

#1

Answer True or False for each of the following claims. (Please refer to the definitions, theorems, and corollaries in Sipser §5.3).

1. Let A be the set of (binary representations of) natural numbers and B be the set of (binary representations of) signed integers. Then $A \leq_m B$ under the function $f(n) = 1 \circ n$. ($f(n)$ is implemented by adding a leading one to the binary string n . You may presume f leaves malformed inputs as is.)
2. $A_{\text{TM}} \leq_m \text{HALT}_{\text{TM}}$.
3. $\text{HALT}_{\text{TM}} \leq_m A_{\text{TM}}$. (*Proof not necessary, but what is your hunch?*)
4. If $A \leq_m B$ and A is decidable then B is decidable.
5. If $A \leq_m B$ and B is decidable then A is decidable.
6. If $A \leq_m B$ and B is undecidable then A is undecidable.
7. If $A \leq_m B$ and A is undecidable then B is undecidable.

#2

Reduce each of the following languages from known, undecidable languages. *N.b. Please refer to Prof. Osera's writeup on reductions for a great guide. See course schedule for link.*

You do not need to give a proof of correctness.

1. $L_1 = \{ M \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w \}$.
(Recall that w^R is the reversal of w .)
2. $L_2 = \{ (M, w) \mid M \text{ is a TM that, on input } w, \text{ writes a '$' on the tape} \}$.

#3

Find a match in the following instance of the Post Correspondence Problem:

$$\left\{ \left[\begin{array}{c} \text{ab} \\ \text{abab} \end{array} \right], \left[\begin{array}{c} \text{b} \\ \text{a} \end{array} \right], \left[\begin{array}{c} \text{aba} \\ \text{b} \end{array} \right], \left[\begin{array}{c} \text{aa} \\ \text{a} \end{array} \right] \right\}.$$