# Longitudinal Data: Repeated Measures

## Mixed Models 2

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# Read in data

```
dat = read_csv("phco2.csv")
```

```
## Parsed with column specification:
## cols(
##   id = col_integer(),
##   co2 = col_integer(),
##   ph = col_double(),
##   prph = col_double(),
##   visit = col_integer(),
##   eye = col_integer()
## )
```

```
tbl_df(dat)
```

```
## # A tibble: 72 x 6
##       id   co2    ph  prph visit   eye
##    <int> <int> <dbl> <dbl> <int> <int>
##  1     1     0  7.43  76.4     1     1
##  2     1     5  6.96  59.4     1     2
##  3     1     3  7.23  80.3     2     1
##  4     1     7  6.98  74.2     2     2
##  5     2     0  7.59  67.8     1     1
##  6     2     7  6.93  61.1     1     2
##  7     2     3  7.23  73.4     2     1
##  8     2     5  7.07  65.9     2     2
##  9     3     3  7.33  87.6     1     1
## 10     3     7  6.96  55.8     1     2
## # ... with 62 more rows
```

# Plot patterns among subjects

# Set of models to be fit

1. $$Y_{ijk} = \beta_0 + \beta_1 X_{ijk} + \beta_2 X_{ijk}^2 + e_{ijk}$$
2. $$Y_{ijk} = \beta_0 + \beta_{0i} + \beta_1 X_{ijk} + \beta_2 X_{ijk}^2 + e_{ijk}$$
3. $$Y_{ijk} = \beta_0 + \beta_{0i} + \beta_{0ij} + \beta_1 X_{ijk} + \beta_2 X_{ijk}^2 + e_{ijk}$$
4. $$Y_{ijk} = \beta_0 + \beta_{0i} + (\beta_1 + \beta_{1i}) X_{ijk} + \beta_2 X_{ijk}^2 + e_{ijk}$$

# Model 1

```r
# Make a co2^2 variable
dat <- dat %>% mutate(co22 = co2^2)
# OLS
lm_1 <- glm(ph ~ co2 + co22, data = dat,
    family = "gaussian")
summary(lm_1)
```

```
##
## Call:
## glm(formula = ph ~ co2 + co22, family = "gaussian", data = dat)
##
## Deviance Residuals:
##       Min         1Q     Median         3Q        Max
## -0.216127  -0.059279   0.000826   0.042823   0.239671
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.646126   0.021202 360.635  < 2e-16 ***
## co2         -0.137314   0.013800  -9.951 7.84e-15 ***
## co22         0.007350   0.001911   3.847 0.000269 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 0.007799385)
##
##     Null deviance: 4.13819  on 69  degrees of freedom
## Residual deviance: 0.52256  on 67  degrees of freedom
##   (2 observations deleted due to missingness)
## AIC: -136.17
##
## Number of Fisher Scoring iterations: 2
```

```r
# Residual SD
sigma(lm_1)
```

# Estimates of changes in mean for different changes in CO2

```
# Note, if co2 is x, then co22 is x^2
comps = rbind(c(0, 3, 9), c(0, 5, 25), c(0,
    7, 49))
labels = c("3 vs 0", "5 vs. 0", "7 vs. 0")
source("glm_post_estimate.R")
post1 <- glm.post.estimate(lm_1, comps, labs = labels)
post1
```

```
##          Est      CI                    pvalue
## 3 vs 0  "-0.346" "-0.396 - -0.296" "0.000"
## 5 vs. 0 "-0.503" "-0.556 - -0.449" "0.000"
## 7 vs. 0 "-0.601" "-0.657 - -0.545" "0.000"
```

# Model 2

```
lme_2 <- lmer(ph ~ co2 + co22 + (1 | id),
    data = dat)
summary(lme_2)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: ph ~ co2 + co22 + (1 | id)
##    Data: dat
##
## REML criterion at convergence: -153.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -1.9844 -0.5218 -0.0148  0.6545  1.6826
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  id       (Intercept) 0.005116 0.07153
##  Residual             0.002639 0.05137
## Number of obs: 70, groups:  id, 18
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  7.645670   0.020982 364.390
## co2         -0.137277   0.008029 -17.097
## co22         0.007355   0.001112   6.616
##
## Correlation of Fixed Effects:
##      (Intr) co2
## co2  -0.388
## co22  0.268 -0.954
```

```
AIC(lme_2)
```

# Model 2 Stats

```
library(sjstats)
# ICC - var(b0i)/(var(b0i)+var(eij))
icc(lme_2)
```

```
##
## Linear mixed model
##
## Family : gaussian (identity)
## Formula: ph ~ co2 + co22 + (1 | id)
##
##   ICC (id): 0.6597
```

```
AIC(lme_2)
```

```
## [1] -143.2945
```

```
# Now do by grabbing relevant objects
rand_vars2 <- re_var(lme_2)
rand_vars2
```

```
##       Within-group-variance:    0.003
##      Between-group-variance:    0.005 (id)
```

```
rand_vars2[2]/(rand_vars2[1] + rand_vars2[2])
```

```
## id_tau.00
## 0.6596925
```

# Estimates of changes in mean for different changes in CO2

```
source("glmer_post_estimate.R")
post2 <- glmer.post.estimate(lme_2, comps,
    labs = labels, exponentiate = F)
post2
```

```
##          Est      CI                    pvalue
## 3 vs 0  "-0.346" "-0.375 - -0.317" "0.000"
## 5 vs. 0 "-0.503" "-0.534 - -0.471" "0.000"
## 7 vs. 0 "-0.601" "-0.634 - -0.568" "0.000"
```

