行基本運算、列基本運算 題目:

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How will A^{-1} be affected if

(a) the ith and jth rows of A are interchanged;

(b) the ith row of A is multiplied by a nonzero scalar, c;

(c)
 c times the ith row of A is added to the jth row?

Ans.

(a)
$$(R_{ij} A)^{-1} = A^{-1} R_{ij} = A^{-1} C_{ij} \Rightarrow A^{-1} 的第i行和第j行交換。$$

(b)
$$(R_i^{(c)} A)^{-1} = A^{-1} R_i^{(\frac{1}{c})} = A^{-1} C_i^{(\frac{1}{c})} \Rightarrow A^{-1} 的第i行乘以 \frac{1}{c}。$$

(c)
$$(R_{ij}^{(c)} A)^{-1} = A^{-1} R_{ij}^{(-c)} = A^{-1} C_{ji}^{(-c)} \Rightarrow A^{-1} 的第j行乘上(-c)$$
$$加到第i行。$$

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Let $A = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ -1 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix}$, and $B = PAP^T = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$ with

P being a permutation matrix. Find P.

Ans.

$$A \xrightarrow{r_{23}} \begin{bmatrix} 2 & 0 & -1 & 0 \\ -1 & 0 & 2 & 0 \\ 0 & 2 & 0 & -1 \\ 0 & -1 & 0 & 2 \end{bmatrix} \xrightarrow{c_{23}} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$