a_{n-2} 型

1. $\sum_{n=2}^{\infty} a_{n-2}x^n = a_0x^2 + a_1x^3 + \cdots + a_{n-2}x^n + \cdots$

$= x^2(a_0 + a_1x^1 + \cdots + a_{n-2}x^{n-2} + \cdots)$

$= x^2\left(\sum_{n=0}^{\infty} a_n x^n\right)$

2. $\sum_{n=3}^{\infty} a_{n-2}x^n = a_1x^3 + a_2x^4 + \cdots$

$= x^2(a_1x^1 + a_2x^2 + \cdots)$

$= x^2\left(\sum_{n=0}^{\infty} a_n x^n - a_0\right)$

3. $\sum_{n=4}^{\infty} a_{n-2}x^n = a_2x^4 + a_3x^5 + \cdots$

$= x^2(a_2x^2 + a_3x^3 + \cdots)$

$= x^2\left(\sum_{n=0}^{\infty} a_n x^n - a_0 - a_1x^1\right)$

$a_n = a_{n-1} + a_{n-2}$ $n \geq 2$, $f(x) = \sum_{n=0}^{\infty} a_n x^n$

$\Rightarrow \sum_{n=2}^{\infty} a_n x^n = \sum_{n=2}^{\infty} a_{n-1}x^n + \sum_{n=2}^{\infty} a_{n-2}x^n$

~~$\Rightarrow f(x) - a_0 - a_1x =$~~

$\Rightarrow f(x) - a_0x^0 - a_1x^1 + a_0x$

$= x(f(x) - a_0) + x^2(f(x))$

$= xf(x) - \cancel{a_0x} + x^2f(x) - f(x)$

$= f(x)[x^2 + x - 1]$

$\sum_{n=2}^{\infty} a_n x^n = a_2x^2 + a_3x^3 + \cdots$

$= \sum_{n=0}^{\infty} a_n x^n - a_0x^0 - a_1x^1$

$\sum_{n=2}^{\infty} a_{n-1}x^n = a_1x^2 + a_2x^3 + \cdots$

$= x(a_1x + a_2x^2 + \cdots)$

$= x\left(\sum_{n=0}^{\infty} a_n x^n - a_0\right)$

$\sum_{n=2}^{\infty} a_{n-2}x^n = a_0x^2 + \cdots$

$= x^2(a_0x^0 + a_1x^1 + \cdots)$

$= x^2\left(\sum_{n=0}^{\infty} a_n x^n\right)$

$\Rightarrow f(x) = \dfrac{\cancel{-a_0} - a_1x + \cancel{a_0x}}{x^2 + x - 1} = \dfrac{-x}{x^2 + x - 1}$

求 a_n 系數

a_n 꼴

1. $\sum_{n=1}^{\infty} a_n x^n$

$= a_1 x^1 + a_2 x^2 + \cdots + a_n x^n + \cdots$

$= \sum_{n=0}^{\infty} a_n x^n - a_0 x^0 = \sum_{n=0}^{\infty} a_n x^n - a_0$

2. $\sum_{n=2}^{\infty} a_n x^n$

$= a_2 x^2 + a_3 x^3 + \cdots + a_n x^n + \cdots$

$= \sum_{n=0}^{\infty} a_n x^n - a_0 x^0 - a_1 x^1 = \sum_{n=0}^{\infty} a_n x^n - a_0 - a_1 x^1$

3. $\sum_{n=3}^{\infty} a_n x^n = \sum_{n=0}^{\infty} a_n x^n - a_0 x^0 - a_1 x^1 - a_2 x^2$

a_{n-1} 꼴

1. $\sum_{n=1}^{\infty} a_{n-1} x^n = a_0 x^1 + a_1 x^2 + \cdots + a_{n-1} x^n + a_n x^{n+1} + \cdots$

$= x(a_0 x^0 + a_1 x^1 + \cdots + a_n x^n + \cdots)$

$= x \sum_{n=0}^{\infty} a_n x^n$

2. $\sum_{n=2}^{\infty} a_{n-1} x^n = a_1 x^2 + a_2 x^3 + \cdots + a_{n-1} x^n + \cdots$

$= x\left(a_1 x^1 + a_2 x^2 + \cdots + a_{n-1} x^n + \cdots\right)$

$= x\left(\sum_{n=0}^{\infty} a_n x^n - a_0\right)$

3. $\sum_{n=3}^{\infty} a_{n-1} x^n = x\left(\sum_{n=0}^{\infty} a_n x^n - a_0 - a_1 x\right)$