Neural Modeling of Flow Rendering Effectiveness

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It has been previously proposed that understanding the mechanisms of contour perception can provide a theory for why some flow rendering methods allow for better judgments of advection pathways than others. In the present paper we develop this theory through a numerical model of the primary visual cortex of the brain (Visual Area 1) where contour enhancement is understood to occur according to most neurological theories. We apply a two-stage model of contour perception to various visual representations of flow fields evaluated using the advection task of Laidlaw et al. In the first stage, contour *enhancement* is modeled based on Li's cortical model. In the second stage, a model of streamline *tracing* is proposed, designed to support the advection task. We examine the predictive power of the model by comparing its performance to that of human subjects on the advection task with four different visualizations. The results show the same overall pattern for humans and the model. In both cases, the best performance was obtained with an aligned streamline based method, which tied with a LIC-based method. Using a regular or jittered grid of arrows produced worse results. The model yields insights into the relative strengths of different flow visualization methods for the task of visualizing advection pathways.

Categories and Subject Descriptors: H.1.2 [**User/Machine SystemsHuman Factors**]: Human Information Processing—*Models and Principles*

General Terms: Theory

Additional Key Words and Phrases: Contour perception, flow visualization, perceptual theory, visual cortex, visualization

1. INTRODUCTION

Many techniques for 2D flow visualization have been developed and applied. These include grids of little arrows, still the most common for many applications, equally spaced streamlines [Turk and Banks 1996] [Jobard and Lefer 1997], and line integral convolution (LIC) [Cabral and Leedom 1993]. But which is best and why? Laidlaw et al [Laidlaw et al. 2001] showed that the "which is best" question can be answered by means of user studies in which participants are asked to carry out tasks such as tracing advection pathways or finding critical points in the flow field. (Note: an advection pathway is the same as a streamline in a steady flow field.) [Ware 2008] proposed that the "why" question may be answered through the application of recent theories of the way contours in the environment are processed in the visual cortex of the brain. But Ware only provided a descriptive sketch with minimal

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detail and no formal expression. In the present paper, we show, through a numerical model of neural processing in the cortex, how the theory predicts which methods will be best for an advection path tracing task.

Our basic rational is as follows: Tracing an advection pathway for a particle dropped in a flow field is a perceptual task that can be carried out with the aid of a visual representation of the flow. The task requires that an individual attempts to trace a continuous contour from some designated starting point in the flow until some terminating condition is realized. This terminating condition might be the edge of the flow field or the crossing of some designated boundary. If we can produce a neurologically plausible model of contour perception then this may be the basis of a rigorous theory of flow visualization efficiency.

The mechanisms of contour perception have been studied by psychologists for at least 80 years, starting with the Gestalt psychologists. A major breakthrough occurred with the work of Hubel and Wiesel [Hubel and Wiesel 1962] [Hubel and Wiesel 1968] since which time, neurological theories of contour perception have begun to develop. In the present paper, we show that a model of neural processing in the visual cortex can be used to predict which flow representation methods will be better. Our model has two stages. The first is a contour enhancement model. Contour enhancement is achieved through lateral connections between nearby local edge detectors. This produces a neural map in which continuous contours have an enhanced representation. The model or cortical processing we chose to apply is adapted from Li [Li 1998]. The second stage is a contour integration model. This represents a higher level cognitive process whereby a pathway is traced.

We apply the model to a set of 2D flow visualization methods that were previously studied by Laidlaw et al. [Laidlaw et al. 2001]. This allows us to carry out a qualitative comparison between the model and how humans actually performed. We evaluated the model against human performance in an experiment in which humans and the model performed the same task.

Our paper is organized as follows. First we summarize what is known about the cortical processing of contours and introduce Li's model of the cortex. Next we show how a slightly modified version of Li's model differentially enhances various flow rendering methods. Following this we develop a perceptual model of advection tracing and show how it predicts different outcomes for an advection path tracing task based on Laidlaw et als prior work. Finally we discuss how this work relates to other work that has applied perceptual modeling to data visualization and suggest other uses of the general method.

