

1 Contest

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Contest (1)

sol.cpp

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for (int i = (a); i < (b); i++)
#define all(x) begin(x), end(x)
#define sz(x) int ((x).size())
using ll = long long;
using pii = pair<int, int>;
using vi = vector<int>;

#ifdef LOCAL
auto& operator<< (auto& o, pair<auto, auto> x) {
    auto operator<< (auto& o, auto x) -> decltype(x.end(), o) {
        o << '{';
        for (int i = 0; auto y : x) o << ", " + !i++ * 2 << y;
        return o << '}' ;
    }
    auto& operator<< (auto& o, pair<auto, auto> x) {
        return o << '{' << x.first << ", " << x.second << '}' ;
    }
}
void __print(auto... x) { ((cerr << ' ' << x), ...) << endl; }
#define debug(x...) cerr << "[" #x "]:", __print(x)
#else
#define debug(...) 2137
#endif

int main() {
    cin.tie(0)->sync_with_stdio(0);
}
```

.vimrc

```
set nu et ts=2 sw=2
filetype indent on
syntax on
colorscheme habamax
hi MatchParen ctermfg=66 ctermbg=234 cterm=underline
nnoremap ; :
nnoremap ; :
inoremap {<cr> {<cr>}<esc>O <bs>
```

Makefile

```
CXXFLAGS=-std=c++20 -Wall -Wextra -Wshadow
sol: sol.cpp
g++ $(CXXFLAGS) -fsanitize=address,undefined -g -DLOCAL \
    sol.cpp -o sol
```

```
1 fast: sol.cpp
   g++ $(CXXFLAGS) -O2 sol.cpp -o fast

1 test.sh

1 #!/bin/bash
   for ((i=1;i>0;i++)) do
3     echo "$i"
     echo "$i" | ./gen > int
     diff -w <(.sol < int) <(.slow < int) || break
done

4 hash.sh

4 #!/bin/bash
   cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum |cut -c-6

.bashrc

alias rm='trash'
alias mv='mv -i'
alias cp='cp -i'
```

Struktury danych (2)

```
OrderedSet.h
Opis: s.find_by_order(k) i s.order_of_key(k).
Czas:  $\mathcal{O}(\log n)$ 

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template<class T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
                        tree_order_statistics_node_update>;

LineSet.h
Opis: Znajduje maksimum funkcji liniowych online. Dla doubli inf = 1/.0, div(a,b) = a/b.
Czas:  $\mathcal{O}(\log n)$ 
```

```
struct line {
    mutable ll k, m, p;
    bool operator<(const line& o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};

struct line_set : multiset<line, less<>> {
    static const ll inf = LLONG_MAX;
    ll div(ll a, ll b) {
        return a / b - ((a ^ b) < 0 && a % b);
    }
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
```

```
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};

Grafy (3)

3.1 Przepływy

Dinic.h
Opis: Dinic ze skalowaniem. Należy ustawić zakres it w flow zgodnie z U.
Czas:  $\mathcal{O}(nm \log U)$ 

struct dinic {
    struct edge {
        int to, rev;
        ll cap;
    };
    vi lvl, ptr, q;
    vector<vector<edge>> adj;
    dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
    void add_edge(int u, int v, ll cap, ll rcap = 0) {
        int i = sz(adj[u]), j = sz(adj[v]);
        adj[u].push_back({v, j + (u == v), cap});
        adj[v].push_back({u, i, rcap});
    }
    ll dfs(int v, int t, ll f) {
        if (v == t || !f) return f;
        for (int& i = ptr[v]; i < sz(adj[v]); i++) {
            edge& e = adj[v][i];
            if (lvl[e.to] == lvl[v] + 1)
                if (ll p = dfs(e.to, t, min(f, e.cap))) {
                    e.cap -= p, adj[e.to][e.rev].cap += p;
                    return p;
                }
        }
        return 0;
    }
    ll flow(int s, int t) {
        ll f = 0; q[0] = s;
        for (int it = 29; it >= 0; it--) do {
            lvl = ptr = vi(sz(q));
            int qi = 0, qe = lvl[s] = 1;
            while (qi < qe && !lvl[t]) {
                int v = q[qi++];
                for (edge e : adj[v])
                    if (!lvl[e.to] && e.cap >> it)
                        q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
            }
            while (lvl[p = dfs(s, t, LLONG_MAX)] f += p;
        } while (lvl[t]);
        return f;
    }
};

