列運算的行列式問題

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If B is a matrix obtained from a square matrix A by adding k times row i to row j, then $\det(B) = k\det(A)$.

Ans

取 $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $k \in F$, 則 $B=\begin{bmatrix} a+kc & b+kd \\ c & d \end{bmatrix}$

$\det(A) = ad-bc$, $\det(B) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} kc & kd \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + k\begin{vmatrix} c & d \\ c & d \end{vmatrix}$

$= \begin{vmatrix} a & b \\ c & d \end{vmatrix} + 0 = ad-bc$

$\therefore \det(B) \neq k\det(A)$ ※