Contest (1)

sol.cpp

```
#include <bits/stdc++.h>
using namespace std;
using ll = long long;

#ifdef LOCAL
auto& operator<<(auto&, pair<auto, auto>);
auto& operator<<(auto& o, auto x) {
  o << '{';
  for (int i = 0; auto y : x) o << ", " + !i++ * 2 << y;
  return o << '}';
}
auto& operator<<(auto& o, pair<auto, auto> x) {
  return o << '(' << x.first << ", " << x.second << ')';
}
void __print(auto... x) { ((cerr << ' ' << x), ...) << endl; }
#define debug(x...) cerr << "[" #x "]:", __print(x)
#else
#define debug(...) 2137
#endif

int main() {
  ios_base::sync_with_stdio(false);
  cin.tie(nullptr);
}
```

.vimrc

```
set nu et ts=2 sw=2
filetype indent on
syntax on
colorscheme habamax
hi MatchParen ctermfg=66 ctermbg=234 cterm=underline
nnoremap ; :
nnoremap : ;
inoremap {<cr> {<cr>}<esc>O <bs>
```

Makefile

```
CXXFLAGS=-std=c++20 -Wall -Wextra -Wshadow

sol: sol.cpp
  g++ $(CXXFLAGS) -fsanitize=address,undefined -g -DLOCAL \
      sol.cpp -o sol
fast: sol.cpp
  g++ $(CXXFLAGS) -O2 sol.cpp -o fast
```

test.sh

```
#!/bin/bash

for((i=1;i>0;i++)) do
  echo "$i"
  echo "$i" | ./gen > int
  diff -w <(./sol < int) <(./slow < int) || break
done
```

Struktury danych (2)

wavelet-tree

Stosowanie: st – początek, ed – koniec, sst – posortowany początek.
Czas: $\mathcal{O}((n+q)\log n)$

```
struct node {
  int lo, hi;
  vector<int> s;
  node *l = 0, *r = 0;
  node(auto st, auto ed, auto sst) {
    int n = ed - st;
    lo = sst[0];
    hi = sst[n - 1] + 1;
    if (lo + 1 < hi) {
      int mid = sst[n / 2];
      if (mid == sst[0]) mid = *upper_bound(sst, sst + n, mid);
      s.reserve(n + 1);
      s.push_back(0);
      for (auto it = st; it != ed; it++) {
        s.push_back(s.back() + (*it < mid));
      }
      auto k = stable_partition(st, ed, [&](int x) {
        return x < mid;
      });
      auto sm = lower_bound(sst, sst + n, mid);
      if (k != st) l = new node(st, k, sst);
      if (k != ed) r = new node(k, ed, sm);
    }
  }
  int kth(int a, int b, int k) {
    if (lo + 1 == hi) return lo;
    int x = s[a], y = s[b];
    return k < y - x ? l->kth(x, y, k)
                     : r->kth(a - x, b - y, k - (y - x));
  }
  int count(int a, int b, int k) {
    if (lo >= k) return 0;
    if (hi <= k) return b - a;
    int x = s[a], y = s[b];
    return (l ? l->count(x, y, k) : 0) +
           (r ? r->count(a - x, b - y, k) : 0);
  }
  int freq(int a, int b, int k) {
    if (k < lo || hi <= k) return 0;
    if (lo + 1 == hi) return b - a;
    int x = s[a], y = s[b];
    return (l ? l->freq(x, y, k) : 0) +
           (r ? r->freq(a - x, b - y, k) : 0);
  }
};
```

ordered-set

Stosowanie: s.find_by_order(k) i s.order_of_key(k).
Czas: $\mathcal{O}(\log n)$

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
                         tree_order_statistics_node_update>;
```

treap

Czas: $\mathcal{O}(\log n)$

