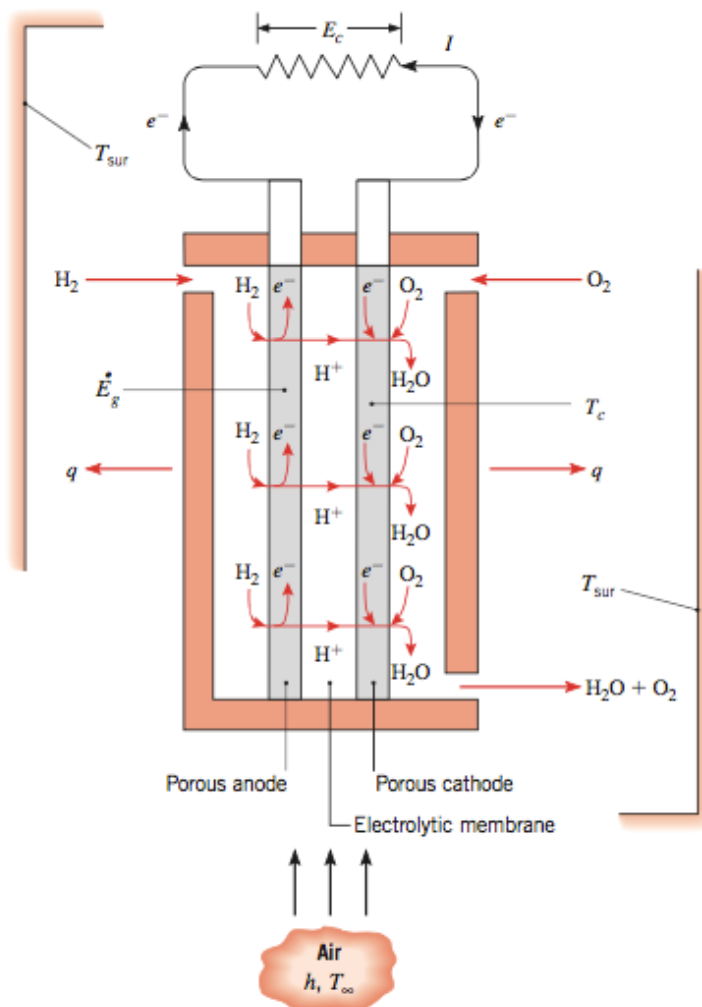




## Heat Transfer Question Sheet

Q1. A hydrogen-air Proton Exchange Membrane (PEM) fuel cell is illustrated below. It consists of an electrolytic membrane sandwiched between porous cathode and anode materials, forming a very thin, three layer membrane electrode assembly (MEA). At the anode, protons and electrons are generated ( $2\text{H}_2 \rightarrow 4\text{H}^+ + 4\text{e}^-$ ); at the cathode, the protons and electrons recombine to form water ( $\text{O}_2 + 4\text{e}^- + 4\text{H}^+ \rightarrow 2\text{H}_2\text{O}$ ). The overall reaction is then  $2\text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O}$ . The dual role of the electrolytic membrane is to transfer hydrogen ions and serve as a barrier to electron transfer, forcing the electrons to the electrical load that is external to the fuel cell.



The membrane must operate in a moist state in order to conduct ions. However, the presence of liquid water in the cathode material may block the oxygen from reaching the cathode reaction sites, resulting in the failure of the fuel cell. Therefore, it is critical to control the temperature of the fuel cell,  $T_c$ , so that the cathode side contains saturated water vapor.

