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Linear Algebra-Sheet 1 on Basics of Linear Algebra

Q1. Determine the values of α, β, γ when $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal.

Answer: $\boldsymbol{\alpha = \pm\frac{1}{\sqrt{2}}, \beta = \pm\frac{1}{\sqrt{6}}, \gamma = \pm\frac{1}{\sqrt{3}}}$

Q2. If A is real skew symmetric matrix such that $A^2 + I = 0$, show that A is orthogonal and is of even order.

Q3. Find the inverse of the matrix $S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show that the transform of the matrix $A = \begin{bmatrix} b+c & c+a & b-c \\ c-b & c+b & a-b \\ b-c & a-c & a+b \end{bmatrix}$ by S, i.e. SAS^{-1} is a diagonal matrix.

Q4. P, Q are non singular matrices. Show that if

$$A = \begin{bmatrix} P & O \\ O & Q \end{bmatrix}, then\ A^{-1} = \begin{bmatrix} P^{-1} & O \\ O & Q^{-1} \end{bmatrix}$$

Q5. If $f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix}$ then show that f(x) is linear in x. Also deduce that $f(0) = \frac{bg(a)-ag(b)}{(b-a)}$, where $g(x) = (c_1 - x)(c_2 - x)(c_3 - x)$.

Q6. Find the value of $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$ where $l_1^2 + m_1^2 + n_1^2 = 1, etc. and\ l_1 l_2 + m_1 m_2 + n_1 n_2 = 0\ etc.$

Answer: $\Delta = \pm\mathbf{1}$

Q7. If $f(x) = \begin{vmatrix} sin^5 x & \log sinx & \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{cos x}} \\ n & \sum_{k=1}^{n} k & \prod_{k=1}^{n} k \\ 8/15 & \frac{\pi}{2}\log 2 & \frac{\pi}{4} \end{vmatrix}$. Then find the value of $\int_0^{\pi/2} f(x)dx$

Answer: 0

Q8. Let n be a positive integer and $\Delta_r = \begin{vmatrix} 2r-1 & n_{C_r} & 1 \\ n^2-1 & 2^n & n+1 \\ tan^2(n^2) & tan^2(n) & tan^2(n+1) \end{vmatrix}$. Then prove that $\sum_{r=0}^{n} \Delta_r = 0$.