```
mt19937_64 rng(2137);
struct node {
  int val, sz = 1;
  uint64_t pr;
  node *l = 0, *r = 0;
  node(int x) {
    val = x;
    pr = rng();
  }
  void pull() {
    sz = 1 + size(l) + size(r);
  }
  friend int size(node* a) {
    return a ? a->sz : 0;
  }
  friend pair<node*, node*> split(node* a, int k) {
    if (!a) return {0, 0};
    if (k <= size(a->l)) {
      auto [la, lb] = split(a->l, k);
      a->l = lb;
      a->pull();
      return {la, a};
    } else {
      auto [ra, rb] = split(a->r, k - size(a->l) - 1);
      a->r = ra;
      a->pull();
      return {a, rb};
    }
  }
  friend node* merge(node* a, node* b) {
    if (!a || !b) return a ? a : b;
    if (a->pr > b->pr) {
      a->r = merge(a->r, b);
      a->pull();
      return a;
    } else {
      b->l = merge(a, b->l);
      b->pull();
      return b;
    }
  }
};
```

Grafy (3)

3.1 Przepływy

dinic

Czas: $\mathcal{O}(nm\log U)$

```
struct dinic {
  struct edge {
    int to, rev;
    int cap;
  };
  int n;
  vector<vector<edge>> adj;
  vector<int> q, lvl, it;
  dinic(int _n) {
    n = _n;
    adj.resize(n);
    q.reserve(n);
    lvl.resize(n);
    it.resize(n);
  }
  void add_edge(int u, int v, int cap) {
    int i = ssize(adj[u]), j = ssize(adj[v]) + (u == v);
    adj[u].push_back({v, j, cap});
    adj[v].push_back({u, i, 0});
  }
  bool bfs(int s, int t, int r) {
    q.clear();
    lvl.assign(n, -1);
    lvl[s] = 0;
    q.push_back(s);
    for (int i = 0; i < ssize(q); i++) {
      int u = q[i];
      for (edge& e : adj[u]) {
        if (e.cap >= r && lvl[e.to] == -1) {
          lvl[e.to] = lvl[u] + 1;
          q.push_back(e.to);
          if (e.to == t) return true;
        }
      }
    }
    return false;
  }
  ll dfs(int u, int t, ll cap) {
    if (u == t) return cap;
    ll f = 0;
    for (int& i = it[u]; i < ssize(adj[u]); i++) {
      edge& e = adj[u][i];
      if (e.cap > 0 && lvl[u] + 1 == lvl[e.to]) {
        ll add = dfs(e.to, t, min(cap - f, (ll)e.cap));
        e.cap -= add;
        adj[e.to][e.rev].cap += add;
        f += add;
      }
      if (f == cap) return f;
    }
    lvl[u] = -1;
    return f;
  }
  ll flow(int s, int t, ll cap) {
    ll f = 0;
    for (int i = 29; i >= 0; i--) {
      while (f < cap && bfs(s, t, 1 << i)) {
        it.assign(n, 0);
        f += dfs(s, t, cap - f);
      }
    }
    return f;
  }
};
```

mcmf

Stosowanie: Jeżeli są ujemne krawędzie, przed puszczeniem `flow` w `dst` trzeba policzyć najkrótsze ścieżki z `s` i puścić `reduce(t)`.
Czas: $\mathcal{O}(Fm \log n)$

