Gram-Schmidt 正交化例題：

$$S = \left\{ v_1 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \subseteq V$$

Gram-Schmidt process begins from v_1 :

formulae :

$$u_k = v_k - \sum_{i=1}^{k-1} \frac{\langle v_k, u_i \rangle}{\langle u_i, u_i \rangle} u_i$$

補充：
如果要得
orthonormal set
$$\Rightarrow \left\{ \frac{u_1}{\|u_1\|}, \frac{u_2}{\|u_2\|}, \frac{u_3}{\|u_3\|} \right\}$$

1. Let $u_1 = v_1 = (2, 1, 2)$

2. $$u_2 = v_2 - \frac{\langle v_2, u_1 \rangle}{\langle u_1, u_1 \rangle} u_1 = (1, 0, -1)$$

3. $$u_3 = v_3 - \frac{\langle v_3, u_2 \rangle}{\langle u_2, u_2 \rangle} u_2 - \frac{\langle v_3, u_1 \rangle}{\langle u_1, u_1 \rangle} u_1 = \frac{1}{18} \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix} \#$$

計算：

$\langle v_2, u_1 \rangle = 0$　　$\langle v_3, u_1 \rangle = 2$

$\langle u_1, u_1 \rangle = 9$

$\langle v_3, u_2 \rangle = -1$

$\langle u_2, u_2 \rangle = 2$

$$u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \frac{-1}{2} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} - \frac{2}{9} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{18} \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$$