利用特徵值計算遞迴關係

Example:

A restaurant provides different ways to accept reservation of ordered seats. For any individual (1 person), he/she can make reservation by email or through the web page. For any party of 2 person, they can use phone, email, or web page to reserve. How many different ways for this restaurant to make reservation of n ordered seats?

Ans. 問 n 個座位有幾種訂法?

1. 又有一個座位 $a_1 = 2$ 種 (email or web)

2. 二個座位 $a_2 = \underline{2a_1} + \underline{3} = 2\times 2 + 3 = 7$

   $2a_1$: 1個人有2種訂法 × 剩1個位的訂法

   3: 2個人的訂法有3種

⇒ n 個座位 a_n 討論1個人訂那就剩 a_{n-1} 訂法，2個人訂的話，剩 a_{n-1} 訂法。

⇒ $a_n = 2\times a_{n-1} + 3\times a_{n-2}$

⇒ $a_n - 2a_{n-1} - 3a_{n-2} = 0$

特徵值解法:

$\alpha^2 - 2\alpha - 3 = 0$

$\alpha = 3, -1$

⇒ $a_n = C_1\cdot 3^n + C_2\cdot(-1)^n$, $C_1, C_2 \in R$

⇒ $\begin{cases} a_1 = 2 = 3C_1 - C_2 \\ a_2 = 7 = 3^2C_1 + C_2 \end{cases}$ ⇒ $C_1 = \frac{3}{4}$, $C_2 = \frac{1}{4}$

代回 a_n 的式子

⇒ $a_n = \frac{3}{4}\cdot 3^n + \frac{1}{4}\cdot(-1)^n$ ※