```
#include <ext/pb_ds/priority_queue.hpp>
ll INF64 = 2e18;
struct MCMF {
  struct edge {
    int to, rev;
    int cap;
    ll cost;
  };
  struct cmp {
    bool operator()(const auto& l, const auto& r) const {
      return l.second > r.second;
    }
  };
  int n;
  vector<vector<edge>> adj;
  vector<ll> dst;
  ll c = 0;
  __gnu_pbds::priority_queue<pair<int, ll>, cmp> q;
  vector<decltype(q)::point_iterator> its;
  vector<int> id;
  MCMF(int _n) {
    n = _n;
    adj.resize(n);
    id.resize(n);
  }
  void add_edge(int u, int v, int cap, int cost) {
    int i = ssize(adj[u]), j = ssize(adj[v]) + (u == v);
    adj[u].push_back({v, j, cap, cost});
    adj[v].push_back({u, i, 0, -cost});
  }
  void reduce(int t) {
    for (int i = 0; i < n; i++) {
      for (edge& e : adj[i]) {
        if (dst[i] != INF64 && dst[e.to] != INF64) {
          e.cost += dst[i] - dst[e.to];
        }
      }
    }
    c += dst[t];
  }
  bool dijkstra(int s, int t) {
    dst.assign(n, INF64);
    its.assign(n, q.end());
    dst[s] = 0;
    q.push({s, 0});
    while (!q.empty()) {
      int u = q.top().first;
      q.pop();
      for (edge& e : adj[u]) {
        if (e.cap > 0) {
          ll d = dst[u] + e.cost;
          if (d < dst[e.to]) {
            dst[e.to] = d;
            if (its[e.to] == q.end()) {
              its[e.to] = q.push({e.to, dst[e.to]});
            } else {
              q.modify(its[e.to], {e.to, dst[e.to]});
            }
            id[e.to] = e.rev;
          }
        }
      }
    }
    reduce(t);
    return dst[t] != INF64;
  }
  pair<ll, ll> flow(int s, int t, ll cap) {
    ll ff = 0;
    ll cc = 0;
    while (ff < cap && dijkstra(s, t)) {
      ll f = cap - ff;
      for (int i = t; i != s;) {
        edge& e = adj[i][id[i]];
        f = min(f, (ll)adj[e.to][e.rev].cap);
        i = e.to;
      }
      for (int i = t; i != s;) {
        edge& e = adj[i][id[i]];
        e.cap += f;
        adj[e.to][e.rev].cap -= f;
        i = e.to;
      }
      ff += f;
      cc += f * c;
    }
    return {ff, cc};
  }
};
```

3.1.1 Przepływy z wymaganiami

Szukamy przepływu $\leq F$ takiego, że $f_i \geq d_i$ dla każdej krawędzi. Tworzymy nowe źródło s' i ujście t'. Następnie dodajemy krawędzie

- (u_i, t', d_i), (s', v_i, d_i), $(u_i, v_i, c_i - d_i)$ zamiast (u_i, v_i, c_i, d_i)
- (t, s, F)

Przepływ spełnia wymagania jeżeli maksymalnie wypełnia wszystkie krawędzie s'.

3.2 Grafy dwudzielne

matching

Czas: $\mathcal{O}(m\sqrt{n})$

```
struct matching {
  int n, m;
  vector<vector<int>> adj;
  vector<int> pb, pa;
  vector<int> lvl, it;
  matching(int _n, int _m) {
    n = _n;
    m = _m;
    adj.resize(n);
    pb.resize(n, -1);
    pa.resize(m, -1);
    it.resize(n);
  }
  void add_edge(int u, int v) {
    adj[u].push_back(v);
  }
  bool bfs() {
    bool res = false;
    lvl.assign(n, -1);
    queue<int> q;
    for (int i = 0; i < n; i++) {
      if (pb[i] == -1) {
        q.push(i);
        lvl[i] = 0;
      }
    }
    while (!q.empty()) {
      int u = q.front();
      q.pop();
```

