```
1 Contest
2 Struktury danych
                                              1
3 Grafv
4 Matma
                                              3
5 Teksty
                                              4
6 Geometria
                                              5
7 Inne
```

Contest (1)

sol.cpp

```
#include <bits/stdc++.h>
using namespace std;
using 11 = long long;
#ifdef LOCAL
auto& operator<<(auto&, pair<auto, auto>);
auto& operator<<(auto& o, auto x) {
 0 << '{';
  for (int i = 0; auto y : x) \circ << ", " + !i++ * 2 << y;
 return o << '}';
auto& operator<<(auto& o, pair<auto, auto> x) {
 return o << '(' << x.first << ", " << x.second << ')';
void __print(auto... x) { ((cerr << ' ' << x), ...) << endl; }</pre>
#define debug(x...) cerr << "[" #x "]:", __print(x)
#define debug(...) 2137
#endif
int main() {
 ios_base::sync_with_stdio(false);
 cin.tie(nullptr);
```

.vimrc

```
set nu et ts=2 sw=2
filetype indent on
syntax on
colorscheme habamax
hi MatchParen ctermfg=66 ctermbg=234 cterm=underline
nnoremap : :
nnoremap : ;
inoremap {<cr> {<cr>}<esc>0 <bs>
```

Makefile

```
CXXFLAGS=-std=c++20 -Wall -Wextra -Wshadow
 g++ $(CXXFLAGS) -fsanitize=address, undefined -g -DLOCAL \
      sol.cpp -o sol
```

```
fast: sol.cpp
  q++ $(CXXFLAGS) -02 sol.cpp -o fast
test.sh
#!/bin/bash
for((i=1;i>0;i++)) do
  echo "$i"
  echo "$i" | ./gen > int
  diff -w < (./sol < int) < (./slow < int) || break
```

Struktury danych (2)

```
wavelet-tree
Stosowanie: st - początek, ed - koniec, sst - posortowany początek.
Czas: \mathcal{O}((n+q)\log n)
```

```
struct node {
 int lo, hi;
  vector<int> s;
  node *1 = 0, *r = 0;
  node (auto st, auto ed, auto sst) {
   int n = ed - st;
    lo = sst[0];
    hi = sst[n - 1] + 1;
    if (lo + 1 < hi) {
      int mid = sst[n / 2];
      if (mid == sst[0]) mid = *upper_bound(sst, sst + n, mid);
      s.reserve(n + 1);
      s.push_back(0);
      for (auto it = st; it != ed; it++) {
        s.push_back(s.back() + (*it < mid));
      auto k = stable_partition(st, ed, [&](int x) {
        return x < mid;</pre>
      auto sm = lower_bound(sst, sst + n, mid);
      if (k != st) l = new node(st, k, sst);
      if (k != ed) r = new node(k, ed, sm);
 int kth(int a, int b, int k) {
    if (lo + 1 == hi) return lo;
    int x = s[a], y = s[b];
    return k < y - x ? 1 \rightarrow kth(x, y, k)
                     : r - kth(a - x, b - y, k - (y - x));
  int count(int a, int b, int k) {
    if (10 >= k) return 0;
    if (hi <= k) return b - a;</pre>
    int x = s[a], v = s[b];
    return (1 ? 1->count(x, y, k) : 0) +
           (r ? r->count(a - x, b - y, k) : 0);
  int freq(int a, int b, int k) {
    if (k < lo || hi <= k) return 0;</pre>
    if (lo + 1 == hi) return b - a;
    int x = s[a], y = s[b];
    return (1 ? 1->freq(x, y, k) : 0) +
           (r ? r - > freq(a - x, b - y, k) : 0);
};
ordered-set
```

```
Stosowanie: s.find_by_order(k) i s.order_of_key(k).
Czas: \mathcal{O}(\log n)
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
                           tree order statistics node update>;
treap
Czas: \mathcal{O}(\log n)
mt19937_64 rng(2137);
```

```
struct node {
 int val, sz = 1;
 uint64_t pr;
  node *1 = 0, *r = 0;
  node(int x) {
    val = x;
    pr = rng();
  void pull() {
    sz = 1 + size(1) + size(r);
  friend int size(node* a) {
    return a ? a->sz : 0;
  friend pair<node*, node*> split(node* a, int k) {
    if (!a) return {0, 0};
    if (k <= size(a->1)) {
      auto [la, lb] = split(a->1, k);
      a -> 1 = 1b;
      a->pull();
      return {la, a};
      auto [ra, rb] = split(a->r, k - size(a->1) - 1);
      a->r = ra:
      a->pull();
      return {a, rb};
  friend node* merge(node* a, node* b) {
   if (!a || !b) return a ? a : b;
    if (a->pr > b->pr) {
      a->r = merge(a->r, b);
      a->pull();
      return a;
    } else {
      b->1 = merge(a, b->1);
      b->pull();
      return b;
};
```

