2,6] + 35\times30\times30$

$m[1,2] + m[3,6] + 18$ ..

$m[1,3] + m[4,6] + 5$ .. ✓

$m[1,4] + m[5,6] + 10$ ..

$m[1,5] + m[6,6] + 25$ ..

計算錯誤 ↓