2. CORTICAL PROCESSING OF CONTOURS

Visual information passes along the optic nerve from the retina of the eye where it is relayed, via a set of synaptic junctions in the midbrain lateral geniculate nucleus, to the primary visual cortex at the back or the brain (Visual Area 1 or V1). It has been known since the Hubel and Wiesels work in the 60s that the visual cortex contains billions of neurons that are sensitive to oriented edges and contours in the light falling on the retina. Such neurons have localized receptive fields each responding to the orientation information contained within the light imaged in a small patch of retina. A widely used mathematical model of a V1 neuron's receptive

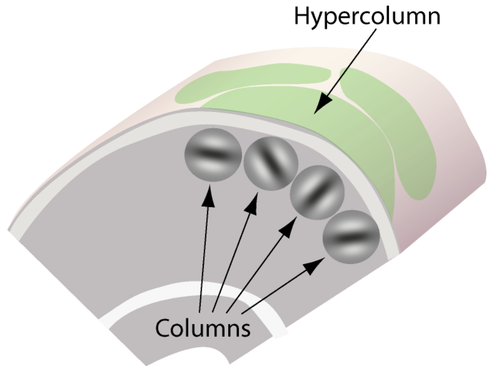

Fig. 1. Neurons are arranged in V1 in a column architecture. Neurons in a particular column respond preferentially to the same edge orientation. Moving across the cortex (by a minute amount) yields columns responding to edges having different orientations. A hypercolumn is a section of cortex that represents a complete set of orientations for a particular location in space.

field is the Gabor function [Daugman 1985]:

$$Gabor(u, v, \lambda, \theta, \phi, \sigma, \gamma) = e^{-\frac{u'^2+\gamma^2 v'^2}{2\sigma^2}} cos(2\pi \frac{u'}{\lambda} + \phi) \quad (1)$$

Hubel and Wiesel found that neurons responding to similar orientations were clustered together in a structure they called a column which extended from the surface of the visual cortex to the white matter. (See Figure 1) Later, they and other researchers discovered hypercolumn structures consisting of thousands of neurons all responding to the same area of visual space and selecting for a range of orientations. Overall, V1 contains a topographic map of the visual field having the property that every part of the retinal image is processed in parallel for all orientations. These orientation selective neurons have provided the basis for all subsequent theories of contour and edge detection.

There remains the problem of how the output of orientation sensitive neurons, each responding to different parts of a visual contour, becomes combined to represent the whole contour. Part of the solution appears to be a contour enhancement mechanism. Field, Hayes, and Hess [Field et al. 1993] examined the human's ability to perceive a contour composed of discrete oriented elements. They placed a contour composed of separated Gabor patches, among a field of randomly orientated Gabor patches. Contours were detected when the patches were smoothly aligned. They were not detected when there was misalignment. This work suggests that there is some manner of lateral coupling among the visual elements involved in perceiving the Gabor patches in the contour. They and other researchers have suggested that similarly oriented aligned contours mutually excite one another, whereas they inhibit other neurons that are nearby. (Figure 2).

3. LI'S V1 MODEL

Based on the observed organization of the neurons in the visual cortex by Hubel and Wiesel [Hubel and Wiesel 1962] [Hubel and Wiesel 1968], and the experimental

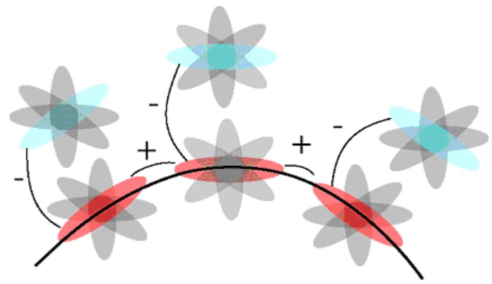

Fig. 2. Neurons whose receptive fields are aligned along a continuous contour mutually reinforce each other. They inhibit nearby neurons with a similar orientation sensitivity.

evidence by Field, Hayes, and Hess [Field et al. 1993], Zhaoping Li constructed a simplified model of the behavior of V1 neurons and examined the model's ability to integrate contours across multiple V1 neurons. The model is introduced briefly here, and described in more detail in [Li 1998]. In Li's model the cortex is approximated by a set of hypercolumns arranged in a hexagonal grid. Each hexagonal cell has 12 orientation selective neuron pairs oriented in 15-degree increments. One of the main simplifications embodied in Li's model is that it fails to incorporate the way the mammalian visual systems scales with respect to the fovea. Real neural architectures have much smaller receptive fields near the fovea at the center of vision than at the edges of the visual field. The neurons in each hex cell were grouped into excitatory and inhibitory pairs responding to an edge of a particular orientation at that location. Thus there were a total of 24 neurons per cell. The firing rates of both the inhibitory and excitatory neurons were modeled with real values. The neuron pairs affected neighboring neuron pairs via a transfer function that depended on the alignment of the edge selectivity orientations. Neuron pairs that were aligned with one another exhibited an excitatory effect on each other, whereas pairs that were not aligned inhibited each other. Finally, Li's model also contains feedback pathways for higher-level visual areas to influence individual neurons.

In our implementation, the mapping of the hexagonal grid to the image space was such that the hex centers were separated by 10 pixels. For the V1 neuron response, we used the Gabor function (equation 1) with a wavelength, λ, of 21 pixels, a σ of 7 pixels, and an aspect ratio, γ, of 1.