Opis: Znajduje maksimum funkcji liniowych online. Dla doubli div (a, b) = a/b oraz INF = 1/.0.Czas: $\mathcal{O}(\log n)$

```
struct line {
  mutable ll a, b, p;
  bool operator<(const line& o) const { return a < o.a; }</pre>
  bool operator<(ll x) const { return p < x; }</pre>
struct line_set : multiset<line, less<>>> {
  static const 11 INF = LLONG MAX;
```

```
11 div(ll a, ll b) {
    return a / b - ((a ^ b) < 0 && a % b);
  bool inter(iterator x, iterator y) {
    if (y == end()) return x->p = INF, false;
    if (x->a == y->a) x->p = x->b > y->b ? INF : -INF;
    else x -> p = div(y -> b - x -> b, x -> a - y -> a);
   return x->p >= y->p;
  void add(ll a, ll b) {
    auto z = insert({a, b, 0}), y = z++, x = y;
    while (inter(y, z)) z = erase(z);
    if (x != begin() \&\& inter(--x, y)) inter(x, y = erase(y));
    while ((y = x) != begin() \&\& (--x)->p >= y->p) {
     inter(x, erase(y));
  11 get(11 x) {
    line l = *lower_bound(x);
    return l.a * x + l.b;
};
```

Grafy (3)

3.1 Przepływy

dinic Czas: $\mathcal{O}(nm \log U)$

```
struct dinic {
  struct edge {
   int to, rev;
   int cap;
  };
  int n;
  vector<vector<edge>> adj;
  vector<int> q, lvl, it;
  dinic(int _n) {
   n = _n;
   adj.resize(n);
   q.reserve(n);
   lvl.resize(n);
   it.resize(n);
  void add_edge(int u, int v, int cap) {
   int i = ssize(adj[u]), j = ssize(adj[v]) + (u == v);
    adj[u].push_back({v, j, cap});
   adj[v].push_back({u, i, 0});
  bool bfs(int s, int t, int r) {
    q.clear();
    lvl.assign(n, -1);
   lvl[s] = 0;
    q.push back(s);
    for (int i = 0; i < ssize(q); i++) {</pre>
     int u = q[i];
     for (edge& e : adj[u]) {
       if (e.cap >= r && lvl[e.to] == -1) {
         lvl[e.to] = lvl[u] + 1;
          q.push_back(e.to);
          if (e.to == t) return true;
```

```
return false;
 11 dfs(int u, int t, ll cap) {
   if (u == t) return cap;
   11 f = 0;
    for (int& i = it[u]; i < ssize(adj[u]); i++) {</pre>
      edge& e = adj[u][i];
      if (e.cap > 0 && lvl[u] + 1 == lvl[e.to]) {
        11 \text{ add} = dfs(e.to, t, min(cap - f, (11)e.cap));
        e.cap -= add;
       adj[e.to][e.rev].cap += add;
        f += add;
     if (f == cap) return f;
    lvl[u] = -1;
    return f;
 11 flow(int s, int t, ll cap) {
   11 f = 0;
    for (int i = 29; i >= 0; i--) {
     while (f < cap && bfs(s, t, 1 << i)) {
       it.assign(n, 0);
        f += dfs(s, t, cap - f);
    return f;
};
```

mcmf

Stosowanie: Jeżeli są ujemne krawędzie, przed pusczeniem flow w dst trzeba policzyć najkrótsze ścieżki z s i puścić reduce (t).

