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Contest (1)

```
sol.cpp

#include <bits/stdc++.h>
using namespace std;
using ll = long long;

#ifdef LOCAL
auto& operator<<(auto&, pair<auto, auto>);
auto& operator<<(auto& o, auto x) {
    o << '{';
    for (int i = 0; auto y : x) o << ", " + !i++ * 2 << y;
    return o << '}'';
}
auto& operator<<(auto& o, pair<auto, auto> x) {
    return o << '(' << x.first << ", " << x.second << ')'';
}
void __print(auto... x) { ((cerr << ' ' << x), ...) << endl; }
#define debug(x...) cerr << "[" #x "]:", __print(x)
#else
#define debug(...) 2137
#endif

int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);
}

.vimrc

set nu et ts=2 sw=2
filetype indent on
syntax on
colorscheme habamax
hi MatchParen ctermfg=66 ctermbg=234 cterm=underline
nnoremap ; :
nnoremap ; :
innoremap {<cr> {<cr>}<esc>O <bs>

Makefile

CXXFLAGS=-std=c++20 -Wall -Wextra -Wshadow

sol: sol.cpp
    g++ $(CXXFLAGS) -fsanitize=address,undefined -g -DLOCAL \
        sol.cpp -o sol
```

```
1 fast: sol.cpp
    g++ $(CXXFLAGS) -O2 sol.cpp -o fast

1 test.sh

1 #!/bin/bash

3 for((i=1;i>0;i++)) do
    echo "$i"
    echo "$i" | ./gen > int
    diff -w <(. /sol < int) <(. /slow < int) || break
4 done

4

5
```

Struktury danych (2)

```
WaveletTree.h
Stosowanie: st – początek, ed – koniec, sst – posortowany początek.
Czas:  $\mathcal{O}((n+q)\log n)$ 

struct node {
    int lo, hi;
    vector<int> s;
    node *l = 0, *r = 0;
    node(auto st, auto ed, auto sst) {
        int n = ed - st;
        lo = sst[0];
        hi = sst[n - 1] + 1;
        if (lo + 1 < hi) {
            int mid = sst[n / 2];
            if (mid == sst[0]) mid = *upper_bound(sst, sst + n, mid);
            s.reserve(n + 1);
            s.push_back(0);
            for (auto it = st; it != ed; it++) {
                s.push_back(s.back() + (*it < mid));
            }
            auto k = stable_partition(st, ed, [&](int x) {
                return x < mid;
            });
            auto sm = lower_bound(sst, sst + n, mid);
            if (k != st) l = new node(st, k, sst);
            if (k != ed) r = new node(k, ed, sm);
        }
    }
    int kth(int a, int b, int k) {
        if (lo + 1 == hi) return lo;
        int x = s[a], y = s[b];
        return k < y - x ? l->kth(x, y, k)
            : r->kth(a - x, b - y, k - (y - x));
    }
    int count(int a, int b, int k) {
        if (lo >= k) return 0;
        if (hi <= k) return b - a;
        int x = s[a], y = s[b];
        return (l ? l->count(x, y, k) : 0) +
            (r ? r->count(a - x, b - y, k) : 0);
    }
    int freq(int a, int b, int k) {
        if (k < lo || hi <= k) return 0;
        if (lo + 1 == hi) return b - a;
        int x = s[a], y = s[b];
        return (l ? l->freq(x, y, k) : 0) +
            (r ? r->freq(a - x, b - y, k) : 0);
    }
};

OrderedSet.h
```

```
Stosowanie: s.find_by_order(k) i s.order_of_key(k).
Czas:  $\mathcal{O}(\log n)$ 

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

Treap.h
Czas:  $\mathcal{O}(\log n)$ 

mt19937_64 rng(2137);
struct node {
    int val, sz = 1;
    uint64_t pr;
    node *l = 0, *r = 0;
    node(int x) {
        val = x;
        pr = rng();
    }
    void pull() {
        sz = 1 + size(l) + size(r);
    }
    friend int size(node* a) {
        return a ? a->sz : 0;
    }
    friend pair<node*, node*> split(node* a, int k) {
        if (!a) return {0, 0};
        if (k <= size(a->l)) {
            auto [la, lb] = split(a->l, k);
            a->l = lb;
            a->pull();
            return {la, a};
        } else {
            auto [ra, rb] = split(a->r, k - size(a->l) - 1);
            a->r = ra;
            a->pull();
            return {a, rb};
        }
    }
    friend node* merge(node* a, node* b) {
        if (!a || !b) return a ? a : b;
        if (a->pr > b->pr) {
            a->r = merge(a->r, b);
            a->pull();
            return a;
        } else {
            b->l = merge(a, b->l);
            b->pull();
            return b;
        }
    }
};

Grafy (3)

3.1 Przepływy

Dinic.h
Czas:  $\mathcal{O}(nm \log U)$ 
```

```

struct dinic {
    struct edge {
        int to, rev;
        int cap;
    };
    int n;
    vector<vector<edge>> adj;
    vector<int> q, lvl, it;
    dinic(int _n) {
        n = _n;
        adj.resize(n);
        q.reserve(n);
        lvl.resize(n);
        it.resize(n);
    }
    void add_edge(int u, int v, int cap) {
        int i = ssize(adj[u]), j = ssize(adj[v]) + (u == v);
        adj[u].push_back({v, j, cap});
        adj[v].push_back({u, i, 0});
    }
    bool bfs(int s, int t, int r) {
        q.clear();
        lvl.assign(n, -1);
        lvl[s] = 0;
        q.push_back(s);
        for (int i = 0; i < ssize(q); i++) {
            int u = q[i];
            for (edge& e : adj[u]) {
                if (e.cap >= r && lvl[e.to] == -1) {
                    lvl[e.to] = lvl[u] + 1;
                    q.push_back(e.to);
                    if (e.to == t) return true;
                }
            }
        }
        return false;
    }
    ll dfs(int u, int t, ll cap) {
        if (u == t) return cap;
        ll f = 0;
        for (int& i = it[u]; i < ssize(adj[u]); i++) {
            edge& e = adj[u][i];
            if (e.cap > 0 && lvl[u] + 1 == lvl[e.to]) {
                ll add = dfs(e.to, t, min(cap - f, (ll)e.cap));
                e.cap -= add;
                adj[e.to][e.rev].cap += add;
                f += add;
            }
            if (f == cap) return f;
        }
        lvl[u] = -1;
        return f;
    }
    ll flow(int s, int t, ll cap) {
        ll f = 0;
        for (int i = 29; i >= 0; i--) {
            while (f < cap && bfs(s, t, 1 << i)) {
                it.assign(n, 0);
                f += dfs(s, t, cap - f);
            }
        }
        return f;
    }
};