GomoryHu.h
Opis: Tworzy drzewo gdzie min cut to minimum na ścieżce.
Czas:  $\mathcal{O}(n)$  przepływów
```

```
struct edge { int u, v; ll w; };
vector<edge> gomory_hu(int n, const vector<edge>& ed) {
    vector<edge> t; vi p(n);
    rep(i, 1, n) {
        dinic d(n);
        for (edge e : ed) d.add_edge(e.u, e.v, e.w, e.w);
```

```
t.push_back({i, p[i], d.flow(i, p[i])});
rep(j, i + 1, n) if (p[j] == p[i] && d.lvl[j]) p[j] = i;
}
return t;
}
```

MCMF.h

Opis: MCMF z Dijkstrą. Jeżeli są ujemne krawędzie to przed pusczeniem flow w pi trzeba policzyć najkrótsze ścieżki z s.
Czas: $O(Fm \log n)$

```
#include <ext/pb_ds/priority_queue.hpp>
const ll INF = 2e18;
struct MCMF {
    struct edge {
        int from, to, rev;
        ll cap, cost;
    };
    int n;
    vector<vector<edge>> adj;
    vector<ll> dst, pi;
    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> it;
    vector<edge*> p;
    MCMF(int _n) : n(_n), adj(n), pi(n), p(n) {}
    void add_edge(int u, int v, ll cap, ll cost) {
        int i = sz(adj[u]), j = sz(adj[v]);
        adj[u].push_back({u, v, j + (u == v), cap, cost});
        adj[v].push_back({v, u, i, 0, -cost});
    }
    bool path(int s, int t) {
        dst.assign(n, INF); it.assign(n, q.end());
        q.push({dst[s] = 0, s});
        while (!q.empty()) {
            int u = q.top().second; q.pop();
            for (edge& e : adj[u]) {
                ll d = dst[u] + pi[u] + e.cost - pi[e.to];
                if (e.cap && d < dst[e.to]) {
                    dst[e.to] = d, p[e.to] = &e;
                    if (it[e.to] == q.end())
                        it[e.to] = q.push({-dst[e.to], e.to});
                    else
                        q.modify(it[e.to], {-dst[e.to], e.to});
                }
            }
        }
        rep(i, 0, n) pi[i] = min(pi[i] + dst[i], INF);
        return pi[t] != INF;
    }
    pair<ll, ll> flow(int s, int t, ll cap) {
        ll f = 0, c = 0;
        while (f < cap && path(s, t)) {
            ll d = cap - f;
            for (edge* e = p[t]; e; e = p[e->from])
                d = min(d, e->cap);
            for (edge* e = p[t]; e; e = p[e->from])
                e->cap -= d, adj[e->to][e->rev].cap += d;
            f += d, c += d * pi[t];
        }
        return {f, c};
    }
};
```

3.2 DFS

SCC.h

Opis: Znajduje SCC w kolejności topologicznej.

Czas: $O(n + m)$

```
struct SCC {
    int n, t = 0, cnt = 0;
    vector<vi> adj;
    vi val, p, st;
    SCC(int _n) : n(_n), adj(n), val(n), p(n, -1) {}
    void add_edge(int u, int v) { adj[u].push_back(v); }
    int dfs(int u) {
        int low = val[u] = ++t; st.push_back(u);
        for (int v : adj[u]) if (p[v] == -1)
            low = min(low, val[v] ?: dfs(v));
        if (low == val[u]) {
            for (int x = -1; x != u;)
                p[x = st.back()] = cnt, st.pop_back();
            cnt++;
        }
        return low;
    }
    void build() {
        rep(i, 0, n) if (!val[i]) dfs(i);
        rep(i, 0, n) p[i] = cnt - 1 - p[i];
    }
};
```

TwoSat.h

Opis: 2-SAT.
Czas: $O(n + m)$