```
      for (int j : adj[u]) {
        if (pa[j] == -1) {
          res = true;
        } else if (lvl[pa[j]] == -1) {
          lvl[pa[j]] = lvl[u] + 1;
          q.push(pa[j]);
        }
      }
    }
    return res;
  }
  bool dfs(int u) {
    for (auto& i = it[u]; i < ssize(adj[u]); i++) {
      int v = adj[u][i];
      if (pa[v] == -1 ||
          (lvl[pa[v]] == lvl[u] + 1 && dfs(pa[v]))) {
        pb[u] = v;
        pa[v] = u;
        return true;
      }
    }
    return false;
  }
  int match() {
    int ans = 0;
    while (bfs()) {
      it.assign(n, 0);
      for (int i = 0; i < n; i++) {
        if (pb[i] == -1 && dfs(i)) ans++;
      }
    }
    return ans;
  }
};
```

3.2.1 Twierdzenie Königa

W grafie dwudzielnym zachodzi
- $\mathrm{nk} = \mathrm{pw}$
- $\mathrm{nk} + \mathrm{pk} = n$
- $\mathrm{pw} + \mathrm{nw} = n$

oraz
- pw to zbiór wierzchołków na brzegu min-cut
- nw to dopełnienie pw
- pk to nk z dodanymi pojedynczymi krawędziami każdego nieskojarzonego wierzchołka

3.2.2 Twierdzenie Gale'a-Rysera

Ciągi stopni $a_1 \geq \ldots \geq a_n$ oraz $b_1, \ldots, b_n$ opisują prosty graf dwudzielny wtw gdy $\sum a_i = \sum b_i$ oraz dla każdego $1 \leq k \leq n$ zachodzi

$$\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{n} \min(b_i, k).$$

3.3 Grafy skierowane

scc
Czas: $\mathcal{O}(n+m)$

```
struct SCC {
  int n, cnt = 0;
  vector<vector<int>> adj;
  vector<int> p, low, in;
  stack<int> st;
  int tour = 0;
  SCC(int _n) {
    n = _n;
    adj.resize(n);
    p.resize(n, -1);
    low.resize(n);
    in.resize(n, -1);
  }
  void add_edge(int u, int v) {
    adj[u].push_back(v);
  }
  void dfs(int u) {
    low[u] = in[u] = tour++;
    st.push(u);
    for (int v : adj[u]) {
      if (in[v] == -1) {
        dfs(v);
        low[u] = min(low[u], low[v]);
      } else {
        low[u] = min(low[u], in[v]);
      }
    }
    if (low[u] == in[u]) {
      int v = -1;
      do {
        v = st.top();
        st.pop();
        in[v] = n;
        p[v] = cnt;
      } while (v != u);
      cnt++;
    }
  }
  void build() {
    for (int i = 0; i < n; ++i) {
      if (in[i] == -1) dfs(i);
    }
    for (int i = 0; i < n; i++) p[i] = cnt - 1 - p[i];
  }
};
```

3.4 Grafy nieskierowane

3.4.1 Twierdzenie Erdősa-Gallaia

Ciąg stopni $d_1 \geq \ldots \geq d_n$ opisuje prosty graf wtw gdy $\sum d_i$ jest parzysta oraz dla każdego $1 \leq k \leq n$ zachodzi

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

Matma (4)

4.1 Arytmetyka modularna

mint

```
template<int M, int R>
struct mod {
  static const int MOD = M, ROOT = R;
  int x;
  mod(ll y = 0) : x(y % M) { x += (x < 0) * M; }
  mod operator+=(const mod& o) {
    if ((x += o.x) >= M) x -= M;
    return *this;
  }
  mod operator-=(const mod& o) {
    if ((x -= o.x) < 0) x += M;
    return *this;
  }
  mod operator*=(const mod& o) {
    x = 1ll * x * o.x % M;
    return *this;
  }
  mod operator/=(const mod& o) {
    return (*this) *= o.inv();
  }
  friend mod operator+(mod a, const mod& b) { return a += b; }
  friend mod operator-(mod a, const mod& b) { return a -= b; }
  friend mod operator*(mod a, const mod& b) { return a *= b; }
  friend mod operator/(mod a, const mod& b) { return a /= b; }
  auto operator<=>(const mod&) const = default;
  mod pow(ll n) const {
    mod a = x, b = 1;
    while (n > 0) {
      if (n % 2 == 1) b *= a;
      a *= a;
      n /= 2;
    }
    return b;
  }
  mod inv() const {
    return pow(M - 2);
  }
};
using mint = mod<998244353, 3>;
```

4.2 Wielomiany

ntt
Stosowanie: Jeżeli $\mathrm{MOD} = 998244353$ to $n + m \leq 2^{23}$.
Czas: $\mathcal{O}((n+m)\log(n+m))$

