Give an $O(n^2)$-time algorithm to find the longest monotonically increasing subsequence of a sequence of n numbers. And illustrate your algorithm by the following example, (66, 92, 123, 31, 83, 53, 48, 17, 9, 57, 75)

【91 年台大資工所、92 年交大資工所年、99 年中央資工所】

Ans.

X: (66, 92, 123, 31, 83, 53, 48, 17, 9, 57, 75)

Y: (9, 17, 31, 48, 53, 57, 66, 75, 83, 92, 123)

LIS 比對：

	∅	9	17	31	48	53	57	66	75	83	92	123
∅	0	0	0	0	0	0	0	0	0	0	0	0
66	0	↑0	↑0	↑0	↑0	↑0	↑0	↖1	←1	←1	←1	←1
92	0	↑0	↑0	↑0	↑0	↑0	↑0	↑1	↑1	↑1	↖2	←2
123	0	↑0	↑0	↑0	↑0	↑0	↑0	↑1	↑1	↑1	↑2	↖3
31	0	↑0	↑0	(↖1)	←1	←1	←1	↑1	↑1	↑1	↑2	↑3
83	0	↑0	↑0	↑1	↑1	↑1	↑1	↑1	↑1	↖2	↑2	↑3
53	0	↑0	↑0	↑1	↑1	(↖2)	←2	←2	←2	↑2	↑2	↑3
48	0	↑0	↑0	↑1	↖2	↑2	↑2	↑2	↑2	↑2	↑2	↑3
17	0	↑0	↖1	↑1	↑2	↑2	↑2	↑2	↑2	↑2	↑2	↑3
9	0	↖1	↑1	↑1	↑2	↑2	↑2	↑2	↑2	↑2	↑2	↑3
57	0	↑1	↑1	↑1	↑2	↑2	(↖3)	←3	←3	←3	←3	↑3
75	0	↑1	↑1	↑1	↑2	↑2	↑3	↑3	(↖4)	←4	←4	←4

經 Backtracking 得 <31, 53, 57, 75> 為序列的最長遞增子序列，且長度為 4。