



**Controlling Infectious Diseases**  
(vaccination, isolation & quarantine, & physical distancing)

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Math 6115/Bonne Bay 2023

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**How do we control SARS-CoV-2 with a vaccine?**

$R_0 = c \times p \times D$   
c = contact rate  
p = probability of transmission given an infected contact  
D = duration of infectiousness

WHAT IS PHYSICAL DISTANCING?



If you must go out, keep a distance of 6 feet from others.

Public health remains a top priority. Continue to:

- physical distancing
- wear a face covering when physical distancing is a challenge or where required
- wash hands frequently and thoroughly
- avoid touching your eyes, nose and mouth.

Combined with rapid test, trace, isolate.

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**And now some vaccine math...**

The effective reproduction number ( $R_e$ ) varies depending on the basic reproduction number ( $R_0$ ), the proportion of the population that is fully vaccinated ( $x$ ), and the effectiveness of the vaccine ( $V$ ).

$R_e = R_0(1-x*V)$

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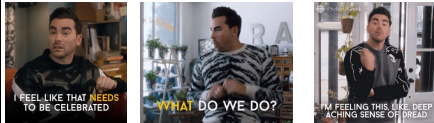
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$R_e$  for **OG** =  
 $2.50 \cdot (1 - 0.76 \cdot 0.94)$   
 $R_e$  (vaccine alone) = 0.73

$R_e$  for **alpha** =  
 $3.75 \cdot (1 - 0.76 \cdot 0.94)$   
 $R_e$  (vaccine alone) = 1.1

$R_e$  for **delta** =  
 $7.5 \cdot (1 - 0.76 \cdot 0.85)$   
 $R_e$  (vaccine alone) = 2.6



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$R_e$  for omicron (25 weeks after dose 2) =  
 $12 \cdot (1 - 0.76 \cdot 0.09)$   
 $R_e$  (vaccine alone) = 11.6

$R_e$  for omicron (10 weeks after dose 3) =  
 $12 \cdot (1 - 0.76 \cdot 0.46)$   
 $R_e$  (vaccine alone) = 7.8



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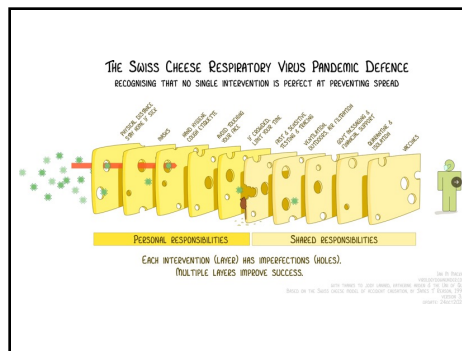
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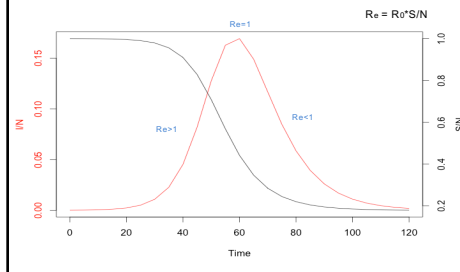
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## Anatomy of an Epidemic



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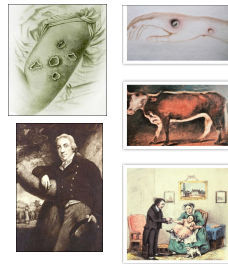
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## Edward Jenner



Source: National Library of Medicine & the Jenner Museum

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## Smallpox Eradication



- Intensive global campaign by WHO results in smallpox eradication in 1977 (last case, Somalia).
- Key factors: human reservoir, effective (live) vaccine, cases easily identified.

