Alex Hubers CSC:341 Spring 2024	HW #7	Due: Mar 12 2024

Instructions. You are allowed to collaborate with others, however you should write up solutions independently. Copying an answer from another source (e.g. the Web) or from another student may yield few or zero points. Write solutions neatly and legibly, or type your solutions in LaTeX. Be sure to number each problem, and indicate a final solution (if relevant). Answers to problems should include justification (show your work).

Acknowledgments. Problems from this homework come from published sources. The specific sources are withheld due to the nature of this assignment.

Academic Honesty. Include the following information at the top of your submission, along with your name.

- Written sources used: (Include textbook(s), complete citations for web or other written sources. Write none if no sources used)
- Help obtained: (Include names of anyone other than the instructor.)

Exam questions. Harder (or, more involved) questions are marked with an **asterisk (*)**. You can expect an exam to consist of four to five questions of about this difficulty. That is, your exam will have at least one question per learning outcome that is similar in difficulty to an asterisk question on your homework.

Come back to this tomorrow. I am rushing. I am going to pick questions I hate because I am tired.

#1 (5 pts)

Suppose that language A is recognized by $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, a single-tape TM that can only move its head right.

1. Show that A must be regular. *Hint: Describe a DFA that also recognizes A. I do not need a proof of correctness for your construction, but please argue informally (1-2 sentences) why the DFA you construct will accept the same set of strings as M.*

2. Suppose that B is recognized by M', which has all the capabilities of a single-tape Turing machine—that is, M' can move its head left, right, and write on the input tape. Is B regular? Why or why not?

   N.b. a formal proof is unnecessary. You should be able to use your intuition about the expressivity of DFAs and Turing machines to form a 1-2 sentence answer.

#2 (5 pts)

Prove the following claim.

Claim 1. *Let language L_1 be a language recognized by TM M_1 and L_2 be recognized by M_2. Then there exists Turing machine $M_\cap$ that recognizes $L_1 \cap L_2$.*

#3 (5 pts)

Give a high-level description of a Turing machine M that decides the following language.

$$\text{CONNECTED} = \{\langle G \rangle \mid \forall u, v \in V,\ (u, v) \in E\}$$

That is to say, a graph G is in CONNECTED if all of its vertices are connected to every other vertex.

N.b. As this a high-level description, you do not need to worry about the underlying encoding of the graph input. Rather, describe the Turing machine as you would an algorithm that solves this problem. For example, you are allowed to write a sentence like "For each vertex v, do ..." without worrying about the particular representation of vertices.

#4 (5 pts)*

Let $\Sigma = \{0, 1\}$ and suppose we have a TM M_{DFA} that decides the language

$$A_{DFA} = \{\langle D, w \rangle \mid D \text{ accepts input string w}\}$$

Use M_{DFA} to build some Turing machine M that decides the following language:

$$A = \{\langle D, k \rangle \mid \exists s \in \Sigma^* \text{ s.t. } |s| \leq k \text{ and } D \text{ accepts } s\}$$

That is, given an arbitrary DFA D and length k, does D accept a string with length k?

Describe M at a high- or implementation-level (whatever you find helpful). You may also suppose we live in a world where all DFAs have alphabet $\{0, 1\}$.

Hint: Remember that $\Sigma = \{0, 1\}$, so you can enumerate all possible strings of length $\leq k$. Nondeterminism may be helpful.