Medical Needle Steering under Motion and Sensor Noises using Feedback-based Information Roadmaps

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Problem Elements

Medical needle steering under motion uncertainty and under noisy sensor measurements is an instance of the sequential decision making problem in partially-observable environments

- Motion model: Steerable needles can be modeled as non-holonomic systems with constraints on the curvature
- Controls: Two available controls on steerable needles are needle insertion speed and its twisting speed (angular velocity)
- Constraints: Usually there exist a set of sensitive regions and anatomical obstacles that the needle has to avoid, while it is approaching its target.
- Motion uncertainty: There are different sources such as tissue deformation, control signal noise, etc. that result in an uncertain and noisy motion.
- Sensing uncertainty: Usually sensory readings are not perfect, and they are also noisy, whose noise can vary depending on the needle position in tissue and the sensor model.

Objective

The main objective in our framework for needle steering is to compute a feedback law (no need for replanning) to control the needle under uncertainty such that the collision probability with obstacles is minimized, while the method is robust to deviations in needle position.

Feedback Motion Planning

- Solution to the feedback motion planning is:
  - ~~A sequence of states that connect start to goal~~
  - ~~A sequence of actions~~
  - A function that returns the best action for any given state

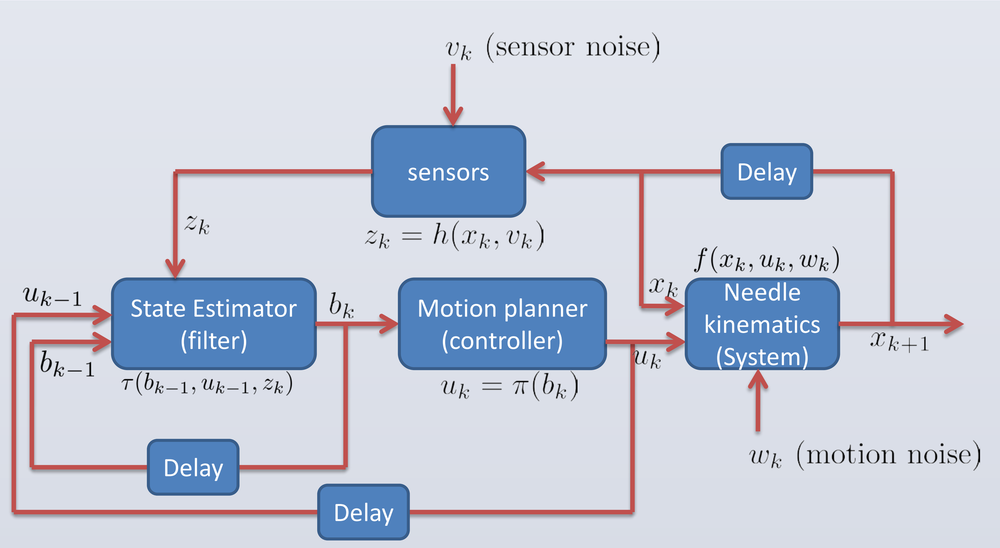
$$action = \pi(state)$$

$\pi(\cdot)$ is called "feedback" or "closed-loop policy"

- With noisy measurements, the only available information is the probability distribution over the state, which is called "belief" or "information-state": $b(x_k) = p(x_k|z_{0:k}, u_{0:k-1})$

In this case, the feedback is a mapping from the belief space into action space: $action = \pi(belief)$

Control with imperfect state information (POMDP)



$$J^{\pi}(b_0) = \mathbb{E}\Big[\sum_{k=0}^{\infty} c(b_k, u_k)\Big] = \mathbb{E}\Big[\sum_{k=0}^{\infty} c(b_k, \pi(b_k))\Big]$$

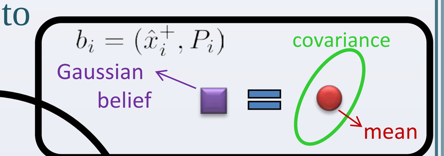
$$J(b_0) = \min_{\pi \in \Pi} J^{\pi}(b_0) \qquad \pi^* = \arg\min_{\pi\in\Pi} J^{\pi}(b_0)$$