Czas: $\mathcal{O}(Fm \log n)$

```
#include <ext/pb_ds/priority_queue.hpp>
11 \text{ INF } 64 = 2e18;
struct MCMF {
  struct edge {
   int to, rev;
   int cap;
   11 cost;
 };
 struct cmp {
   bool operator()(const auto& 1, const auto& r) const {
      return 1.second > r.second;
  };
 int n;
  vector<vector<edge>> adj;
 vector<ll> dst:
  11 c = 0;
  __gnu_pbds::priority_queue<pair<int, ll>, cmp> q;
  vector<decltype(q)::point_iterator> its;
  vector<int> id;
 MCMF (int _n) {
   n = _n;
    adj.resize(n);
    id.resize(n);
  void add_edge(int u, int v, int cap, int cost) {
    int i = ssize(adj[u]), j = ssize(adj[v]) + (u == v);
    adj[u].push_back({v, j, cap, cost});
    adj[v].push_back({u, i, 0, -cost});
 void reduce(int t) {
    for (int i = 0; i < n; i++) {
      for (edge& e : adj[i]) {
        if (dst[i] != INF64 && dst[e.to] != INF64) {
```

```
e.cost += dst[i] - dst[e.to];
    c += dst[t];
 bool dijkstra(int s, int t) {
    dst.assign(n, INF64);
    its.assign(n, q.end());
    dst[s] = 0;
    q.push({s, 0});
    while (!q.empty()) {
      int u = q.top().first;
      q.pop();
      for (edge& e : adj[u]) {
       if (e.cap > 0) {
          11 d = dst[u] + e.cost;
          if (d < dst[e.to]) {
            dst[e.to] = d;
            if (its[e.to] == q.end()) {
              its[e.to] = q.push({e.to, dst[e.to]});
              q.modify(its[e.to], {e.to, dst[e.to]});
            id[e.to] = e.rev;
    reduce(t);
    return dst[t] != INF64;
  pair<11, 11> flow(int s, int t, 11 cap) {
    11 \text{ ff} = 0;
    11 cc = 0;
    while (ff < cap && dijkstra(s, t)) {
     11 f = cap - ff;
      for (int i = t; i != s;) {
        edge& e = adj[i][id[i]];
        f = min(f, (ll)adj[e.to][e.rev].cap);
        i = e.to;
      for (int i = t; i != s;) {
        edge& e = adj[i][id[i]];
        e.cap += f;
        adj[e.to][e.rev].cap -= f;
        i = e.to;
      ff += f;
      cc += f * c;
    return {ff, cc};
};
```

3.1.1 Przepływy z wymaganiami

Szukamy przepływu $\leq F$ takiego, że $f_i \geq d_i$ dla każdej krawędzi. Tworzymy nowe źródło s' i ujście t'. Następnie dodajemy krawedzie

- $(u_i, t', d_i), (s', v_i, d_i), (u_i, v_i, c_i d_i)$ zamiast (u_i, v_i, c_i, d_i)
- (t, s, I

Przepływ spełnia wymagania jeżeli maksymalnie wypełnia wszystkie krawedzie $s^\prime.$

