判斷時間複雜度問題:

Assume a set G whose elements are real numbers and the size of G is equal to 2^k, where k is a positive integer. We just want to find the maximum value and the minimum value in this set G and develop an algorithm in the following.

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1. FindMaxMin(G)
2.     IF G contains only two number. DO
3.         Compare these two numbers, set M to be
               the larger one, and set m to be the
               smaller one.
4.     ELSE
5.         Divide G into two subsets with equal size
               G1 and G2.
6.         Apply FindMaxMin(G1) to get M1 and m1.
7.         Apply FindMaxMin(G2) to get M2 and m2.
8.         M := max(M1, M2), m := min(m1, m2)
9.     RETURN M, m
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We use $T(N)$ to represent the number of comparisons when the set size is equal to N. $N=2^k$.

Please find $T(N)$ in terms of N.

Ans.

可明顯看出，當 $N=2$ 時，只有執行 1 次第3行的判斷 M, m 大小，即 $T(2)=1$

如果 $N>2$ 時，會將 N 分成兩塊一樣大小，一塊再呼叫自己，另一塊呼叫自己，成本為 $2T(\frac{N}{2})$，接著判斷 $M=\max(M_1,M_2)$，$m=\min(m_1,m_2)$，要固定執行 2 次，此外將 N 分成兩塊，並不一定要花費成本，因為呼叫 Function 時，直接把一半 Index 傳去就行了。

例如：

FindMaxMin({34, 2, 3, 7})

⋮

ELSE

⋮

M_1, m_1 := FindMaxMin({34, 2})

M_2, m_2 := FindMaxMin({3, 7})

⋮

最後得時間複雜度為

$$\begin{cases} T(N) = 2T(\frac{N}{2}) + 2 & , N > 2 \\ T(2) = 1 \end{cases}$$

題目問 $T(N)$ in term N.

題目有給 $N=2^k$，即使用變數變換解題：

$$\Rightarrow T(N) = T(2^k) = 2T(2^{k-1}) + 2 = 2T(2T(2^{k-2})+2)+2 = \cdots$$

變數變換

$$\Rightarrow x_k = T(2^k), \quad x_{k-1} = T(2^{k-1}), \quad x_{k-2} = T(2^{k-2}), \cdots$$

$$x_1 = T(2^1) = 1$$

$$\Rightarrow x_k = 2x_{k-1} + 2 = 2(2x_{k-2}+2)+2 \overset{整理}{=} 2^2 x_{k-2} + 2^2 + 2^1$$

$$= 2^2(2x_{k-3}+2) + 2^2 + 2^1 \overset{整理}{=} 2^3 x_{k-3} + 2^3 + 2^2 + 2^1 = \cdots\cdots$$

$$= 2^{k-1}x_1 + 2^{k-1} + 2^{k-2} + \cdots + 2^2 + 2^1$$

因 $x_1 = T(2) = 1$，代入

$$= (2^{k-1}) + (2^{k-1} + 2^{k-2} + \cdots\cdots + 2^2 + 2^1)$$

Hint:

等比級數公式

當倍數大於1時，假設倍率為c，初值為a

則 $a\cdot\frac{1-c^n}{1-c} = a\cdot\frac{c^n-1}{c-1}$

$$\Rightarrow \left(2^{k-1}\right) + 2\times\frac{2^{k-1}-1}{\underset{1}{\cancel{2-1}}} = 2^{k-1}+2^{k}-2$$

$$= \frac{2^{k}}{2}+2^{k}-2 = \frac{N}{2}+N-2 = \frac{3}{2}N-2$$

即 $T(N)=\frac{3}{2}N-2$ #