Image source: CDC Public Health Image Library. <http://nhil.cdc.gov>

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### Vaccination: Concept

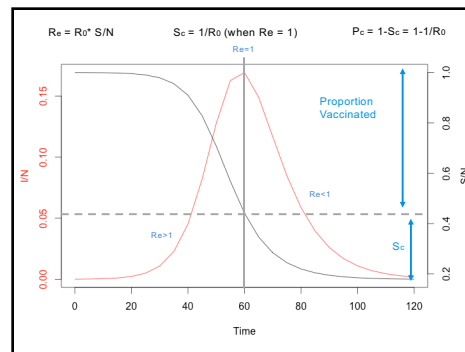
- Vaccines are biological substances or microorganisms that confer **immunity** to infectious disease that could otherwise only be obtained through natural infection.
  - Move directly to immune state without having to risk morbidity and (sometimes) mortality associated with natural infection.
- **Utilitarian** framework:
  - Vaccines not risk free, societal adoption of vaccination presumes **net** reduction in mortality & morbidity (and sometimes costs).

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### What Are We Trying To Achieve Through Vaccination?

- Protection of an **individual** who encounters a source of infection.
  - Modification of clinical illness, if vaccination fails.
- Elimination of conditions that permit **disease transmission** in the population ("herd immunity").
  - Elimination (from geographic area) or eradication (extinction) of infectious disease.

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### Conditions for Herd Immunity (Vaccine with 100% Efficacy)

For no epidemic,  $R_e < 1$

$$R_e = (1 - P_c) \times R_0$$

$$(1 - P_c) \times R_0 < 1$$

$$1 - P_c < (1/R_0)$$

$$1 - (1/R_0) < P_c$$

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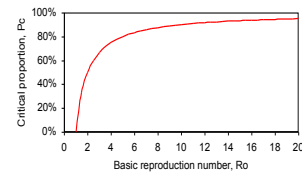
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### Example: Critical Fraction to Vaccinate

- For a disease with  $R_0 = 3$
- $1 - 1/R_0 = 1 - 0.33 = 0.67$
- $R_e < 1$  when  $P_c > 0.67$  or 67%.



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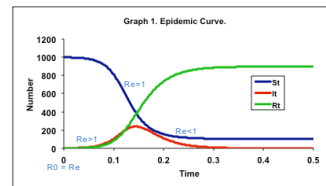
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### Why does the incidence of an immunizing infection cycle over time?

1. What can we say about the  $R_e$  when the disease incidence is 1) increasing, 2) decreasing, 3) at the peak?



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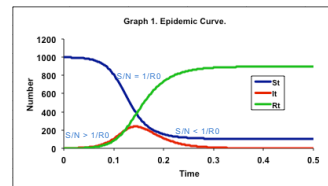
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1. Given that  $R_e$  is related to the proportion of the population susceptible

$$R_e = R_0 * (\text{proportion Susceptible})$$

What can we say about the proportion of the population susceptible when disease incidence is 1) increasing, 2) decreasing, and 3) at the peak?



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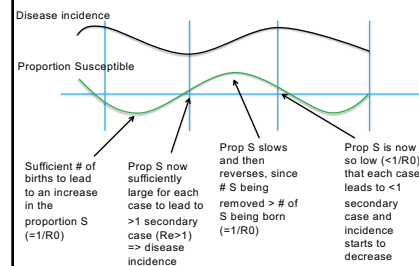
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### Entry of new susceptibles



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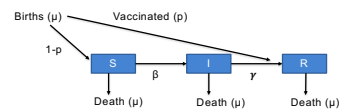
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### Measures to reduce S/N

- Pediatric immunization (treated as fraction,  $p$ , of newborns vaccinated)



Write out the equations!

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$$\frac{dS}{dt} = \mu(1-p) - \beta SI - \mu S$$

$$\frac{dI}{dt} = \beta SI - (\mu + \gamma)I$$

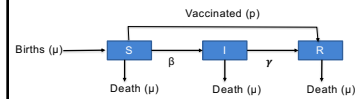
$$\frac{dR}{dt} = \mu p + \gamma I - \mu R$$

Eradication requires that  $p = 1-1/R_0$   
This is the fraction of newborns to be immunized for (eventual) control.