```

## MCMF.h

**Stosowanie:** Jeżeli są ujemne krawędzie, przed pusczeniem flow w dst trzeba policzyć najkrótsze ścieżki z s i puścić reduce(t).

**Czas:**  $\mathcal{O}(Fm \log n)$

```

#include <ext/pb_ds/priority_queue.hpp>
ll INF64 = 2e18;
struct MCMF {
    struct edge {
        int to, rev;
        int cap;
        ll cost;
    };
    struct cmp {
        bool operator()(const auto& l, const auto& r) const {
            return l.second > r.second;
        }
    };
    int n;
    vector<vector<edge>> adj;
    vector<ll> dst;
    ll c = 0;
    __gnu_pbds::priority_queue<pair<int, ll>, cmp> q;
    vector<decltype(q)::point_iterator> its;
    vector<int> id;
    MCMF(int _n) {
        n = _n;
        adj.resize(n);
        id.resize(n);
    }
    void add_edge(int u, int v, int cap, int cost) {
        int i = ssize(adj[u]), j = ssize(adj[v]) + (u == v);
        adj[u].push_back({v, j, cap, cost});
        adj[v].push_back({u, i, 0, -cost});
    }
    void reduce(int t) {
        for (int i = 0; i < n; i++) {
            for (edge& e : adj[i]) {
                if (dst[i] != INF64 && dst[e.to] != INF64) {
                    e.cost += dst[i] - dst[e.to];
                }
            }
        }
        c += dst[t];
    }
    bool dijkstra(int s, int t) {
        dst.assign(n, INF64);
        its.assign(n, q.end());
        dst[s] = 0;
        q.push({s, 0});
        while (!q.empty()) {
            int u = q.top().first;
            q.pop();
            for (edge& e : adj[u]) {
                if (e.cap > 0) {
                    ll d = dst[u] + e.cost;
                    if (d < dst[e.to]) {
                        dst[e.to] = d;
                        if (its[e.to] == q.end()) {
                            its[e.to] = q.push({e.to, dst[e.to]});
                        } else {
                            q.modify(its[e.to], {e.to, dst[e.to]});
                        }
                        id[e.to] = e.rev;
                    }
                }
            }
        }
        reduce(t);
        return dst[t] != INF64;
    }
    pair<ll, ll> flow(int s, int t, ll cap) {
        ll ff = 0;
        ll cc = 0;

