

#1 Membership

Show that each of the following languages are in NP.

1. $A_1 = \{ \langle P \rangle \mid P \text{ is a satisfiable boolean formula} \}$.
2. $A_2 = \{ \langle S, t \rangle \mid S \subseteq \mathbb{Z} \text{ and there exists } x_1, \dots, x_k \in S \text{ such that } \sum_i^k x_i = t \}$.
3. $A_3 = \{ \langle G, k \rangle \mid G = (V, E) \text{ is a graph and there exists some } V' \subseteq V \text{ is a } k\text{-covering of } G \}$.
A k -cover of a graph is a subset of vertices that includes at least one endpoint of every edge in the graph.

#2 Completeness

Define the following languages:

$$\text{CLIQUE} = \{ \langle G, k \rangle \mid G \text{ is a graph and } G \text{ contains a } k\text{-clique} \}$$
$$\text{ISO} = \{ \langle G, H \rangle \mid G \text{ and } H \text{ are graphs and there is a subset } G' \subseteq G \text{ isomorphic to } H \}$$

Call graphs G_1 and G_2 isomorphic if the vertices and edges of G may be relabeled to be identical to G_2 . That is, there exists a function. See https://en.wikipedia.org/wiki/Graph_isomorphism for a picture.

Show that $\text{CLIQUE} \leq_p \text{ISO}$.

(Hint: what is the type of the mapping function f in this case?)