```
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```

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# $\underline{\text{Contest}}$ (1)

# sol.cpp

```
#include <bits/stdc++.h>
using namespace std;
#define rep(i, a, b) for (int i = (a); i < (b); i++)
#define all(x) begin(x), end(x)
#define sz(x) int((x).size())
using 11 = long long;
using pii = pair<int, int>;
using vi = vector<int>;
#ifdef LOCAL
auto& operator<<(auto&, pair<auto, auto>);
auto operator<<(auto& o, auto x) -> decltype(x.end(), o) {
  for (int i = 0; auto y : x) \circ << ", " + !i++ * 2 << y;
 return o << ' \';
auto& operator<<(auto& o, pair<auto, auto> x) {
 return o << '(' << x.first << ", " << x.second << ')';</pre>
void __print(auto... x) { ((cerr << ' ' << x), ...) << endl; }</pre>
#define debug(x...) cerr << "[" #x "]:", __print(x)
#else
#define debug(...) 2137
#endif
int main() {
 cin.tie(0)->sync_with_stdio(0);
```

#### .vimrc

```
set nu et ts=2 sw=2
filetype indent on
syntax on
colorscheme habamax
hi MatchParen ctermfg=66 ctermbg=234 cterm=underline
nnoremap;:
nnoremap;
inoremap {<cr> {<cr> }<cr> }<sc> <br/>inoremap {<cr> }<cr> }<sc> <br/>colorscheme
```

## Makefile

```
CXXFLAGS=-std=c++20 -Wall -Wextra -Wshadow
sol: sol.cpp
g++ $(CXXFLAGS) -fsanitize=address,undefined -g -DLOCAL \
sol.cpp -o sol
```

```
fast: sol.cpp
     q++ $(CXXFLAGS) -02 sol.cpp -o fast
1
    test.sh
    #!/bin/bash
1
   for((i=1;i>0;i++)) do
     echo "$i"
     echo "$i" | ./gen > int
     diff -w < (./sol < int) < (./slow < int) || break
3
   hash.sh
    #!/bin/bash
    cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum |cut -c-6
    .bashrc
   alias rm='trash'
   alias mv='mv -i'
```

# Struktury danych (2)

# Grafy (3)

alias cp='cp -i'

# 3.1 Przepływy

# Dinic.h

Opis: Dinic ze skalowaniem. Należy ustawić zakres it w flow zgodnie z U. Czas:  $\mathcal{O}(nm\log U)$ 

```
struct dinic {
 struct edge {
   int to, rev;
   11 cap;
 };
 vi lvl, ptr, q;
 vector<vector<edge>> adi;
 dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
 void add_edge(int u, int v, ll cap, ll rcap = 0) {
   int i = sz(adj[u]), j = sz(adj[v]);
   adj[u].push_back(\{v, j + (u == v), cap\});
   adj[v].push_back({u, i, rcap});
 11 dfs(int v, int t, 11 f) {
    if (v == t || !f) return f;
   for (int& i = ptr[v]; i < sz(adj[v]); i++) {</pre>
     edge& e = adj[v][i];
```

```
if (lvl[e.to] == lvl[v] + 1)
       if (ll p = dfs(e.to, t, min(f, e.cap))) {
         e.cap -= p, adj[e.to][e.rev].cap += p;
          return p;
    return 0;
 11 flow(int s, int t) {
   11 f = 0; q[0] = s;
    for (int it = 29; it >= 0; it--) do {
     lvl = ptr = vi(sz(q));
      int qi = 0, qe = lvl[s] = 1;
      while (qi < qe && !lvl[t]) {
       int v = q[qi++];
       for (edge e : adj[v])
         if (!lvl[e.to] && e.cap >> it)
            q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
      while (ll p = dfs(s, t, LLONG_MAX)) f += p;
    } while (lvl[t]);
    return f;
};
```

### GomorvHu.h

Opis: Tworzy drzewo gdzie min cut to minimum na ścieżce.

Czas:  $\mathcal{O}(n)$  przepływów

