

1 Contest

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Contest (1)

sol.cpp

```
#include <bits/stdc++.h>
using namespace std;
using ll = long long;

#ifdef LOCAL
auto& operator<<(auto&, pair<auto, auto>);
template <typename T, typename = T::value_type>
auto& operator<<(auto& o, T x) {
    o << "{";
    string s;
    for (auto i : x) {
        o << s << i;
        s = ", ";
    }
    return o << "}";
}
auto& operator<<(auto& o, pair<auto, auto> x) {
    return o << "(" << x.first << ", " << x.second << ")";
}
#define debug(x...) \
    cerr << "[" #x "]:", \
    [] (auto... y) { ((cerr << " " << y), ...) << endl; } (x)
#else
#define debug(...) 2137
#endif
int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);
}
```

.vimrc

```
set autoindent expandtab tabstop=2 shiftwidth=2 nu ic
    timeoutlen=1000 ttimeoutlen=10 hidden smartindent
syntax enable
colorscheme retrobox
inoremap {<cr> {<cr>}<esc>O <bs>
nnoremap ; :
nnoremap : ;
nnoremap <space>` :b#<cr>
filetype indent on
```

Makefile

```
CXXFLAGS=-std=c++20 -Wall -Wextra -Wshadow
```

```
sol: sol.cpp
    g++ $(CXXFLAGS) -fsanitize=address,undefined -g -DLOCAL sol.
        cpp -o sol
```

```
1 fast: sol.cpp
    g++ $(CXXFLAGS) -O2 sol.cpp -o fast

1 test.sh

2 #!/bin/bash

3 for((i=1;i>0;i++)) do
    echo "$i"
    echo "$i" | ./gen > int
    diff -w <(. /sol < int) <(. /slow < int) || break
3 done
```

Struktury danych (2)

Drzewo falkowe

Opis: Obsługuje zapytania typu *podaj k-ty najmniejszy na przedziale* itp. na statycznej tablicy. Jeżeli czas albo pamięć są ciasne warto przeskalować liczby. Niszczy tablicę.
Czas: $\mathcal{O}(\log A)$

```
struct node {
    int lo, hi;
    vector<int> s;
    node* l = 0;
    node* r = 0;
    node(int _lo, int _hi, auto st, auto ed) {
        lo = _lo;
        hi = _hi;
        if (lo + 1 < hi) {
            int mid = (lo + hi) / 2;
            s.reserve(ed - st + 1);
            s.push_back(0);
            for (auto it = st; it != ed; it++) {
                s.push_back(s.back() + (*it < mid));
            }
            auto k = stable_partition(
                st, ed, [&](int x) { return x < mid; });
            if (k != st) l = new node(lo, mid, st, k);
            if (k != ed) r = new node(mid, hi, k, ed);
        }
    }
    int kth(int a, int b, int k) {
        if (lo + 1 == hi) return lo;
        int x = s[a];
        int y = s[b];
        return k < y - x ? l->kth(x, y, k)
            : r->kth(a - x, b - y, k - (y - x));
    }
    int count(int a, int b, int k) {
        if (lo >= k) return 0;
        if (hi <= k) return b - a;
        int x = s[a];
        int y = s[b];
        return (l ? l->count(x, y, k) : 0) +
            (r ? r->count(a - x, b - y, k) : 0);
    }
    int freq(int a, int b, int k) {
        if (k < lo || hi <= k) return 0;
        if (lo + 1 == hi) return b - a;
        int x = s[a];
        int y = s[b];
        return (l ? l->freq(x, y, k) : 0) +
            (r ? r->freq(a - x, b - y, k) : 0);
    }
};
```

Ordered set

Opis: Alternatywnie można użyć treapa albo trie.
Stosowanie: `s.find_by_order(k)` i `s.order_of_key(k)`.
Czas: $\mathcal{O}(\log n)$ z dużą stałą.

