sol: sol.cpp

sol .vimrc Makefile test WaveletTree

1 Contest	1
2 Wzory	1
3 Struktury danych	1
4 Grafy	2
5 Matma	4
6 Teksty	4
7 Geometria	5
8 Inne	5
$\frac{\mathrm{Contest}}{\mathrm{sol.cpp}}$ (1)	
<pre>#include <bits stdc++.h=""> using namespace std; using l1 = long long;</bits></pre>	
<pre>#ifdef LOCAL auto& operator<<(auto&, pair<auto, auto="">); auto& operator<<(auto& o, auto x) { o << '{';} for (int i = 0; auto y : x) o << ", " + !i++ * 2 << y; return o << '}';</auto,></pre>	
<pre>} auto& operator<<(auto& o, pair<auto, auto=""> x) { return o << '(' << x.first << ", " << x.second << ')'; }</auto,></pre>	
<pre>voidprint(auto x) { ((cerr << ' ' << x),) << endl; #define debug(x) cerr << "[" #x "]:",print(x) #else #define debug() 2137 #endif</pre>	}
<pre>int main() { ios_base::sync_with_stdio(false); cin.tie(nullptr); }</pre>	
.vimre	
<pre>set nu et ts=2 sw=2 filetype indent on syntax on colorscheme habamax hi MatchParen ctermfg=66 ctermbg=234 cterm=underline nnoremap;: nnoremap;; inoremap {<cr> {<cr>}<esc>0 <bs></bs></esc></cr></cr></pre>	
Makefile	
CXXFLAGS=-std=c++20 -Wall -Weytra -Wshadow	

```
#!/bin/bash
for((i=1;i>0;i++)) do
    echo "$i"
    echo "$i" | ./gen > int
    diff -w <(./sol < int) <(./slow < int) || break
done</pre>
```

Wzory (2)

2.1 Kombinatoryka

2.1.1 Lemat Burnside'a

Niech G oznacza grupę symetrii reprezentacji, S zbiór obiektów, a $I(\pi)$ ilość punktów stałych π . Wtedy

$$|S| = \frac{1}{|G|} \sum_{\pi \in G} I(\pi).$$

2.2 Grafy

2.2.1 Twierdzenie Königa

W grafie dwudzielnym zachodzi

- nk = pw
- nk + pk = n
- pw + nw = n

oraz

- pw to zbiór wierzchołków na brzegu min-cut
- nw to dopełnienie pw
- pk to nk z dodanymi pojedynczymi krawędziami każdego nieskojarzonego wierzchołka

2.2.2 Twierdzenie Erdősa-Gallaia

Ciąg stopni $d_1 \geq \ldots \geq d_n$ opisuje prosty graf w
tw gdy $\sum d_i$ jest parzysta oraz dla każdego $1 \leq k \leq n$ zachodzi

$$\sum_{i=1}^{k} d_i \le k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

2.2.3 Twierdzenie Gale'a-Rysera

Ciągi stopni $a_1 \geq \ldots \geq a_n$ oraz b_1,\ldots,b_n opisują prosty graf dwudzielny wtw gdy $\sum a_i = \sum b_i$ oraz dla każdego $1 \leq k \leq n$ zachodzi

$$\sum_{i=1}^{k} a_i \le \sum_{i=1}^{n} \min(b_i, k).$$

2.2.4 Przepływy z wymaganiami

Szukamy przepływu $\leq F$ takiego, że $f_i \geq d_i$ dla każdej krawędzi. Tworzymy nowe źródło s' i ujście t'. Następnie dodajemy krawedzie

- $(u_i, t', d_i), (s', v_i, d_i), (u_i, v_i, c_i d_i)$ zamiast (u_i, v_i, c_i, d_i)
- \bullet (t, s, F)

Przepływ spełnia wymagania jeżeli maksymalnie wypełnia wszystkie krawedzie $s^\prime.$

2.3 Analiza

2.3.1 Mnożniki Lagrange'a

Jeżeli optymalizujemy $f(x_1, \ldots, x_n)$ przy ograniczeniach typu $g_k(x_1, \ldots, x_n) = 0$ to x_1, \ldots, x_n jest ekstremum lokalnym tylko jeżeli gradient $\nabla f(x_1, \ldots, x_n)$ jest kombinacją liniową gradientów $\nabla g_k(x_1, \ldots, x_n)$.