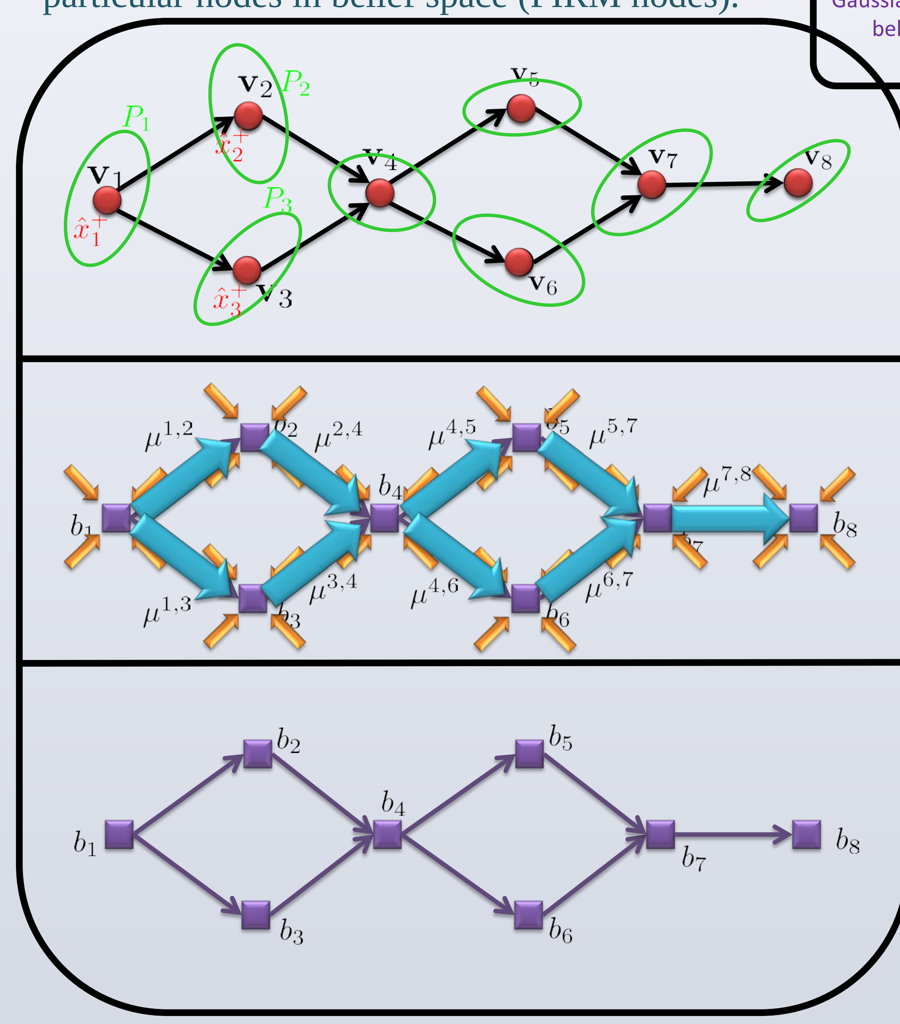
$$J(b) = \min_u \{c(b,u) + \int_{\mathbb{B}} p(b'|b,u) J(b') db'\}, \quad \forall b \in \mathbb{B}$$

$$u^* = \pi(b) = \arg\min_u \{c(b,u) + \int_{\mathbb{B}} p(b'|b,u) J(b') db'\}$$

FIRM: Feedback-based Information Roadmap

The main idea in FIRM is to generate a "graph" in the belief space. The key elements in this procedure are belief stabilizers, corresponding to every PRM node, that guarantee the reachability to particular nodes in belief space (FIRM nodes).





FIRM nodes (unique reachable belief nodes corresponding to each PRM node)

We use belief stabilizers to induce reachable nodes in the belief space.

Resulting structure in the belief space is a graph.

