

Instructions. You are allowed to collaborate with others, however you should write up solutions independently. Copying an answer from another source (e.g. the Web) or from another student may yield few or zero points. Write solutions neatly and legibly, or type your solutions in LaTeX. Be sure to number each problem, and indicate a final solution (if relevant). Answers to problems should include justification (show your work).

Acknowledgments. Problems from this homework come from published sources. The specific sources are withheld due to the nature of this assignment.

Academic Honesty. Include the following information at the top of your submission, along with your name.

- Written sources used: (Include textbook(s), complete citations for web or other written sources. Write none if no sources used)
- Help obtained: (Include names of anyone other than the instructor.)

#1 (5 pts)

Recall that a language B is called NP-hard if, for all $A \in \text{NP}$, $A \leq_p B$.

1. (4 pts). Prove that HALT_{TM} is NP-hard.
2. (1 pt). Describe in one sentence why HALT_{TM} is not NP-complete.

#2 (5 pts)

Show that if $P = \text{NP}$, then every *nontrivial* language $A \in P$ is NP-complete. Specifically, let $B \in P$ be nontrivial. Show that:

1. $B \in \text{NP}$, and
2. B is NP-hard.

#3 (5 pts)

We can generalize k -SAT to any combination of sizes of cnf-formula as follows:

$$\text{CSAT} = \{ \langle \phi \rangle \mid \phi \text{ is a satisfiable formula in CNF form} \}.$$

Show that $\text{CSAT} \leq_p 3\text{SAT}$. (Hint: each sub-formula of ϕ can have arbitrary size. Make sure that your reduction covers every possible size of formula.)

Approach (Hints)

The following subproblems should inspire your approach to the problem above. Do not hand in answers to these questions.

1. Suppose that you have a 2cnf-formula clause of the form $(x_1 \vee x_2)$. Describe how you would turn this clause into a logically equivalent 3cnf-formula clause. Argue in a sentence or two why your transformation is correct.
2. With this fact, prove that $2\text{SAT} \leq_p 3\text{SAT}$.
3. Suppose that you have a 4cnf-formula clause of the form $(x_1 \vee x_2 \vee x_3 \vee x_4)$. Describe how you would turn this clause into a logically equivalent 3cnf-formula clause. Argue in a sentence or two why your transformation is correct. (Hint: you will want to split up the 4cnf clause into, e.g., $(x_1 \vee x_2 \vee \dots) \wedge (\dots \vee x_3 \vee x_4)$. How do you complete these two 3cnf clauses so that only one of x_1, \dots, x_4 is necessary to be true to make both clauses true? You should consider making a new variable distinct from x_1, \dots, x_4 to help you with this transformation.)
4. With this fact, prove that $4\text{SAT} \leq_p 3\text{SAT}$.