Struktury danych (3)

WaveletTree.h

Stosowanie: st – początek, ed – koniec, sst – posortowany początek. Czas: $\mathcal{O}((n+q)\log n)$

```
struct node {
 int lo, hi;
 vector<int> s;
 node *1 = 0, *r = 0;
 node (auto st, auto ed, auto sst) {
    int n = ed - st;
    lo = sst[0];
   hi = sst[n - 1] + 1;
   if (lo + 1 < hi) {
      int mid = sst[n / 2];
      if (mid == sst[0]) mid = *upper_bound(sst, sst + n, mid);
      s.reserve(n + 1);
      s.push_back(0);
      for (auto it = st; it != ed; it++) {
        s.push back(s.back() + (*it < mid));
      auto k = stable_partition(st, ed, [&](int x) {
       return x < mid:
      auto sm = lower_bound(sst, sst + n, mid);
     if (k != st) l = new node(st, k, sst);
      if (k != ed) r = new node(k, ed, sm);
 int kth(int a, int b, int k) {
```

if (lo + 1 == hi) **return** lo;

return $k < y - x ? 1 \rightarrow kth(x, y, k)$

: r - kth(a - x, b - y, k - (y - x));

int x = s[a], y = s[b];

```
int count(int a, int b, int k) {
    if (10 >= k) return 0;
    if (hi <= k) return b - a;</pre>
    int x = s[a], y = s[b];
    return (1 ? 1->count(x, y, k) : 0) +
           (r ? r - > count(a - x, b - y, k) : 0);
  int freq(int a, int b, int k) {
    if (k < lo | | hi <= k) return 0;</pre>
    if (lo + 1 == hi) return b - a;
    int x = s[a], y = s[b];
    return (1 ? 1->freq(x, y, k) : 0) +
           (r ? r - > freq(a - x, b - y, k) : 0);
};
OrderedSet.h
Stosowanie: s.find_by_order(k) i s.order_of_key(k).
Czas: \mathcal{O}(\log n)
#include <ext/pb ds/assoc container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename T>
using ordered set = tree<T, null type, less<T>, rb tree tag,
                          tree_order_statistics_node_update>;
Treap.h
Czas: \mathcal{O}(\log n)
mt19937 64 rng(2137);
struct node {
  int val, sz = 1;
  uint64 t pr;
  node *1 = 0, *r = 0;
  node(int x) {
   val = x:
   pr = rng();
  void pull() {
    sz = 1 + size(1) + size(r);
  friend int size(node* a) {
    return a ? a->sz : 0;
  friend pair<node*, node*> split(node* a, int k) {
    if (!a) return {0, 0};
    if (k <= size(a->1)) {
      auto [la, lb] = split(a->1, k);
      a -> 1 = 1b;
      a->pull();
      return {la, a};
    } else {
      auto [ra, rb] = split(a->r, k - size(a->l) - 1);
      a->r = ra;
      a->pull();
      return {a, rb};
  friend node* merge(node* a, node* b) {
    if (!a || !b) return a ? a : b;
   if (a->pr > b->pr) {
     a->r = merge(a->r, b);
```

```
a->pull();
      return a;
    } else {
      b->1 = merge(a, b->1);
      b->pull();
      return b;
 }
};
LineSet.h
Opis: Znajduje maksimum funkcji liniowych online. Dla doubli div (a, b) =
a/b \text{ oraz INF} = 1/.0.
Czas: \mathcal{O}(\log n)
struct line {
  mutable ll a, b, p;
 bool operator<(const line& o) const { return a < o.a; }</pre>
 bool operator<(11 x) const { return p < x; }</pre>
struct line_set : multiset<line, less<>>> {
  static const 11 INF = LLONG MAX;
  ll div(ll a, ll b) {
    return a / b - ((a ^ b) < 0 && a % b);
  bool inter(iterator x, iterator y) {
    if (y == end()) return x->p = INF, false;
    if (x->a == y->a) x->p = x->b > y->b ? INF : -INF;
    else x->p = div(y->b - x->b, x->a - y->a);
    return x->p >= y->p;
  void add(ll a, ll b) {
    auto z = insert({a, b, 0}), y = z++, x = y;
    while (inter(y, z)) z = erase(z);
    if (x != begin() \&\& inter(--x, y)) inter(x, y = erase(y));
    while ((y = x) != begin() \&\& (--x)->p >= y->p) {
      inter(x, erase(y));
 11 get(11 x) {
   line l = *lower bound(x);
    return 1.a * x + 1.b;
};
4.1 Przepływy
```