```
struct edge { int u, v; ll w; };
vector<edge> gomory_hu(int n, const vector<edge>& ed) {
  vector<edge> t; vi p(n);
  rep(i, 1, n) {
    dinic d(n);
    for (edge e : ed) d.add_edge(e.u, e.v, e.w, e.w);
      t.push_back({i, p[i], d.flow(i, p[i])});
      rep(j, i + 1, n) if (p[j] == p[i] && d.lvl[j]) p[j] = i;
  }
  return t;
}
```

### MCMF.h

 ${\bf Opis:}\ {\rm MCMF}$ z Dijkstrą. Jeżeli są ujemne krawędzie to przed puszczeniem flow w pi trzeba policzyć najkrótsze ścieżki z s.

Czas:  $\mathcal{O}(Fm \log n)$ 

```
#include <ext/pb_ds/priority_queue.hpp>
const 11 INF = 2e18;
struct MCMF {
  struct edge {
    int from, to, rev;
    11 cap, cost;
  };
 int n;
  vector<vector<edge>> adj;
  vector<ll> dst, pi;
  __gnu_pbds::priority_queue<pair<ll, int>> q;
  vector<decltype(q)::point_iterator> it;
  vector<edge*> p;
  MCMF(int _n) : n(_n), adj(n), pi(n), p(n) {}
  void add_edge(int u, int v, ll cap, ll cost) {
    int i = sz(adj[u]), j = sz(adj[v]);
    adj[u].push_back({u, v, j + (u == v), cap, cost});
    adj[v].push_back({v, u, i, 0, -cost});
 bool path(int s, int t) {
    dst.assign(n, INF); it.assign(n, q.end());
    q.push(\{dst[s] = 0, s\});
    while (!q.empty()) {
```

## SCC TwoSat RDSU DMST GCD CRT

```
int u = q.top().second; q.pop();
      for (edge& e : adj[u]) {
       11 d = dst[u] + pi[u] + e.cost - pi[e.to];
       if (e.cap && d < dst[e.to]) {</pre>
         dst[e.to] = d, p[e.to] = &e;
         if (it[e.to] == q.end())
           it[e.to] = q.push({-dst[e.to], e.to});
          else
            q.modify(it[e.to], {-dst[e.to], e.to});
     }
    rep(i, 0, n) pi[i] = min(pi[i] + dst[i], INF);
   return pi[t] != INF;
  pair<ll, ll> flow(int s, int t, ll cap) {
   11 f = 0, c = 0;
    while (f < cap && path(s, t)) {
     11 d = cap - f;
     for (edge* e = p[t]; e; e = p[e->from])
       d = min(d, e->cap);
     for (edge* e = p[t]; e; e = p[e->from])
       e->cap -= d, adj[e->to][e->rev].cap += d;
      f += d, c += d * pi[t];
    return {f, c};
};
      DFS
SCC.h
Opis: Znajduje SCC w kolejności topologicznej.
```

Czas:  $\mathcal{O}(n+m)$ 

```
struct SCC {
 int n, t = 0, cnt = 0;
  vector<vi> adj;
 vi val, p, st;
  SCC(int _n) : n(_n), adj(n), val(n), p(n, -1) {}
  void add_edge(int u, int v) { adj[u].push_back(v); }
  int dfs(int u) {
   int low = val[u] = ++t; st.push_back(u);
    for (int v : adj[u]) if (p[v] == -1)
     low = min(low, val[v] ?: dfs(v));
    if (low == val[u]) {
     for (int x = -1; x != u;)
       p[x = st.back()] = cnt, st.pop_back();
     cnt++;
    return low;
  void build() {
   rep(i, 0, n) if (!val[i]) dfs(i);
    rep(i, 0, n) p[i] = cnt - 1 - p[i];
};
TwoSat.h
```

```
struct two_sat {
  int n;
  vector<pii> ed;
  vector<bool> b;
  two_sat(int _n) : n(_n) {}
```

Opis: 2-SAT. Czas:  $\mathcal{O}(n+m)$ 

