Problem Definition:

We are given a set of n sequences $S = \left\{s^{(1)}, .., s^{(n)}\right\}$, each sequence has m vertices (numbers) $s^{(i)} = \left(v_1^{(i)}, .., v_m^{(i)}\right)$, each vertex colored in one out of 10 colors (digits).

We are given a secret sequence $s^* = \left(v_1^*, .., v_m^*\right)$ for which the colors of the vertices are unknown.

A vertex $v_j^{(i)}$ is **correct** iff its color of is equal to v_j^*.

We are given the number of correct vertices in each sequence in S, denoted by $\left\{d^{(1)}, .., d^{(n)}\right\}$, and are promised that given S and D, the colors of s^* are unique.

Our goal is to discover the colors of the vertices in s^*.

Problem Example:

- ▷ Number of sequences given: $n = 6$
- ▷ Length of each sequence: $m = 5$
- ▷ The colors of the vertices in S are:

$$s^{(1)} = (9, 0, 3, 4, 2), s^{(2)} = (7, 0, 7, 9, 4), s^{(3)} = (3, 9, 4, 5, 8)$$
$$s^{(4)} = (3, 4, 1, 0, 9), s^{(5)} = (5, 1, 5, 4, 5), s^{(6)} = (1, 2, 5, 3, 1)$$

- ▷ The number of correct vertices in S is given to be $\{2, 0, 2, 1, 2, 1\}$.

Modeling as an ILP (Integer Linear Program):

Let $\{x_{i,c}\}_{i\in[m], c\in[10]}$ be binary variables indicating the color of the digits in s^*, that is:

$$x_{j,c} = 1 \iff v_j^* = c$$

Since each vertex in s^* has only one color, we get the constraints:

$$\left\{\sum_{c=0}^{9} x_{j,c} = 1\right\}_{j\in[m]}$$

And from the input of correct vertices we get the constraints:

$$\left\{\sum_{j\in[m]} x_{j,v_j^{(i)}} = d^{(i)}\right\}_{i\in[n]}$$

Since we are guarantied that only one solution exists, we can put a dummy objective function of our choice, and solve the ILP to get the unique solution.