

Euler 推廣的費馬小定理:

ex. (97 靜宜資工)

Find $3^{33} \bmod 14$.

Ans.

$$\begin{cases} \gcd(3, 14) = 1 \end{cases}$$

$$\begin{cases} \phi(14) = 14 \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{7}\right) = 14 \times \frac{1}{2} \times \frac{6}{7} = 6 \end{cases}$$

$$\Rightarrow 3^6 \equiv 1 \pmod{14}.$$

$$\Rightarrow (3^6)^5 \cdot 3^3 \equiv 1 \cdot 3^3 \equiv 13 \pmod{14}$$

$$\therefore 3^{33} \bmod 14 = 13 \quad *$$