For a given set of  $\text{H}_2$  and air inlet flow rates and use of a 50 mm X 50 mm MEA, the fuel cell generates  $P = 9 \text{ W}$  of electrical power. Saturated vapor conditions exist in the fuel cell,



corresponding to  $T_c = T_{sat} = 56.4^\circ\text{C}$ . The overall electrochemical reaction is exothermic, and the corresponding thermal generation rate of  $\dot{E}_g = 11.25 \text{ W}$  must be removed from the fuel cell by convection and radiation. The ambient and surrounding temperatures are  $T_\infty = T_{surr} = 25^\circ\text{C}$ , and the relationship between the cooling air velocity and the convection heat transfer coefficient  $h$  is

$$h = 10.9 \text{ W} \cdot \frac{\text{s}^{0.8}}{\text{m}^{2.8}} \cdot \text{K} \times V^{0.8}$$

Where  $V$  has units of  $\text{m/s}$ . The exterior surface of the fuel cell has an emissivity of  $\varepsilon = 0.88$ . Determine the value of the cooling air velocity needed to maintain steady-state operating conditions. Assume the edges of the fuel cell are well insulated.

Ans: **9.4 m/s**

Q2. Humans are able to control their heat production rate and heat loss rate to maintain a nearly constant core temperature of  $T_c = 37^\circ\text{C}$  under a wide range of environmental conditions. This process is called *thermoregulation*. From the perspective of calculating heat transfer between a human body and its surroundings, we focus on a layer of skin and fat, with its outer surface exposed to the environment and its inner surface at a temperature slightly less than the core temperature,  $T_i = 308 \text{ K}$ . Consider a person with a skin/fat layer of thickness  $L = 3 \text{ mm}$  and effective thermal conductivity  $k = 0.3 \text{ W/mK}$ . The person has a surface area  $A = 1.8 \text{ m}^2$  and is dressed in a bathing suit. The emissivity of the skin is  $\varepsilon = 0.95$ .

1. When the person is in still air at  $T_\infty = 297 \text{ K}$ , what is the skin surface temperature and rate of heat loss to the environment? Convection heat transfer to the air is characterized by a free convection coefficient of  $h = 2 \text{ W/m}^2\text{K}$ .
2. When the person is in water at  $T_\infty = 297 \text{ K}$ , what is the skin surface temperature and heat loss rate? Heat transfer to the water is characterized by a convection coefficient of  $h = 200 \text{ W/m}^2\text{K}$ .

Ans: **(1) 146 W, (2) 1320 W**

Q3. In a large steam power plant, the combustion of coal provides a heat rate of  $\dot{q}_{in} = 2500 \text{ MW}$  at a flame temperature of  $T_h = 1000 \text{ K}$ . Heat is rejected from the plant to a river flowing at  $T_c = 300 \text{ K}$ . Heat is transferred from the combustion products to the exterior of large tubes in the boiler by way of radiation and convection, through the boiler tubes by conduction, and then from the interior tube surface to the working fluid (water) by convection. On the cold side, heat is extracted from the power plant by condensation of steam on the exterior condenser tube surface, through the condenser tube walls by conduction, and from the interior of the condenser tubes to the river water by convection. Hot and cold side thermal resistances account for the combined effects of conduction, and radiation and under design conditions, they are  $R_{t,h} = 8 \times 10^{-8} \text{ K/W}$  and  $R_{t,c} = 2 \times 10^{-8} \text{ K/W}$ , respectively.



1. Determine the efficiency and power output of the power plant, accounting for heat transfer effects to and from the cold and hot reservoirs. Treat the power plant as an internally reversible heat engine.
2. Over time, coal slag will accumulate on the combustion side of the boiler tubes. This fouling process increases the hot side resistance to  $R_{t,h} = 2.2 \times 10^{-8} \text{ K/W}$ . Find the efficiency and power output of the plant under fouled conditions

