Coordinate vector: $[v]_\beta$ $v \in R^2$, $\beta = \{v_1, v_2\}$ 為 V 的一組有序基底

$v \in V$

$$v = av_1 + bv_2 \Rightarrow [v]_\beta = \begin{bmatrix} a \\ b \end{bmatrix}$$

自己理解：

$$[v_i]_\beta = [I]^\beta_{v_i}$$

$v_i = a\beta_1 + b\beta_1 + \cdots$

ex:

$$v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \beta = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

$$v = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow [v]_\beta = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \#$$

Transition Matrix: $[I]^\gamma_\beta$ or $P_{\gamma \leftarrow \beta}$ $\beta = \{v_1, v_2, \cdots, v_n\}$, $\gamma = \{u_1, u_2, \cdots, u_n\}$ 皆為 V 的有序基底

$$[I]^\gamma_\beta = [I_V]^\gamma_\beta = P_{\gamma \leftarrow \beta} = \begin{bmatrix} [v_1]_\gamma & [v_2]_\gamma & \cdots & [v_n]_\gamma \end{bmatrix} \Rightarrow \text{為 } F^{n \times n}$$

ex: $E = \left\{ \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$, $F = \left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$, find $P_{F \leftarrow E} = [I]^F_E$

$$\begin{bmatrix} 5 \\ 2 \end{bmatrix} = a\begin{bmatrix} 3 \\ 2 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow (a, b) = (3, -4)$$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix} = c\begin{bmatrix} 3 \\ 2 \end{bmatrix} + d\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow (c, d) = (4, -5)$$

$$P_{F \leftarrow E} = \begin{bmatrix} 3 & 4 \\ -4 & -5 \end{bmatrix} \#$$