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CSC:341 Spring 2024

Lab #9B

Due: Apr 6, 2024

#1

Answer True or False for each of the following claims. (Please refer to the definitions, theorems, and corollaries in Sipser §5.3).

1. Let A be the set of (binary representations of) natural numbers and B be the set of (binary representations of) signed integers. Then $A \leq_m B$ under the function $f(n) = 1 \circ n$. ($f(n)$ is implemented by adding a leading one to the binary string n. You may presume f leaves malformed inputs as is.)
2. $A_{\text{TM}} \leq_m \mathsf{HALT}_{\text{TM}}$.
3. $\mathsf{HALT}_{\text{TM}} \leq_m A_{\text{TM}}$. (*Proof not necessary, but what is your hunch?*)
4. If $A \leq_m B$ and A is decidable then B is decidable.
5. If $A \leq_m B$ and B is decidable then A is decidable.
6. If $A \leq_m B$ and B is undecidable then A is undecidable.
7. If $A \leq_m B$ and A is undecidable then B is undecidable.

#2

Reduce each of the following languages from known, undecidable languages.

1. $L_1 = \{ M \mid M$ is a TM that accepts w^R whenever it accepts $w \}$.
   (Recall that w^R is the reversal of w.)
2. $L_2 = \{ (M, w) \mid M$ is a TM that, on input w, writes a '\$' on the tape $\}$.

#3

Find a match in the following instance of the Post Correspondence Problem:

$$\left\{ \left[\frac{\texttt{ab}}{\texttt{abab}}\right], \left[\frac{\texttt{b}}{\texttt{a}}\right], \left[\frac{\texttt{aba}}{\texttt{b}}\right], \left[\frac{\texttt{aa}}{\texttt{a}}\right] \right\}.$$