```
int add var() { return n++; }
 void either(int x, int y) {
   x = max(2 * x, -1 - 2 * x), y = max(2 * y, -1 - 2 * y);
   ed.push_back({x, y}); }
 void implies(int x, int y) { either(~x, y); }
 void must(int x) { either(x, x); }
 void at most one(const vi& v) {
   if (sz(v) <= 1) return;</pre>
    int cur = ~v[0];
    rep(i, 2, sz(v)) {
     int nxt = add_var();
     either(cur, ~v[i]); either(cur, nxt);
     either(~v[i], nxt); cur = ~nxt;
    either(cur, ~v[1]);
 bool solve() {
   SCC scc(2 * n);
    for (auto [u, v] : ed)
     scc.add_edge(u ^ 1, v), scc.add_edge(v ^ 1, u);
    scc.build(); b.resize(n, 1);
   rep(i, 0, n) {
     if (scc.p[2 * i] == scc.p[2 * i + 1]) return 0;
     if (scc.p[2 * i] < scc.p[2 * i + 1]) b[i] = 0;</pre>
    return 1;
};
```

# 3.3 **DSU**

## RDSU.h

Opis: DSU z rollbackami.

Czas:  $\mathcal{O}(\log n)$ 

```
struct RDSU {
 vi e; vector<pii> st;
 RDSU(int n) : e(n, -1) {}
 int size(int x) { return -e[find(x)]; }
 int find(int x) { return e[x] < 0 ? x : find(e[x]); }</pre>
 int time() { return sz(st); }
 void rollback(int t) {
    for (int i = time(); i-- > t;)
     e[st[i].first] = st[i].second;
    st.resize(t);
 bool join(int a, int b) {
   a = find(a), b = find(b);
   if (a == b) return false;
   if (e[a] > e[b]) swap(a, b);
   st.push_back({a, e[a]});
   st.push_back({b, e[b]});
   e[a] += e[b]; e[b] = a;
   return true;
};
```

# 3.4 Drzewa

# DMST.h

```
Opis: Skierowane MST z r. Zwraca {-1, {}} gdy nie istnieje.
Czas: \mathcal{O}(m \log m)
```

```
struct edge { int a, b; ll w; };
struct node {
  edge kev;
  node *1 = 0, *r = 0;
  11 delta = 0;
  void prop() {
   kev.w += delta;
   if (1) 1->delta += delta;
   if (r) r->delta += delta;
    delta = 0:
 edge top() { prop(); return key; }
node* merge(node* a, node* b) {
 if (!a || !b) return a ?: b;
 a->prop(), b->prop();
 if (a->key.w > b->key.w) swap(a, b);
 swap(a->1, (a->r = merge(b, a->r)));
 return a:
void pop(node*\& a) { a->prop(); a = merge(a->1, a->r); }
pair<11, vi> dmst(int n, int r, const vector<edge>& g) {
 RDSU uf(n);
  vector<node*> heap(n);
  for (edge e : g) heap[e.b] = merge(heap[e.b], new node{e});
 11 \text{ res} = 0;
 vi seen(n, -1), path(n), par(n);
 seen[r] = r;
 vector<edge> Q(n), in(n, \{-1, -1, -1\}), comp;
  deque<tuple<int, int, vector<edge>>> cycs;
  rep(s, 0, n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {</pre>
     if (!heap[u]) return {-1, {}};
      edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
       node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
    rep(i, 0, qi) in[uf.find(Q[i].b)] = Q[i];
  for (auto& [u, t, cmp] : cycs) {
   uf.rollback(t);
    edge inedge = in[u];
    for (auto& e : cmp) in[uf.find(e.b)] = e;
    in[uf.find(inedge.b)] = inedge;
 rep(i, 0, n) par[i] = in[i].a;
 return {res, par};
```

# Matma (4)

# 4.1 Arytmetyka modularna

# GCD.h

Opis: Rozszerzony algorytm Euklidesa.