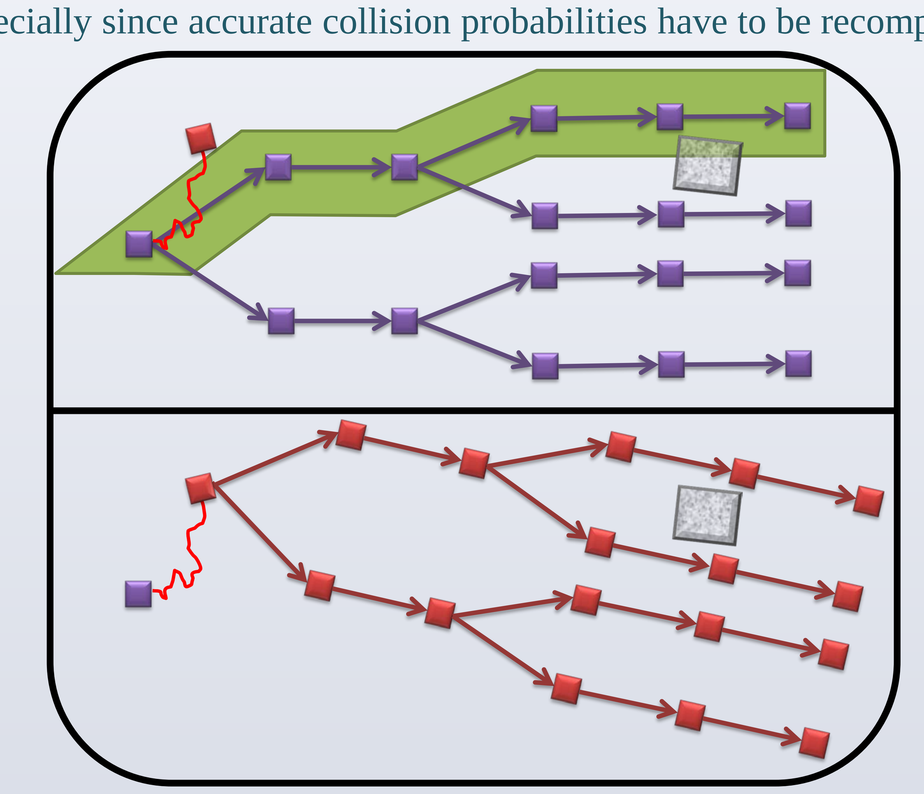
This construction breaks the curse of history and transforms the intractable POMDP to a tractable MDP on FIRM nodes:

$$J^g(B_i) = \min_{\mathbb{M}(i)} C^g(B_i, \mu^{ij}) + J^g(F)\mathbb{P}^g(F|B_i, \mu^{ij}) + \sum_{l=1}^{N} J^g(B_l)\mathbb{P}^g(B_l|B_i, \mu^{il}),$$

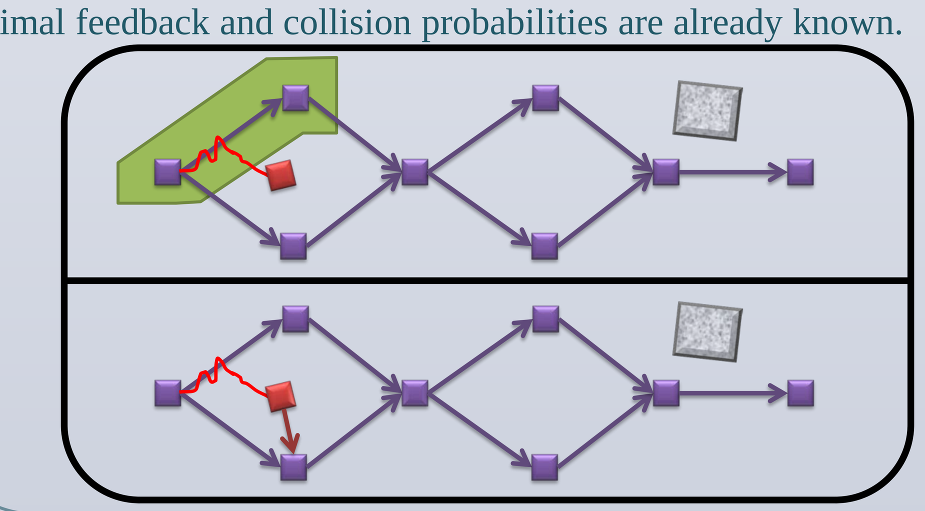
$$\pi^g(B_i) = \arg\min_{\mathbb{M}(i)} C^g(B_i, \mu^{ij}) + J^g(F)\mathbb{P}^g(F|B_i, \mu^{ij}) + \sum_{l=1}^{N} J^g(B_l)\mathbb{P}^g(B_l|B_i, \mu^{il}).$$

Robustness of FIRM plans

FIRM is robust to deviations from the path. No replanning is needed. In state-of-art methods, if a large deviation happens, replanning has to be performed from the new belief, which can be a very expensive procedure, especially since accurate collision probabilities have to be recomputed.