```

```

        while (ff < cap && dijkstra(s, t)) {
            ll f = cap - ff;
            for (int i = t; i != s; ) {
                edge& e = adj[i][id[i]];
                f = min(f, (ll)adj[e.to][e.rev].cap);
                i = e.to;
            }
            for (int i = t; i != s; ) {
                edge& e = adj[i][id[i]];
                e.cap += f;
                adj[e.to][e.rev].cap -= f;
                i = e.to;
            }
            ff += f;
            cc += f * c;
        }
        return {ff, cc};
    }
};

```

### 3.1.1 Przepływy z wymaganiami

Szukamy przepływu  $\leq F$  takiego, że  $f_i \geq d_i$  dla każdej krawędzi. Tworzymy nowe źródło  $s'$  i ujście  $t'$ . Następnie dodajemy krawędzie

- $(u_i, t', d_i), (s', v_i, d_i), (u_i, v_i, c_i - d_i)$  zamiast  $(u_i, v_i, c_i, d_i)$
- $(t, s, F)$

Przepływ spełnia wymagania jeżeli maksymalnie wypełnia wszystkie krawędzie  $s'$ .

## 3.2 Grafy dwudzielne

### Matching.h

**Czas:**  $\mathcal{O}(m\sqrt{n})$

```

struct matching {
    int n, m;
    vector<vector<int>> adj;
    vector<int> pb, pa;
    vector<int> lvl, it;
    matching(int _n, int _m) {
        n = _n;
        m = _m;
        adj.resize(n);
        pb.resize(n, -1);
        pa.resize(m, -1);
        it.resize(n);
    }
    void add_edge(int u, int v) {
        adj[u].push_back(v);
    }
    bool bfs() {
        bool res = false;
        lvl.assign(n, -1);
        queue<int> q;
        for (int i = 0; i < n; i++) {
            if (pb[i] == -1) {
                q.push(i);
                lvl[i] = 0;
            }
        }
        while (!q.empty()) {
            int u = q.front();
            q.pop();

```

```
for (int j : adj[u]) {
    if (pa[j] == -1) {
        res = true;
    } else if (lvl[pa[j]] == -1) {
        lvl[pa[j]] = lvl[u] + 1;
        q.push(pa[j]);
    }
}
}
return res;
}
bool dfs(int u) {
    for (auto& i = it[u]; i < ssize(adj[u]); i++) {
        int v = adj[u][i];
        if (pa[v] == -1 ||
            (lvl[pa[v]] == lvl[u] + 1 && dfs(pa[v]))) {
            pb[u] = v;
            pa[v] = u;
            return true;
        }
    }
    return false;
}
int match() {
    int ans = 0;
    while (bfs()) {
        it.assign(n, 0);
        for (int i = 0; i < n; i++) {
            if (pb[i] == -1 && dfs(i)) ans++;
        }
    }
    return ans;
}
};
```

3.2.1 Twierdzenie Königa

W grafie dwudzielnym zachodzi

- $n_k = p_w$
- $n_k + p_k = n$
- $p_w + n_w = n$

oraz

- $p_w$  to zbiór wierzchołków na brzegu min-cut
- $n_w$  to dopełnienie  $p_w$
- $p_k$  to  $n_k$  z dodanymi pojedynczymi krawędziami każdego nieskojarzonego wierzchołka

3.2.2 Twierdzenie Gale’a-Rysera

Ciągi stopni  $a_1 \geq \dots \geq a_n$  oraz  $b_1, \dots, b_n$  opisują prosty graf dwudzielnym wtw gdy  $\sum a_i = \sum b_i$  oraz dla każdego  $1 \leq k \leq n$  zachodzi

