The *matrix-chain multiplication problem* can be stated as follows: given a chain $\langle A_1, A_2, \ldots, A_n \rangle$ of matrices, where for $i = 1,2,\ldots,n$, matrix A_i has dimension $p_{i-1} * p_i$, fully parenthesize the product $A_1, A_2, \ldots, A_n$ in a way that minimizes the number of scalar multiplications. Suppose you have 6 matrices: A_1 has dimesion 30x35, A_2 has dimension 35x15, A_3 has dimension 15x5, A_4 has dimension 5x10, A_5 has dimension 10x20, A_6 has dimension 20x30. Please calculate the minimum number of scalar multiplications. 【104 年成大資工所】

Ans.

$P_0 = 30$

$P_1 = 35$

$P_2 = 15$

$P_3 = 5$

$P_4 = 10$

$P_5 = 20$

$P_6 = 30$

m	1	2	3	4	5	6
1	0	15750	~~6825~~ 78125	~~8325~~ 93125	~~13875~~ 118125	~~15375~~ 163125
2		0	~~15375~~ 2625	~~3325~~ 4375	~~6075~~ 7125	~~10875~~ 11875
3			0	1750	2500	6250
4				0	1000	4000
5					0	6000
6						0

2022.4.4

下午 22:34

$30 \times 35 \times 15 = 15750$

35
450
1750
140
15750

$35 \times 15 \times 5 = $ ~~15375~~ 2625

35
45
175
140
1575

35
x 75
175
245
2625

$m[1,3] = 6825$ ~~2625~~

$m[1,3] = m[1,1] + m[2,3] + 30 \times 5 \times 35$ ~~= 6825~~

$m[1,2] + m[3,3] + \text{〃} \times 15$

1175
30
5250
+15375
6825

5250
+2625
7875

$m[2,4] = 3325$

$m[2,2] + m[3,4] + 15 \times 35 \times 10$

~~15375~~ 2625

$m[2,3] + m[4,4] + 5 \times$ 〃

11750
+2625
4375

350
x 5
1750
+15375
3325

$m[3,5] = 2500$

$m[3,3] + m[4,5] + 5 \times 15 \times 20$

$m[3,4] + m[5,5] + 10 \times$ 〃

$m[4,6] = 4000$

$m[4,4] + m[5,6] + 10 \times 5 \times 30$

$m[4,5] + m[6,6] + 20 \times 5 \times 30$

$m[1,4] =$ ~~8525~~ $78125 + 15000 = 93125$

$m[1,1] + m[2,4] + 35 \times 30 \times 10$

$m[1,2] + m[3,4] + 15$ 〃

~~6825~~ 78125

$m[1,3] + m[4,4] + 5$ 〃

3500
+3625
7125

3000
+2575
6075

$m[2,5]$

$m[2,2] + m[3,5] + 15 \times 35 \times 20$

~~13875~~ 2625

$m[2,3] + m[4,5] + 5 \cdot$ 〃

$m[2,4] + m[5,5] + 10$ 〃

$m[3,6]$

$m[3,3] + m[4,6] + 5 \times 15 \times 30 = 6250$

$m[3,4] + m[5,6] + 10$ 〃

$m[3,5] + m[6,6] + 20$ 〃

$m[1.5] = 11875$

$m[1.1] + m[2.5] + 35 \times 600$ (0, 6075)

$m[1.2] + m[3.5] + 15$ "

$m[1.3] + m[4.5] + 5$ "

$m[1.4] + m[5.5] + 10$ "

78125 11875 = 4000 + 7825 = 10825

$m[2.6] = 11875$

$m[2.2] + m[3.6] + 15 \times 35 \times 30$ (0, 6250)

$m[2.3] + m[4.6] + 5 \times$ " (3625, 4000) $= 6625 + 5250 = 11875$

$m[2.4] + m[5.6] + 10 \times$ " (3325, 6000)

$m[2.5] + m[6.6] + 20 \times$ " (6075, 0)

6625 + 5250 = 11875

11875 1050 / 5250

5575 + 5250 = 10825

$m[1.6] = 16375$

$m[1.1] + m[2.6] + 35 \times 900$ (0, 10825)

$m[1.2] + m[3.6] + 15$ " (15750, 6250)

$m[1.3] + m[4.6] + 5$ " (7875, 4000)

$m[1.4] + m[5.6] + 10$ " (8325, 6000)

$m[1.5] + m[6.6] + 20$ " (10825, 0)

10825 + 4500 = 15325

11875 + 4500 = 16375