#1 P ≠NP?!

Describe the error in the following fallacious "proof" that P ≠ NP.

Proof. Assume that P = NP and obtain a contradiction. If P = NP, then SAT ∈P and so for some k, SAT ∈ TIME(n^k). Because every language in NP is polynomial time reducible to SAT, you have NP ⊆ TIME(n^k). Therefore, P ⊆ TIME(n^k). But by the time hierarchy theorem, TIME(n^{k+1}) contains a language that isn't in TIME(n^k), which contradicts P ⊆ TIME(n^k). Therefore, P ≠ NP. □

#2 Complexity Review

Recall each of the complexity classes we have studied thus far (time and space). For each:

- Give the definition.
- Give a classic example (or examples) of languages that have that particular complexity.
- Give an outline of how to prove that a language is in that given complexity class.

#3 Complexity relationships

Describe, as completely as possible, what we know about the relationships between the complexity classes you listed in the previous problem. Additionally, clearly state which relationships are currently unknown. For these unknown relationship, give an informed hypothesis.