Linear Model

1. Linear Regression

1.1 Simple Linear Regression

- ***model formulation***

$$
\begin{aligned}
\hat{y}_i &= wx_i + b \\
J(w,b) &= \frac{1}{m}\sum_{i=1}^{m}(\hat{y}_i - y_i)^2 \\
(w^*, b^*) &= \underset{(w,b)}{\operatorname{argmin}} J(w,b)
\end{aligned}
$$

- ***parameter estimation***

$$
\begin{aligned}
\frac{\partial J}{\partial w} &= \frac{2}{m}\sum_i(\hat{y}_i - y_i)\frac{\partial}{\partial w}(\hat{y}_i - y_i) = \frac{2}{m}\sum_i(\hat{y}_i - y_i)\,x_i = \frac{2}{m}\left(w\sum_i x_i^2 - \sum_i (y_i - b)x_i\right) \\
\frac{\partial J}{\partial b} &= \frac{2}{m}\sum_i(\hat{y}_i - y_i)\frac{\partial}{\partial b}(\hat{y}_i - y_i) = \frac{2}{m}\sum_i(\hat{y}_i - y_i) \; = \frac{2}{m}\left(mb - \sum_i (y_i - wx_i)\right)
\end{aligned}
$$

- ***closed-form solution***

$$
\begin{aligned}
w^* &= \frac{\sum y_i(x_i - x)}{\sum x_i^2 - \frac{1}{m}\left(\sum x_i\right)^2} \\
b^* &= \frac{1}{m}\sum_{i=1}^{m}(y_i - wx_i)
\end{aligned}
$$

1.2 Multiple Linear Regression

$$
\hat{w} = (\mathbf{w}; b)\mathbf{X} = \begin{pmatrix} \mathbf{x_1} & 1 \\ \boldsymbol{x_2} & \end{pmatrix}
$$

$$
\mathbf{w}^T x
$$

2. Logistic Regression