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### Measures to reduce S/N

- What if we can't/don't vaccinate newborns?  
Need continuous vaccination instead. Is there anything this figure doesn't account for?  
Simplifying assumptions?



Write out the equations!

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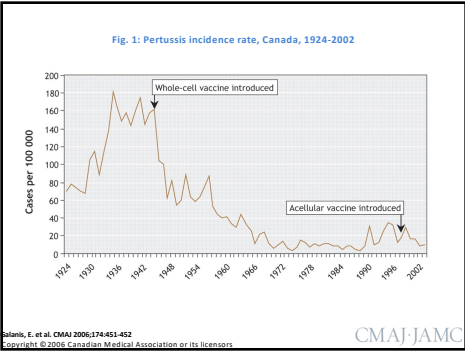
$$\frac{dS}{dt} = \mu - \beta SI - \mu S - pS$$

$$\frac{dI}{dt} = \beta SI - (\mu + \gamma)I$$

$$\frac{dR}{dt} = pS + \gamma I - \mu R$$

Eradication requires that  $p \geq \mu(R_0 - 1)$   
This is the rate of susceptibles to be immunized for (eventual) control.

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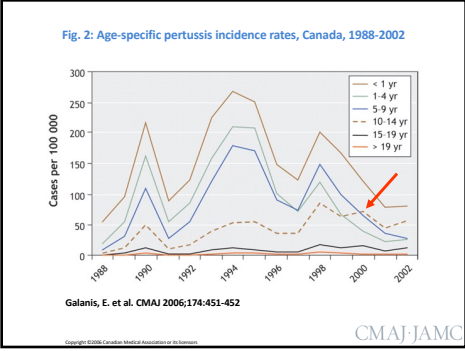
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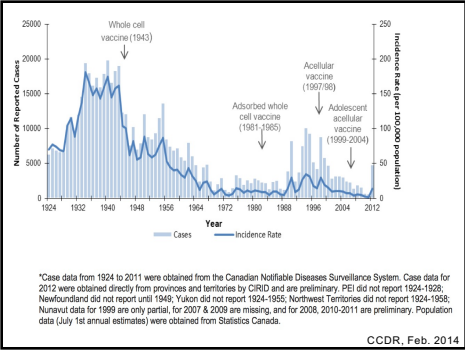
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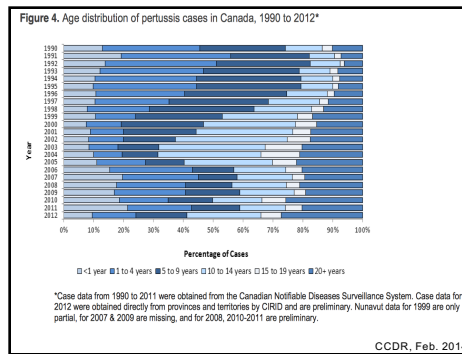
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### Age Structure and Partial Vaccination

- Can build **age-structured** models by subdividing model “compartments” to reflect different age-groups.
- Using age-structured model can derive the relationship:

$$R_0 \approx L/A$$

- L=life expectancy, and A = average age at infection.

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### With Vaccination

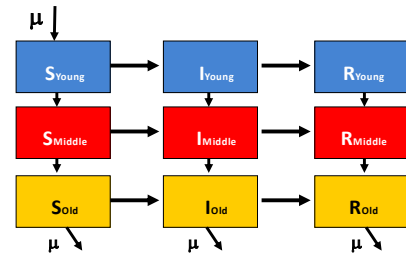
$$\downarrow R \approx L/A \uparrow$$

- When might age at first infection be relevant?
- How might partial vaccination of the herd be **harmful**?

$$A \approx \frac{1}{\mu R_0} \Rightarrow A \approx \frac{L}{R_0} \Rightarrow R_0 \approx \frac{L}{A}$$