4. STREAMLINE TRACING ALGORITHM

Laidlaw et. al. compared the effectiveness of visualization techniques by presenting test subjects with the task of estimating where a particle placed in the center of a flow field would exit a circle. Six different flow field visualization methods were assessed by comparing the difference between the actual exit numerically calculated and the estimation of the exit by the human subjects. Laidlaws experiment was carried out on humans, but in our work we apply this evaluation technique to humans as well as to our model of the human visual system and use a streamline tracing algorithm to trace the path of the particle.

We use the term streamline tracing to describe the higher level process that must exist for people to judge a streamline pathway. We call it streamline tracing because the task seems to require the user to make a series of judgments, starting at the center, whereby the path of a particle dropped in the center is integrated in a stepwise pattern to the edge of the field. Though many algorithms exist in the machine vision literature for contour tracing, we found these to be inappropriate for use in this application. Contour tracing algorithms are generally designed to trace out the boundary of some shape, but a streamline tracing algorithm must also be able able to produce a streamline in a field of disconnected contours, such as is the case with the regular arrows. The streamline to be traced will often not follow a visible contour, but instead be locate between contours, and will sometimes pass through areas devoid of visual elements. Thus we developed a specialized algorithm that is capable of tracing streamlines that do not necessarily correspond to the boundary of any shape, but can pass between visual contours.

Perception is a combination of top-down and bottom up processes. Bottom up processes are driven by information on the retina and are what is simulated by Lis model. Top down processes are much more varied and are driven in the brain by activation from regions in the frontal and temporal cortex that are known to be involved in the control of pattern identification and attention [Lund 2001]. All of the flow visualizations evaluated by Laidlaw et al, except for LIC, contain symbolic information regarding the direction of flow along the contour elements (e.g. an arrowhead). In a perpetual/cognitive process this would be regarded as a top-down influence. At present our model does not deal with symbolic direction information but it does do streamline tracing once set in the right general direction.

Streamline tracing is a combination of top down and bottom up processes. Broadly speaking, top down processes reflect task demands and the bottom up processes reflect environmental information. In our case, the bottom up information comes from the different types of visualization while the top down information is an attempt to model the cognitive process of streamline pathway tracing. Contour integration was modeled using the following iterative algorithm:

$current\_position \leftarrow$ center
$current\_direction \leftarrow$ up
$current\_position$ is inside circle
while $current\_position$ is inside circle, do:
    $neighborhood \leftarrow$ all grid hexes within two hexes from $current\_position$
    for each hex in $neighborhood$, do:
        for each $neuron$ in hex
        convert $neuron\_orientation$ to $vector$
        scale $vector$ by $neuron\_excitation$
        $vector\_sum \leftarrow vector\_sum + vector$
    normalize $vector\_sum$
    $current\_position \leftarrow current\_position + vector\_sum$
    $current\_direction \leftarrow vector\_sum$
return $current\_position$

The algorithm maintains a context that contains a current position and direction.

Initially, the position is the center, and the direction set to upward. This context models the higherorder, top-down influence on the algorithm that results from the task requirements (tracing from the center dot) and the directionality which in our experiment was set to be always in an upwardly trending direction.

The algorithm traces the contour by repeatedly estimating the flow direction at the *current_position* and moving the position a small distance (.5 hex radii) in that direction. The flow direction is calculated from the neural responses in the local neighborhood of the *current_position*. The excitation of each neuron is used to generate a vector whose length is proportional to the strength of the response and whose orientation is given by the receptive field orientation. Because receptive field orientations are ambiguous as to direction (for any vector aligned with the receptive field, its negative is similarly aligned). The algorithm chose the vector most closely corresponding to the vector computed on the previous iteration. Vectors are computed for all neurons in hypercolumns within a 2 hexes radius of the current position; they are summed and normalized to generate the next *current_direction*.

Some changes were made from the method published by Pineo and Ware [Pineo and Ware 2008]. Previously, the algorithm considered only a single hex cell at each iteration of the algorithm. We found that this would occasionally cause unrealistically large errors in streamline tracing. For example, on visualizations with arrowheads, the neural network might yield a very strong edge orthogonal to the flow field positioned at the back of an arrowhead. If the algorithm considered only the edges at this point, it may make a significant error, despite the edges in nearby positions indicating the correct direction. We felt that creating an average over *neighborhood* was the more correct approach, and we found closer agreement with human performance with this change.