```
Czas: \mathcal{O}(\log \min(a, b))
```

```
ll gcd(ll a, ll b, ll &x, ll &v) {
 if (!b) return x = 1, y = 0, a;
 11 d = gcd(b, a % b, y, x);
 return y -= a / b * x, d;
CRT.h
Opis: Chińskie twierdzenie o resztach.
Czas: \mathcal{O}(\log \min(m, n))
11 crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
 11 x, y, g = gcd(m, n, x, y);
  assert((a - b) % q == 0); // no solution
 x = (b - a) % n * x % n / q * m + a;
```

#### ModMul.h

**return** x < 0 ? x + m \* n / q : x;

Opis: Mnożenie i potegowanie dwóch long longów modulo. Jest to wyraźnie szybsze niż zamiana na int128.

```
using ull = uint64 t;
ull modmul(ull a, ull b, ull M) {
 11 ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (11)M);
ull modpow(ull b, ull e, ull mod) {
 ull ans = 1:
  for (; e; b = modmul(b, b, mod), e /= 2)
   if (e & 1) ans = modmul(ans, b, mod);
  return ans;
```

### ModInt.h

```
template<int M, int R>
struct mod {
  static const int MOD = M, ROOT = R;
  int x;
  mod(11 y = 0) : x(y % M) { x += (x < 0) * M; }
  mod operator += (mod o) {
   if ((x += 0.x) >= M) x -= M;
   return *this; }
  mod operator-= (mod o) {
   if ((x -= 0.x) < 0) x += M;
   return *this: }
  mod operator *= (mod o) {
   x = 111 * x * 0.x % M;
   return *this; }
  mod operator/=(mod o) { return (*this) *= o.inv(); }
  friend mod operator+(mod a, mod b) { return a += b; }
  friend mod operator-(mod a, mod b) { return a -= b; }
  friend mod operator*(mod a, mod b) { return a *= b; }
  friend mod operator/(mod a, mod b) { return a /= b; }
  auto operator<=>(const mod&) const = default;
  mod pow(ll n) const {
   mod a = x, b = 1;
   for (; n; n /= 2, a *= a) if (n & 1) b *= a;
   return h:
 mod inv() const { return pow(M - 2); }
using mint = mod<998244353, 3>;
```

# 4.2 Liczby pierwsze

## MillerRabin.h

Opis: Test pierwszości Millera-Rabina.

```
bool prime(ull n) {
 if (n < 2 | | n % 6 % 4 != 1) return (n | 1) == 3;
 ull A[] = \{2, 325, 9375, 28178, 450775, 9780504, 1795265022\},
     s = \underline{builtin_ctzll(n - 1)}, d = n >> s;
  for (ull a : A) {
   ull p = modpow(a % n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
     p = modmul(p, p, n);
    if (p != n - 1 && i != s) return 0;
 return 1;
```

#### PollardRho.h

Opis: Algorytm faktoryzacji rho Pollarda.

Czas:  $\mathcal{O}(n^{1/4})$ 

```
ull pollard(ull n) {
 ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
 auto f = [&](ull x) { return modmul(x, x, n) + i; };
 while (t++ % 40 || __gcd(prd, n) == 1) {
   if (x == y) x = ++i, y = f(x);
   if ((q = modmul(prd, max(x, y) - min(x, y), n))) prd = q;
   x = f(x), y = f(f(y));
 return __gcd(prd, n);
void factor(ull n, map<ull, int>& cnt) {
 if (n == 1) return;
 if (prime(n)) { cnt[n]++; return; }
 ull x = pollard(n);
 factor(x, cnt); factor(n / x, cnt);
```

# 4.3 Wielomiany

```
NTT.h
```

```
Czas: \mathcal{O}((n+m)\log(n+m))
```

```
template<class T>
void ntt(vector<T>& a, bool inv) {
 int n = sz(a); vector<T> b(n);
 for (int i = n / 2; i; i /= 2, swap(a, b)) {
   T w = T(T::ROOT).pow((T::MOD - 1) / n * i), m = 1;
   for (int j = 0; j < n; j += 2 * i, m *= w) rep(k, 0, i) {
    T u = a[j + k], v = a[j + k + i] * m;
     b[j / 2 + k] = u + v, b[j / 2 + k + n / 2] = u - v;
 if (inv) {
   reverse(1 + all(a));
   T z = T(n).inv(); rep(i, 0, n) a[i] *= z;
template<class T>
vector<T> conv(vector<T> a, vector<T> b) {
 a.resize(n); ntt(a, 0); b.resize(n); ntt(b, 0);
 rep(i, 0, n) a[i] *= b[i];
 ntt(a, 1); a.resize(s);
 return a;
```

