排列矩陣的行列式問題

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Let B be obtained from $A \in \mathbb{C}^{5\times 5}$ by moving the i-th row to the $(i+1)$-th row, $i = 1, \dots, 4$, and moving the fifth row to the first row. Then $\det(A) = \det(B)$.

Ans.

假設 $A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix}$，則 $B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = PA$，其中 $P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

$$\Rightarrow \det(B) = \det(PA) = \det(P)\det(A) = (-1)^4 \det(A) = \det(A) \#$$