```
struct two_sat {
    int n;
    vector<pii> ed;
    vector<bool> b;
    two_sat(int _n) : n(_n) {}
    int add_var() { return n++; }
    void either(int x, int y) {
        x = max(2 * x, -1 - 2 * x), y = max(2 * y, -1 - 2 * y);
        ed.push_back({x, y});
    }
    void implies(int x, int y) { either(~x, y); }
    void must(int x) { either(x, x); }
    void at_most_one(const vi& v) {
        if (sz(v) <= 1) return;
        int cur = ~v[0];
        rep(i, 2, sz(v)) {
            int nxt = add_var();
            either(cur, ~v[i]); either(cur, nxt);
            either(~v[i], nxt); cur = ~nxt;
        }
        either(cur, ~v[1]);
    }
    bool solve() {
        SCC scc(2 * n);
        for (auto [u, v] : ed)
            scc.add_edge(u ^ 1, v), scc.add_edge(v ^ 1, u);
        scc.build(); b.resize(n, 1);
        rep(i, 0, n) {
            if (scc.p[2 * i] == scc.p[2 * i + 1]) return 0;
            if (scc.p[2 * i] < scc.p[2 * i + 1]) b[i] = 0;
        }
        return 1;
    }
};
```

3.3 DSU

RDSU.h

Opis: DSU z rollbackami.

Czas: $O(\log n)$

```
struct RDSU {
    vi e; vector<pii> st;
    RDSU(int n) : e(n, -1) {}
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : find(e[x]); }
    int time() { return sz(st); }
    void rollback(int t) {
        for (int i = time(); i-- > t;)
            e[st[i].first] = st[i].second;
        st.resize(t);
    }
    bool join(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        st.push_back({a, e[a]});
        st.push_back({b, e[b]});
        e[a] += e[b]; e[b] = a;
        return true;
    }
};
```

3.4 Drzewa

DMST.h

Opis: Skierowane MST z r. Zwraca {-1, {}} gdy nie istnieje.
Czas: $O(m \log m)$

```
struct edge { int a, b; ll w; };
struct node {
    edge key;
    node *l = 0, *r = 0;
    ll delta = 0;
    void prop() {
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    }
    edge top() { prop(); return key; }
};
node* merge(node* a, node* b) {
    if (!a || !b) return a ?: b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
}
void pop(node&& a) { a->prop(); a = merge(a->l, a->r); }
pair<ll, vi> dmst(int n, int r, const vector<edge>& g) {
    RDSU uf(n);
    vector<node*> heap(n);
    for (edge e : g) heap[e.b] = merge(heap[e.b], new node(e));
    ll res = 0;
    vi seen(n, -1), path(n), par(n);
    seen[r] = r;
    vector<edge> Q(n), in(n, {-1, -1, -1}), comp;
    deque<tuple<int, int, vector<edge>>> cyscs;
    rep(s, 0, n) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            if (!heap[u]) return {-1, {}};
            edge e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
            Q[qi] = e, path[qi++] = u, seen[u] = s;
```

```
res += e.w, u = uf.find(e.a);
if (seen[u] == s) {
    node* cyc = 0;
    int end = qi, time = uf.time();
    do cyc = merge(cyc, heap[w = path[--qi]]);
    while (uf.join(u, w));
    u = uf.find(u), heap[u] = cyc, seen[u] = -1;
    cycs.push_front({u, time, {&Q[qi], &Q[end]}});
}
}
rep(i, 0, qi) in[uf.find(Q[i].b)] = Q[i];
}
for (auto& [u, t, cmp] : cycs) {
    uf.rollback(t);
    edge inedge = in[u];
    for (auto& e : cmp) in[uf.find(e.b)] = e;
    in[uf.find(inedge.b)] = inedge;
}
rep(i, 0, n) par[i] = in[i].a;
return {res, par};
}
```

Matma (4)

4.1 Arytmetyka modularna

GCD.h

Opis: Rozszerzony algorytm Euklidesa.

Czas: $\mathcal{O}(\log \min(a, b))$