Grafy (4)

Dinic.h Czas: $\mathcal{O}(nm \log U)$

```
struct dinic {
 struct edge {
    int to, rev;
   int cap;
 int n;
 vector<vector<edge>> adj;
 vector<int> q, lvl, it;
 dinic(int _n) {
   n = _n;
   adj.resize(n);
    q.reserve(n);
   lvl.resize(n);
```

```
it.resize(n);
 void add edge(int u, int v, int cap) {
    int i = ssize(adj[u]), j = ssize(adj[v]) + (u == v);
    adj[u].push_back({v, j, cap});
    adj[v].push_back({u, i, 0});
 bool bfs(int s, int t, int r) {
    q.clear();
    lvl.assign(n, -1);
    lvl[s] = 0;
    q.push back(s);
    for (int i = 0; i < ssize(q); i++) {</pre>
      int u = q[i];
      for (edge& e : adj[u]) {
        if (e.cap >= r && lvl[e.to] == -1) {
          lvl[e.to] = lvl[u] + 1;
          q.push_back(e.to);
          if (e.to == t) return true;
    return false;
 11 dfs(int u, int t, ll cap) {
    if (u == t) return cap;
    11 f = 0;
    for (int& i = it[u]; i < ssize(adj[u]); i++) {</pre>
      edge& e = adj[u][i];
      if (e.cap > 0 && lvl[u] + 1 == lvl[e.to]) {
       11 \text{ add} = dfs(e.to, t, min(cap - f, (11)e.cap));
        e.cap -= add;
        adj[e.to][e.rev].cap += add;
        f += add;
      if (f == cap) return f;
    lvl[u] = -1;
    return f;
 ll flow(int s, int t, ll cap) {
    for (int i = 29; i >= 0; i--) {
      while (f < cap && bfs(s, t, 1 << i)) {</pre>
       it.assign(n, 0);
        f += dfs(s, t, cap - f);
    return f;
};
```