3.2 Grafy dwudzielne

matching Czas: $\mathcal{O}(m\sqrt{n})$

```
struct matching {
  int n, m;
  vector<vector<int>> adj;
  vector<int> pb, pa;
  vector<int> lvl, it;
  matching(int _n, int _m) {
   n = _n;
   m = _m;
    adj.resize(n);
   pb.resize(n, -1);
    pa.resize(m, -1);
    it.resize(n);
  void add_edge(int u, int v) {
    adj[u].push_back(v);
  bool bfs() {
   bool res = false;
   lvl.assign(n, -1);
    queue<int> q;
    for (int i = 0; i < n; i++) {</pre>
     if (pb[i] == -1) {
        q.push(i);
        lvl[i] = 0;
    while (!q.empty()) {
     int u = q.front();
      q.pop();
      for (int j : adj[u]) {
        if (pa[j] == -1) {
          res = true;
        } else if (lvl[pa[j]] == -1) {
          lvl[pa[j]] = lvl[u] + 1;
          q.push(pa[j]);
    return res;
  bool dfs(int u) {
    for (auto& i = it[u]; i < ssize(adj[u]); i++) {</pre>
      int v = adj[u][i];
      if (pa[v] == -1 ||
          (lvl[pa[v]] == lvl[u] + 1 && dfs(pa[v]))) {
        pb[u] = v;
        pa[v] = u;
        return true;
    return false;
  int match() {
    int ans = 0:
    while (bfs()) {
     it.assign(n, 0);
     for (int i = 0; i < n; i++) {</pre>
       if (pb[i] == -1 && dfs(i)) ans++;
   return ans;
};
```

3.2.1 Twierdzenie Königa

W grafie dwudzielnym zachodzi

- nk = pw
- nk + pk = n
- pw + nw = n

oraz

- pw to zbiór wierzchołków na brzegu min-cut
- nw to dopełnienie pw
- pk to nk z dodanymi pojedynczymi krawędziami każdego nieskojarzonego wierzchołka

3.2.2 Twierdzenie Gale'a-Rysera

Ciągi stopni $a_1 \geq \ldots \geq a_n$ oraz b_1,\ldots,b_n opisują prosty graf dwudzielny wtw gdy $\sum a_i = \sum b_i$ oraz dla każdego $1 \leq k \leq n$ zachodzi

$$\sum_{i=1}^k a_i \le \sum_{i=1}^n \min(b_i, k).$$

3.3 Grafy skierowane

SCC Czas: $\mathcal{O}(n+m)$

```
struct SCC {
 int n, cnt = 0;
 vector<vector<int>> adj;
 vector<int> p, low, in;
 stack<int> st;
 int tour = 0;
 SCC(int n) {
   n = n;
   adj.resize(n);
   p.resize(n, -1);
   low.resize(n);
   in.resize(n, -1);
 void add_edge(int u, int v) {
   adj[u].push_back(v);
 void dfs(int u) {
   low[u] = in[u] = tour++;
   st.push(u);
   for (int v : adj[u]) {
     if (in[v] == -1) {
       dfs(v);
       low[u] = min(low[u], low[v]);
     } else {
       low[u] = min(low[u], in[v]);
   if (low[u] == in[u]) {
     int v = -1;
       v = st.top();
       st.pop();
       in[v] = n;
       p[v] = cnt;
     } while (v != u);
     cnt++;
```

```
void build() {
  for (int i = 0; i < n; ++i) {
    if (in[i] == -1) dfs(i);
  }
  for (int i = 0; i < n; i++) p[i] = cnt - 1 - p[i];
}
};</pre>
```

3

3.4 Grafy nieskierowane

3.4.1 Twierdzenie Erdősa-Gallaia

Ciąg stopni $d_1 \geq \ldots \geq d_n$ opisuje prosty graf wtw gdy $\sum d_i$ jest parzysta oraz dla każdego $1 \leq k \leq n$ zachodzi

$$\sum_{i=1}^{k} d_i \le k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

Matma (4)

4.1 Arytmetyka modularna

mint

```
template<int M, int R>
struct mod {
  static const int MOD = M, ROOT = R;
 mod(11 y = 0) : x(y % M) { x += (x < 0) * M; }
 mod operator+= (const mod& o) {
   if ((x += 0.x) >= M) x -= M;
    return *this;
 mod operator -= (const mod& o) {
    if ((x -= 0.x) < 0) x += M;
    return *this;
 mod operator *= (const mod& o) {
    x = 111 * x * o.x % M;
    return *this;
 mod operator/=(const mod& o) {
    return (*this) *= o.inv();
  friend mod operator+(mod a, const mod& b) { return a += b;
  friend mod operator-(mod a, const mod& b) { return a -= b;
 friend mod operator*(mod a, const mod& b) { return a *= b;
  friend mod operator/(mod a, const mod& b) { return a /= b; }
  auto operator<=>(const mod&) const = default;
 mod pow(ll n) const {
   mod a = x, b = 1;
    while (n > 0) {
     if (n % 2 == 1) b \star= a;
      a *= a;
      n /= 2;
    return b;
 mod inv() const {
```

```
return pow (M - 2);
using mint = mod<998244353, 3>;
```

