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Lab #7A

Due: March 9, 2024

#1

Let N be a 2-tape Computing NDTM with the following transition functions:

δ_0	δ_1
$(q_s, \triangleright, \triangleright) \to (q_{scan}, \triangleright, \triangleright, R, R)$	$(q_s, \triangleright, \triangleright) \to (q_{scan}, \triangleright, \triangleright, R, R)$
$(q_{scan}, 0, \square) \to (q_{scan}, 0, \square, R, S)$	$(q_{scan}, 0, \square) \to (q_{scan}, 0, \square, R, S)$
$(q_{scan}, 1, \square) \to (q_{scan}, 0, \square, R, R)$	$(q_{scan}, 1, \square) \to (q_{scan}, 1, \square, R, R)$
$(q_{scan}, \square, \square) \to (q_H, \square, \square, S, S)$	$(q_{scan}, \square, \square) \to (q_H, \square, \square, S, S)$

(Observe that N starts with $\triangleright$ as the first character on each tape).

1. Describe the set of strings generated by each input string below.
   (a) 0
   (b) 1
   (c) 10
   (d) 11
2. Consider the following derivation that $(q_s, 11, 0, 0) \Rightarrow^* (q_H, 11, 10, 3, 3)$. Note that I ignore the $\triangleright$ symbol when describing the contents of each tape in a given configuration.

$$
\begin{aligned}
&(q_s, 11, \square, 0, 0) &(1)\\
\Rightarrow\ &(q_{scan}, 11, \square, 1, 1) &(2)\\
\Rightarrow\ &(q_{scan}, 11, 1, 2, 2) &(3)\\
\Rightarrow\ &(q_{scan}, 11, 10, 3, 3) &(4)\\
\Rightarrow\ &(q_H, 11, 10, 3, 3) &(5)
\end{aligned}
$$

   For each labeled step above (greater than 1), describe which δ function was used to yield that step. Write "either" if either δ_0 or δ_1 could have been used.
3. Will this Turing machine halt on all inputs for **all** branches of nondeterminism taken?

#2 Sipser vs. Arora

In lecture, we gave two definitions of nondeterministic Turing machines.

Definition 1 (NDTM (Sipser)). *A nondeterministic Turing machine, or NDTM, is a Turing machine with all components equal except for δ, which has type*

$$\delta : Q \times \Gamma \to \mathcal{P}(Q \times \Gamma \times \{L, R, S\})$$

Definition 2 (NDTM (Arora & Barak version)). *An NDTM is a Turing Machine with at least 2 transition functions δ_0 and δ_1 (not necessarily unequal).*

Convert the Arora and Barak NDTM of #1 to a Sipser NDTM. Specifically, please define δ as is done above (as input-to-output mappings) but with sets as outputs. You may omit cases you believe are unnecessary to describe.