UW

MCMF.h

Stosowanie: Jeżeli są ujemne krawędzie, przed pusczeniem flow w dst trzeba policzyć najkrótsze ścieżki z s i puścić reduce (t). Czas: $\mathcal{O}(Fm \log n)$

```
#include <ext/pb_ds/priority_queue.hpp>
11 \text{ INF } 64 = 2e18;
struct MCMF {
  struct edge {
    int to, rev;
    int cap;
    11 cost;
  };
  struct cmp {
    bool operator()(const auto& 1, const auto& r) const {
      return 1.second > r.second;
```

```
};
int n:
vector<vector<edge>> adi;
vector<ll> dst;
11 c = 0;
__gnu_pbds::priority_queue<pair<int, ll>, cmp> q;
vector<decltype(q)::point_iterator> its;
vector<int> id;
MCMF (int _n) {
 n = _n;
 adj.resize(n);
 id.resize(n);
void add_edge(int u, int v, int cap, int cost) {
 int i = ssize(adj[u]), j = ssize(adj[v]) + (u == v);
  adj[u].push_back({v, j, cap, cost});
 adj[v].push_back({u, i, 0, -cost});
void reduce(int t) {
  for (int i = 0; i < n; i++) {</pre>
   for (edge& e : adj[i]) {
     if (dst[i] != INF64 && dst[e.to] != INF64) {
        e.cost += dst[i] - dst[e.to];
   }
  c += dst[t];
bool dijkstra(int s, int t) {
  dst.assign(n, INF64);
 its.assign(n, q.end());
 dst[s] = 0;
  q.push({s, 0});
  while (!q.empty()) {
   int u = q.top().first;
   q.pop();
   for (edge& e : adj[u]) {
     if (e.cap > 0) {
       11 d = dst[u] + e.cost;
       if (d < dst[e.to]) {
          dst[e.to] = d;
          if (its[e.to] == q.end()) {
           its[e.to] = g.push({e.to, dst[e.to]});
           q.modify(its[e.to], {e.to, dst[e.to]});
          id[e.to] = e.rev;
  reduce(t);
  return dst[t] != INF64;
pair<ll, ll> flow(int s, int t, ll cap) {
  11 \text{ ff} = 0;
  11 cc = 0;
  while (ff < cap && dijkstra(s, t)) {
   11 f = cap - ff;
   for (int i = t; i != s;) {
     edge& e = adj[i][id[i]];
      f = min(f, (ll)adj[e.to][e.rev].cap);
     i = e.to;
    for (int i = t; i != s;) {
     edge& e = adj[i][id[i]];
     e.cap += f;
     adj[e.to][e.rev].cap -= f;
     i = e.to;
```

```
ff += f;
     cc += f * c;
   return {ff, cc};
};
```

4.2Skojarzenia

```
Matching.h
Czas: \mathcal{O}(m\sqrt{n})
```

```
struct matching {
 int n, m;
 vector<vector<int>> adj;
 vector<int> pb, pa;
 vector<int> lvl, it;
 matching(int n, int m) {
   n = _n;
   m = m;
   adj.resize(n);
   pb.resize(n, -1);
   pa.resize(m, -1);
   it.resize(n);
 void add edge(int u, int v) {
   adiful.push back(v);
 bool bfs() {
   bool res = false:
   lvl.assign(n, -1);
   queue<int> q;
   for (int i = 0; i < n; i++) {</pre>
     if (pb[i] == -1) {
       q.push(i);
       lvl[i] = 0;
    while (!q.empty()) {
     int u = q.front();
     q.pop();
      for (int j : adj[u]) {
       if (pa[j] == -1) {
         res = true;
       } else if (lvl[pa[j]] == -1) {
         lvl[pa[j]] = lvl[u] + 1;
         q.push(pa[j]);
   return res;
 bool dfs(int u) {
   for (auto& i = it[u]; i < ssize(adj[u]); i++) {</pre>
     int v = adj[u][i];
     if (pa[v] == -1 ||
          (lvl[pa[v]] == lvl[u] + 1 && dfs(pa[v]))) {
       pb[u] = v;
       pa[v] = u;
       return true;
   return false;
 int match() {
   int ans = 0;
```

```
while (bfs()) {
      it.assign(n, 0);
      for (int i = 0; i < n; i++) {</pre>
        if (pb[i] == -1 && dfs(i)) ans++;
    return ans;
};
```

Grafy skierowane

```
SCC.h
```

Czas: $\mathcal{O}(n+m)$

```
struct SCC {
  int n, cnt = 0;
  vector<vector<int>> adj;
  vector<int> p, low, in;
  stack<int> st;
  int tour = 0;
  SCC(int _n) {
    n = _n;
    adj.resize(n);
    p.resize(n, -1);
    low.resize(n);
    in.resize(n, -1);
  void add_edge(int u, int v) {
    adj[u].push_back(v);
  void dfs(int u) {
    low[u] = in[u] = tour++;
    st.push(u);
    for (int v : adj[u]) {
      if (in[v] == -1) {
        dfs(v);
        low[u] = min(low[u], low[v]);
        low[u] = min(low[u], in[v]);
    if (low[u] == in[u]) {
      int v = -1;
       v = st.top();
        st.pop();
        in[v] = n;
        p[v] = cnt;
      } while (v != u);
      cnt++;
  void build() {
    for (int i = 0; i < n; ++i) {</pre>
      if (in[i] == -1) dfs(i);
    for (int i = 0; i < n; i++) p[i] = cnt - 1 - p[i];</pre>
};
```

