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BRANCH: 3CS6

→ Parameter Estimation

1) $$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$X_1, X_2, X_3 \ldots X_n$ Sample of size n

$$L(X_1, X_2, X_3 \ldots X_n) = f(X_1) \cdot f(X_2) \ldots f(X_n)$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{-(x_1-\mu)^2}{2\sigma^2}} \right) \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)}{2\sigma^2}} \right)$$

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left| \frac{(x_i-\mu)^2}{2\sigma^2} \right|$$

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} - \left(\frac{2(x_i-\mu)}{2\sigma^2} \right) = 0$$

$$\sum_{i=1}^{n} (x_i - \mu) = 0$$

$$n\bar{X} - n\mu = 0 \qquad \boxed{\bar{X} = \mu}$$

$$\frac{\partial \ln(L)}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \sum_{i=1}^{n} -\frac{(x_i-\mu)}{2\sigma^2} = 0$$

$$n = \sum_{i=1}^{n} \left(\frac{(x-\mu)^2}{\sigma^2} \right) = 0$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

2) Binomial Distribution

$$^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

$$L = \prod_{i=1}^{n} {}^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

$$\log(L) = \sum_{i=1}^{n} \log(^nC_{x_i}) + \log\theta \sum_{i=1}^{n} x_i + \log(1-\theta) \sum_{i} (n-x_i)$$

$$\frac{d}{d\theta} \log(L) = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{1}{1-\theta} \sum (n - x_i) = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \sum x_i = 0$$

$$\boxed{\theta = \frac{\sum x_i}{n^2}}$$