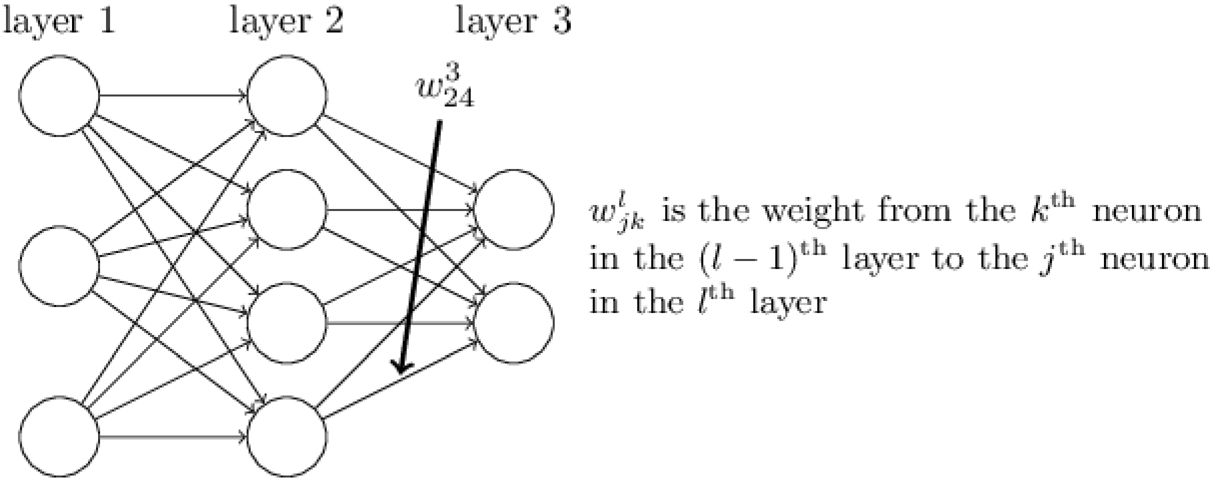
Derive backpropagation

1 Forward Propagation



$$\boldsymbol{a}^l = \sigma(\boldsymbol{z}^l)$$

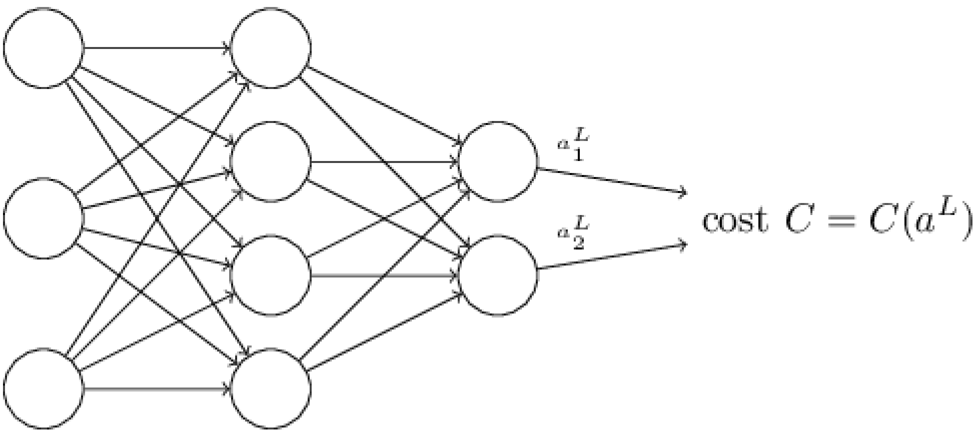
$$\boldsymbol{z}^l = \boldsymbol{w}^l \boldsymbol{a}^{l-1} + \boldsymbol{b}^l$$

non-vectored form

$$a^l_j = \sigma(z^l_j)$$

$$z^l_j = \sum_k w^l_{jk} a^{l-1}_k + b^l_j$$

2 Backward Propagation



$$\delta^l_j \equiv \frac{\partial C}{\partial z^l_j}$$

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delta衡量的是对神经元输出激励值的误差
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$$\boldsymbol{\delta}^L = \frac{\partial C}{\partial \boldsymbol{a}^L} \odot \sigma'(\boldsymbol{z}^L) = \nabla_a C \odot \sigma'(\boldsymbol{z}^L) \tag{1}$$

$$\boldsymbol{\delta}^l = ((\boldsymbol{w}^{l+1})^T \boldsymbol{\delta}^{l+1}) \odot \sigma'(\boldsymbol{z}^l) \tag{2}$$

$$\frac{\partial C}{\partial b^l} = \delta^l \tag{3}$$

$$\frac{\partial C}{\partial w^l_{jk}} = \delta^l_j a^{l-1}_k \tag{4}$$

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a(in)是输入给权重w的神经元的激活值，δ(out)是输出自权重w的神经元的误差，那么代价函数对参数w的偏导如下：
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$$\frac{\partial C}{\partial w} =$$

3 Proof and Derivation

- *3.1 The first equation*

$$\begin{aligned}
\delta^L_j &= \sum_k \frac{\partial C}{\partial a^L_k} \cdot \frac{\partial a^L_k}{\partial z^L_j} \\
&= \frac{\partial C}{\partial a^L_j} \cdot \frac{\partial a^L_j}{\partial z^L_j} \\
&= \frac{\partial C}{\partial a^L_j} \sigma'(z^L_j)
\end{aligned}$$

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当j不等于k时，a(k)对z(j)偏导为0
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- *3.2 The second equation*

$$\begin{aligned}
\delta_j^l &= \frac{\partial C}{\partial z_j^l} \\
&= \frac{\partial C}{\partial a_j^l} \cdot \frac{\partial a_j^l}{\partial z_j^l} \\
&= \frac{\partial C}{\partial a_j^l} \sigma'(z_j^l) \\
&= \sum_k \frac{\partial C}{\partial z_k^{l+1}} \cdot \frac{\partial z_k^{l+1}}{\partial a_j^l} \cdot \sigma'(z_j^l) \\
&= \sum_k \delta_k^{l+1} w_{kj} \sigma'(z_j^l)
\end{aligned}$$

- *3.3 The third equation*

$$\frac{\partial C}{\partial b_j^l} = \frac{\partial C}{\partial z_j^l} \cdot \frac{\partial z_j^l}{\partial b_j^l} = \frac{\partial C}{\partial z_j^l} \cdot 1 = \delta_j^l$$

- *3.4 The fourth equation*

$$\begin{aligned}
\frac{\partial C}{\partial w_{jk}^l} &= \frac{\partial C}{\partial z_j^l} \cdot \frac{\partial z_j^l}{\partial w_{jk}^l} \\
&= \delta_j^l \cdot \frac{\partial z_j^l}{\partial w_{jk}^l} \\
&= \delta_j^l \cdot \frac{\partial(\sum_k a_k^{l-1} w_{jk}^l + b_j^l)}{\partial w_{jk}^l} \\
&= \delta_j^l a_k^{l-1}
\end{aligned}$$