```
ll gcd(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll d = gcd(b, a % b, y, x);
    return y -= a / b * x, d;
}
```

CRT.h

Opis: Chińskie twierdzenie o resztach.

Czas: $\mathcal{O}(\log \min(m, n))$

```
ll crt(ll a, ll m, ll b, ll n) {
    if (n > m) swap(a, b), swap(m, n);
    ll x, y, g = gcd(m, n, x, y);
    assert((a - b) % g == 0); // no solution
    x = (b - a) % n * x % n / g * m + a;
    return x < 0 ? x + m * n / g : x;
}
```

ModMul.h

Opis: Mnożenie i potęgowanie dwóch long longów modulo. Jest to wyraźnie szybsze niż zamiana na __int128.

```
using ull = uint64_t;
ull modmul(ull a, ull b, ull M) {
    ll ret = a * b - M * ull(1.L / M * a * b);
    return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
    ull ans = 1;
    for (; e; b = modmul(b, b, mod), e /= 2)
        if (e & 1) ans = modmul(ans, b, mod);
    return ans;
}
```

ModInt.h

```
template<int M, int R>
struct mod {
    static const int MOD = M, ROOT = R;
    int x;
    mod(ll y = 0) : x(y % M) { x += (x < 0) * M; }
    mod operator+=(mod o) {
        if ((x += o.x) >= M) x -= M;
        return *this;
    }
    mod operator-=(mod o) {
        if ((x -= o.x) < 0) x += M;
        return *this;
    }
    mod operator*=(mod o) {
        x = ll1 * x * o.x % M;
        return *this;
    }
    mod operator/=(mod o) { return (*this) *= o.inv(); }
    friend mod operator+(mod a, mod b) { return a += b; }
    friend mod operator-(mod a, mod b) { return a -= b; }
    friend mod operator*(mod a, mod b) { return a *= b; }
    friend mod operator/(mod a, mod b) { return a /= b; }
    auto operator<=> (const mod&) const = default;
    mod pow(ll n) const {
        mod a = x, b = 1;
        for (; n; n /= 2, a *= a) if (n & 1) b *= a;
        return b;
    }
    mod inv() const { return pow(M - 2); }
};
using mint = mod<998244353, 3>;
```

4.2 Liczby pierwsze

MillerRabin.h

Opis: Test pierwszości Millera-Rabina.

```
bool prime(ull n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n - 1), d = n >> s;
    for (ull a : A) {
        ull p = modpow(a % n, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--)
            p = modmul(p, p, n);
        if (p != n - 1 && i != s) return 0;
    }
    return 1;
}
```

PollardRho.h

Opis: Algorytm faktoryzacji rho Pollarda.

Czas: $\mathcal{O}(n^{1/4})$

```
ull pollard(ull n) {
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    auto f = [&](ull x) { return modmul(x, x, n) + i; };
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) x = ++i, y = f(x);
        if ((q = modmul(prd, max(x, y) - min(x, y), n)) prd = q;
            x = f(x), y = f(f(y));
        }
        return __gcd(prd, n);
    }
}
void factor(ull n, map<ull, int>& cnt) {
    if (n == 1) return;
    if (prime(n)) { cnt[n]++; return; }
    ull x = pollard(n);
    factor(x, cnt); factor(n / x, cnt);
}
```

4.3 Wielomiany

NTT.h

Czas: $\mathcal{O}((n + m) \log(n + m))$

