Determinant by cofactors

$A \in F^{3\times 3}$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & & \\ & a_{22} & a_{23} \\ & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} & a_{12} & \\ a_{21} & & a_{23} \\ a_{31} & & a_{33} \end{vmatrix} + \begin{vmatrix} & & a_{13} \\ a_{21} & a_{22} & \\ a_{31} & a_{32} & \end{vmatrix}$$

The entries $a_{ij}$ (row 1 in this) will create $(-1)^{(i+j)}$ multiple submatrix $M_{ij}$.

The signs pattern: $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$.

$$\Rightarrow a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$ #

Cofactor formula: $\det A = a_{i1} C_{i1} + a_{i2} C_{i2} + \cdots + a_{in} C_{in}$

Cofactor $= C_{ij} = \cancel{(-1)^{(i+j)} \det(M_{ij})}$

$(-1)^{(i+j)} \cdot \det(M_{ij})$