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CSC:341 Spring 2024

Lab #2A

Due: February 3, 2024

In this lab, we will go over the basic patterns of automata design that you will see through your study of the theory of computation. Studying these patterns should:

- Reinforce your knowledge of how finite automata operate.
- Give you a toolbox of fundamental patterns for designing automata.
- Give you a sense of what is possible and what is not possible with finite automata.

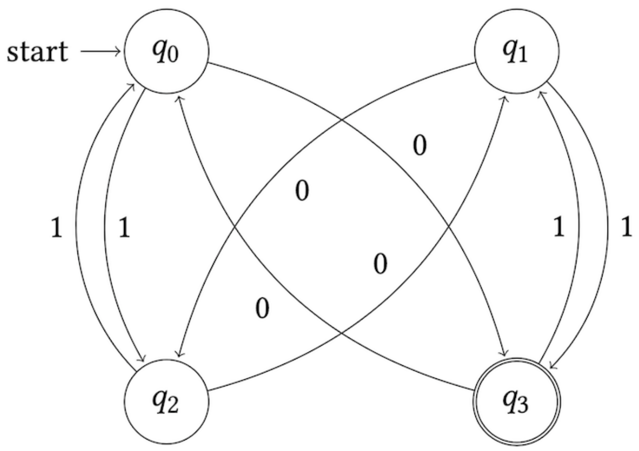
#1 Connect the Dots

Answer the following questions about the fundamental definitions that connect languages, machines, and computation.

1. What do the metavariables Σ and L traditionally represent? Give a concrete example that covers each.
2. What are the types of the five components of a finite automata: $(Q, \Sigma, \delta, q_0, F)$?
3. What does $L(M)$ conventionally mean? If $L(M) = A$, what is the relationship between M and A?
4. If we want to prove that "M accepts string w," what we must we do?

#2 Tracing the Lines

Consider the following sample deterministic finite automata, D, over the alphabet $\Sigma = \{0, 1\}$:



1. Give execution traces, i.e., the series of states that the DFA steps through on the following input strings. Also state whether D accepts the given input string. Make sure to give each of your group members a chance to take the lead in creating these traces.
   (a) 01100
   (b) 101101
   (c) 11111
   (d) ϵ (the empty string)

(e) 1010101

2. Importantly, we can each think of state of a finite automata as encoding a property about the current computation, i.e., some property of the set of characters read so far. Based on the traces you wrote above, give an appropriate property for each of q_0, q_1, q_2, and q_3 solely in terms of the input read so far after arriving at that state, not about the various parts of the DFA.

3. Using these traces and the properties you wrote down for each state, give a formal description of $L(D)$.

#3 DFA Design

For each language L_1 through L_7, build a DFA that recognizes that language. Throughout, $\Sigma = \{0, 1\}$. *Please formally specify δ and describe which states are accept states. You may specify δ as a table. I also encourage you to draw first each as a state diagram, but this needn't be typeset and turned in.*

1. **Emptiness**: $L_1 = \emptyset$, i.e. the empty set of strings.
2. **Everything**: $L_2 = \Sigma^*$, i.e., the set of all possible strings.
3. **Constants**: $L_3 = \{0110\}$, the singleton set containing the string 0110.
4. **Prefixes and Suffixes**: $L_4 = \{01x10 \mid x \in \Sigma^*\}$.
5. **Parity**: $L_5 = \{x \mid x \in \Sigma^*, x \text{ has an even number of 0s}\}$.
6. **Counts**: $L_6 = \{x \mid x \in \Sigma^*, x \text{ contains } \textit{at least} \text{ three 1s}\}$.
7. **Constraints**: $L_7 = \{x \mid x \in \Sigma^*, x \text{ does not contain } 01\}$.