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Quiz #1A

This quiz is designed to familiarize me with the degree of your mathematical literacy. Full credit is given upon participation. Because it will be submitted anonymously, I will instruct you on how to receive participatory credit on the day it is given. (You are allowed to put your name in the top-right corner if you wish to de-anonymize your submission.)

This quiz is *not* designed to trick, judge, or challenge you. It just helps me avoid either (i) over-explaining content you understand well or (ii) under-explaining content you do not understand well. For this reason, I ask that you answer questions on which you are unsure according to the scale below.

The *Not-Clear (NC)* Scale. In the case that you are unsure on an answer, please do not guess. Instead, write the code, defined below, that most appropriately describes in what way you are confused by the problem.

- **NC0**—I just have no clue.
- **NC1**—This looks familiar but I couldn't really tell right from wrong.
- **NC2**—This looks familiar and I know I would have gotten this right in the past, but can't recall now.

If you find yourself on the fence between an NC code and a guess, please write both. In the case that this quiz is timed, please leave answers you did not get to blank.

#1 Sets and Set Operations

Define the sets R, S, and T as follows. Note that S is a set of functions.

$$\begin{aligned} R =& \{0, 1, 2\} \\ S =& \{n^2, 2^n, n!\} \\ T =& \{a, 42, R\} \end{aligned}$$

Label each of the following claims as either `T` (True) or `F` (False).

- ______ $42 \in R$.
- ______ $42 \notin T$.
- ______ $\{0, 1\} \subseteq R$.
- ______ $\{0, 1\} \supset R$.
- ______ The function f, defined by $f(x) = x!$, is an element of S.
- ______ $R \cap S = \{0, 1, 2\}$.
- ______ $R \cup S = \{0, 1, 2, n^2, 2^n, n!\}$.
- ______ $|R| = |S| = |T|$.
- ______ The difference between sets R and S (written $R \setminus S$) is the empty set.

- ______ The power set of R, denoted $\mathcal{P}(R)$, is the set of all subsets of R.

Write below the notation that describes "the ordered product of sets A, B, and C".

#2 Important Sets

Please draw arrows between the following sets and their descriptions. We will follow Sipser in letting the set of natural numbers ($\mathbb{N}$) start at 1. If unsure, write your NC code close to the left or right of the thing that confuses you.

$\emptyset$	$\mathbb{Q} \cup \{z \mid z \text{ is irrational}\}$	(1)
$\mathbb{N}$	$\{\frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{N}\}$	(2)
$\mathbb{R}$	$\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$	(3)
$\mathbb{Q}$	$\{1, 2, 3, ...\}$	(4)
$\mathbb{Z}$	the empty set	(5)

#3 Logical Quantification

What do $\forall$ and $\exists$ stand for?

Translate the following mathematical statement to plain English.

$$\forall x \in \mathbb{Z}. \exists y \in \mathbb{Z}. x + y = 0$$

#4 Properties of Functions

Please fill in the blanks and answer the question below.

- A function $f : X \to Y$ is called __________ if, for all $y \in Y$, there exists $x \in X$ such that $f(x) = y$.
- A function $g : X \to Y$ is called __________ if, for all $x_1, x_2 \in X$, if $f(x_1) = f(x_2)$ then $x_1 = x_2$.

A function $f : X \to Y$ is called bijective if ...

#5 Properties of Relations

The binary relations below are defined below so that exactly one of each is *reflexive*, *symmetric*, and *transitive*.

$$
\begin{aligned}
A =&\{(0,1),(1,0),(1,1)\}\\
B =&\{(a,a),(b,b),(a,b)\}\\
C =&\{(x,y),(y,z),(x,z)\}
\end{aligned}
$$

- Which relation is *reflexive*?
- Which relation is *symmetric*?
- Which relation is *transitive*?

#6 Boolean Logic

Let the notation

- $\wedge$ denote Boolean conunction (`AND`);
- $\vee$ denote Boolean disjunction (`OR`);
- $\neg$ denote Boolean negation (`NOT`);
- $\oplus$ denote Boolean Exclusive-Or (`XOR`); and
- $\Rightarrow$ denote Boolean implication.

Label each of the following boolean statements as either `T` (True) or `F` (False).

- ______ $\neg T \vee T$.
- ______ $\neg(T \wedge F)$.
- ______ $T \oplus F$.
- ______ $F \Rightarrow T$.