```
template<typename T>
void ntt(vector<T>& a, bool inv) {
  int n = ssize(a);
  vector<T> b(n);
  for (int i = n / 2; i > 0; i /= 2, swap(a, b)) {
    T w = T(T::ROOT).pow((T::MOD - 1) / n * i), m = 1;
    for (int j = 0; j < n; j += 2 * i, m *= w) {
      for (int k = 0; k < i; k++) {
        T u = a[j + k], v = a[j + k + i] * m;
        b[j / 2 + k] = u + v;
        b[j / 2 + k + n / 2] = u - v;
      }
    }
  }
  if (inv) {
    reverse(a.begin() + 1, a.end());
    T ni = T(n).inv();
    for (int i = 0; i < n; i++) a[i] *= ni;
  }
}
template<typename T>
```

```
vector<T> conv(vector<T> a, vector<T> b) {
  int s = ssize(a) + ssize(b) - 1;
  int n = 1 << (__lg(2 * s - 1));
  a.resize(n); b.resize(n);
  ntt(a, false); ntt(b, false);
  for (int i = 0; i < n; i++) a[i] *= b[i];
  ntt(a, true);
  a.resize(s);
  return a;
}
```

ntt3
Stosowanie: $n + m \leq 2^{24}$.
Czas: $\mathcal{O}((n+m)\log(n+m))$

```
template<typename T>
vector<T> mconv(const auto& a, const auto& b) {
  auto cp = [&](const auto& v) {
    vector<T> vv(ssize(v));
    for (int i = 0; i < ssize(v); i++) vv[i] = T(v[i].x);
    return vv;
  };
  return conv(cp(a), cp(b));
}
template<typename T>
vector<T> conv3(const vector<T>& a, const vector<T>& b) {
  using m0 = mod<754974721, 11>; auto c0 = mconv<m0>(a, b);
  using m1 = mod<167772161, 3>; auto c1 = mconv<m1>(a, b);
  using m2 = mod<469762049, 3>; auto c2 = mconv<m2>(a, b);
  m1 r01 = m1(m0::MOD).inv();
  m2 r02 = m2(m0::MOD).inv(), r12 = m2(m1::MOD).inv();
  vector<T> d(ssize(c0));
  for (int i = 0; i < ssize(c0); i++) {
    int a = c0[i].x;
    int b = ((c1[i] - a) * r01).x;
    int c = (((c2[i] - a) * r02 - b) * r12).x;
    d[i] = (T(c) * m1::MOD + b) * m0::MOD + a;
  }
  return d;
}
```

4.3 Sploty bitowe

fst
Stosowanie: n musi być potęgą dwójki.
Czas: $\mathcal{O}(n \log n)$

```
void fst(vector<mint>& a, bool inv) {
  int n = ssize(a);
  for (int i = 1; i < n; i *= 2) {
    for (int j = 0; j < n; j += 2 * i) {
      for (int k = 0; k < i; k++) {
        mint u = a[j + k], v = a[j + k + i];
        a[j + k] = u + v, a[j + k + i] = u - v; // XOR
        // a[j + k] = inv ? u - v : u + v; // AND
        // a[j + k + i] = inv ? v - u : u + v; // OR
      }
    }
  }
  // XOR
  if (inv) {
    mint ni = mint(n).inv();
    for (int i = 0; i < n; i++) a[i] = a[i] * ni;
  }
}
vector<mint> conv(vector<mint> a, vector<mint> b) {
  int n = ssize(a);
  fst(a, false); fst(b, false);
  for (int i = 0; i < n; i++) a[i] = a[i] * b[i];
  fst(a, true);
  return a;
}
```

4.4 Optymalizacja

4.4.1 Mnożniki Lagrange'a

Jeżeli optymalizujemy $f(x_1, \ldots, x_n)$ przy ograniczeniach typu $g_k(x_1, \ldots, x_n) = 0$ to $x_1, \ldots, x_n$ jest ekstremum lokalnym tylko jeżeli gradient $\nabla f(x_1, \ldots, x_n)$ jest kombinacją liniową gradientów $\nabla g_k(x_1, \ldots x_n)$.

Teksty (5)

kmp
Czas: $\mathcal{O}(n)$

