Flipping a difference estimate in confidence intervals

Consider two estimators $\hat{a}, \hat{b}$. For example, these might be the estimates of regression coefficients.

Suppose you want a confidence interval for $(\hat{a} - \hat{b})$. Then the interval will be of the form

$$\Big((\hat{a} - \hat{b}) - \Delta \ \mathrm{se}(\hat{a} - \hat{b}), \ \ (\hat{a} - \hat{b}) + \Delta \ \mathrm{se}(\hat{a} - \hat{b})\Big)$$

for Δ dependent on whether you are using an indvidual interval or one of the joint interval forms.

What happen if you now want the interval for $(\hat{b} - \hat{a})$? Then the form is

$$\Big((\hat{b} - \hat{a}) - \Delta \ \mathrm{se}(\hat{b} - \hat{a}), \ \ (\hat{b} - \hat{a}) + \Delta \ \mathrm{se}(\hat{b} - \hat{a})\Big)$$

Though not immediately obvious, this is simply an inverted form of the first interval. To prove this, we first observe that

$$\mathrm{Var}(\hat{b} - \hat{a}) = \mathrm{Var}((-1)(\hat{a} - \hat{b})) = (-1)^2 \mathrm{Var}(\hat{a} - \hat{b}) = \mathrm{Var}(\hat{a} - \hat{b}).$$

Since the variances are equal, the standard errors are equal:

$$\mathrm{se}(\hat{b} - \hat{a}) = \sqrt{\mathrm{Var}(\hat{b} - \hat{a})} = \sqrt{\mathrm{Var}(\hat{a} - \hat{b})} = \mathrm{se}(\hat{a} - \hat{b})$$

Thus,

$$\begin{gathered}
\Big((\hat{b} - \hat{a}) - \Delta \ \mathrm{se}(\hat{b} - \hat{a}), \ \ (\hat{b} - \hat{a}) + \Delta \ \mathrm{se}(\hat{b} - \hat{a})\Big) \\
\Big((\hat{b} - \hat{a}) - \Delta \ \mathrm{se}(\hat{a} - \hat{b}), \ \ (\hat{b} - \hat{a}) + \Delta \ \mathrm{se}(\hat{a} - \hat{b})\Big) \\
\Big(-(\hat{a} - \hat{b}) - \Delta \ \mathrm{se}(\hat{a} - \hat{b}), \ \ -(\hat{a} - \hat{b}) + \Delta \ \mathrm{se}(\hat{a} - \hat{b})\Big) \\
\Big(-\Big[(\hat{a} - \hat{b}) + \Delta \ \mathrm{se}(\hat{a} - \hat{b})\Big], \ \ -\Big[(\hat{a} - \hat{b}) - \Delta \ \mathrm{se}(\hat{a} - \hat{b})\Big]\Big) \\
\Big(-[\text{upper bound for } \hat{a} - \hat{b}], \ \ -[\text{lower bound for } \hat{a} - \hat{b}]\Big).
\end{gathered}$$

In other words, in flipping our estimate of interest from $\hat{a} - \hat{b}$ to $\hat{b} - \hat{a}$, we have transformed our resulting interval from

$$(x, y) \to (-y, -x)$$

where x, y are the appropriate values.