

A.) 1.) $R(Y, \theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2.) $R(z, \theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3.) IT can be written as the following with a vector $n = (-Dy/Dr, Dx/Dr, 0)$. This vector and rotation angle $\theta = Dr / R$ are used to do 3d rotation

4.) $P1 = (1,1,1)$ First we want to Translate $P0$ to the origin. $P0 = (1,1,0)$ $T(p0) = \begin{bmatrix} 1 & 0 & 0 & -X0 \\ 0 & 1 & 0 & -Y0 \\ 0 & 0 & 1 & -Z0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Now we rotate about the X axis

$$L = \sqrt{P1x^2 + P1y^2 + P1z^2}$$

$$V = \sqrt{P1y^2 + P1z^2} = \sqrt{2} \quad \sin(\theta) = P1y/V \quad \cos(\theta) = P1z/V$$

So the matrix is

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & P1z/V & -P1y/V & 0 \\ 0 & P1y/V & P1z/V & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sin(\theta) = -P1x/L$$

$$\cos(\theta) = V/L$$

Now we do the same thing about the Y axis

$$R_y = \begin{bmatrix} V/L & 0 & -P1x/L & 0 \\ 0 & 1 & 0 & 0 \\ P1x/L & 0 & V/L & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we rotate by an angle θ about the Z axis

$$R(z, \theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally. We take the inverse of rotation R_y then the inverse of rotation R_x and then finally the inverse of our translation $T(p0)$

So our overall ModelView matrix looks like this

$$[M] = [T(P0)]^{-1} * [R_x]^{-1} * [R_y]^{-1} * [R_z(\theta)] * [R_y] * [R_x] * [T(P0)]$$

B.)

1. We have the following defined 3D coordinates:

$$u = x / [(1 / f) (D - z)] = x' / w'$$

$$v = y / [(1 / f) (D - z)] = y' / w'$$

$$\lambda = D - f = z' / w'$$

We get this matrix using this

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -(1/f)(D-f) & (D/f) * (D-f) \\ 0 & 0 & -(1/f) & D/f \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$2. u = (f \cdot x) / -z$$

$$v = (f \cdot y) / -z$$

3. This would be:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -(1/f) & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

If at $z = -f$, we plug $-f$ into the 2nd formulas for u and v for z . This results in being $-(-f)$, which gives the equations

$$u = (f \cdot x) / f$$

$$v = (f \cdot y) / f$$