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
```

Treap

Opis: Randomizowane drzewo binarne.
Czas: $\mathcal{O}(\log n)$

```
mt19937_64 rng(2137);
struct treap {
    struct node {
        int val, sz;
        uint64_t pr;
        int l = -1, r = -1;
    };
    vector<node> t;
    int make(int val) {
        int a = ssize(t);
        node& x = t.emplace_back();
        x.val = val;
        x.pr = rng();
        pull(x);
        return a;
    }
    int size(int x) { return x != -1 ? t[x].sz : 0; }
    void pull(int x) {
        if (x != -1) t[x].sz = 1 + size(t[x].l) + size(t[x].r);
    }
    int merge(int x, int y) {
        if (x == -1 || y == -1) return x != -1 ? x : y;
        int a = -1;
        if (t[x].pr > t[y].pr) {
            t[x].r = merge(t[x].r, y);
            a = x;
        } else {
            t[y].l = merge(x, t[y].l);
            a = y;
        }
        pull(a);
        return a;
    }
    pair<int, int> split(int x, int k) {
        if (x == -1) return {-1, -1};
        auto a = pair(-1, -1);
        if (k <= size(t[x].l)) {
            auto [aa, bb] = split(t[x].l, k);
            t[x].l = bb;
            a = {aa, x};
        } else {
            auto [aa, bb] = split(t[x].r, k - size(t[x].l) - 1);
            t[x].r = aa;
            a = {x, bb};
        }
        pull(a.first);
        pull(a.second);
        return a;
    }
};
```

Grafy (3)

3.1 Przepływy

Dinic

Opis: Znajduje maksymalny przepływ.

Czas: $\mathcal{O}(n^2m)$, ale w rzeczywistości szybszy.

```
struct dinic {
    struct edge {
        int to, rev;
        int cap;
    };
    int n;
    vector<vector<edge>> adj;
    vector<int> lvl, it;
    dinic(int _n) {
        n = _n;
        adj.resize(n);
        lvl.resize(n);
        it.resize(n);
    }
    void add_edge(int u, int v, int cap) {
        int i = ssize(adj[u]);
        int j = ssize(adj[v]);
        if (u == v) j++;
        adj[u].push_back({v, j, cap});
        adj[v].push_back({u, i, 0});
    }
    bool bfs(int s, int t) {
        lvl.assign(n, -1);
        queue<int> q;
        lvl[s] = 0;
        q.push(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (edge& e : adj[u]) {
                if (e.cap > 0 && lvl[e.to] == -1) {
                    lvl[e.to] = lvl[u] + 1;
                    q.push(e.to);
                    if (e.to == t) return true;
                }
            }
        }
        return false;
    }
    int dfs(int u, int t, int cap) {
        if (u == t) return cap;
        int ans = 0;
        for (int& i = it[u]; i < ssize(adj[u]); i++) {
            edge& e = adj[u][i];
            if (e.cap > 0 && lvl[u] + 1 == lvl[e.to]) {
                int add = dfs(e.to, t, min(cap - ans, e.cap));
                e.cap -= add;
                adj[e.to][e.rev].cap += add;
                ans += add;
            }
            if (ans == cap) return ans;
        }
        lvl[u] = -1;
        return ans;
    }
    int flow(int s, int t, int cap) {
        int ans = 0;
        while (ans < cap && bfs(s, t)) {
            it.assign(n, 0);
            ans += dfs(s, t, cap - ans);
        }
        return ans;
    }
};
```

```
        ans += dfs(s, t, cap - ans);
    }
    return ans;
}
};
```

Matching

Opis: Dinic uproszczony do szukania największego skojarzenia.

Czas: $\mathcal{O}(m\sqrt{n})$

```
struct matching {
    int n, m;
    vector<vector<int>> adj;
    vector<int> pb, pa;
    vector<int> lvl, it;
    matching(int _n, int _m) {
        n = _n;
        m = _m;
        adj.resize(n);
        pb.resize(n, -1);
        pa.resize(m, -1);
        it.resize(n);
    }
    void add_edge(int u, int v) { adj[u].push_back(v); }
    bool bfs() {
        bool res = false;
        lvl.assign(n, -1);
        queue<int> q;
        for (int i = 0; i < n; i++) {
            if (pb[i] == -1) {
                q.push(i);
                lvl[i] = 0;
            }
        }
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int j : adj[u]) {
                if (pa[j] == -1) {
                    res = true;
                } else if (lvl[pa[j]] == -1) {
                    lvl[pa[j]] = lvl[u] + 1;
                    q.push(pa[j]);
                }
            }
        }
        return res;
    }
    bool dfs(int u) {
        for (auto& i = it[u]; i < ssize(adj[u]); i++) {
            int v = adj[u][i];
            if (pa[v] == -1 ||
                (lvl[pa[v]] == lvl[u] + 1 && dfs(pa[v]))) {
                pb[u] = v;
                pa[v] = u;
                return true;
            }
        }
        return false;
    }
    int match() {
        int ans = 0;
        while (bfs()) {
            it.assign(n, 0);
            for (int i = 0; i < n; i++) {
                if (pb[i] == -1 && dfs(i)) ans++;
            }
        }
        return ans;
    }
};
```

