HW #3

Due: February 13, 2024

Instructions. You are allowed to collaborate with others, however you should write up solutions independently. Copying an answer from another source (e.g. the Web) or from another student may yield few or zero points. Write solutions neatly and legibly, or type your solutions in LaTeX. Be sure to number each problem, and indicate a final solution (if relevant). Answers to problems should include justification (show your work).

Acknowledgments. Problems from this homework come from published sources. The specific sources are withheld due to the nature of this assignment.

Academic Honesty. Include the following information at the top of your submission, along with your name.

- Written sources used: (Include textbook(s), complete citations for web or other written sources. Write *none* if no sources used)
- Help obtained: (Include names of anyone other than the instructor.)

Rubric. Problems will be graded on a 4-point *EMBN rubric*. Graded answers will be assigned an appropriate letter descriptor (below) and provided some justification of this assignment. Points will be awarded in the range associated with each descriptor.

- **E (Excellent)**. 4 pts. Complete understanding of the material is evident; exhibits no errors and can serve as an exemplar solution for the course.
- **M (Meets Expectation)**. $[3, 4)$ pts. Complete understanding of the material is evident, but exhibits some minor errors that warrant revision.
- **B (Below Expectation)**. $[1, 3)$ pts. Limited understanding of the material is evident; exhibits many minor errors or one or more major errors that necessitate revision.
- **N (Not Completed)**. 0 pts. Not completed to a degree where understanding is evident.

The next set of questions are designed to give you more practice with *nondeterminism*, *regular expressions*, and *closure properties of the regular languages*. Each question has the aim of satisfying one or more of the following learning outcomes.

- 1-1: Represent a problem using a regular model of computation.
- 1-2: Prove closure and algorithmic properties of the regular languages.

Exam questions. Harder (or, more involved) questions are marked with an **asterisk (*)**. You can expect an exam to consist of four to five questions of about this difficulty. That is, your exam will have at least one question per learning outcome that is similar in difficulty to an asterisk question on your homework.

#1 (4 pts)

Give a regular expression (as defined in def. 1.52 of Sipser) for a language L of strings drawn from the alphabet $\Sigma = \{0, 1, -, .\}$ that obey the following properties:

1. Each string in L must contain at least one binary digit.
2. Each string may optionally be preceded by a single $(-)$.
3. Each string may optionally contain a single period (.).

For example, the strings 0, 1, 10.1010101, and -0.10 are all in the language L.

N.b. you may use the syntactic sugar described in Lab 3A. In particular, I found it helpful to use the extended operators R^+ and $R^?$.

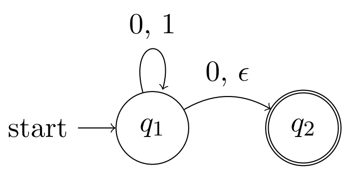
#2 (4 pts)

In the form of a diagram, give an NFA recognizing the language $(01 \cup 110)^*$.

#3 (4 pts)

Convert the following NFA into a DFA using the construction given in Theorem 1.39. Please describe your NFA formally by telling me each component Q, Σ, q_0, F, and δ. Give δ in the form of a table.

N.b. Remember to take not only the ϵ-closure of δ but also of the start state (See Sipser Pg. 56). That is, we have to start the DFA not just at the start state q_1 but also any state reachable by 0 or more ϵ-arrows from q_1. *(I skipped over this detail in lecture, so I give it to you now.)*



#4 (8 pts)*

The *reverse* of a language A, written A^R, is defined as follows:

$$A^R = \{\, x_k \cdots x_1 \mid x_1 \cdots x_k \in A \,\}.$$

Formally prove that if A is regular, then A^R is also regular. Specifically do so using the following proof technique.

1. (4 pts). Suppose $D = (Q, \Sigma, \delta, q_0, F)$ is a DFA that recognizes A. Describe an NFA N that recognizes A^R. Please be specific in your description: give me each component in $N = (Q', \Sigma', \delta', q_0', F')$.

2. (4 pts). Prove that N recognizes A^R, i.e., show that N recognizes $x_k ... x_1$ for all $x_1 ... x_k \in A$.