判斷線性獨立集 86 台大資工

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題目

Let $W = \{v_1, v_2, ..., v_k\} \subseteq \mathbb{R}^n$, $A \in \mathbb{R}^{n \times n}$, If $\{Av_1, Av_2, ..., Av_k\}$ is a linearly independent set, then W is a linearly independent set.

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解:

題目說 $\{Av_1, Av_2, \dots, Av_k\}$ 爲線性獨立集, 則純在k個純量都爲0視爲唯一解 $\Rightarrow \alpha_1 Av_1 + \alpha_2 Av_2 + \dots + \alpha_k Av_k = 0, \alpha_1 = \alpha_2 = \dots = \alpha_k = 0$ $\Rightarrow (\alpha_1 A)v_1 + (\alpha_2 A)v_2 + \dots + (\alpha_k A)v_k = 0$ $\Rightarrow 0 \cdot v_1 + 0 \cdot v_2 + \dots + 0 \cdot v_k = 0$ 所以 $\{v_1, v_2, \dots, v_k\}$ 爲線性獨立集。