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### Simple Age-Structured Model



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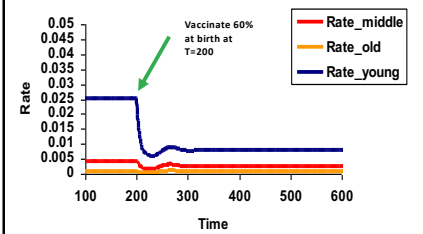
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Rate of Infection per Unit Time



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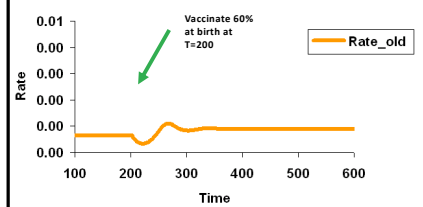
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Rate of Infection per Unit Time, Oldest Only



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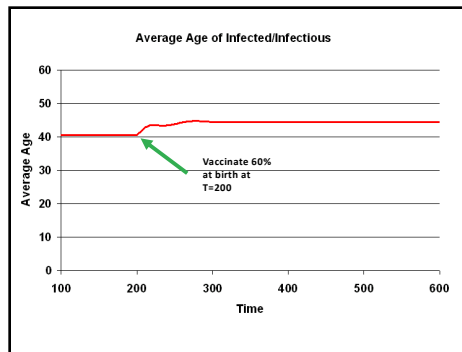
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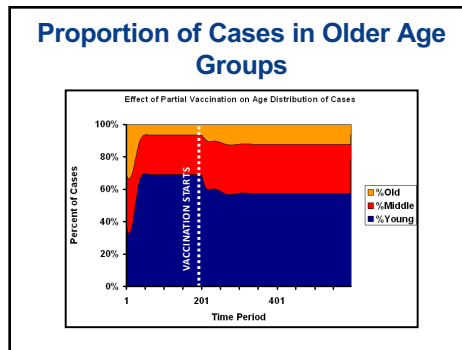
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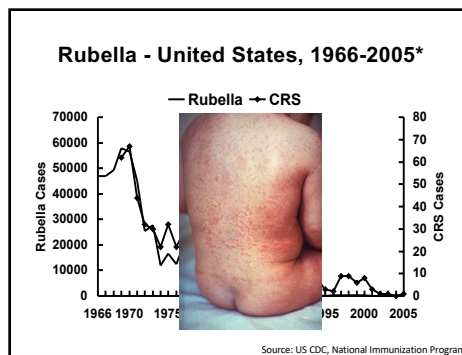
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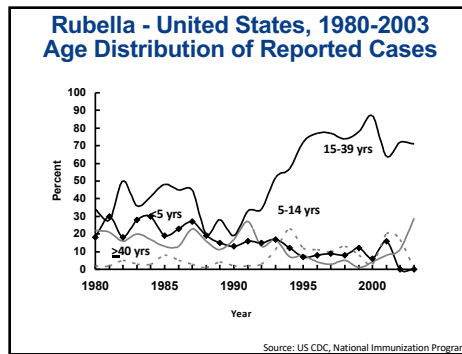
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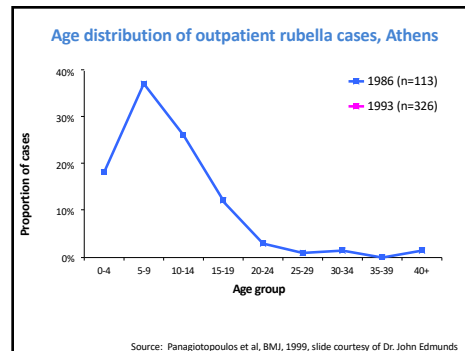
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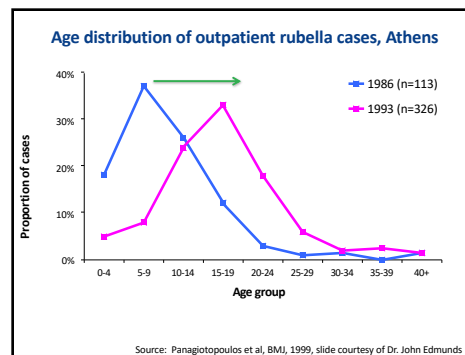
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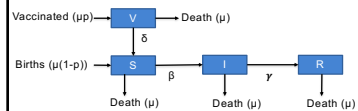
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### Imperfect vaccines

- In some cases, vaccine induced immunity wanes over time.