Ans: (1) 60% and 1500 MW, (2) 58.3% and 1460 MW

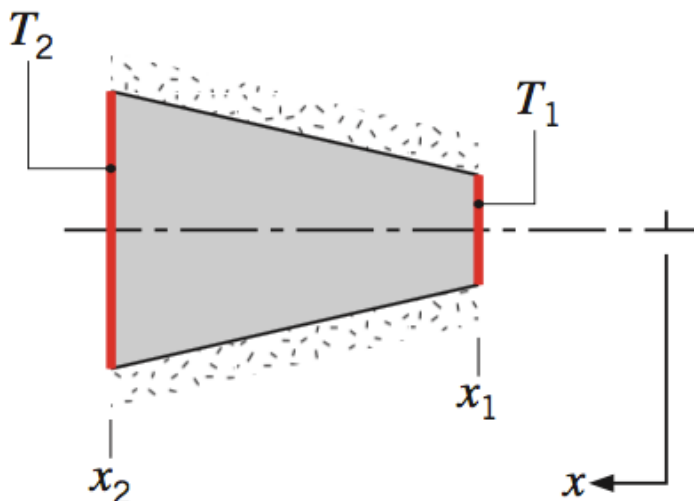
Q4. In Q2. of this sheet, we calculate the heat loss rate from a human body in air and water environments. Now we consider the same conditions except that the surroundings (air or water) are at  $10^\circ\text{C}$ . To reduce the heat loss rate, the person wears special sporting gear (snow suit and wet suit) made from a nanostructured silica aerogel insulation with an extremely low thermal conductivity of  $0.014 \text{ W/m.K}$ . The emissivity of the outer surface of the snow and wet suits is 0.95. What thickness of aerogel insulation is needed to reduce the heat loss rate to 100W (a typical metabolic heat generation rate) in air and water? What are the resulting skin temperatures?

Ans: Air = 4.4 mm, Water = 6.1 mm,  $T_s = 34.4^\circ\text{C}$

Q5. A thin silicon chip and an 8 mm thick aluminium substrate are separated by a 0.02 mm thick epoxy joint. The chip and substrate are each 10 mm on a side, and their exposed surfaces are cooled by air, which is at a temperature of  $25^\circ\text{C}$  and provides a convection coefficient of  $100 \text{ W/m}^2\text{K}$ . If the chip dissipates  $10^4 \text{ W/m}^2$  under normal conditions, will it operate below a maximum allowable temperature of  $85^\circ\text{C}$ ?

Ans:  $75.3^\circ\text{C}$ , Yes it will operate

Q6. The diagram shows a conical section fabricated from pyroceram. It is of circular cross section with the diameter  $D = ax$ , where  $a = 0.25$ . The small end is at  $x_1 = 50 \text{ mm}$  and the large end is at  $x_2 = 250 \text{ mm}$ . The end temperatures are  $T_1 = 400 \text{ K}$  and  $T_2 = 600 \text{ K}$ , while the lateral surface is well insulated.





1. Derive an expression for the temperature distribution  $T(x)$  in symbolic form, assuming one-dimensional conditions. Sketch the temperature distribution.
2. Calculate the heat rate through the cone

Ans: **-2.12 W**

Q7. A very long rod of 5 mm in diameter has one end maintained at  $100^\circ\text{C}$ . The surface of the rod is exposed to ambient air at  $25^\circ\text{C}$  with a convection heat transfer coefficient of  $100 \text{ W/m}^2\text{K}$ .

1. Determine the temperature distributions along the rods constructed from pure copper, 2024 aluminium alloy, and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?
2. Estimate how long the rods must be for the assumption of infinite length to yield an accurate estimate of the heat loss.

Ans: **(1) 5.6 W and 1.6 W, (2) 0.13 m and 0.04 m**

Q8. The engine cylinder of a motorcycle is constructed of 2024-T6 aluminium alloy and is of height  $H = 0.15 \text{ m}$  and outside diameter  $D = 50 \text{ mm}$ . Under typical operating conditions the outer surface of the cylinder is at a temperature of  $500 \text{ K}$  and is exposed to ambient air at  $300 \text{ K}$ , with a convection coefficient of  $50 \text{ W/m}^2\text{K}$ . Annular fins are integrally cast with the cylinder to increase heat transfer to the surroundings. Consider five such fins, which are of thickness  $t=6\text{mm}$ , length  $L = 20 \text{ mm}$  and equally spaced. What is the increase in heat transfer due to use of the fins?

Ans: **454 W**