$$\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k).$$

3.3 Grafy skierowane

SCC.h  
Czas:  $\mathcal{O}(n + m)$

```
struct SCC {
    int n, cnt = 0;
    vector<vector<int>>> adj;
    vector<int> p, low, in;
    stack<int> st;
    int tour = 0;
    SCC(int _n) {
        n = _n;
        adj.resize(n);
        p.resize(n, -1);
        low.resize(n);
        in.resize(n, -1);
    }
    void add_edge(int u, int v) {
        adj[u].push_back(v);
    }
    void dfs(int u) {
        low[u] = in[u] = tour++;
        st.push(u);
        for (int v : adj[u]) {
            if (in[v] == -1) {
                dfs(v);
                low[u] = min(low[u], low[v]);
            } else {
                low[u] = min(low[u], in[v]);
            }
        }
        if (low[u] == in[u]) {
            int v = -1;
            do {
                v = st.top();
                st.pop();
                in[v] = n;
                p[v] = cnt;
            } while (v != u);
            cnt++;
        }
    }
    void build() {
        for (int i = 0; i < n; ++i) {
            if (in[i] == -1) dfs(i);
        }
        for (int i = 0; i < n; i++) p[i] = cnt - 1 - p[i];
    }
};
```

3.4 Grafy nieskierowane

3.4.1 Twierdzenie Erdős’a-Gallaia

Ciąg stopni  $d_1 \geq \dots \geq d_n$  opisuje prosty graf wtw gdy  $\sum d_i$  jest parzysta oraz dla każdego  $1 \leq k \leq n$  zachodzi

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k).$$

Matma (4)

4.1 Arytmetyka modularna

Mint.h

```
#define NORM(x) x >= MOD ? x - MOD : x
const int MOD = 998244353;
struct mint {
    int x;
    mint(ll y = 0) : x((y % MOD) < 0 ? y + MOD : y) {}
    mint operator+(mint o) const { return NORM(x + o.x); }
    mint operator-(mint o) const { return NORM(x + MOD - o.x); }
    mint operator*(mint o) const { return 1ll * x * o.x % MOD; }
    mint operator/(mint o) const { return *this * o.inv(); }
    auto operator<=>(const mint&) const = default;
    mint pow(ll n) const {
        mint a = x, b = 1;
        while (n > 0) {
            if (n % 2 == 1) b = b * a;
            a = a * a;
            n /= 2;
        }
        return b;
    }
    mint inv() const {
        return pow(MOD - 2);
    }
};
```

4.2 Sploty

NTT.h  
Stosowanie: Musi zachodzić  $n + m \leq 2^{23}$ .  
Czas:  $\mathcal{O}((n + m) \log(n + m))$

```
const int ROOT = 3;
void ntt(vector<mint>& a) {
    int n = ssize(a), d = __lg(n);
    vector<mint> w(n);
    mint ww = 1, r = mint(ROOT).pow((MOD - 1) / n);
    for (int i = 0; i < n / 2; i++) {
        w[i + n / 2] = ww;
        ww = ww * r;
    }
    for (int i = n / 2 - 1; i > 0; i--) w[i] = w[2 * i];
    vector<int> rev(n);
    for (int i = 0; i < n; i++) {
        rev[i] = (rev[i] >> 1) | ((i & 1) << d) >> 1;
        if (i < rev[i]) swap(a[i], a[rev[i]]);
    }
    for (int i = 1; i < n; i *= 2) {
        for (int j = 0; j < n; j += 2 * i) {
            for (int k = 0; k < i; k++) {
                mint z = w[i + k] * a[j + k + i];
                a[j + k + i] = a[j + k] - z;
                a[j + k] = a[j + k] + z;
            }
        }
    }
}
vector<mint> conv(vector<mint> a, vector<mint> b) {
    int n = 1, s = ssize(a) + ssize(b) - 1;
    while (n < s) n *= 2;
    a.resize(n); b.resize(n);
    ntt(a); ntt(b);
    for (int i = 0; i < n; i++) a[i] = a[i] * b[i];
    ntt(a);
    reverse(a.begin() + 1, a.end());
    a.resize(s);
    mint ni = mint(n).inv();
    for (int i = 0; i < s; i++) a[i] = a[i] * ni;
    return a;
}
```

```

}
```