```
    }
};
```

MCMF

Opis: Znajduje przepływ o minimalnym koszcie.

Stosowanie: Trzeba w init znaleźć najkrótsze ścieżki. Jeżeli są ujemne krawędzie trzeba puścić SPFA albo dynamika (jeżeli graf jest DAGiem).

Czas: $\mathcal{O}(Fm \log n)$

```
const int INF = 1e9;
struct MCMF {
    struct edge {
        int to, rev;
        int cap, cost;
    };
    struct ds {
        int u, val;
        friend bool operator<(const ds& lhs, const ds& rhs) {
            return lhs.val > rhs.val;
        }
    };
    int n;
    vector<vector<edge>> adj;
    vector<array<int, 3>> f;
    int c = 0;
    MCMF(int _n) {
        n = _n;
        adj.resize(n);
        f.resize(n);
    }
    void add_edge(int u, int v, int cap, int cost) {
        if (u == v) {
            assert(cost >= 0);
            return;
        }
        int i = adj[u].size();
        int j = adj[v].size();
        adj[u].push_back({v, j, cap, cost});
        adj[v].push_back({u, i, 0, -cost});
    }
    void reduce(const vector<int>& dst, int t) {
        for (int i = 0; i < n; i++) {
            for (edge& e : adj[i]) {
                if (dst[i] < INF && dst[e.to] < INF) {
                    e.cost += dst[i] - dst[e.to];
                }
            }
            c += dst[t];
        }
    }
    bool init(int s, int t) {
        vector<int> dst(n, INF);
        queue<int> q;
        vector<bool> inq(n);
        dst[s] = 0;
        q.push(s);
        inq[s] = true;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            inq[u] = false;
            for (edge& e : adj[u]) {
                if (e.cap > 0 && dst[u] + e.cost < dst[e.to]) {
                    dst[e.to] = dst[u] + e.cost;
                    if (!inq[e.to]) {
                        q.push(e.to);
                        inq[e.to] = true;
                    }
                }
            }
        }
    }
};
```

```
    }
}
if (dst[t] == INF) return false;
reduce(dst, t);
return true;
}
bool dijkstra(int s, int t) {
    vector<int> dst(n, INF);
    priority_queue<ds> q;
    dst[s] = 0;
    q.push({0, s});
    while (!q.empty()) {
        auto [u, d] = q.top();
        q.pop();
        if (d != dst[u]) continue;
        int i = 0;
        for (edge& e : adj[u]) {
            if (e.cap > 0) {
                int dd = d + e.cost;
                if (dd < dst[e.to]) {
                    dst[e.to] = dd;
                    f[e.to] = {u, i, e.rev};
                    q.push({e.to, dd});
                }
            }
            i++;
        }
    }
    if (dst[t] == INF) return false;
    reduce(dst, t);
    return true;
}
pair<int, int> build(int s, int t, int cap) {
    if (!init(s, t)) return {0, 0};
    int flow = 0;
    int cost = 0;
    while (flow < cap && dijkstra(s, t)) {
        int add = cap - flow;
        for (int i = t; i != s; i = f[i][0]) {
            add = min(add, adj[f[i][0]][f[i][1]].cap);
        }
        flow += add;
        cost += c * add;
        for (int i = t; i != s; i = f[i][0]) {
            adj[f[i][0]][f[i][1]].cap -= add;
            adj[i][f[i][2]].cap += add;
        }
    }
    return {flow, cost};
}
};
```

3.2 Grafy skierowane

SCC
Opis: Znajduje silne spójne składowe w kolejności topologicznej.
Czas: $\mathcal{O}(n + m)$

