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Recursive Definition

Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \cdots$, if

(a)

$$a_n = 1 + (-1)^n$$

Ans.

$$a_n - a_{n-1} = \left[1 + (-1)^n\right] - \left[1 + (-1)^{n-1}\right]$$

$$= 1 + (-1)^n - 1 - 1 \cdot (-1)^{n-1}$$

$$= (-1)^n + (-1)^n = 2(-1)^n$$

$$\Rightarrow \begin{cases} a_n - a_{n-1} = 2(-1)^n, & n \geq 2 \\ a_1 = 0 \end{cases}$$

(b)

$$a_n = n(n+1).$$

Ans.

$$a_n - a_{n-1} = \left[n(n+1)\right] - \left[(n-1)(n-1+1)\right] = n^2 + n - (n^2 - n)$$

$$= 2n$$

$$\Rightarrow \begin{cases} a_n - a_{n-1} = 2n, & n \geq 2 \\ a_1 = 2 \end{cases}$$