4.1 Qualitative Evaluation

Four different flow visualization methods were used in our evaluation of the theory. These were implementations of four of the six used by Laidlaw et al. [Laidlaw et al. 2001]. We chose to investigate a regular arrow grid because it is still the most commonly used in practice and a jittered arrow grid because of the arguments that have been made that this should improve perceptual aliasing problems [Turk and Banks 1996]. We added Line Integral Convolution (LIC) because of its widespread advocation by the visualization community [Cabral and Leedom 1993] and head-to-tail aligned streaklets because of Laidlaw et. al.s finding that is was the best and the theoretical arguments in support of this method [Ware 2008]. Note that Laidlaw used Turk and Banks algorithm to achieve aligned arrows on equally spaced streamlines whereas we used Jobard and Lefers [Jobard and Lefer 1997] method to achieve the same effect and we used streaklets without an arrowhead [Fowler and Ware 1989].

V1 is known to have detectors at different scales. However, to make the problem computationally tractable we chose only a single scale for the V1 and designed the data visualizations with elements scaled such that they were effectively detected by the gabor filter used by the model. The widths of the arrows and streaklets were chosen to be smaller than the central excitatory band of the gabor filter. This allowed the edge to be detected even if not precisely centered on the receptive field of

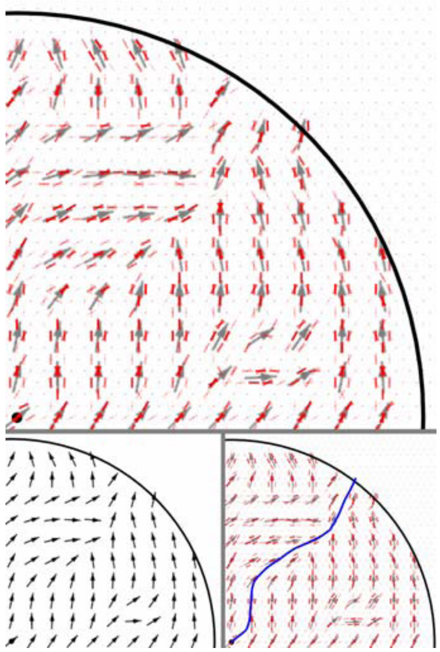

Fig. 3. Regular Arrows

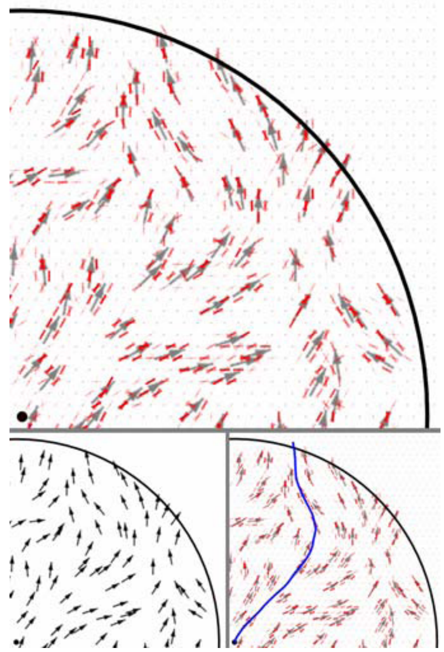

Fig. 4. Jittered Arrows

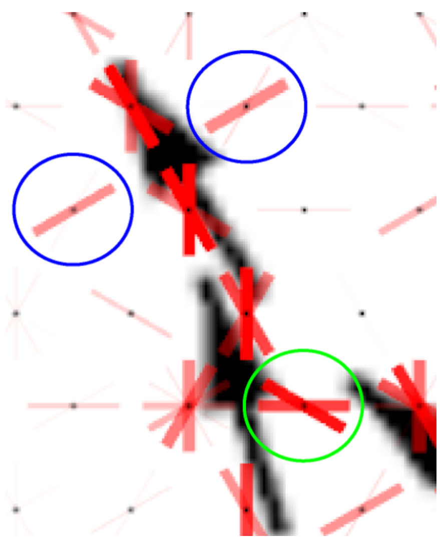

Fig. 5. Closeup of neural response to arrowheads

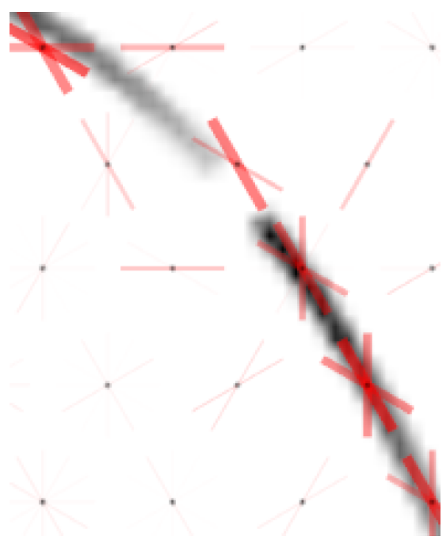

Fig. 6. Closeup of neural response to aligned streaklets

the neuron. The spatial frequency of the LIC visualization is defined by the texture over which the vector field is convoluted. Our texture was created by generating a texture of random white noise of one-third the necessary size and scaling it up