Write out the equations!

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$$\frac{dS}{dt} = \mu(1-p) - \beta SI - \mu S + \delta V$$

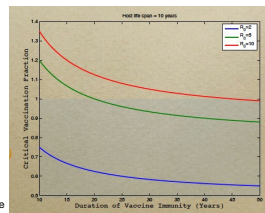
Eradication requires that:

$$\frac{dI}{dt} = \beta SI - (\mu + \gamma)I$$

$$p = \left(1 - \frac{1}{R_0}\right)\left(1 + \frac{\delta}{\mu}\right)$$

$$\frac{dV}{dt} = \mu p - (\mu + \delta)V$$

Eradication is going to require boosters!



From Rohani and Drake

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### “Catch Up” Vaccination and Boosting

- Catch-up vaccination:** Means of overcoming increased age at infection due to decreased FOI when new vaccine introduced.
  - E.g., U.K. introduces rubella vaccine *simultaneously* for infants and 12 year-olds.
    - No late spike in CRS in U.K. following rubella vaccination, unlike U.S.
- Boosting** when immunity wanes: diminished secondary failures.
  - E.g., pertussis boosting now advocated for pre-teens in Canada.

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### Summary

- Partial vaccination of herd predicted to increase average age at infection via decreased FOI, cohort effect, and advanced age at secondary failure.
  - May be desirable if disease is more dangerous to young, undesirable if more dangerous to older individuals.
- Overcome through “catch-up” vaccination and **boosting**.

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### Non-pharmaceutical interventions

- Physical distancing
- Test, trace, isolate
- Quarantine

What are some of the challenges with these types of interventions?

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### Pandemic planning as a case study

- Emergence of a novel pathogen
- Entire population is Susceptible
- No pharmaceutical interventions
- NPI are the only option
- How long?
- To what extent?

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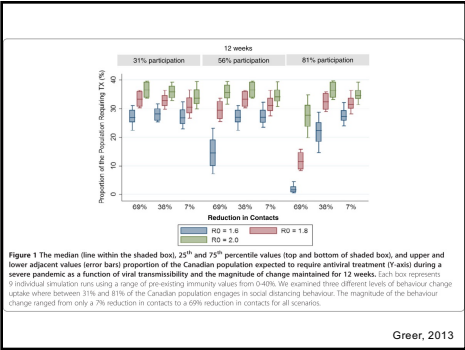
Table 1 Parameter values and assumptions used for the Canadian antiviral stockpile model			
Item	Strain	Value (range)	Reference(s)
Transmissibility	1951/1958	1.6 (attack rate = 37%)	[8,26,34-36]
R0	1968/1969	1.8 (attack rate = 41%)	[8,26,34,35,37,38]
	1978	2.0 (attack rate = 45%)	[8,26,34,35,39,40]
Natural history			
Latent period	Seasonal	2.1 days	[41]
Duration of infection	Seasonal	4.8 days	[42]
Pre-existing immunity in individuals > years		0% (0 - 40%)	Assumption
Clinical characteristics			
Proportion symptomatic	1957	65%	[43]
Proportion of symptomatic cases seeking medical attention	1957	70%	[43]