Matma (5)

5.1 Arytmetyka modularna

```
ModInt.h
```

```
template<int M, int R>
struct mod {
  static const int MOD = M, ROOT = R;
  mod(11 y = 0) : x(y % M) { x += (x < 0) * M; }
  mod operator+= (const mod& o) {
   if ((x += 0.x) >= M) x -= M;
   return *this;
  mod operator = (const mod& o) {
   if ((x -= 0.x) < 0) x += M;
   return *this;
  mod operator *= (const mod& o) {
   x = 111 * x * o.x % M;
   return *this;
  mod operator/=(const mod& o) {
    return (*this) *= o.inv();
  friend mod operator+(mod a, const mod& b) { return a += b; }
  friend mod operator-(mod a, const mod& b) { return a -= b; }
  friend mod operator*(mod a, const mod& b) { return a *= b; }
  friend mod operator/(mod a, const mod& b) { return a /= b; }
  auto operator<=>(const mod&) const = default;
  mod pow(ll n) const {
   mod a = x, b = 1;
   while (n > 0) {
     if (n % 2 == 1) b \star= a;
     a *= a;
     n /= 2;
   return b;
  mod inv() const {
    return pow (M - 2);
using mint = mod<998244353, 3>;
GCD.h
Opis: Znajduje x i y takie, że ax + by = \gcd(a, b).
Czas: \mathcal{O}(\log(\min(a,b)))
ll gcd(ll a, ll b, ll& x, ll& y) {
 if (!b) return x = 1, y = 0, a;
 11 q = qcd(b, a % b, y, x);
 return y = x * (a / b), g;
```

}

5.2 Sploty

NTT 1

Stosowanie: Jeżeli MOD = 998244353 to $n + m \le 2^{23}$. Czas: $\mathcal{O}((n+m)\log(n+m))$

```
template<typename T>
void ntt(vector<T>& a, bool inv) {
  int n = ssize(a);
```

```
vector<T> b(n);
  for (int i = n / 2; i > 0; i /= 2, swap(a, b)) {
   T w = T(T::ROOT).pow((T::MOD - 1) / n * i), m = 1;
    for (int j = 0; j < n; j += 2 * i, m *= w) {
      for (int k = 0; k < i; k++) {
       T u = a[j + k], v = a[j + k + i] * m;
        b[i / 2 + k] = u + v;
       b[j / 2 + k + n / 2] = u - v;
 if (inv) {
   reverse(a.begin() + 1, a.end());
   T ni = T(n).inv();
    for (int i = 0; i < n; i++) a[i] *= ni;</pre>
template<typename T>
vector<T> conv(vector<T> a, vector<T> b) {
 int s = ssize(a) + ssize(b) - 1;
 int n = 1 << (__1g(2 * s - 1));
 a.resize(n); b.resize(n);
 ntt(a, false); ntt(b, false);
  for (int i = 0; i < n; i++) a[i] *= b[i];</pre>
 ntt(a, true);
 a.resize(s);
 return a;
NTT3.h
Stosowanie: n+m \leq 2^{24}.
Czas: \mathcal{O}((n+m)\log(n+m))
template<typename T>
vector<T> mconv(const auto& a, const auto& b) {
 auto cp = [&](const auto& v) {
    vector<T> vv(ssize(v));
    for (int i = 0; i < ssize(v); i++) vv[i] = T(v[i].x);
    return vv;
  return conv(cp(a), cp(b));
template<tvpename T>
vector<T> conv3(const vector<T>& a, const vector<T>& b) {
  using m0 = mod<754974721, 11>; auto c0 = mconv<m0>(a, b);
  using m1 = mod<167772161, 3>; auto c1 = mconv<m1>(a, b);
  using m2 = mod<469762049, 3>; auto c2 = mconv < m2 > (a, b);
  m1 r01 = m1(m0::MOD).inv();
  m2 r02 = m2 (m0::MOD).inv(), r12 = m2 (m1::MOD).inv();
  vector<T> d(ssize(c0));
  for (int i = 0; i < ssize(c0); i++) {</pre>
    int a = c0[i].x;
    int b = ((c1[i] - a) * r01).x;
    int c = (((c2[i] - a) * r02 - b) * r12).x;
   d[i] = (T(c) * m1::MOD + b) * m0::MOD + a;
 return d;
FST.h
Stosowanie: n musi być potega dwójki.
Czas: \mathcal{O}(n \log n)
void fst(vector<mint>& a, bool inv) {
 int n = ssize(a);
  for (int i = 1; i < n; i *= 2) {
    for (int j = 0; j < n; j += 2 * i) {
```