## FST.h

**Stosowanie:**  $n$  musi być potęgą dwójki.

**Czas:**  $\mathcal{O}(n \log n)$

```

void fst(vector<mint>& a, bool inv) {
    int n = ssize(a);
    for (int i = 1; i < n; i *= 2) {
        for (int j = 0; j < n; j += 2 * i) {
            for (int k = 0; k < i; k++) {
                mint u = a[j + k], v = a[j + k + i];
                a[j + k] = u + v, a[j + k + i] = u - v; // XOR
                // a[j + k] = inv ? u - v : u + v; // AND
                // a[j + k + i] = inv ? v - u : u + v; // OR
            }
        }
    }
    // XOR
    if (inv) {
        mint ni = mint(n).inv();
        for (int i = 0; i < n; i++) a[i] = a[i] * ni;
    }
}

vector<mint> conv(vector<mint> a, vector<mint> b) {
    int n = ssize(a);
    fst(a, false); fst(b, false);
    for (int i = 0; i < n; i++) a[i] = a[i] * b[i];
    fst(a, true);
    return a;
}
```

## 4.3 Optymalizacja

### 4.3.1 Mnożniki Lagrange’a

Jeżeli optymalizujemy  $f(x_1, \dots, x_n)$  przy ograniczeniach typu  $g_k(x_1, \dots, x_n) = 0$  to  $x_1, \dots, x_n$  jest ekstremum lokalnym tylko jeżeli gradient  $\nabla f(x_1, \dots, x_n)$  jest kombinacją liniową gradientów  $\nabla g_k(x_1, \dots, x_n)$ .

## Teksty (5)

### KMP.h

**Czas:**  $\mathcal{O}(n)$

```

vector<int> kmp(const string& s) {
    int n = ssize(s);
    vector<int> p(n);
    for (int i = 1; i < n; i++) {
        int j = p[i - 1];
        while (j > 0 && s[i] != s[j]) j = p[j - 1];
        p[i] = j + (s[i] == s[j]);
    }
    return p;
}
```

### Manacher.h

**Stosowanie:**  $p[2 * i]$  – środek w  $i$ ,  $p[2 * i + 1]$  – środek między  $i$  a  $i + 1$ .

**Czas:**  $\mathcal{O}(n)$

```

vector<int> manacher(const string& s) {
    int n = ssize(s);
    string t(2 * n, '.');
    for (int i = 0; i < n; i++) {
        t[2 * i] = s[i];
        t[2 * i + 1] = '#';
    }
    vector<int> p(2 * n - 1);
    for (int i = 0, l = -1, r = -1; i < 2 * n - 1; i++) {
        if (i <= r) p[i] = min(r - i + 1, p[l + r - i]);
        while (p[i] < min(i + 1, 2 * n - 1 - i) &&
            t[i - p[i]] == t[i + p[i]]) {
            p[i]++;
        }
        if (i + p[i] - 1 > r) {
            l = i - p[i] + 1;
            r = i + p[i] - 1;
        }
    }
    for (int i = 0; i < 2 * n - 1; i++) {
        if (t[i - p[i] + 1] == '#') p[i]--;
        p[i] = (p[i] + (1 - i % 2)) / 2;
    }
    return p;
}
```

### SuffixArray.h

**Stosowanie:** Jeżeli tekst ma znaki inne niż a-z trzeba zmienić inicjalizację.

**Czas:**  $\mathcal{O}(n \log n)$

```

vector<int> suffix_array(const string& s) {
    int n = ssize(s);
    vector<int> p(n), cnt(26);
    for (int i = 0; i < n; i++) cnt[s[i] - 'a']++;
    for (int i = 1; i < 26; i++) cnt[i] += cnt[i - 1];
    for (int i = 0; i < n; i++) p[--cnt[s[i] - 'a']] = i;
    vector<int> rnk(n);
    for (int i = 1; i < n; i++) {
        rnk[p[i]] = s[p[i]] == s[p[i - 1]] ? rnk[p[i - 1]] : i;
    }
    cnt.resize(n);
    vector<int> np(n), nrnk(n);
    for (int len = 1; len < n; len *= 2) {
        iota(cnt.begin(), cnt.end(), 0);
        for (int i = n - len; i < n; i++) np[cnt[rnk[i]]++] = i;
        for (int i = 0; i < n; i++) {
            if (p[i] - len >= 0) {
                np[cnt[rnk[p[i] - len]]++] = p[i] - len;
            }
        }
        nrnk[np[0]] = 0;
        for (int i = 1; i < n; i++) {
            int a = np[i - 1];
            int b = np[i];
            if (max(a, b) + len < n && rnk[a] == rnk[b] &&
                rnk[a + len] == rnk[b + len]) {
                nrnk[b] = nrnk[a];
            } else {
                nrnk[b] = i;
            }
        }
        swap(p, np);
        swap(rnk, nrnk);
    }
    return p;
}

vector<int> build_lcp(const string& s, const vector<int>& sa) {
    int n = ssize(s);
    vector<int> pos(n);
```

```

    for (int i = 0; i < n; i++) pos[sa[i]] = i;
    vector<int> lcp(n - 1);
    int k = 0;
    for (int i = 0; i < n; i++) {
        if (pos[i] == 0) continue;
        while (i + k < n && s[i + k] == s[sa[pos[i] - 1] + k]) k++;
        lcp[pos[i] - 1] = k;
        k = max(0, k - 1);
    }
    return lcp;
}
```