```
vector<int> kmp(const string& s) {
  int n = ssize(s);
  vector<int> p(n);
  for (int i = 1; i < n; i++) {
    int j = p[i - 1];
    while (j > 0 && s[i] != s[j]) j = p[j - 1];
    p[i] = j + (s[i] == s[j]);
  }
  return p;
}
```

manacher
Stosowanie: Zwraca długość najdłuższego palindromu. `p[2 * i]` – środek w i, `p[2 * i + 1]` – środek między i a $i+1$.
Czas: $\mathcal{O}(n)$

```
vector<int> manacher(const string& s) {
  int n = ssize(s);
  string t(2 * n - 1, '#');
  for (int i = 0; i < n; i++) t[2 * i] = s[i];
  vector<int> p(2 * n - 1);
  for (int i = 0, l = -1, r = -1; i < 2 * n - 1; i++) {
    if (i <= r) p[i] = min(r - i + 1, p[l + r - i]);
    while (p[i] < min(i + 1, 2 * n - 1 - i)) {
      if (t[i - p[i]] != t[i + p[i]]) break;
      p[i]++;
    }
    if (i + p[i] - 1 > r) {
      l = i - p[i] + 1;
      r = i + p[i] - 1;
    }
  }
  for (int i = 0; i < 2 * n - 1; i++) {
    p[i] -= t[i - p[i] + 1] == '#';
  }
  return p;
}
```

suffix-array
Stosowanie: Jeżeli tekst ma znaki inne niż a-z trzeba zmienić inicjalizację.
Czas: $\mathcal{O}(n \log n)$

```
vector<int> suffix_array(const string& s) {
  int n = ssize(s);
  vector<int> p(n), cnt(26);
  for (int i = 0; i < n; i++) cnt[s[i] - 'a']++;
  for (int i = 1; i < 26; i++) cnt[i] += cnt[i - 1];
  for (int i = 0; i < n; i++) p[--cnt[s[i] - 'a']] = i;
  vector<int> rnk(n);
  for (int i = 1; i < n; i++) {
    rnk[p[i]] = s[p[i]] == s[p[i - 1]] ? rnk[p[i - 1]] : i;
  }
  cnt.resize(n);
  vector<int> np(n), nrnk(n);
  for (int len = 1; len < n; len *= 2) {
    iota(cnt.begin(), cnt.end(), 0);
    for (int i = n - len; i < n; i++) np[cnt[rnk[i]]++] = i;
    for (int i = 0; i < n; i++) {
      if (p[i] - len >= 0) {
        np[cnt[rnk[p[i] - len]]++] = p[i] - len;
      }
    }
    nrnk[np[0]] = 0;
    for (int i = 1; i < n; i++) {
      int a = np[i - 1];
      int b = np[i];
      if (max(a, b) + len < n && rnk[a] == rnk[b] &&
          rnk[a + len] == rnk[b + len]) {
        nrnk[b] = nrnk[a];
      } else {
        nrnk[b] = i;
      }
    }
    swap(p, np);
    swap(rnk, nrnk);
  }
  return p;
};
vector<int> build_lcp(const string& s, const vector<int>& sa) {
  int n = ssize(s);
  vector<int> pos(n);
  for (int i = 0; i < n; i++) pos[sa[i]] = i;
  vector<int> lcp(n - 1);
  int k = 0;
  for (int i = 0; i < n; i++) {
    if (pos[i] == 0) continue;
    while (i + k < n && s[i + k] == s[sa[pos[i] - 1] + k]) k++;
    lcp[pos[i] - 1] = k;
    k = max(0, k - 1);
  }
  return lcp;
}
```

z
Czas: $\mathcal{O}(n)$