```
template<class T>
void ntt(vector<T>& a, bool inv) {
    int n = sz(a); vector<T> b(n);
    for (int i = n / 2; i; i /= 2, swap(a, b)) {
        T w = T(T::ROOT).pow((T::MOD - 1) / n * i), m = 1;
        for (int j = 0; j < n; j += 2 * i, m *= w) rep(k, 0, i) {
            T u = a[j + k], v = a[j + k + i] * m;
            b[j / 2 + k] = u + v, b[j / 2 + k + n / 2] = u - v;
        }
    }
    if (inv) {
        reverse(1 + all(a));
        T z = T(n).inv(); rep(i, 0, n) a[i] *= z;
    }
}
template<class T>
vector<T> conv(vector<T> a, vector<T> b) {
    int s = sz(a) + sz(b) - 1, n = 1 << __lg(2 * s - 1);
    a.resize(n); ntt(a, 0); b.resize(n); ntt(b, 0);
    rep(i, 0, n) a[i] *= b[i];
    ntt(a, 1); a.resize(s);
    return a;
}
```

Conv3.h

Opis: NTT z Garnerem. Działa dla $n + m \leq 2^{24}$ i $c_k \leq 5 \cdot 10^{25}$.

Czas: $\mathcal{O}((n + m) \log(n + m))$

```
template<class T>
vector<T> mconv(const auto& x, const auto& y) {
    auto con = [&](const auto& v) {
        vector<T> w(sz(v)); rep(i, 0, sz(v)) w[i] = v[i].x;
        return w; };
    return conv(con(x), con(y));
}
template<class T>
vector<T> conv3(const vector<T>& a, const vector<T>& b) {
    using m0 = mod<754974721, 11>; auto c0 = mconv<m0>(a, b);
    using m1 = mod<167772161, 3>; auto c1 = mconv<m1>(a, b);
    using m2 = mod<469762049, 3>; auto c2 = mconv<m2>(a, b);
    int n = sz(c0); vector<T> d(n); m1 r01 = m1(m0::MOD).inv();
    m2 r02 = m2(m0::MOD).inv(), r12 = m2(m1::MOD).inv();
    rep(i, 0, n) {
        int x = c0[i].x, y = ((c1[i] - x) * r01).x,
            z = (((c2[i] - x) * r02 - y) * r12).x;
        d[i] = (T(z) * m1::MOD + y) * m0::MOD + x;
    }
    return d;
}
```

4.4 Inne

SameDiv.h

Opis: Dzieli przedział $[1, \infty)$ na przedziały równości dzielenia.
Czas: $\mathcal{O}(\sqrt{n})$

```
vector<ll> same_floor(ll n) {
    vector<ll> v;
    for (ll i = 1; i <= n; i = n / (n / i) + 1) v.push_back(i);
    return v.push_back(n + 1), v;
}

vector<ll> same_ceil(ll n) {
    vector<ll> v;
    for (ll i = 1, j; i < n; i = (n + j - 2) / (j - 1)) {
        j = (n + i - 1) / i;
        v.push_back(i);
    }
    return v.push_back(n), v;
}
```

Teksty (5)

5.1 Podstawy

KMP.h

Opis: $p[k]$ – najdłuższy ścisły sufixs $s[0, k]$ który jest prefiksem s .
Czas: $\mathcal{O}(n)$

```
vi kmp(const string& s) {
    vi p(sz(s));
    rep(i, 1, sz(s)) {
        int g = p[i - 1];
        while (g && s[i] != s[g]) g = p[g - 1];
        p[i] = g + (s[i] == s[g]);
    }
    return p;
}
```

Z.h

Opis: $f[k]$ – najdłuższy prefiks $s[k, n]$ który jest prefiksem s .
Czas: $\mathcal{O}(n)$

```
vi z(const string& s) {
    int n = sz(s), l = -1, r = -1;
    vi f(n); f[0] = n;
    rep(i, 1, sz(s)) {
        if (i < r) f[i] = min(r - i, f[i - 1]);
        while (i + f[i] < n && s[i + f[i]] == s[f[i]]) f[i]++;
        if (i + f[i] > r) l = i, r = i + f[i];
    }
    return f;
}
```

Manacher.h

Opis: $p[1][k]$ – środek w k , $p[0][k]$ – środek między $k - 1$ a k .
Czas: $\mathcal{O}(n)$

