University of Waterloo
Midterm Examination
Spring 2015

Student Name:	
Student ID Number:	
Course Section:	**MTE 140**

Instructor:	Igor Ivkovic
Date of Exam	June 18th 2015
Time Period	11:00am-1:00pm
Duration of Exam	2 hours
Pages (including cover)	10 pages
Exam Type	Closed Book

NOTE: No calculators or other electronic devices are allowed. Do not leave the exam room during the first 30 minutes and the last 15 minutes of the exam. Plan your time wisely to finish the exam on time.

Question 1: (10 marks)	Question 2: (12 marks)	Question 3: (8 marks)	Question 4: (10 marks)	Question 5: (15 marks)
Total: (55 marks)				

Useful Formulas

- For $S = a_1 + (a_1 + d) + ... + (a_n - d) + a_n$
  (S is a series that goes from a_1 to a_n in d-size increments),

$$S = n\left(\frac{a_1 + a_n}{2}\right) \tag{1}$$

- $$\sum_{i=k}^{n} 1 = (n - k + 1) \tag{2}$$

- $$\sum_{i=1}^{n} i = \left(\frac{n(n+1)}{2}\right) \tag{3}$$

- $$\sum_{i=1}^{n} i^2 = \left(\frac{n(n+1)(2n+1)}{6}\right) \tag{4}$$

- $$\sum_{i=0}^{n} r^i = \left(\frac{r^{n+1} - 1}{r - 1}\right) \tag{5}$$

- $$\log_b x = \frac{\log_c x}{\log_c b} \tag{6}$$

1 Question 1. Algorithm Complexity (10 marks)

a. (5 marks)

Order the following functions from smallest to largest based on their order of growth in terms of the Big-O notation:

1. $1 + 2 + 3 + 4 + 5 + \cdots + (n-1) + n$
2. $n \log(\log(\log(42n)))$
3. $1 + 2 + 4 + 8 + 16 + \cdots + 2^{n-1} + 2^n$
4. $n \log(4242n)$
5. $444n^4 + 333n^3 + 222n^2$

b. (5 marks)

Suppose that the runtime efficiency of an algorithm is defined as:
$T(n) = 4log_2(n) + 8log_2(n) + 12log_2(n) + ... + 4(n-1)log_2(n) + 4nlog_2(n)$.
Determine the algorithm's order of growth in terms of the Big-O notation, and prove that this is indeed the true order of growth using the Big-O formal definition. Show all steps in your proof.

2 Question 2. Algorithm Design (12 marks)

Design a linear-time $O(n)$ algorithm that iterates through a sorted sequential list L of size n. The list contains distinct positive integers in increasing order that represent x coordinates in a plane, such as the list $[1, 3, 11, 16, 21, 35]$ of size 6. The algorithm outputs the coordinate for which the maximum distance from any other coordinate in the list is minimal. In $[1, 3, 11, 16, 21, 35]$, the coordinate 16 is the closest to all other coordinates, and for 16 the maximum distance to any other coordinate is 19. The second-best coordinate is 21 for which the maximum distance to any other coordinate is 20. You may write your function in pseudocode (e.g., "for each element in L"), but you need to specify each step unambiguously (e.g., "return L[i];").

Once you have designed the algorithm, state its runtime efficiency in terms of the Big-O notation as a function of n, and briefly explain how you have derived this classification. For full marks, your algorithm needs to run in linear-time $O(n)$. For partial marks, your algorithm may grow faster than linear time $O(n)$.

3 Question 3. Pointers and Dynamic Memory (8 marks)

Examine the following code fragment. For each line marked with the [*Question∗*] tag, state what is the output, and briefly explain why that output occurs. You can assume that each line that crashes the program will be commented out or skipped, so all of the given lines will be attempted at least once.

```
    int* p = new int(15);
    int* q = new int(25);
    *q = 42;
    *p = *q;
    cout << *p << " and " << *q << endl; // [Question1]

    cout << p + q << endl; // [Question2]

    *q = 1;
    cout << p << endl; // output 0x555111
    cout << p + *q << endl; // [Question3]

    Node *node = new Node(5); // Node includes the data item of int type
    delete node;
    Node *node2 = new Node(7);
    cout << node->getData() << endl; //[Question4]
                // For this last line, we intentionally wrote node-> and not node2->
                // Also note that this line will not crash the program in Dev-C++
    ...
}
```