# Model 3

```
lme_3 <- lmer(ph ~ co2 + co22 + (1 | id/visit),
    data = dat)
summary(lme_3)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: ph ~ co2 + co22 + (1 | id/visit)
##    Data: dat
##
## REML criterion at convergence: -153.9
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -1.85077 -0.48073 -0.02356  0.63698  1.74510
##
## Random effects:
##  Groups   Name        Variance  Std.Dev.
##  visit:id (Intercept) 0.0004291 0.02071
##  id       (Intercept) 0.0049795 0.07057
##  Residual             0.0023540 0.04852
## Number of obs: 70, groups:  visit:id, 35; id, 18
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  7.644453   0.020910 365.593
## co2         -0.136463   0.007963 -17.137
## co22         0.007266   0.001101   6.597
##
## Correlation of Fixed Effects:
##      (Intr) co2
## co2  -0.386
## co22  0.270 -0.956
```

```
AIC(lme_3)
```

# Model 3 Stats

```r
# ICC
icc(lme_3)
```

```
##
## Linear mixed model
##
## Family : gaussian (identity)
## Formula: ph ~ co2 + co22 + (1 | id/visit)
##
##   ICC (visit:id): 0.0553
##         ICC (id): 0.6415
```

```r
# Now do by grabbing relevant objects
rand_vars3 <- re_var(lme_3)
rand_vars3
```

```
##       Within-group-variance:    0.002
##      Between-group-variance:    0.000 (visit:id)
##      Between-group-variance:    0.005 (id)
```

```r
# ICC id
rand_vars3[3]/(sum(rand_vars3))
```

```
## id_tau.00
## 0.6414773
```

```r
# ICC visit in id
rand_vars3[2]/(sum(rand_vars3))
```

```
## visit:id_tau.00
##      0.05527483
```

# Estimates of changes in mean for different changes in CO2

```
post3 <- glmer.post.estimate(lme_3, comps,
    labs = labels, exponentiate = F)
post3
```

```
##          Est      CI                    pvalue
## 3 vs 0  "-0.344" "-0.373 - -0.315" "0.000"
## 5 vs. 0 "-0.501" "-0.532 - -0.470" "0.000"
## 7 vs. 0 "-0.599" "-0.631 - -0.567" "0.000"
```

# Model 4

```
lme_4 <- lmer(ph ~ co2 + (co2 | id) + co22,
    data = dat)
summary(lme_4)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: ph ~ co2 + (co2 | id) + co22
##    Data: dat
##
## REML criterion at convergence: -160.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -2.12176 -0.40438  0.01508  0.61809  1.79975
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  id       (Intercept) 9.389e-03 0.096897
##           co2         4.297e-05 0.006555 -1.00
##  Residual             2.259e-03 0.047525
## Number of obs: 70, groups:  id, 18
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)  7.645652   0.025695 297.550
## co2         -0.137245   0.007597 -18.065
## co22         0.007350   0.001028   7.149
##
## Correlation of Fixed Effects:
##      (Intr) co2
## co2  -0.472
## co22  0.204 -0.933
```

```
AIC(lme_4)
```

# Model 4 Stats

```
# ICC
icc(lme_4)
```

```
## Caution! ICC for random-slope-intercept models usually not meaningful. See 'Note' in `?icc`.
```

```
##
## Linear mixed model
##
## Family : gaussian (identity)
## Formula: ph ~ co2 + (co2 | id) + co22
##
##   ICC (id): 0.8061
```

```
# Now do by grabbing relevant objects
rand_vars4 <- re_var(lme_4)
rand_vars4
```

```
##       Within-group-variance:    0.002
##      Between-group-variance:    0.009 (id)
##       Random-slope-variance:    0.000 (id.co2)
##  Slope-Intercept-covariance:   -0.001 (id.(Intercept))
## Slope-Intercept-correlation:   -1.000 (id)
```

```
# ICC id at Xijk=0
rand_vars4[2]/(rand_vars4[1] + rand_vars4[2])
```

```
## id_tau.00
## 0.8060881
```

# Estimates of changes in mean for different changes in CO2

```
# Note, if co2 is x, then co22 is x^2
post4 <- glmer.post.estimate(lme_4, comps,
    labs = labels, exponentiate = F)
post4
```

```
##          Est      CI                    pvalue
## 3 vs 0  "-0.346" "-0.374 - -0.317" "0.000"
## 5 vs. 0 "-0.502" "-0.535 - -0.470" "0.000"
## 7 vs. 0 "-0.601" "-0.638 - -0.563" "0.000"
```

# Summary stats

Table 1: Estimates(SE) and Akaike's Information Criterion (AIC) of four hierarchical linear mixed effects models for pH versus $C0_2$

| | Model 1 | | Model 2 | | Model 3 | | Model 4 | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Est | SE | Est | SE | Est | SE | Est | SE |
| $\beta_0$ | 7.6 | .021 | 7.6 | .021 | 7.6 | .021 | 7.6 | .026 |
| $\beta_1$ | -.14 | .014 | -.14 | .008 | -.14 | .0080 | -.14 | .0076 |
| $\beta_2$ | .0073 | .0019 | .0074 | .0011 | .0073 | .0011 | .0073 | .0010 |
| $\sigma_{\beta_{0i}}$ | | | .071 | | .071 | | .097 | |
| $\sigma_{\beta_{0ij}}$ | | | | | .021 | | | |
| $\sigma_{\beta_{1i}}$ | | | | | | | .0065 | |
| $\sigma_e$ | .088 | | .051 | | .048 | | .047 | |
| *AIC* | -136 | | -143 | | -142 | | -147 | |

# Model vs. Association of Interest