```
array<vi, 2> manacher(const string& s) {
    int n = sz(s);
    array<vi, 2> p = {vi(n + 1), vi(n)};
    rep(z, 0, 2) for (int i = 0, l = 0, r = 0; i < n; i++) {
        int t = r - i + !z;
        if (i < r) p[z][i] = min(t, p[z][l + t]);
        int L = i - p[z][i], R = i + p[z][i] - !z;
```

```
        while (L >= 1 && R + 1 < n && s[L - 1] == s[R + 1])
            p[z][i]++, L--, R++;
        if (R > r) l = L, r = R;
    }
    return p;
}
```

Duval.h

Opis: Rozkłada słowo na nierosnący ciąg podsłów mniejszych od swoich wszystkich nietrywialnych sufiksów.
Czas: $\mathcal{O}(n)$

```
vi duval(const string& s) {
    int n = sz(s); vi f;
    for (int i = 0; i < n; i) {
        int j = i + 1, k = i;
        for (; j < n && s[k] <= s[j]; j++) {
            if (s[k] < s[j]) k = i;
            else ++k;
        }
        for (; i <= k; i += j - k) f.push_back(i);
    }
    return f.push_back(n), f;
}
```

5.2 Struktury sufiksowe

SuffixArray.h

Opis: Zawiera pusty sufiks. $lcp[k]$ – najdłuższy wspólny prefiks $k - 1$ i k .
Czas: $\mathcal{O}(n \log n)$

```
struct suffix_array {
    vi sa, lcp;
    suffix_array(const string& s, int lim = 128) {
        int n = sz(s) + 1, k = 0, a, b;
        vi x(all(s) + 1), y(n), ws(max(n, lim)), rank(n);
        sa = lcp = y, iota(all(sa), 0);
        for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
            p = j, iota(all(y), n - j);
            rep(i, 0, n) if (sa[i] >= j) y[p++] = sa[i] - j;
            fill(all(ws), 0);
            rep(i, 0, n) ws[x[i]]++;
            rep(i, 1, lim) ws[i] += ws[i - 1];
            for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            rep(i, 1, n) a = sa[i - 1], b = sa[i], x[b] =
                (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
        }
        rep(i, 1, n) rank[sa[i]] = i;
        for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
            for (k && k--, j = sa[rank[i] - 1];
                s[i + k] == s[j + k]; k++);
    }
};
```

Geometria (6)

6.1 Podstawy

Point.h

Opis: Podstawowy szablon do geometrii.

```
using D = long double;
const D EPS = 1e-9l;
int sgn(D x) { return (x > EPS) - (x < -EPS); }
struct P {
    D x, y;
    P operator+(P o) const { return {x + o.x, y + o.y}; }
    P operator-(P o) const { return {x - o.x, y - o.y}; }
    P operator*(D a) const { return {x * a, y * a}; }
    P operator/(D a) const { return {x / a, y / a}; }
    auto operator<=(P o) const {
        return pair(sgn(x - o.x), sgn(y - o.y)) <= pair(0, 0); }
    bool operator==(P o) const {
        return !sgn(x - o.x) && !sgn(y - o.y); }
};
D cross(P a, P b) { return a.x * b.y - a.y * b.x; }
D cross(P p, P a, P b) { return cross(a - p, b - p); }
D dot(P a, P b) { return a.x * b.x + a.y * b.y; }
D dot(P p, P a, P b) { return dot(a - p, b - p); }
D abs2(P a) { return a.x * a.x + a.y * a.y; }
D abs(P a) { return sqrt(abs2(a)); }
auto& operator<<(auto& o, P a) {
    return o << "(" << a.x << ", " << a.y << ")"; }
```

AngleCmp.h

Opis: Sortuje punkty rosnąco po kącie z przedziału $(-\pi, \pi]$. Punkt $(0, 0)$ ma kąt 0.