Wielomiany

```
Stosowanie: Jeżeli MOD = 998244353 to n + m < 2^{23}.
Czas: \mathcal{O}((n+m)\log(n+m))
```

```
template<typename T>
void ntt(vector<T>& a, bool inv) {
  int n = ssize(a);
  vector<T> b(n);
  for (int i = n / 2; i > 0; i /= 2, swap(a, b)) {
   T w = T(T::ROOT).pow((T::MOD - 1) / n * i), m = 1;
    for (int j = 0; j < n; j += 2 * i, m *= w) {
      for (int k = 0; k < i; k++) {
       T u = a[j + k], v = a[j + k + i] * m;
       b[j / 2 + k] = u + v;
       b[i / 2 + k + n / 2] = u - v;
  if (inv) {
   reverse(a.begin() + 1, a.end());
   T ni = T(n).inv();
   for (int i = 0; i < n; i++) a[i] *= ni;</pre>
template<typename T>
vector<T> conv(vector<T> a, vector<T> b) {
  int s = ssize(a) + ssize(b) - 1;
  int n = 1 \ll (\underline{\ } \lg(2 * s - 1));
  a.resize(n); b.resize(n);
  ntt(a, false); ntt(b, false);
  for (int i = 0; i < n; i++) a[i] *= b[i];</pre>
  ntt(a, true);
  a.resize(s);
 return a:
ntt3
```

Stosowanie: $n+m < 2^{24}$. Czas: $\mathcal{O}((n+m)\log(n+m))$

```
template<typename T>
vector<T> mconv(const auto& a, const auto& b) {
  auto cp = [&] (const auto& v) {
    vector<T> vv(ssize(v));
   for (int i = 0; i < ssize(v); i++) vv[i] = T(v[i].x);</pre>
   return vv;
  return conv(cp(a), cp(b));
template<typename T>
vector<T> conv3(const vector<T>& a, const vector<T>& b) {
  using m0 = mod<754974721, 11>; auto c0 = mconv<m0>(a, b);
  using m1 = mod<167772161, 3>; auto c1 = mconv<m1>(a, b);
  using m2 = mod<469762049, 3>; auto c2 = mconv < m2 > (a, b);
  m1 r01 = m1 (m0::MOD).inv();
  m2 r02 = m2 (m0::MOD).inv(), r12 = m2 (m1::MOD).inv();
  vector<T> d(ssize(c0));
  for (int i = 0; i < ssize(c0); i++) {</pre>
   int a = c0[i].x;
    int b = ((c1[i] - a) * r01).x;
```

```
int c = (((c2[i] - a) * r02 - b) * r12).x;
  d[i] = (T(c) * m1::MOD + b) * m0::MOD + a;
return d;
```

Sploty bitowe

Stosowanie: n musi być potęgą dwójki. Czas: $\mathcal{O}(n \log n)$

```
void fst (vector<mint>& a, bool inv) {
 int n = ssize(a);
 for (int i = 1; i < n; i *= 2) {
   for (int j = 0; j < n; j += 2 * i) {
     for (int k = 0; k < i; k++) {
       mint u = a[j + k], v = a[j + k + i];
       a[j + k] = u + v, a[j + k + i] = u - v; // XOR
       // a[j + k] = inv ? u - v : u + v; // AND
        // a[j + k + i] = inv ? v - u : u + v; // OR
 // XOR
 if (inv) {
   mint ni = mint(n).inv();
   for (int i = 0; i < n; i++) a[i] = a[i] * ni;</pre>
vector<mint> conv(vector<mint> a, vector<mint> b) {
 int n = ssize(a);
 fst(a, false); fst(b, false);
 for (int i = 0; i < n; i++) a[i] = a[i] * b[i];</pre>
 fst(a, true);
 return a;
```