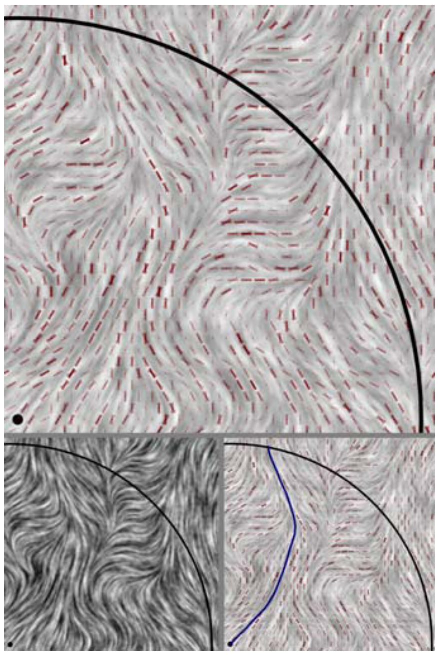

Fig. 7. Line Integral Convolution

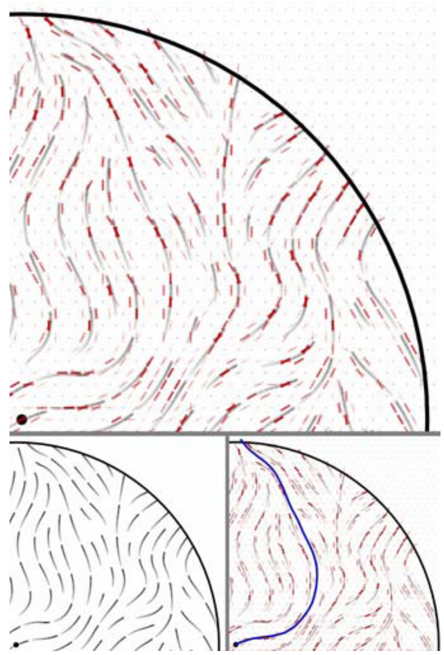

Fig. 8. Aligned Streaklets

via. interpolation. The resulting spacial frequency of the LIC visualization was of a scale that was effectively detected by the gabor filters of the model.

Figures 3, 4, 7, and 8 show samples of how the algorithm performed with the various visualization methods. For greater clarity we only show a section of each image although the application of the algorithm to the whole image was computed. In each example the original visualization is shown in the lower left. The top center panel shows the effect of the Li algorithm on the image following 10 feedback iterations, at which point the neural excitation values had stabilized. The small bars show how strongly each neuron responds, with redder meaning stronger. The lower right panel shows the path traced out by the contour integration algorithm.

4.1.1 *Regular Arrows (Figure 3).* This visualization is produced by placing arrow glyphs at regular spacings. The magnitude of the vector field is indicated by the arrow length, and the flow direction by the arrow head. The grid underlying the regular arrows is apparent to humans, but the edge weights of the model show no obvious signs of being negatively affected. In fact, the regularity ensures that the arrows are well spaced, preventing any false edge responses that might be produced by the interference of multiple arrows. We can expect that non-tangential edge responses will be produced by the arrowheads and these will lead to errors in the streamline advection task.

4.1.2 *Jittered Arrows (Figure 4).* This visualization is similar to the regular arrows, but the arrows are moved a small random distance from the regular locations. While composed of the same basic elements as the regular grid, we see instances where nearby arrows interfere with each other and produce edge responses non-tangential to the flow direction. Also, as with gridded arrows, the arrowheads will excite neurons with orientation selectivity non-tangential to the flow. This can be seen in figure 5. In this figure, we can see orthogonal neural excitation to each side of the upper arrow, caused by the back edge of the arrowhead (blue circles). We can also see excitation caused by the interference of two arrows at the bottom right (green circle). These non-tangential responses are much stronger than those found in the aligned streaklets visualization (figure 6)

4.1.3 *LIC (Figure 7).* This is formed by integrating a texture of random noise along the flow direction. The neurons of the model are not strongly excited by the LIC visualization. Elongated patches of black or white produce the strongest responses, but these are still weak compared to other visualizations. However, we note the lack of responses that are not tangential to the flow direction. A major shortcoming of LIC is that it is completely ambiguous as to flow direction which could be in either of two directions at any point. In Laidlaw et als experiment this lead to very poor performance for LIC. However, because our algorithm did not take symbolic direction into account for any of the visualization methods (which we remedied with the upward bias), LIC could be expected to perform better in our experiment than it would in real-world practice

4.1.4 *Aligned Streaklets (Figure 8).* In this visualization, streaklets are aligned such that the head of one points to the tail of the next. This visualization produced strong neural responses within the model. As Ware argued, theory predicts that head to tail placement of arrows should produce good results. Perceptual theory suggests that evenly spaced streamlines should provide the best stimulus for coherent chains of excited neurons to develop.

