1 Backround

Consider the PDE and the ODE

$$\frac{\partial}{\partial t}T(x,t) = S(x,t)\frac{\partial^2}{\partial x^2}T(x,t), \quad \frac{\partial}{\partial t}S(x,t) = T(x,t) - S(x,t), \quad \forall x, t$$

Then with symmetric differences

$$\frac{\partial}{\partial t}T(x,t) = S(x,t)\frac{T(x-h,t) - 2T(x,t) + T(x+h,t)}{h^2} + \mathcal{O}(h^4).$$

We now discretize the state space Ω by $[x_0, ..., x_{n+1}]$ and define $s_i(t) := S(x_i, t)$ and $T_i(t) := S_(x_i, t)$ and then get

$$\frac{d}{dt}\begin{pmatrix} T_1 \\ S_1 \\ \dots \\ T_n \\ S_n \end{pmatrix} = \begin{pmatrix} S_1(t)\frac{T_0(t) - 2T_1(t) + T_2(t)}{h^2} + \mathcal{O}(h^4) \\ T_1(t) - S_1(t) \\ \dots \\ S_n(t)\frac{T_{n-1}(t) - 2T_n(t) + T_{n+1}(t)}{h^2} + \mathcal{O}(h^4) \\ T_n(t) - S_n(t) \end{pmatrix}$$

which is a system of $2n$ ODEs.