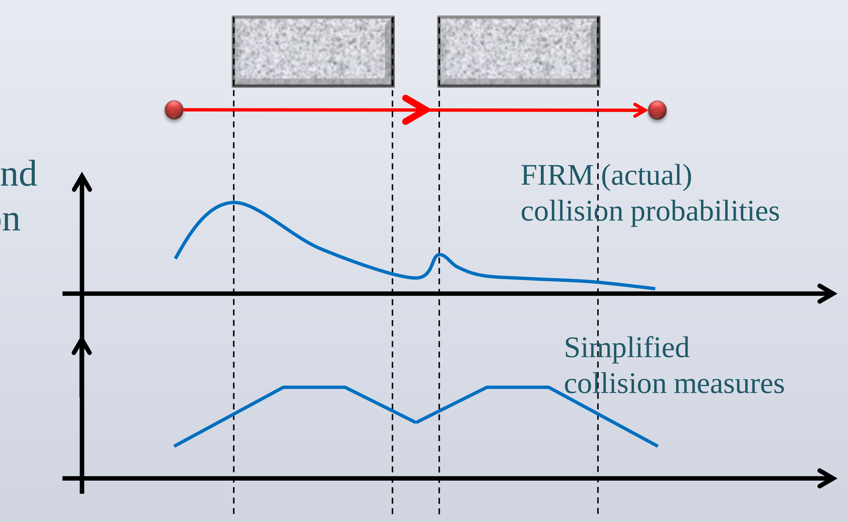
In FIRM-based methods, no replanning is needed, since in case of any deviation the belief can be driven to a FIRM node and from thereon optimal feedback and collision probabilities are already known.



Reliability of FIRM plans

FIRM generates more reliable plans as it is able to compute the true collision probabilities, rather than simplified collision measures.

Top curve in this figure shows the true collision probability and the second curve shows the collision probability with simplifying assumption that "the collisions in different steps are independent".



Concrete instantiations of the FIRM framework

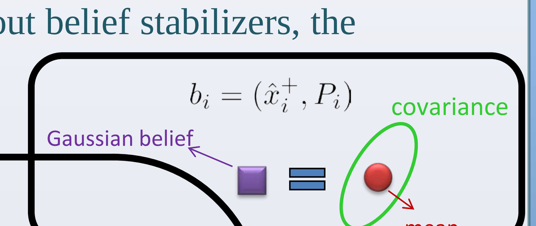
FIRM is an abstract framework and any choice of edge controllers and node controllers (belief stabilizers) can generate a concrete instantiation of the FIRM framework. Several examples of FIRM are:

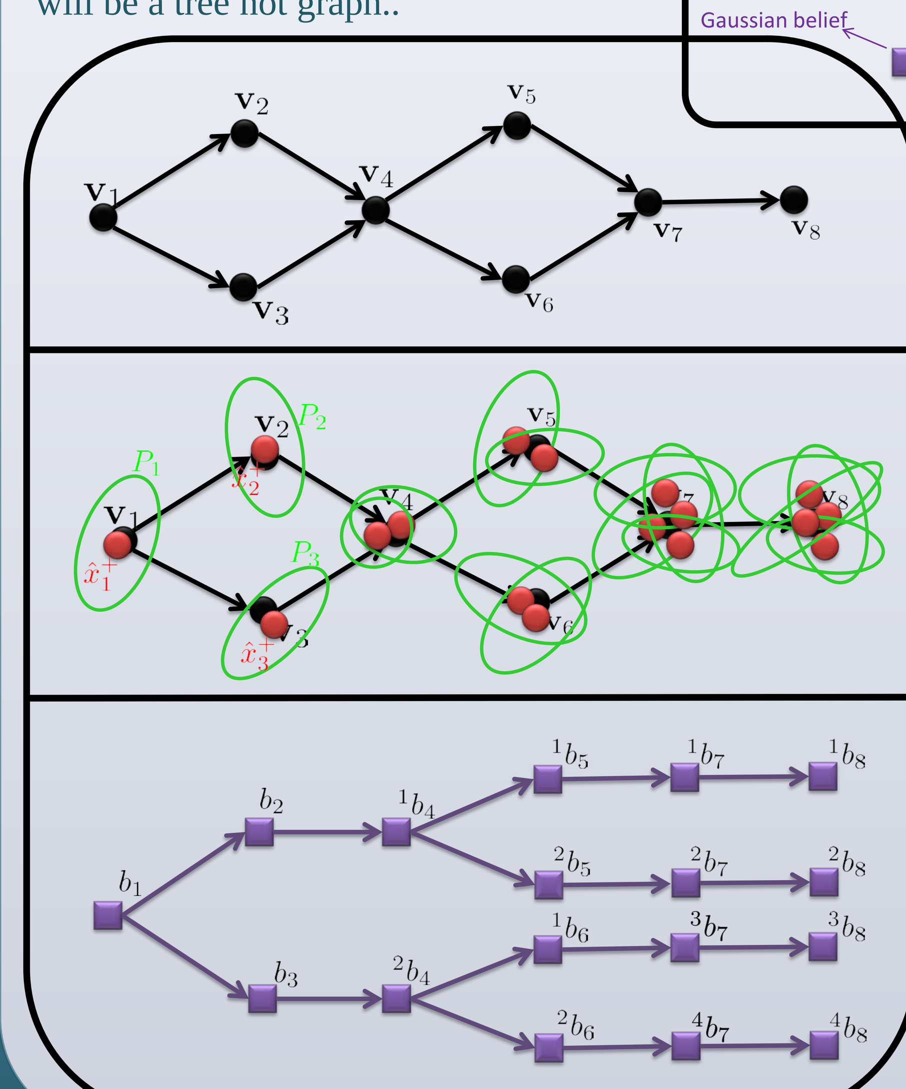
- SLQG-based FIRM [1], [2]
  - Node-controller = Stationary LQG controller
- DFL-based FIRM [3]
  - Node-controller = Dynamic Feedback linearization-based controller along with Kalman filter
- PLQG-based FIRM [4]
  - Node-controller = Periodic LQG controller