5. EVALUATION

By comparing the performance of humans to the model on the streamline advection task, we can attempt to understand in what cases the model adequately describes contour enhancement and extraction, and in what cases other perceptual mechanics may need to be incorporated. In order to compare how well the cortical model predicted human performance, we conducted an experiment where human subjects and the model completed the same task with the same set of flow representations. We chose Laidlaws streamline advection task.

5.0.5 *Participants.* Six human subjects were used in this experiment. They were a mix of volunteers and paid undergraduate students. There were four males and two females.

5.0.6 *Vector Field Generation.* Artificial flow fields were generated by interpolating a regular 8x8 grid of vectors. Each vector is a pseudo-randomly generated, normalized vector with a positive y component. This produced a vector field with an upward trend. This trend is important for ensuring that all streamline paths eventually exit the circle. To vary the overall trending direction of the vector field, a pseudorandom angle was produced between 45 degrees, and all vectors were rotated by this value. The resulting vector fields thus trended in the upper 90 degree quadrant. The motivation for this was the fact that the LIC method produces a visualization that is ambiguous; two possible flow directions may be inferred from any standard LIC image. In the experiment, we asked the subject to find pathways that trended upward. Likewise, the algorithm was also set to look for upwardly trending solutions.

5.0.7 *Instructions.* Subjects were asked to click where they felt that a particle deposited in the middle of the circle would exit the circle. Due to the directional ambiguity of the LIC visualization, the subjects were informed that the general trend of the flow fields would always be upward.

5.0.8 *Procedure.* The subjects were presented with each flow field visualization on a 15.1 inch, 133dpi LCD screen. The diameter of the circle was 4.5 inches and the viewing distance was approximately 57 cm. We evaluated four different visualization methods: regular arrows, jittered arrows, LIC, and aligned arrows. The test subjects were given as long as they needed to select with a mouse the point where they estimated that the particle in the center would exit the circle. Each test subject was first allowed to practice the task to minimize learning effects. Following this they participated in 5 experiment blocks. Each block was constructed by generating 10 random flow fields and rendering them using the four visualization methods. The resulting 40 visualizations were then presented to the test subject in a random order. Following each block, the subjects was allowed to take a short break to minimize fatigue effects. Each test subject performed 200 tests, for a total of 1200 tests throughout the experiment.

5.0.9 *Computer Trials.* The computer carried out exactly the same set of trials with exactly the same stimulus pattern generation algorithm. Raster images generated by the same four visualization methods were used as inputs.

6. RESULTS

Figures 10 and 11 illustrate the distribution of errors for the human participants and computer model, respecitively. In both cases most of the errors are less than three degrees. Because the error data were highly skewed we applied a log transform to the raw data before further analysis. This transform produced a roughly normal distribution (Figure 9), allowing the use of analysis of variance techniques. The geometric means for aggregated human performance and model performance are summarized in Figure 12. We conducted a two-way analysis of variance (ANOVA) with the two factors being aggregate human data and model output. This revealed that the model was significantly more accurate than the human participants [$F(1, 2966) = 128.22,\ p < 0.001$]. The mean error of the model was .92 degrees, outperforming the human average of 1.5 degrees. There was no interaction between the visualization type and whether the subject was the human or model. There was also a highly significant main effect for the type of visualization [$F(3, 2966) = 23.43,\ p < 0.001$]. A Tukey HSD test indicated that the difference between LIC and aligned streaklets was not significant, but that there were significant differences between all other visualizations. A linear fit of the error over the block number yielded that the block number was not significant on the human trials [$F(1, 1125) = 2.29,\ p = .13$], indicating that fatigue and learning effects were