```
struct SCC {
    int n, cnt = 0;
    vector<vector<int>> adj;
    vector<int> p, low, in;
    stack<int> st;
    int tour = 0;
    SCC(int _n) {
        n = _n;
        adj.resize(n);
    }
};
```

```
p.resize(n, -1);
low.resize(n);
in.resize(n, -1);
}
void add_edge(int u, int v) { adj[u].push_back(v); }
void dfs(int u) {
    low[u] = in[u] = tour++;
    st.push(u);
    for (int v : adj[u]) {
        if (in[v] == -1) {
            dfs(v);
            low[u] = min(low[u], low[v]);
        } else {
            low[u] = min(low[u], in[v]);
        }
    }
    if (low[u] == in[u]) {
        int v = -1;
        do {
            v = st.top();
            st.pop();
            in[v] = n;
            p[v] = cnt;
        } while (v != u);
        cnt++;
    }
}
void build() {
    for (int i = 0; i < n; ++i) {
        if (in[i] == -1) dfs(i);
    }
    for (int i = 0; i < n; i++) p[i] = cnt - 1 - p[i];
}
vector<vector<int>> groups() {
    vector<vector<int>> res(cnt);
    for (int i = 0; i < n; i++) res[p[i]].push_back(i);
    return res;
}
};
```

Matma (4)

4.1 Wielomiany

FFT
Opis: Mnoży dwa wielomiany o sumarycznej długości 2^{23} modulo 998244353.
Czas: $\mathcal{O}((n + m) \log(n + m))$

```
struct FFT {
    int N;
    vector<int> rev;
    vector<mint> w;
    FFT(int k) {
        N = 1 << k;
        rev.resize(N);
        for (int i = 1; i < N; i++) {
            rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (k - 1));
        }
    }
    #warning MOD = 998244353
    mint W = mint(3).pow(119 * ((1 << 23) / N));
    w.resize(N);
    mint ww = 1;
    for (int i = 0; i < N / 2; i++) {
        w[i + N / 2] = ww;
        ww *= W;
    }
};
```

```
    }
    for (int i = N / 2 - 1; i > 0; i--) w[i] = w[2 * i];
}
void fft(vector<mint>& a) {
    int n = ssize(a);
    int s = __lg(N / n);
    for (int i = 0; i < n; i++) {
        int r = rev[i] >> s;
        if (i < r) swap(a[i], a[r]);
    }
    for (int i = 1; i < n; i *= 2) {
        for (int j = 0; j < n; j += 2 * i) {
            for (int k = 0; k < i; k++) {
                mint z = w[i + k] * a[j + k + i];
                a[j + k + i] = a[j + k] - z;
                a[j + k] += z;
            }
        }
    }
}
vector<mint> conv(vector<mint> a, vector<mint> b) {
    int n = ssize(a);
    int m = ssize(b);
    int k = 1;
    while (k < n + m - 1) k *= 2;
    a.resize(k);
    b.resize(k);
    fft(a);
    fft(b);
    for (int i = 0; i < k; i++) a[i] *= b[i];
    fft(a);
    reverse(a.begin() + 1, a.end());
    a.resize(n + m - 1);
    mint inv = mint(k).inv();
    for (int i = 0; i < n + m - 1; i++) a[i] *= inv;
    return a;
}
};
```

4.2 Mnożniki Lagrange’a

Jeżeli optymalizujemy $f(x_1, \dots, x_n)$ przy ograniczeniach typu $g_k(x_1, \dots, x_n) = 0$ to x_1, \dots, x_n jest ekstremum lokalnym tylko jeżeli gradient $\nabla f(x_1, \dots, x_n)$ jest kombinacją liniową gradientów $\nabla g_k(x_1, \dots, x_n)$.

Teksty (5)

KMP
Opis: Znajduje funkcje prefiksową. Można z niej skonstruować automat w czasie $\mathcal{O}(nA)$.
Czas: $\mathcal{O}(n)$