In all these cases, it is proved that the belief node reachability is achieved.

Challenges in motion planning under uncertainty (which are addressed by FIRM)

Direct transformation of the sampling-based methods to the belief space is a challenge. The main reason is that without belief stabilizers, the constructed structure in the belief space will be a tree not graph..





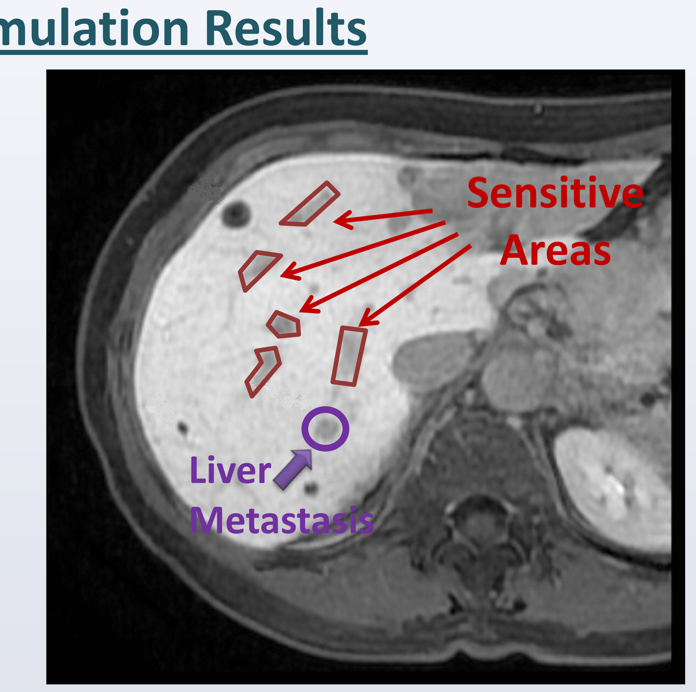
A simple PRM with eight nodes

Propagation of Gaussian uncertainty on the PRM graph, without belief stabilizers..

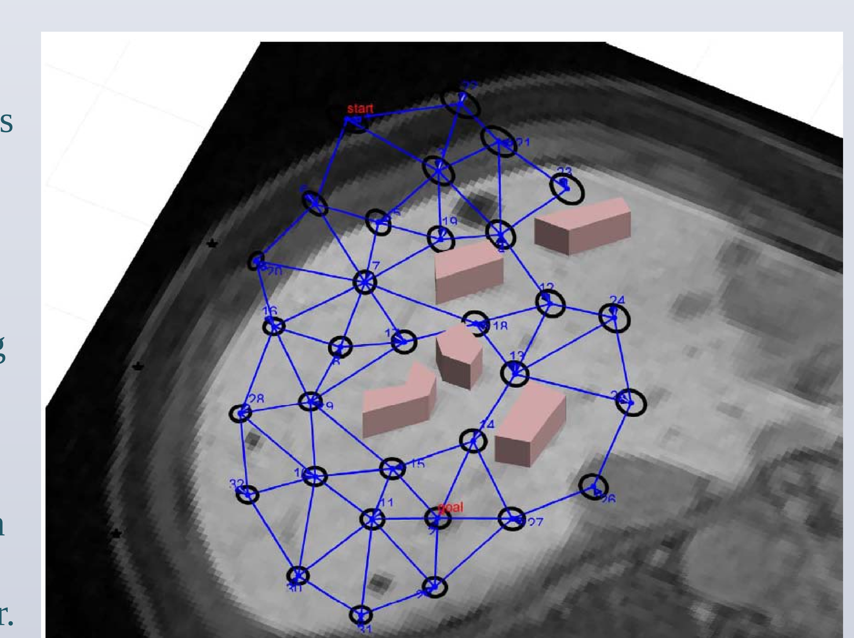
Resulting structure in the belief space is a tree, not a graph.

