Alex Hubers
CSC:341 Spring 2024

HW #4

Due: ? 2022

Instructions. You are allowed to collaborate with others, however you should write up solutions independently. Copying an answer from another source (e.g. the Web) or from another student may yield few or zero points. Write solutions neatly and legibly, or type your solutions in LaTeX. Be sure to number each problem, and indicate a final solution (if relevant). Answers to problems should include justification (show your work).

Acknowledgments. Problems from this homework come from published sources. The specific sources are withheld due to the nature of this assignment.

Academic Honesty. Include the following information at the top of your submission, along with your name.

- Written sources used: (Include textbook(s), complete citations for web or other written sources. Write none if no sources used)
- Help obtained: (Include names of anyone other than the instructor.)

This week's questions are designed to give you more practice with identifying and proving irregularity with the use of the Pumping Lemma and also context-free grammars. Recall Learning Outcomes 1-3 and 1-4 as stated in the syllabus:

- 1-3: Prove the irregularity of a problem.
- 1-4: Represent a problem using a context-free model of computation.

Exam questions. Harder (or, more involved) questions are marked with an **asterisk (*)**. You can expect an exam to consist of four to five questions of about this difficulty. That is, your exam will have at least one question per learning outcome that is similar in difficulty to an asterisk question on your homework.

#1 (4 pts)

Consider the language $A = 0^*1^*$. The following is an erroneous "proof" that A is irregular.

Proof. Assume for the sake of contradiction that A is regular. Then the pumping lemma applies. Let p be the pumping length and choose $s = 0^p1^p \in A$. Observe that $|0^p1^p| = 2p \geq p$. Let $s = xyz$ according to the conditions of the pumping lemma. There are three cases to consider:

- The string y only contains $0s$. Therefore $xyyz \notin A$ because it has has more 0s than 1s.
- The string y only contains 1s. Likewise, $xyyz \notin A$ because it has more 1s than 0s.
- The string y contains a substring of 0s followed by a substring of 1s. The string $xyyz \notin A$ because the substring yy will be of the form $0^a1^b0^a1^b$ for some constants a and b.

In all cases, we arrive at a logical contradiction. Therefore, our original assumption that A is regular must be false; therefore, A is irregular. □

1. (2 pts). In a few sentences, describe the erroneous step(s) of reasoning in the proof.
2. (2 pts). Prove that A is actually regular.

#2 (8 pts)

Use the pumping lemma to show that:

1. The language $A = \{0^n1^n2^n \mid n \geq 0\}$ is irregular, and
2. the language $B = \{\, a^ib^jc^k \mid k = i + j \,\}$ is irregular.

#3 (4 pts)

Consider the following context-free grammar G with start symbol S and alphabet $\Sigma^* = \{\, <, >, - \,\}$, i.e., angle brackets and dashes:

$$
\begin{aligned}
S &\to UT \mid TV \\
T &\to T- \mid -T \mid - \\
U &\to \; < \\
V &\to \; >
\end{aligned}
$$

1. (2 pts). Give a derivation demonstrating that $UTV \stackrel{*}{\Rightarrow} <--->$. Is this string in the language $L(G)$?

2. (2 pts). Give a regular expression that generates $L(G)$. *N.b. it is not always the case that a context-free grammar can be written as a regular expression, but it is the case here.*

#4 (4 pts)

Give a context-free grammar for each of the languages below. For each, let $\Sigma = \{0, 1\}$. Make sure to specify the full tuple (V, Σ, R, S). Please specify R as a series of substitution rules, as is done in Question #3.

1. $\{w \mid w$ contains at least three 1s $\}$
2. $\{w \mid w$ has odd length$\}$
3. $\{w \mid w$ is a palindrome $\}$

N.b. here are the LaTeX commands you will need:

- $\to$ is `\to`
- $\mid$ is `\mid`