一些矩陣觀念

1.

Question:

If A and B are nxn matrices that have the rank, then the rank of A^2 must equal the rank of B^2.

Ans. False,

$$A=\begin{bmatrix}1&0\\0&0\end{bmatrix},\ B=\begin{bmatrix}0&1\\0&0\end{bmatrix},\ A^2=\begin{bmatrix}1&0\\0&0\end{bmatrix},\ B^2=\begin{bmatrix}0&0\\0&0\end{bmatrix}.$$

$$\text{rank}(A)=\text{rank}(B)=1,\ \text{rank}(A^2)=1,\ \text{rank}(B^2)=0$$

2.

If $\{u_1, u_2, \cdots, u_k\}$ is an orthonormal set of vectors in R^n and

$V=\{u_1, u_2, \cdots, u_k\}$

then $VV^T=I_n$ (the nxn identity matrix).

Ans. False.

V要是nxn的矩陣，V^TV or VV^T才能是I_n。

ex. $V=\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

$$VV^T=\begin{bmatrix}1&0\\0&1\\0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&1\\0&1&1\end{bmatrix}\neq I$$