```
vector<int> z(const string& s) {
  int n = ssize(s);
  vector<int> f(n);
  f[0] = n;
  for (int i = 1, l = 0, r = 0; i < n; i++) {
    if (i <= r) f[i] = min(r - i + 1, f[i - l]);
    while (f[i] < n - i && s[i + f[i]] == s[f[i]]) f[i]++;
    if (i + f[i] - 1 > r) {
      l = i;
      r = i + f[i] - 1;
    }
  }
  return f;
}
```

Geometria (6)

6.1 Podstawy

point

```
struct pt {
  ll x, y;
  pt operator+(pt o) const { return {x + o.x, y + o.y}; }
  pt operator-(pt o) const { return {x - o.x, y - o.y}; }
  pt operator*(ll a) const { return {x * a, y * a}; }
  pt operator/(ll a) const { return {x / a, y / a}; }
  auto operator<=>(const pt&) const = default;
  friend ll cross(pt a, pt b) { return a.x * b.y - a.y * b.x; }
  friend ll dot(pt a, pt b) { return a.x * b.x + a.y * b.y; }
  friend ll norm(pt a) { return a.x * a.x + a.y * a.y; }
  friend int half(pt a) {
    if (a.y < 0) return -1;
    if (a.y == 0 && a.x >= 0) return 0;
    return 1;
  }
  friend auto& operator<<(auto& o, pt a) {
    return o << '(' << a.x << ", " << a.y << ')';
  }
};
```

6.2 Wielokąty

convex-hull

Stosowanie: Usuwa punkty współliniowe.
Czas: $\mathcal{O}(n \log n)$

```
vector<pt> convex_hull(vector<pt> p) {
  if (ssize(p) <= 1) return p;
  sort(p.begin(), p.end());
  vector<pt> h(ssize(p) + 1);
  int s = 0, t = 0;
  for (int it = 0; it < 2; it++) {
    for (pt a : p) {
      while (t >= s + 2) {
        pt u = h[t - 2], v = h[t - 1];
        if (cross(v - u, a - v) <= 0) t--;
        else break;
      }
      h[t++] = a;
    }
    reverse(p.begin(), p.end());
    s = --t;
  }
  h.resize(t - (t == 2 && h[0] == h[1]));
  return h;
}
```

polygon-tangents

Stosowanie: Wielokąt musi być CCW i $n \geq 3$. Zwraca najbliższe punkty styczne różne od a.
Czas: $\mathcal{O}(\log n)$

```
pair<pt, pt> tangents(const vector<pt>& p, pt a) {
  int n = ssize(p);
  pt t[2];
  for (int it = 0; it < 2; it++) {
    auto dir = [&](int i) {
      pt u = p[i] - a;
      pt v = p[i < n - 1 ? i + 1 : 0] - a;
      ll c = cross(u, v);
      if (c != 0) return c < 0;
      if (dot(u, v) > 0) return norm(u) > norm(v);
      return true;
    };
    auto dirx = [&](int i) { return dir(i) ^ it; };
    if (dirx(0) == 1 && dirx(n - 1) == 0) {
      t[it] = p[0];
      continue;
    }
    int s[2] = {0, n - 1};
    while (s[1] - s[0] > 2) {
      int mid = (s[0] + s[1]) / 2;
      int x = dirx(mid);
      if (dirx(s[x ^ 1]) == (x ^ 1)) {
        s[x] = mid;
      } else {
        ((cross(p[mid] - a, p[s[1]] - a) < 0) ^ it
            ? s[x]
            : s[x ^ 1]) = mid;
      }
    }
    t[it] = dirx(s[0] + 1) == 0 ? p[s[0] + 2] : p[s[0] + 1];
  }
  return {t[0], t[1]};
}
```

Inne (7)

gcc

```
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
```