```
bool angle_cmp(P a, P b) {
    auto half = [](P p) { return sgn(p.y) ? -sgn(p.x); };
    int A = half(a), B = half(b);
    return A == B ? sgn(cross(a, b)) > 0 : A < B;
}
```

LineDist.h

Opis: Najkrótsza odległość między punktem i prostą/odcinkiem.

```
D line_dist(P p, P a, P b) {
    return abs(cross(p, a, b)) / abs(b - a);
}
D seg_dist(P p, P a, P b) {
    if (sgn(dot(a, p, b)) <= 0) return abs(p - a);
    if (sgn(dot(b, p, a)) <= 0) return abs(p - b);
    return line_dist(p, a, b);
}
```

LineInter.h

Opis: Punkt przecięcia prostych/odcinków.

```
pair<int, P> line_inter(P a, P b, P c, P d) {
    auto u = cross(b - a, d - c);
    if (sgn(u) == 0) // parallel
        return {-(!sgn(cross(a, b, c))), {}};
    auto p = cross(c, b, d), q = cross(c, d, a);
    return {l, (a * p + b * q) / u};
}
vector<P> seg_inter(P a, P b, P c, P d) {
    auto oa = cross(c, d, a), ob = cross(c, d, b),
        oc = cross(a, b, c), od = cross(a, b, d);
    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0) // strict
        return {(a * ob - b * oa) / (ob - oa)};
    set<P> s;
    if (!sgn(seg_dist(a, c, d))) s.insert(a);
    if (!sgn(seg_dist(b, c, d))) s.insert(b);
    if (!sgn(seg_dist(c, a, b))) s.insert(c);
    if (!sgn(seg_dist(d, a, b))) s.insert(d);
    return {all(s)};
}
```

6.2 Wielokąty

ConvexHull.h
Opis: Otoczka wypukła w kierunku CCW.
Czas: $\mathcal{O}(n \log n)$

```
vector<P> convex_hull(vector<P> pts) {
    if (sz(pts) <= 1) return pts;
    sort(all(pts));
    vector<P> h(sz(pts) + 1);
    int s = 0, t = 0;
    for (int it = 2; it--; s = --t, reverse(all(pts)))
        for (P p : pts) {
            while (t >= s + 2 &&
                sgn(cross(h[t - 2], h[t - 1], p)) <= 0) t--;
            h[t++] = p;
        }
    return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

HullTangents.h
Opis: Znajduje punkty styczne różne od a . Wielokąt musi być CCW i $n \geq 3$. Punkt a nie może leżeć w ściśłym wnętrzu wielokąta.
Czas: $\mathcal{O}(\log n)$

```
pii tangents(const vector<P>& p, P a) {
    int n = sz(p), t[2];
    rep(it, 0, 2) {
        auto dir = [&](int i) {
            P u = p[i] - a, v = p[(i + 1) % n] - a;
            D c = cross(u, v);
            if (sgn(c) != 0) return sgn(c) < 0;
            if (sgn(dot(u, v)) <= 0) return true;
            return sgn(abs2(u) - abs2(v)) > 0;
        };
        auto idir = [&](int i) { return dir(i) ^ it; };
        if (idir(0) && !idir(n - 1)) { t[it] = 0; continue; }
        int s[2] = {0, n - 1};
        while (s[1] - s[0] > 2) {
            int mid = (s[0] + s[1]) / 2, x = idir(mid);
            if (idir(s[x ^ 1]) == (x ^ 1)) {
                s[x] = mid;
            } else {
                bool b = sgn(cross(p[mid] - a, p[s[1]] - a)) < 0;
                s[b ^ x ^ it ^ 1] = mid;
            }
        }
        t[it] = s[0] + 1 + (idir(s[0] + 1) == 0);
    }
    return {t[0], t[1]};
}
```