```
Opis: NTT z Garnerem. Działa dla n + m < 2^{24} i c_k < 5 \cdot 10^{25}.
Czas: \mathcal{O}((n+m)\log(n+m))
```

```
template<class T>
vector<T> mconv(const auto& x, const auto& y) {
  auto con = [&](const auto& v) {
    vector<T> w(sz(v)); rep(i, 0, sz(v)) w[i] = v[i].x;
    return w; };
  return conv(con(x), con(y));
template<class T>
vector<T> conv3(const vector<T>& a, const vector<T>& b) {
 using m0 = mod<754974721, 11>; auto c0 = mconv < m0 > (a, b);
 using m1 = mod<167772161, 3>; auto c1 = mconv<m1>(a, b);
  using m2 = mod<469762049, 3>; auto c2 = mconv < m2 > (a, b);
  int n = sz(c0); vector<T> d(n); m1 r01 = m1(m0::MOD).inv();
  m2 r02 = m2 (m0::MOD).inv(), r12 = m2 (m1::MOD).inv();
 rep(i, 0, n) {
    int x = c0[i].x, y = ((c1[i] - x) * r01).x,
       z = (((c2[i] - x) * r02 - y) * r12).x;
    d[i] = (T(z) * m1::MOD + y) * m0::MOD + x;
 return d;
```

# Teksty (5)

# 5.1 Podstawy

#### KMP.h

Opis: p[k] – nadjłuższy ścisły sufiks s[0, k] który jest prefiksem s. Czas:  $\mathcal{O}(n)$ 

```
vi kmp(const string& s) {
 vi p(sz(s));
 rep(i, 1, sz(s)) {
   int g = p[i - 1];
    while (g \&\& s[i] != s[g]) g = p[g - 1];
    p[i] = g + (s[i] == s[g]);
 return p;
```

Opis: f[k] - nadjłuższy prefiks s[k, n) który jest prefiksem s. Czas:  $\mathcal{O}(n)$ 

```
vi z(const string& s) {
 int n = sz(s), 1 = -1, r = -1;
 vi f(n); f[0] = n;
 rep(i, 1, sz(s)) {
   if (i < r) f[i] = min(r - i, f[i - 1]);
    while (i + f[i] < n \&\& s[i + f[i]] == s[f[i]]) f[i]++;
    if (i + f[i] > r) l = i, r = i + f[i];
 return f;
```

#### Manacher.h

**Opis:** p[1][k] – środek w k, p[0][k] – środek między k-1 a k. Czas:  $\mathcal{O}(n)$ 

```
array<vi, 2> manacher(const string& s) {
  int n = sz(s);
  array < vi, 2 > p = {vi(n + 1), vi(n)};
  rep(z, 0, 2) for (int i = 0, l = 0, r = 0; i < n; i++) {
   int t = r - i + !z;
   if (i < r) p[z][i] = min(t, p[z][l + t]);
   int L = i - p[z][i], R = i + p[z][i] - !z;
   while (L >= 1 \&\& R + 1 < n \&\& s[L - 1] == s[R + 1])
     p[z][i]++, L--, R++;
   if (R > r) 1 = L, r = R;
  return p;
```

#### Duval.h

Opis: Rozkłada słowo na nierosnący ciąg podsłów mniejszych od swoich wszystkich nietrywialnych sufiksów.

Czas:  $\mathcal{O}(n)$ 

```
vi duval (const string& s) {
  int n = sz(s); vi f;
  for (int i = 0; i < n;) {</pre>
    int j = i + 1, k = i;
    for (; j < n \&\& s[k] \le s[j]; j++) {
      if (s[k] < s[j]) k = i;
      else ++k;
    for (; i <= k; i += j - k) f.push_back(i);</pre>
  return f.push back(n), f;
```

# Struktury sufiksowe

## SuffixArray.h

**Opis:** Zawiera pusty sufiks. lcp[k] - najdłuższy wspólny prefiks <math>k-1 i k. Czas:  $\mathcal{O}(n \log n)$ 