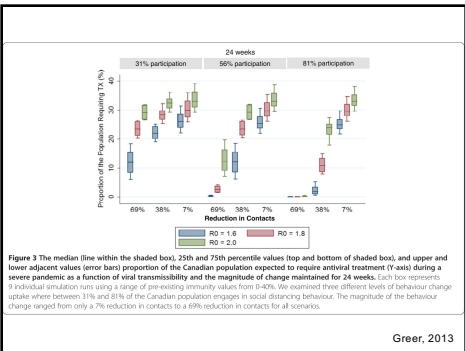
Table 2 Parameter values and assumptions for implementing social distancing into the model			
Social distancing parameters			
Item	Experience	Value (range examined)	Reference
Proportion of the population intending to engage in social distancing	1. Avoiding crowded places	56% (31-81%)	[1,2,31-53]
	2. Avoiding public transportation		
	3. Avoiding public places		
	4. Changing school / work arrangements		
Magnitude of behaviour change	Percent reduction in contacts	38% (7 - 69%)	[13]
Start of social distancing behaviour	Assumption	2 weeks after first imported case	Assumption
Number of weeks that behaviour change is maintained	Assumption	16 weeks (12 - 24 weeks)	[53]
Effect of age on behaviour change	Assumption	No effect	[1,56]

Greer, 2013

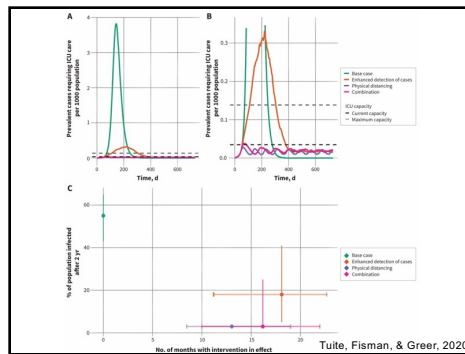
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### Contact Tracing and Isolation

- Assume average contact rate,  $\kappa$
- Transmission probability,  $\nu$
- Infectious individuals immediately symptomatic
- Infectious isolated at rate  $d_I$
- Fraction  $q$  of contacts with infectious individuals quarantined
- Kept in quarantine for average  $\tau_Q$

From Rohani and Drake

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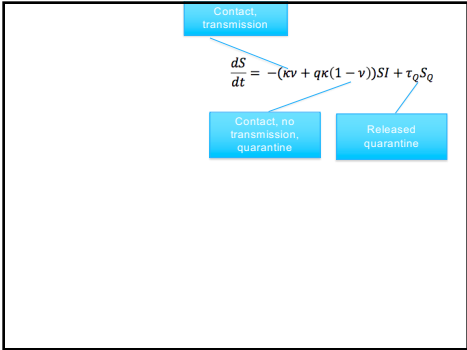
### We now have 5 compartments

- Susceptible
- Susceptible in **Quarantine**
- Infectious
- Infectious in **Isolation**
- Recovered

Based on the previous slide, draw out the compartment model and write the associated equations

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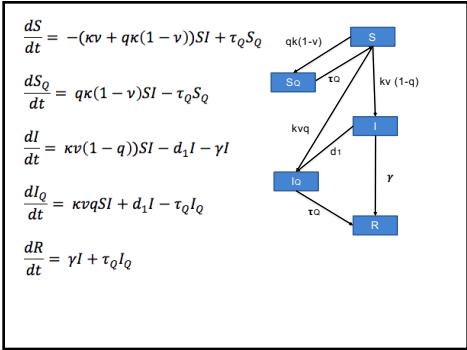
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What does this tell us?

- Can show that control requires

$$S < \frac{d_1 + \gamma}{\kappa v(1 - q)}$$

$$\kappa v(1 - q) > d_1 + \gamma$$

- If  $R_0 = 5$ , then  $\tau_Q = 21$  days

From Rohani and Drake

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But there are some  
challenges...

- Assumed infected individuals are immediately symptomatic
- Uncertainties and delays with identifying and isolating potential contacts
- How do these factors work to complicate our ability to control SARS-CoV-2?

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