Output and explanations:

[*Question*1]

[*Question*2]

[*Question*3]

[*Question*4]

4 Question 4. Divide and Conquer Algorithms (10 marks)

Consider the following function named $woofPlus$:

```
void woofPlus(int K, int firstWoof, int lastWoof) {
    int segment = (lastWoof - firstWoof) / 4;
    if (lastWoof <= firstWoof)
        cout << "WOOF!" << endl;
    else {
        woofPlus(K, firstWoof, firstWoof + segment);
        woofPlus(K, firstWoof + segment + 1, firstWoof + segment * 2);
        woofPlus(K, firstWoof + segment * 2 + 1, firstWoof + segment * 3);
        woofPlus(K, firstWoof + segment * 3 + 1, lastWoof);
    }
}
```

a. (4 marks)

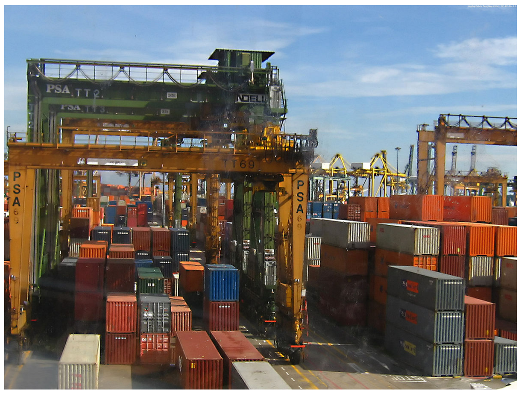
Determine the recurrence relationship for the runtime efficiency $T(n)$ of this function, where $n = lastWoof - firstWoof$ for $n \geq 0$.

c. (6 marks)

Solve the recurrence relationship for this function by unrolling the recurrence (use back substitution), and give the $T(n)$'s order of growth in terms of the Big-O notation as a function of n. You do not have to provide a formal proof using the Big-O formal definition. Show all steps in deriving your solution.

5 Question 5. Data Structure Design (15 marks)

We are tasked with programming a data structure interface for storage of shipping containers at a port. The shipping containers arrive by ships, and are stored via cranes on top of each other. The last container to be stored is the first one to be removed.



Each container will be stored as an instance of *ShippingContainer* class that encapsulates the needed data and behaviour. Each container has an RFID tag, so once it is deposited, we will also store its x, y, and z coordinates as *double* values. For each container, we will also store unique ID, owner's name, country of origin, and date deposited; all of these to be stored as *string* values.

ContainerHandler class will be used to handle *ShippingContainer* objects, and it will include relevant pointers to a collection of container objects.

a. (2 marks) Based on the abstract data types (ADT) presented in class, which ADT would be most appropriate for this problem? Explain your answer.

b. (6 marks) Use doubly linked list from *Assignment* #1 as the basis for your implementation, and provide the declarations for member attributes and methods, including getter/setter methods as needed, that are needed to make *ShippingContainer* and *ContainerHandler* function. For *ShippingContainer*, you should also include at least one parametric constructor.

```
class ShippingContainer {
    string containerID;
    // fill in other required member attributes and methods below
    // members should be declared as public or private as appropriate









    };
```

```
class ContainerHandler {
// fill in other required member attributes and methods below
// members should be declared as public or private as appropriate









    bool insertContainer(ShippingContainer* container);
    bool removeContainer();
    ShippingContainer* findContainer(string containerID);
};
```

c. (7 marks) Write the method $ContainerHandler :: findContainer(string\ containerID)$. The method searches for the container with the matching ID, and returns a pointer reference to it. If such container cannot be found, it returns $NULL$ instead. You can assume that the operators $==$ and $!=$ are available for strings.

You may only use *iostream* and *string* libraries in your implementation, and no other external libraries. However, you may write helper functions of your own. Include appropriate error checking in your code, and adequately document your code. Marks will be deducted for implementations that are difficult to read, or that are not adequately documented.

```
ShippingContainer* ContainerHandler::findContainer(string containerID) {
```

Scrap Paper

You may remove this page from the exam. If you do remove it, write your Student Number on it, and hand it in with your exam.