```
struct suffix_array {
  vi sa, lcp;
  suffix_array(const string& s, int lim = 128) {
    int n = sz(s) + 1, k = 0, a, b;
    vi \times (all(s) + 1), y(n), ws(max(n, lim)), rank(n);
    sa = lcp = v, iota(all(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
     p = j, iota(all(y), n - j);
      rep(i, 0, n) if (sa[i] >= j) y[p++] = sa[i] - j;
      fill(all(ws), 0);
      rep(i, 0, n) ws[x[i]]++;
      rep(i, 1, lim) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      rep(i, 1, n) = sa[i - 1], b = sa[i], x[b] =
        (y[a] == y[b] \&\& y[a + j] == y[b + j]) ? p - 1 : p++;
    rep(i, 1, n) rank[sa[i]] = i;
    for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)</pre>
     for (k \& \& k--, j = sa[rank[i] - 1];
          s[i + k] == s[j + k]; k++);
};
```

# Geometria (6)

# 6.1 Podstawy

## Point.h

Opis: Podstawowy szablon do geometrii.

```
template < class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct pt {
 Тх, у;
  pt operator+(pt o) const { return {x + o.x, y + o.y}; }
  pt operator-(pt o) const { return {x - o.x, y - o.y}; }
  pt operator*(T a) const { return {x * a, y * a}; }
  pt operator/(T a) const { return {x / a, y / a}; }
  friend T cross(pt a, pt b) { return a.x * b.y - a.y * b.x; }
  friend T cross(pt p, pt a, pt b) {
    return cross(a - p, b - p); }
  friend T dot(pt a, pt b) { return a.x * b.x + a.y * b.y; }
  friend T dot(pt p, pt a, pt b) {
    return dot(a - p, b - p); }
  friend T abs2(pt a) { return a.x * a.x + a.y * a.y; }
  friend T abs(pt a) { return sqrt(abs2(a)); }
  auto operator<=>(pt o) const {
   return pair(sgn(x - o.x), sgn(y - o.y)) <=> pair(0, 0); }
  bool operator==(pt o) const {
    return sgn(x - o.x) == 0 && sgn(y - o.y) == 0; }
 friend auto& operator<<(auto& o, pt a) {</pre>
    return o << '(' << a.x << ", " << a.y << ')'; }
using P = pt<11>;
AngleCmp.h
Opis: Sortuje punkty rosnąco po kącie z przedziału (-\pi, \pi]. Punkt (0,0)
```

ma kat 0.

```
bool angle cmp(P a, P b) {
 auto half = [](P p) { return sqn(p.y) ?: -sqn(p.x); };
 int A = half(a), B = half(b);
 return A == B ? sgn(cross(a, b)) > 0 : A < B;
```

Opis: Najkrótsza odległość między punktem i prostą/odcinkiem.

```
auto line_dist(P p, P a, P b) {
 return abs(cross(p, a, b)) / abs(b - a);
auto seq_dist(P p, P a, P b) {
 if (sqn(dot(a, p, b)) <= 0) return abs(p - a);</pre>
 if (sqn(dot(b, p, a)) <= 0) return abs(p - b);</pre>
 return line_dist(p, a, b);
```

#### 6.2Wielokaty

## ConvexHull.h

Opis: Otoczka wypukła w kierunku CCW. Czas:  $\mathcal{O}(n \log n)$ 

```
vector<P> convex_hull(vector<P> pts) {
 if (sz(pts) <= 1) return pts;</pre>
 sort(all(pts));
 vector<P> h(sz(pts) + 1);
 int s = 0, t = 0;
```

```
for (int it = 2; it--; s = --t, reverse(all(pts)))
  for (P p : pts) {
    while (t >= s + 2 \& \&
           sgn(cross(h[t - 2], h[t - 1], p)) \le 0) t--;
    h[t++] = p;
return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
```