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CSC:341 Spring 2024

HW #6

Due: Mar 5 2024

Instructions. You are allowed to collaborate with others, however you should write up solutions independently. Copying an answer from another source (e.g. the Web) or from another student may yield few or zero points. Write solutions neatly and legibly, or type your solutions in LaTeX. Be sure to number each problem, and indicate a final solution (if relevant). Answers to problems should include justification (show your work).

Acknowledgments. Problems from this homework come from published sources. The specific sources are withheld due to the nature of this assignment.

Academic Honesty. Include the following information at the top of your submission, along with your name.

- Written sources used: (Include textbook(s), complete citations for web or other written sources. Write none if no sources used)
- Help obtained: (Include names of anyone other than the instructor.)

Exam questions. Harder (or, more involved) questions are marked with an **asterisk (*)**. You can expect an exam to consist of four to five questions of about this difficulty. That is, your exam will have at least one question per learning outcome that is similar in difficulty to an asterisk question on your homework.

#1 (5 pts)

Give a formal description of a Turing Machine that decides the language

$$A = \{x \in \{0,1\}^* \mid 101 \text{ is a substring of } x\}$$

For example, $011010 \in A$ but $11001 \notin A$.

Hint: I give this question to you in part because you know very well by now how to recognize this language with a DFA! Try to mimic your approach for the DFA but with a Turing machine.

#2 (5 pts)

Give an implementation-level description of a Turing Machine that decides the language

$$A = \{0^n 1^n 0^n \mid n \geq 0\}$$

Hint: I found this problem a lot easier if you use multiple tapes.

#3 (5 pts)

Prove the following claim.

Hint: recall how we used nondeterminism to show this same property for regular languages.

Claim 1. *The collection of Turing-recognizable languages is closed under the union operation.*

Next, argue in no more than one sentence why the claim below follows immediately from the one above.

Claim 2. *The collection of decidable languages is closed under the union operation.*

#4 (5 pts)*

Give a high-level description a 2-tape Turing Machine that on input $x \in \{0,1\}^*$ outputs the binary representation of $|x|$ on the second tape (call this the output tape). For example, if the input is $x = 1010$, then $|x| = 4$ and the TM should halt in a configuration with the string 100 on the output tape.

A note on formality. In question #2, you "graduated" from being asked to give a formal description of a Turing Machine to an implementation-level description. Now, you have graduated to the task of giving a high-level description. This is generally the level of formality we will stay at for the rest of this course. However, you should be confident that, were you asked, you could give an accurate formal description with some thought. In particular, I will expect of you that you are able to translate a high-level description to a formal definition of δ if prompted.