for (int k = 0; k < i; k++) {

mint u = a[j + k], v = a[j + k + i];

Teksty (6)

KMP.h Czas: $\mathcal{O}(n)$

```
vector<int> kmp(const string& s) {
  int n = ssize(s);
  vector<int> p(n);
  for (int i = 1; i < n; i++) {
    int j = p[i - 1];
    while (j > 0 && s[i] != s[j]) j = p[j - 1];
    p[i] = j + (s[i] == s[j]);
  }
  return p;
}
```

Manacher.h

Stosowanie: Zwraca długość najdłuższego palindromu. p[2 * i] – środek w i, p[2 * i + 1] – środek między i a i+1. **Czas:** $\mathcal{O}(n)$

```
vector<int> manacher(const string& s) {
 int n = ssize(s);
 string t(2 * n - 1, '#');
 for (int i = 0; i < n; i++) t[2 * i] = s[i];
 vector<int> p(2 * n - 1);
 for (int i = 0, l = -1, r = -1; i < 2 * n - 1; i++) {
   if (i \le r) p[i] = min(r - i + 1, p[1 + r - i]);
   while (p[i] < min(i + 1, 2 * n - 1 - i)) {
     if (t[i - p[i]] != t[i + p[i]]) break;
     p[i]++;
   if (i + p[i] - 1 > r)  {
     1 = i - p[i] + 1;
     r = i + p[i] - 1;
 for (int i = 0; i < 2 * n - 1; i++) {
   p[i] -= t[i - p[i] + 1] == '#';
 return p;
```

Suffix Array l

Stosowanie: Jeżeli tekst ma znaki inne niż a-z trzeba zmienić inicjalizację. Czas: $\mathcal{O}(n \log n)$

Z Point ConvexHull PolygonTangents GCC

```
vector<int> suffix_array(const string& s) {
  int n = ssize(s);
  vector<int> p(n), cnt(26);
  for (int i = 0; i < n; i++) cnt[s[i] - 'a']++;
  for (int i = 1; i < 26; i++) cnt[i] += cnt[i - 1];</pre>
  for (int i = 0; i < n; i++) p[--cnt[s[i] - 'a']] = i;</pre>
  vector<int> rnk(n);
  for (int i = 1; i < n; i++) {
   rnk[p[i]] = s[p[i]] == s[p[i-1]] ? rnk[p[i-1]] : i;
  cnt.resize(n):
 vector<int> np(n), nrnk(n);
  for (int len = 1; len < n; len *= 2) {</pre>
   iota(cnt.begin(), cnt.end(), 0);
   for (int i = n - len; i < n; i++) np[cnt[rnk[i]]++] = i;</pre>
   for (int i = 0; i < n; i++) {</pre>
     if (p[i] - len >= 0) {
        np[cnt[rnk[p[i] - len]]++] = p[i] - len;
   nrnk[np[0]] = 0;
    for (int i = 1; i < n; i++) {</pre>
     int a = np[i - 1];
     int b = np[i];
     if (max(a, b) + len < n && rnk[a] == rnk[b] &&</pre>
          rnk[a + len] == rnk[b + len]) {
       nrnk[b] = nrnk[a];
     } else {
        nrnk[b] = i;
    swap(p, np);
    swap(rnk, nrnk);
vector<int> build_lcp(const string& s, const vector<int>& sa) {
 int n = ssize(s);
  vector<int> pos(n);
  for (int i = 0; i < n; i++) pos[sa[i]] = i;</pre>
  vector<int> lcp(n - 1);
  int k = 0;
  for (int i = 0; i < n; i++) {
   if (pos[i] == 0) continue;
   while (i + k < n \&\& s[i + k] == s[sa[pos[i] - 1] + k]) k++;
   lcp[pos[i] - 1] = k;
   k = \max(0, k - 1);
  return lcp;
```

