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A hiker faces the knapsack problem. There are 7 items to be packed into the knapsack, each with value v_i and weight w_i as shown in the following table.

i	1	2	3	4	5	6	7
v_i	8	6	15	3	3	5	9
w_i	2	3	5	4	3	2	6

The knapsack, which is initially empty, can hold a maximum weight of 16, so some item(s) must be left behind. The optimality criterion is to maximize the total value of the items that are placed in the knapsack. The hiker fills the knapsack one item at a time. Now consider the following two cases: (a) fractions of items cannot be packed, and (2) fractions of item can be packed. What are the optimal values of the items that are packed in these two cases respectively?

Give the answer in the form of (no-fractions-allowed, fractions-allowed)

(A) (28,30.5) (B) (37,40) (C) (30,34) (D) (38,40) (E) (34,38.5)

【104 年交大資工】

解 (B)

item 單位價值

$i_1 = \frac{8}{2} = 4$

$i_2 = \frac{6}{3} = 2$

$i_3 = \frac{15}{5} = 3$

$i_4 = \frac{3}{4} = 0.75$

$i_5 = \frac{3}{3} = 1$

$i_6 = \frac{5}{2} = 2.5$

$i_7 = \frac{9}{6} = 1.5$

i	1	3	6	2	7	5	4
v	8	15	5	6	9	3	3
w	2	5	2	3	6	3	4

最高負重16

fractional：

拿 i_1, i_3, i_6, i_2 的負重為：$2+5+2+3=12$

i_7 只能拿4，其價值為 $\frac{4}{6} \times 9 = 6$

得背包總價為：$V_1 + V_3 + V_6 + V_2 + 6$

$= 8+15+5+6+6 = 40$

0/1 knapsack

	可負重多少	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\{i_1\}$	i_1	0	0	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
$\{i_1, i_3\}$	i_3	0	0	8	8	8	15	15	23	23	23	23	23	23	23	23	23	23
$\{i_1, i_3, i_6\}$	i_6	0	0	8	8	13	15	15	23	23	28	28	28	28	28	28	28	28
$\{i_1, i_3, i_6, i_2\}$	i_2	0	0	8	8	13	15	15	23	23	28	29	29	34	34	34	34	34
$\{i_1, i_3, i_6, i_2, i_7\}$	i_7	0	0	8	8	13	15	15	23	23	28	29	29	34	34	34	34	34
$\{i_1, i_3, i_6, i_2, i_7, i_5\}$	i_5	0	0	8	8	13	15	15	23	23	28	29	29	34	34	34	34	34
{all}	i_4	0	0	8	8	13	15	15	23	23	28	29	29	34	34	34	34	37