Optymalizacja

4.4.1 Mnożniki Lagrange'a

Jeżeli optymalizujemy $f(x_1, \ldots, x_n)$ przy ograniczeniach typu $g_k(x_1,\ldots,x_n)=0$ to x_1,\ldots,x_n jest ekstremum lokalnym tylko jeżeli gradient $\nabla f(x_1,\ldots,x_n)$ jest kombinacją liniową gradientów $\nabla g_k(x_1,\ldots x_n).$

Teksty (5)

kmp Czas: $\mathcal{O}(n)$

```
vector<int> kmp(const string& s) {
 int n = ssize(s);
 vector<int> p(n);
 for (int i = 1; i < n; i++) {
   int j = p[i - 1];
   while (j > 0 \&\& s[i] != s[j]) j = p[j - 1];
   p[i] = j + (s[i] == s[j]);
```

```
manacher
Stosowanie: Zwraca długość najdłuższego palindromu. p[2 * i] - środek
w i, p[2 * i + 1] – środek między i a i + 1.
Czas: \mathcal{O}(n)
```

```
vector<int> manacher(const string& s) {
 int n = ssize(s);
 string t(2 * n - 1, '#');
 for (int i = 0; i < n; i++) t[2 * i] = s[i];
 vector<int> p(2 * n - 1);
 for (int i = 0, l = -1, r = -1; i < 2 * n - 1; i++) {
   if (i \le r) p[i] = min(r - i + 1, p[1 + r - i]);
   while (p[i] < min(i + 1, 2 * n - 1 - i)) {
     if (t[i - p[i]] != t[i + p[i]]) break;
   if (i + p[i] - 1 > r) {
     1 = i - p[i] + 1;
     r = i + p[i] - 1;
 for (int i = 0; i < 2 * n - 1; i++) {
   p[i] -= t[i - p[i] + 1] == '#';
 return p;
```

suffix-array

return p;

Stosowanie: Jeżeli tekst ma znaki inne niż a-z trzeba zmienić inicjalizację. Czas: $\mathcal{O}(n \log n)$

```
vector<int> suffix_array(const string& s) {
 int n = ssize(s);
 vector<int> p(n), cnt(26);
 for (int i = 0; i < n; i++) cnt[s[i] - 'a']++;</pre>
 for (int i = 1; i < 26; i++) cnt[i] += cnt[i - 1];</pre>
 for (int i = 0; i < n; i++) p[--cnt[s[i] - 'a']] = i;</pre>
 vector<int> rnk(n);
 for (int i = 1; i < n; i++) {</pre>
   rnk[p[i]] = s[p[i]] == s[p[i-1]] ? rnk[p[i-1]] : i;
 cnt.resize(n);
 vector<int> np(n), nrnk(n);
 for (int len = 1; len < n; len *= 2) {
   iota(cnt.begin(), cnt.end(), 0);
    for (int i = n - len; i < n; i++) np[cnt[rnk[i]]++] = i;</pre>
    for (int i = 0; i < n; i++) {</pre>
     if (p[i] - len >= 0) {
        np[cnt[rnk[p[i] - len]]++] = p[i] - len;
   nrnk[np[0]] = 0;
    for (int i = 1; i < n; i++) {</pre>
     int a = np[i - 1];
     int b = np[i];
      if (\max(a, b) + \text{len} < n \&\& rnk[a] == rnk[b] \&\&
          rnk[a + len] == rnk[b + len]) {
        nrnk[b] = nrnk[a];
     } else {
        nrnk[b] = i;
    swap(p, np);
    swap(rnk, nrnk);
 return p;
```