Z.h Czas: $\mathcal{O}(n)$

```
vector<int> z(const string& s) {
  int n = ssize(s);
  vector<int> f(n);
  f[0] = n;
  for (int i = 1, 1 = 0, r = 0; i < n; i++) {
    if (i <= r) f[i] = min(r - i + 1, f[i - 1]);
    while (f[i] < n - i && s[i + f[i]] == s[f[i]]) f[i]++;
    if (i + f[i] - 1 > r) {
        l = i;
        r = i + f[i] - 1;
    }
}
return f;
}
```

Geometria (7)

Point.h

```
struct pt {
 pt operator+(pt o) const { return {x + o.x, y + o.y}; }
 pt operator-(pt o) const { return {x - o.x, y - o.y}; }
 pt operator*(ll a) const { return {x * a, y * a}; }
 pt operator/(ll a) const { return {x / a, y / a}; }
  auto operator<=>(const pt&) const = default;
 friend 11 cross(pt a, pt b) { return a.x * b.y - a.y * b.x; }
 friend 11 dot(pt a, pt b) { return a.x * b.x + a.y * b.y; }
 friend ll norm(pt a) { return a.x * a.x + a.v * a.v; }
 friend int half(pt a) {
   if (a.y < 0) return -1;
    if (a.y == 0 && a.x >= 0) return 0;
    return 1;
 friend auto& operator<<(auto& o, pt a) {</pre>
   return o << '(' << a.x << ", " << a.v << ')';
};
```

ConvexHull.h

Stosowanie: Usuwa punkty współliniowe.

Czas: $\mathcal{O}(n \log n)$

```
vector<pt> convex hull(vector<pt> p) {
 if (ssize(p) <= 1) return p;</pre>
 sort(p.begin(), p.end());
 vector<pt> h(ssize(p) + 1);
 int s = 0, t = 0;
 for (int it = 0; it < 2; it++) {</pre>
   for (pt a : p) {
     while (t >= s + 2) {
       pt u = h[t - 2], v = h[t - 1];
       if (cross(v - u, a - v) <= 0) t--;
       else break;
     h[t++] = a;
   reverse(p.begin(), p.end());
   s = --t:
 h.resize(t - (t == 2 && h[0] == h[1]));
 return h:
```

Polygon Tangents.h

Stosowanie: Wielokąt musi być CCW i $n \geq 3$. Zwraca najbliższe punkty styczne różne od a.

Czas: $\mathcal{O}(\log n)$

```
pair<pt, pt> tangents(const vector<pt>& p, pt a) {
   int n = ssize(p);
   pt t[2];
   for (int it = 0; it < 2; it++) {
      auto dir = [&] (int i) {
        pt u = p[i] - a;
        pt v = p[i < n - 1 ? i + 1 : 0] - a;
        ll c = cross(u, v);
        if (c!= 0) return c < 0;
        if (dot(u, v) > 0) return norm(u) > norm(v);
        return true;
      };
      auto dirx = [&] (int i) { return dir(i) ^ it; };
      if (dirx(0) == 1 && dirx(n - 1) == 0) {
```

Inne (8)

GCC.h

```
#pragma GCC optimize("03,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
```