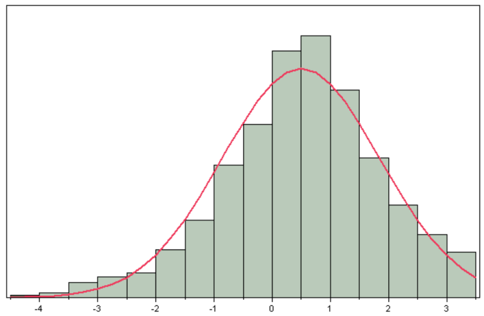

Fig. 9. Histogram of the log transformed data with a normal distribution overlay

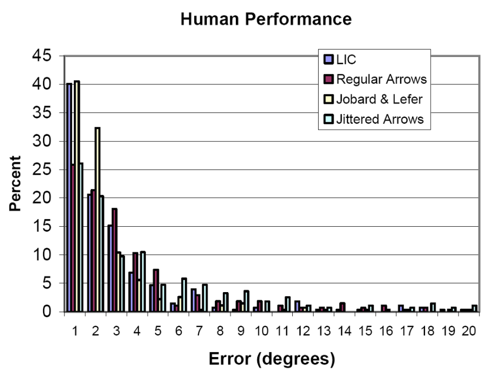

Fig. 10. Histogram showing distribution of errors for humans

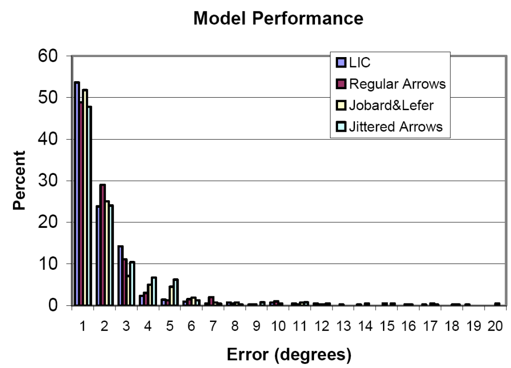

Fig. 11. Histogram showing distribution of errors for the computer model

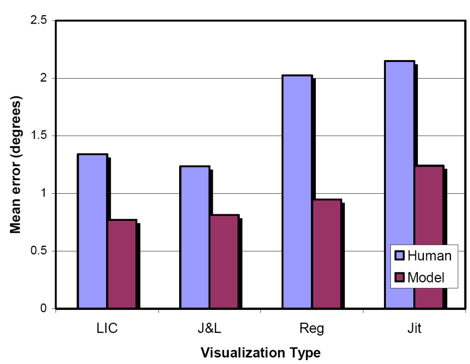

Fig. 12. Mean errors for the four visualization methods for human participants and the model

not significant.

7. DISCUSSION

The overall agreement between the pattern of results for human observers and the V1-based model provides strong support of the perceptual theory we outlined in the introduction. The aligned arrows style of visualization produced clear chains of mutually reinforcing neurons along the flow path in the representation, making the flow pathway easy to trace as predicted by theory.

The fact that LIC produced results as good as the equally spaced streamlines was something of a surprise, and this lends support to its popularity within the visualization community. While it did not produce as much neuron excitation as the aligned arrows method, this was offset by the lack of non-tangential edge responses produced by glyph-based visualizations. However, its good performance was achieved only because our evaluation method ignored the directional ambiguity inherent in this method. Laidlaw et al [Laidlaw et al. 2001] found this method to be the worst and there is little doubt that had we allowed flow in any direction, up or down, human observers would have found pathways with close to 180 degrees of error half of the time.

The performance of both the model and the human test subjects is likely to be highly dependent on the underlying vector field used. As described in section 5.0.6, the vector field was generated by interpolating between an 8x8 grid of random, but generally upward pointing vectors. A consequence of this is that when adjacent vectors in this grid point somewhat toward each other, the vector field forms an area of convergence. This convergence area tends to funnel neighboring streamline paths together, reducing error in streamline tracing (Figure 3 is an example of this). Thus, the overall accuracies of both the model and human subjects may be higher than might be might be observed using a vector field without such convergence zones.

We were surprised that the computer algorithm actually did better at the task than human observers. One reason for this may have been that humans would have to make saccadic eye movements to trace a path, whereas the computer did not. For the patterns we used, it is likely that the observers had to make fixations on several successive parts of a path, and errors may have accumulated as they resumed a trace from a previous fixation. Nevertheless, we feel that the algorithm could easily be adjusted to make it give results closer to human subjects. A more sophisticated approach would be to simulate eye fixations.

The model we applied is a considerable simplification over what actually occurs. It only uses the simplest model of the simplest orientation sensitive neurons, and fails to include cortical magnification, among other shortcomings. Real cortical receptive fields are not arranged in a rigid hexagonal grid as they are in Lis model. Furthermore, the neurons of V1 respond to many frequencies, however our model only uses one in its present form. In addition, besides the so-called 'simple' cells modeled by Li, other neurons in V1 and V2 called complex and hypercomplex cells all have important functions. For example, end-stopped cell respond best to a contour that terminates in the receptive field and understanding these may be important in showing how the direction of flow along a contour can be unambiguously shown. Moreover, visual information is processed through several stages following the primary cortex, including V2, V4 and the IT cortex. Each of these appears to abstract more complex, less localized patterns. Researchers are far from having sufficient information to model the operations of these stages all of which may have a role in tracing contours. Nevertheless, the results are compelling and there are advantages in having a relatively simple model. We have plans to add some of these more complex functions in future versions of the model.