Simulation Results

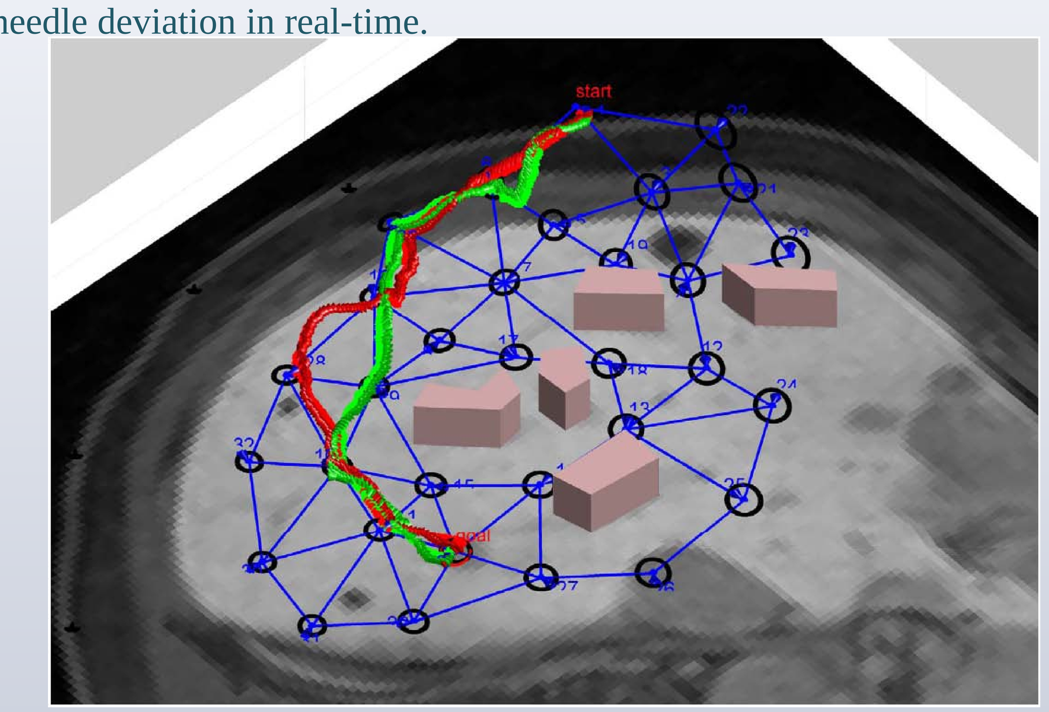
Gd-EOB-DTPA–enhanced 3D gradient-echo MRI of a 64-y-old patient with liver metastases due to pancreatic cancer. The shown lesion (purple oval) corresponds to metastasis as confirmed by intra-operative biopsy. The sensitive areas which have to be avoided by needle are also shown. [5]



The roadmap in the liver is drawn in this figure. It is well spread all over the liver so that it can compensate any noise in the steering procedure. To unclutter the image we show a small amount of nodes. In reality the roadmap will be much denser.



Two different outcomes of executing the optimal plan are shown. It emphasizes that we do not compute a path as the solution of planning, but we compute the feedback, which may result in different paths based on the needle deviation in real-time.



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[2] A. Agha-mohammadi, S. Chakravorty, N. M. Amato. "On the Probabilistic Completeness of the Sampling-based Feedback Motion Planners in Belief Space,"In Proc. IEEE Int. Conf. Robot. Autom. (ICRA12), 2012.

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[4] A. Agha-mohammadi, S. Chakravorty, N. M. Amato, "Periodic-Feedback Motion Planning in Belief Space for Nonholonomic and/or Nonstoppable Robots", Technical Report, TR12-003, Parasol lab., Dept. of CSE, Texas A&M University, Feb 2012.

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