```
vector<int> kmp(const string& s) {
    int n = ssize(s);
    vector<int> p(n);
    for (int i = 1; i < n; i++) {
        int j = p[i - 1];
        while (j > 0 && s[i] != s[j]) j = p[j - 1];
        if (s[i] == s[j]) j++;
        p[i] = j;
    }
    return p;
}
```

Manacher

Opis: Znajduje najdłuższy palindrom o każdym środku.
Stosowanie: $p[2 * i]$ - środek w i , $p[2 * i + 1]$ - środek między i a $i+1$.
Czas: $\mathcal{O}(n)$

```
vector<int> manacher(const string& s) {
    int n = ssize(s);
    string t(2 * n, '.');
    for (int i = 0; i < n; i++) {
        t[2 * i] = s[i];
        t[2 * i + 1] = '#';
    }
    vector<int> p(2 * n - 1);
    for (int i = 0, l = -1, r = -1; i < 2 * n - 1; i++) {
        if (i <= r) p[i] = min(r - i + 1, p[l + r - i]);
        while (p[i] < min(i + 1, 2 * n - 1 - i) &&
            t[i - p[i]] == t[i + p[i]]) {
            p[i]++;
        }
        if (i + p[i] - 1 > r) {
            l = i - p[i] + 1;
            r = i + p[i] - 1;
        }
    }
    for (int i = 0; i < 2 * n - 1; i++) {
        if (t[i - p[i] + 1] == '#') p[i]--;
    }
    return p;
}
```

Tablica sufiksowa

Opis: Sortuje leksykograficznie wszystkie sufiksy słowa. Tablic rnk można użyć do porównywania leksykograficznie podśłów.
Czas: $\mathcal{O}(n \log n)$

```
vector<int> suffix_array(const string& s) {
    int n = ssize(s);
    vector<int> p(n), cnt(26);
    for (int i = 0; i < n; i++) cnt[s[i] - 'a']++;
    for (int i = 1; i < 26; i++) cnt[i] += cnt[i - 1];
    for (int i = 0; i < n; i++) p[--cnt[s[i] - 'a']] = i;
    vector<int> rnk(n);
    for (int i = 1; i < n; i++) {
        rnk[p[i]] = s[p[i]] == s[p[i - 1]] ? rnk[p[i - 1]] : i;
    }
    cnt.resize(n);
    vector<int> np(n), nrnk(n);
    for (int len = 1; len < n; len *= 2) {
        iota(cnt.begin(), cnt.end(), 0);
        for (int i = n - len; i < n; i++) {
            np[cnt[rnk[i]]++] = i;
        }
        for (int i = 0; i < n; i++) {
            if (p[i] - len >= 0) {
                np[cnt[rnk[p[i] - len]]++] = p[i] - len;
            }
        }
        nrnk[np[0]] = 0;
        for (int i = 1; i < n; i++) {
            int a = np[i - 1];
            int b = np[i];
            if (max(a, b) + len < n && rnk[a] == rnk[b] &&
                rnk[a + len] == rnk[b + len]) {
                nrnk[b] = nrnk[a];
            } else {
                nrnk[b] = i;
            }
        }
    }
    swap(p, np);
}
```

```
        swap(rnk, nrnk);
    }
    return p;
};
vector<int> build_lcp(const string& s, const vector<int>& sa) {
    int n = ssize(s);
    vector<int> pos(n);
    for (int i = 0; i < n; i++) pos[sa[i]] = i;
    vector<int> lcp(n - 1);
    int k = 0;
    for (int i = 0; i < n; i++) {
        if (pos[i] == 0) continue;
        while (i + k < n && s[i + k] == s[sa[pos[i] - 1] + k]) {
            k++;
        }
        lcp[pos[i] - 1] = k;
        k = max(0, k - 1);
    }
    return lcp;
}
```

Z

Opis: Znajduje funkcję Z.
Czas: $\mathcal{O}(n)$

```
vector<int> z(const string& s) {
    int n = ssize(s);
    vector<int> f(n);
    for (int i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r) f[i] = min(r - i + 1, f[i - 1]);
        while (f[i] < n - i && s[i + f[i]] == s[f[i]]) f[i]++;
        if (i + f[i] - 1 > r) {
            l = i;
            r = i + f[i] - 1;
        }
    }
    return f;
}
```