HW #13

Instructions. You are allowed to collaborate with others, however you should write up solutions independently. Copying an answer from another source (e.g. the Web) or from another student may yield few or zero points. Write solutions neatly and legibly, or type your solutions in LaTeX. Be sure to number each problem, and indicate a final solution (if relevant). Answers to problems should include justification (show your work).

Acknowledgments. Problems from this homework come from published sources. The specific sources are withheld due to the nature of this assignment.

Academic Honesty. Include the following information at the top of your submission, along with your name.

- Written sources used: (Include textbook(s), complete citations for web or other written sources. Write none if no sources used)
- Help obtained: (Include names of anyone other than the instructor.)

#1 (10 pts)

Prove that the class of PSPACE languages are closed under the following properties:

1. (5 pts). Complement: $\overline{L} = \{w \mid w \notin L\}$
2. (5 pts). Kleene star: L^*

Describe your TMs in as high-level of details as you feel necessary. Make sure to argue that your resulting Turing machines takes a polynomial amount of space!

#2 (10 pts)

Consider the following position in a standard tic-tac-toe game:

X		
	O	
O		X

1. (5 pts). Let's say that it is the x-player's turn to move next. Describe a winning strategy for this player. (Recall that a winning strategy isn't merely the best move to make in the current position. It also includes all the responses that this player must make to win regardless of how the opponent moves.)
2. (5 pts). Define the generalized tic-tac-toe game as:

$$\mathsf{GTAC} = \{(M, m, n, k) \mid M \text{ is a } m \times n \text{ board and player one has the winning strategy}\}$$

   A generalized tic-tac-toe game is played on an $m \times n$ board with some placement of X's (player one's pieces) and O's (player two's pieces). The player wins the game if they can arrange k of their pieces on the board in a line—vertically, horizontally, or diagonally. The problem asks whether for a given board if player one has a strategy that allows them to win given the current board configuration.

   Show that

$$\mathsf{GTAC} \text{ is in PSPACE.}$$

   (Hint: adapt the recursive solutions from section 8.3 from the reading to this setting.)