台大 109 線代

Question:

If the polynomial function $f(x)=ax^4+bx^3+cx^2+dx+e$

satisfies

$$\left|\begin{array}{l} f(-2)=150 \\ f(-1)=16 \\ f(0)=2 \\ f(1)=18 \\ f(2)=166. \end{array}\right.$$

then $a,b,c,d,e$ are ___, ___, ___, ___, ___, respectively.

Ans. 策略:

有5個結果，代入解題

$$16a-8b+4c-2d+e=150 \quad -①$$
$$a-b+c-d+e=16 \quad -②$$
$$e=2 \quad -③$$
$$a+b+c+d+e=18 \quad -④$$
$$16a+8b+4c+2d+e=166 \quad -⑤$$

$$\Rightarrow \begin{bmatrix} 16 & -8 & 4 & -2 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 16 & 8 & 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 150 \\ 16 \\ 2 \\ 18 \\ 166 \end{bmatrix}$$

⇒ argumented matrix

$$\left[\begin{array}{ccccc|c} 16 & -8 & 4 & -2 & 1 & 150 \\ 1 & -1 & 1 & -1 & 1 & 16 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 & 18 \\ 16 & 8 & 4 & 2 & 1 & 166 \end{array}\right] \xrightarrow{R_{35},\, R_{41}} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 18 \\ 1 & -1 & 1 & -1 & 1 & 16 \\ 16 & 8 & 4 & 2 & 1 & 166 \\ 16 & -8 & 4 & -2 & 1 & 150 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow{R_{12}^{(-1)},\, R_{13}^{(-16)},\, R_{14}^{(-16)}} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 18 \\ 0 & -2 & 0 & -2 & 0 & -2 \\ 0 & -8 & -12 & -14 & -15 & -122 \\ 0 & -24 & -12 & -18 & -15 & -138 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{R_{23}^{(-4)},\, R_{24}^{(-12)}} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 18 \\ 0 & -2 & 0 & -2 & 0 & -2 \\ 0 & 0 & -12 & -6 & -15 & -114 \\ 0 & 0 & -12 & 6 & -15 & -114 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow{R_{34}^{(-1)},\, R_{2}^{(-\frac{1}{2})}} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 18 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -12 & -6 & -15 & -114 \\ 0 & 0 & 0 & 12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{R_{4}^{(\frac{1}{12})},\, R_{3}^{(-\frac{1}{3})}} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 18 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 4 & 2 & 5 & 38 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow{R_{53}^{(-5)},\, R_{43}^{(-2)},\, R_{42}^{(-1)}} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 18 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 4 & 0 & 0 & 28 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{R_{3}^{(\frac{1}{4})},\, R_{51}^{(-1)},\, R_{41}^{(-1)},\, R_{31}^{(-1)},\, R_{21}^{(-1)}} \cdots$$

$$\cdots \rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$\therefore a = 8,\ b = 1,\ c = 7,\ d = 0,\ e = 2$