### Z.h

**Czas:**  $\mathcal{O}(n)$

```

vector<int> z(const string& s) {
    int n = ssize(s);
    vector<int> f(n);
    for (int i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r) f[i] = min(r - i + 1, f[i - 1]);
        while (f[i] < n - i && s[i + f[i]] == s[f[i]]) f[i]++;
        if (i + f[i] - 1 > r) {
            l = i;
            r = i + f[i] - 1;
        }
    }
    return f;
}
```

## Geometria (6)

## 6.1 Podstawy

### Point.h

```

struct pt {
    ll x, y;
    pt operator+(pt o) const { return {x + o.x, y + o.y}; }
    pt operator-(pt o) const { return {x - o.x, y - o.y}; }
    pt operator*(ll a) const { return {x * a, y * a}; }
    pt operator/(ll a) const { return {x / a, y / a}; }
    auto operator<=>(const pt&) const = default;
    friend ll cross(pt a, pt b) { return a.x * b.y - a.y * b.x; }
    friend ll dot(pt a, pt b) { return a.x * b.x + a.y * b.y; }
    friend ll norm(pt a) { return a.x * a.x + a.y * a.y; }
    friend int half(pt a) {
        if (a.y < 0) return -1;
        if (a.y == 0 && a.x >= 0) return 0;
        return 1;
    }
    friend auto& operator<<(auto& o, pt a) {
        return o << '(' << a.x << ", " << a.y << ')';
    }
};
```

## 6.2 Wielokąty

### ConvexHull.h

**Stosowanie:** Usuwa punkty współliniowe.

**Czas:**  $\mathcal{O}(n \log n)$

```
vector<pt> convex_hull(vector<pt> p) {
    if (ssize(p) <= 1) return p;
    sort(p.begin(), p.end());
    vector<pt> h(ssize(p) + 1);
    int s = 0, t = 0;
    for (int it = 0; it < 2; it++) {
        for (pt a : p) {
            while (t >= s + 2) {
                pt u = h[t - 2], v = h[t - 1];
                if (cross(v - u, a - v) <= 0) t--;
                else break;
            }
            h[t++] = a;
        }
        reverse(p.begin(), p.end());
        s = --t;
    }
    h.resize(t - (t == 2 && h[0] == h[1]));
    return h;
}
```

### PolygonTangents.h

**Stosowanie:** Wielokąt musi być CCW i  $n \geq 3$ . Zwraca najbliższe punkty styczne różne od  $a$ .

**Czas:**  $\mathcal{O}(\log n)$

```
pair<pt, pt> tangents(const vector<pt>& p, pt a) {
    int n = ssize(p);
    pt t[2];
    for (int it = 0; it < 2; it++) {
        auto dir = [&](int i) {
            pt u = p[i] - a;
            pt v = p[i < n - 1 ? i + 1 : 0] - a;
            ll c = cross(u, v);
            if (c != 0) return c < 0;
            if (dot(u, v) > 0) return norm(u) > norm(v);
            return true;
        };
        auto dirx = [&](int i) { return dir(i) ^ it; };
        if (dirx(0) == 1 && dirx(n - 1) == 0) {
            t[it] = p[0];
            continue;
        }
        int s[2] = {0, n - 1};
        while (s[1] - s[0] > 2) {
            int mid = (s[0] + s[1]) / 2;
            int x = dirx(mid);
            if (dirx(s[x ^ 1]) == (x ^ 1)) {
                s[x] = mid;
            } else {
                ((cross(p[mid] - a, p[s[1]] - a) < 0) ^ it
                 ? s[x]
                 : s[x ^ 1]) = mid;
            }
        }
        t[it] = dirx(s[0] + 1) == 0 ? p[s[0] + 2] : p[s[0] + 1];
    }
    return {t[0], t[1]};
}
```

## Inne (7)

### GCC.h

```
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
```