# #1

Let $N$ be a 2-tape Computing NDTM with the following transition functions:

| $\delta_0$ | $\delta_1$ |
|---|---|
| $(q_s, \triangleright, \triangleright) \to (q_{scan}, \triangleright, \triangleright, R, R)$ | $(q_s, \triangleright, \triangleright) \to (q_{scan}, \triangleright, \triangleright, R, R)$ |
| $(q_{scan}, 0, \square) \to (q_{scan}, 0, \square, R, S)$ | $(q_{scan}, 0, \square) \to (q_{scan}, 0, \square, R, S)$ |
| $(q_{scan}, 1, \square) \to (q_{scan}, 1, 0, R, R)$ | $(q_{scan}, 1, \square) \to (q_{scan}, 1, 1, R, R)$ |
| $(q_{scan}, \square, \square) \to (q_H, \square, \square, S, S)$ | $(q_{scan}, \square, \square) \to (q_H, \square, \square, S, S)$ |

(Observe that $N$ starts with $\triangleright$ as the first character on each tape).

1. Describe the set of strings generated by each input string below.

    (a) 0
    (b) 1
    (c) 10
    (d) 11

2. Consider the following derivation that $(q_s, 11, \square, 0, 0) \Rightarrow^* (q_H, 11, 10, 3, 3)$. Note that I ignore the $\triangleright$ symbol when describing the contents of each tape in a given configuration.

$$(q_s, 11, \square, 0, 0) \tag{1}$$
$$\Rightarrow (q_{scan}, 11, \square, 1, 1) \tag{2}$$
$$\Rightarrow (q_{scan}, 11, 1, 2, 2) \tag{3}$$
$$\Rightarrow (q_{scan}, 11, 10, 3, 3) \tag{4}$$
$$\Rightarrow (q_H, 11, 10, 3, 3) \tag{5}$$

    For each labeled step above (greater than 1), describe which $\delta$ function was used to yield that step. Write "either" if either $\delta_0$ or $\delta_1$ could have been used.

3. Will this Turing machine halt on all inputs for **all** branches of nondeterminism taken?

# #2 Sipser vs. Arora

In lecture, we gave two definitions of nondeterministic Turing machines.

**Definition 1** (NDTM (Sipser)). *A <u>nondeterministic Turing machine</u>, or NDTM, is a Turing machine with all components equal except for $\delta$, which has type*

$$\delta : Q \times \Gamma \to \mathcal{P}(Q \times \Gamma \times \{L, R, S\})$$

**Definition 2** (NDTM (Arora & Barak version)). *An NDTM is a Turing Machine with at least 2 transition functions $\delta_0$ and $\delta_1$ (not necessarily unequal).*

Convert the Arora and Barak NDTM of #1 to a Sipser NDTM. Specifically, please define $\delta$ as is done above (as input-to-output mappings) but with sets as outputs. You may omit cases you believe are unnecessary to describe.