The problem of modeling perception at a higher more cognitive level is even more challenging. It is likely that people adopt different perceptual strategies depending on the visualization. For example, with a regular arrow grid people are likely to rely less on the contour information available in the display, which is weak, and more on strategies, such as looking ahead from a particular arrow, mentally interpolating the flow direction at the look-ahead point and repeating to the edge of the field. Conversely, In the case of aligned streaklets, a simpler contour following finding strategy will be more successful.

The success of even a quite simple artificial V1 model in predicting human performance suggests that our result could have practical as well as theoretical importance. We could, for example, use the model to provide a fitness function in a hill-climbing optimization process with the goal of automatically developing perceptually good visualizations. This is a possibility we are currently exploring.

The application of a neural model of contour perception has the potential to be applied to many visual applications where the perception of form and pattern is critical. We believe that the approach can be fruitfully applied to other application domains including map design and graph layout. In both of these instances it has frequently been observed that "Gestalt" theories of perception may yield insights to the problem (e.g. [Ware 2004]). However, this observation has never been developed into a rigorous mathematical model. The theory of contour integration we have outlined here could, with minor modifications, be used as a model of the Gestalt concept of continuity and thereby be applied to evaluate design alternatives for many common forms of charts and diagrams. A complete and accurate model of the perceptual process for even what seems to be a quite simple task is probably decades away. Nevertheless, we believe that even simple first order models of the kind that we report here can provide important insights into the reasons why certain methods are better than others. Despite the many deficiencies we have outlined having such a model is still better than having no model. By implementing algorithms loose arguments can be carefully examined and properly tested.

REFERENCES

Cabral, B. and Leedom, L. C. 1993. Imaging vector fields using line integral convolution. In *SIGGRAPH '93: Proceedings of the 20th annual conference on Computer graphics and interactive techniques*. ACM, New York, NY, USA, 263–270.

Daugman, J. G. 1985. Uncertainty relation for resolution in space, spatial frequency, and orientation optimized by two-dimensional visual cortical filters. *Journal of the Optical Society of America A: Optics, Image Science, and Vision 2,* 7, 1160–1169.

Field, D. J., Hayes, A., and Hess, R. F. 1993. Contour integration by the human visual system: Evidence for a local "association field". *Vision Research 33,* 2, 173 – 193.

Fowler, D. and Ware, C. 1989. Strokes for representing univariate vector field maps. In *Graphics Interface '89*. 249–253.

Hubel, D. H. and Wiesel, T. N. 1962. Receptive fields, binocular interaction and functional architecture in the cat's visual cortex. *J Physiol 160,* 1, 106–154.

Hubel, D. H. and Wiesel, T. N. 1968. Receptive fields and functional architecture of monkey striate cortex. *J Physiol 195,* 1, 215–243.

Jobard, B. and Lefer, W. 1997. Creating evenly-spaced streamlines of arbitrary density. In *Eurographics Workshop*. Springer Verlag, 43–56.

Laidlaw, D. H., Davidson, J. S., Miller, T. S., da Silva, M., Kirby, R. M., Warren, W. H., and Tarr, M. 2001. Quantitative comparative evaluation of 2d vector field visualization methods. In *VIS '01: Proceedings of the conference on Visualization '01*. IEEE Computer Society, Washington, DC, USA, 143–150.

Li, Z. 1998. A neural model of contour integration in the primary visual cortex. *Neural Comput. 10,* 4, 903–940.

Lund, N. 2001. *Attention and Pattern Recognition*. Routledge.

Pineo, D. and Ware, C. 2008. Neural modeling of flow rendering effectiveness. In *APGV '08: Proceedings of the 5th symposium on Applied perception in graphics and visualization*. ACM, New York, NY, USA, 171–178.

Turk, G. and Banks, D. 1996. Image-guided streamline placement. In *SIGGRAPH '96: Proceedings of the 23rd annual conference on Computer graphics and interactive techniques*. ACM, New York, NY, USA, 453–460.

Ware, C. 2004. *Information Visualization: Perception for Design, 2nd Ed.* Morgan Kaufman.

Ware, C. 2008. Toward a perceptual theory of flow visualization. *IEEE Comput. Graph. Appl. 28,* 2, 6–11.

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