z point convex-hull polygon-tangents gcc

```
};
vector<int> build_lcp(const string& s, const vector<int>& sa) {
  int n = ssize(s);
  vector<int> pos(n);
  for (int i = 0; i < n; i++) pos[sa[i]] = i;</pre>
  vector<int> lcp(n - 1);
  int k = 0;
  for (int i = 0; i < n; i++) {
   if (pos[i] == 0) continue;
   while (i + k < n \&\& s[i + k] == s[sa[pos[i] - 1] + k]) k++;
   lcp[pos[i] - 1] = k;
   k = max(0, k - 1);
  return lcp;
Czas: \mathcal{O}(n)
vector<int> z(const string& s) {
  int n = ssize(s);
  vector<int> f(n);
  f[0] = n;
  for (int i = 1, l = 0, r = 0; i < n; i++) {
   if (i \le r) f[i] = min(r - i + 1, f[i - 1]);
    while (f[i] < n - i \&\& s[i + f[i]] == s[f[i]]) f[i]++;
   if (i + f[i] - 1 > r) {
     1 = i;
      r = i + f[i] - 1;
  return f;
```

Geometria (6)

6.1 Podstawy

point

```
struct pt {
 11 x, y;
  pt operator+(pt o) const { return {x + o.x, y + o.y}; }
  pt operator-(pt o) const { return {x - o.x, y - o.y}; }
  pt operator*(11 a) const { return {x * a, y * a}; }
  pt operator/(ll a) const { return {x / a, y / a}; }
  auto operator<=>(const pt&) const = default;
  friend 11 cross(pt a, pt b) { return a.x * b.y - a.y * b.x; }
  friend 11 dot(pt a, pt b) { return a.x * b.x + a.y * b.y; }
  friend ll norm(pt a) { return a.x * a.x + a.y * a.y; }
  friend int half(pt a) {
   if (a.y < 0) return -1;
   if (a.y == 0 && a.x >= 0) return 0;
  friend auto& operator<<(auto& o, pt a) {</pre>
    return o << '(' << a.x << ", " << a.y << ')';
};
```

6.2 Wielokąty

convex-hull

```
Stosowanie: Usuwa punkty współliniowe. Czas: O(n \log n)
```

```
vector<pt> convex_hull(vector<pt> p) {
 if (ssize(p) <= 1) return p;</pre>
 sort(p.begin(), p.end());
 vector<pt> h(ssize(p) + 1);
 int s = 0, t = 0;
 for (int it = 0; it < 2; it++) {
   for (pt a : p) {
     while (t >= s + 2) {
       pt u = h[t - 2], v = h[t - 1];
       if (cross(v - u, a - v) <= 0) t--;
       else break;
     h[t++] = a;
   reverse(p.begin(), p.end());
   s = --t;
 h.resize(t - (t == 2 \&\& h[0] == h[1]));
 return h;
```

polygon-tangents

Stosowanie: Wielokąt musi być CCW i $n \geq 3$. Zwraca najbliższe punkty styczne różne od a.

Czas: $\mathcal{O}(\log n)$

```
pair<pt, pt> tangents(const vector<pt>& p, pt a) {
 int n = ssize(p);
 pt t[2];
 for (int it = 0; it < 2; it++) {</pre>
   auto dir = [&](int i) {
     pt u = p[i] - a;
     pt v = p[i < n - 1 ? i + 1 : 0] - a;
     11 c = cross(u, v);
     if (c != 0) return c < 0;
     if (dot(u, v) > 0) return norm(u) > norm(v);
     return true;
    auto dirx = [&](int i) { return dir(i) ^ it; };
    if (dirx(0) == 1 && dirx(n - 1) == 0) {
     t[it] = p[0];
      continue;
    int s[2] = \{0, n - 1\};
    while (s[1] - s[0] > 2) {
     int mid = (s[0] + s[1]) / 2;
     int x = dirx(mid);
     if (dirx(s[x ^ 1]) == (x ^ 1)) {
        ((cross(p[mid] - a, p[s[1]] - a) < 0) ^ it
             ? s[x]
             : s[x ^ 1]) = mid;
   t[it] = dirx(s[0] + 1) == 0 ? p[s[0] + 2] : p[s[0] + 1];
 return {t[0], t[1]};
```

Inne (7)

gcc

```
#pragma GCC optimize("03,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
```