Consideration of objects between a camera and the target for infrared measurements

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In making infrared measurements, it is usually the case that objects, including air, exist between the camera and the target that affect the infrared radiation between the two. In addition, all objects reflect some amount of radiation from the environment. A method is detailed here of using summary properties of the objects involved to estimate the actual radiance emitted by the target. Self-emission, transmittances, and reflectances of all objects are considered, also allowing for reflections between objects.

When using infrared imaging to measure object temperatures, there are often objects along the imaging path that may alter the radiation through absorption, reflection, or addition of their own emission. These should all be taken into account when using a measured radiance to estimate that of a target. In this work, the properties of each object are assumed to be single values, and should therefore be calculated with the spectral response of the detector included.

An example system is shown in Figure 1, indicating that each object n out of N objects can have its own radiance M_n, emittance ϵ_n, transmittance τ_n, specular reflectance ρ_n, and diffuse reflectance σ_n. Object radiances are generally taken to be that of a black body at the same temperature, as wavelength dependant emittances are seldom known. Also defined are the properties of the target (with subscribt T), radiance of the background around the system M_B, the emission measured by the camera M_M, and F_n and B_n, being the forward and backward travelling radiation, respectively, between objects n and $n+1$.

From this arrangement, one can begin to define the quantities F_n and B_n in a recursive manner based on the properties of the objects. The quantity F_{n-1} must be the result of the interaction of object n with B_{n-1} and F_n, leading to Equations (1) and (2).

$$F_{n-1} = \rho_n B_{n-1} + \tau_n F_n + \epsilon_n M_n + \sigma_n M_B \tag{1}$$

$$F_n = \frac{F_{n-1} - \rho_n B_{n-1} - \epsilon_n M_n - \sigma_n M_B}{\tau_n} \tag{2}$$

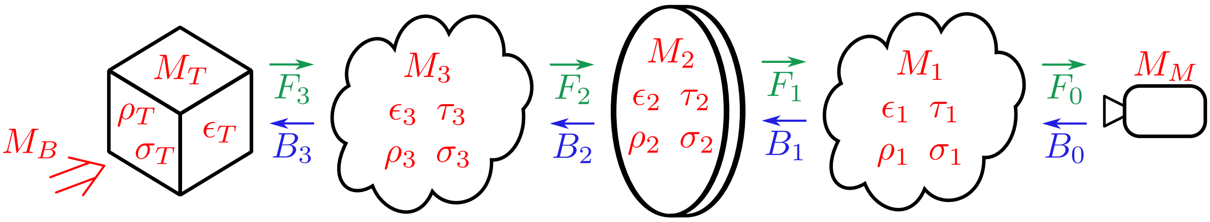

Figure 1: An example system with a window between the camera and the target, showing the atmosphere between each object should also be considered.

Similarly, B_n also results from the same interaction, but in the other direction, as given in Equation (3).

$$
\begin{aligned}
B_n &= \rho_n F_n + \tau_n B_{n-1} + \epsilon_n M_n + \sigma_n M_B \\
&= \rho_n \frac{F_{n-1} - \rho_n B_{n-1} - \epsilon_n M_n - \sigma_n M_B}{\tau_n} + \tau_n B_{n-1} + \epsilon_n M_n + \sigma_n M_B \\
&= \frac{\rho_n}{\tau_n} F_{n-1} + \left(\tau_n - \frac{\rho_n^2}{\tau_n} \right) B_{n-1} + \left(1 - \frac{\rho_n}{\tau_n} \right) (\epsilon_n M_n + \sigma_n M_B)
\end{aligned}
\tag{3}
$$

The pair of coupled recursive equations Equations (2) and (3) are linear in F_{n-1} and B_{n-1}, and can be turned into a single recursive matrix equation, Equation (4), by defining the matrices that follow.

$$
\boldsymbol{X}_n = \begin{bmatrix} F_n \\ B_n \end{bmatrix} \qquad \boldsymbol{A}_n = \begin{bmatrix} \frac{1}{\tau_n} & -\frac{\rho_n}{\tau_n} \\ \frac{\rho_n}{\tau_n} & \tau_n - \frac{\rho_n^2}{\tau_n} \end{bmatrix} \qquad \boldsymbol{b}_n = \begin{bmatrix} -\frac{\epsilon_n M_n + \sigma_n M_B}{\tau_n} \\ \left(1 - \frac{\rho_n}{\tau_n}\right) (\epsilon_n M_n + \sigma_n M_B) \end{bmatrix}
$$

$$
\boldsymbol{X}_n = \boldsymbol{A}_n \boldsymbol{X}_{n-1} + \boldsymbol{b}_n \tag{4}
$$

With the relatively light assumption that $F_0 = M_M$ and that $B_0 = 0$ (i.e. negligible radiation is travelling from the camera back toward the target), $\boldsymbol{X}_0 = [M_M \;\; 0]^T$, and the system can be solved iteratively if the properties of all objects (apart from the target) are known.

To avoid iterative solutions, the recursion can be unravelled into a single formula, given in Equation (5). In this and following expressions, "Big-Pi" notation is used to mean matrix multiplication, with subsequent terms being ordered left to right.

$$
\boldsymbol{X}_n = \left(\prod_{i=1}^{n} \boldsymbol{A}_{n-i} \right) \boldsymbol{X}_0 + \sum_{i=0}^{n-2} \left(\prod_{j=1}^{n-i-1} \boldsymbol{A}_{n-j} \right) \boldsymbol{b}_i + \boldsymbol{b}_n \tag{5}
$$

By isolating the two constant terms, they can be taken out of the expression and evaluated individually to give a single, non-recursive, expression for $\boldsymbol{X}_N$ in Equation (6), from which the value of F_N can be extracted.

$$
\boldsymbol{D}_n = \prod_{i=1}^{n} \boldsymbol{A}_{n-i} \qquad \boldsymbol{C}_n = \sum_{i=0}^{n-2} \left(\prod_{j=1}^{n-i-1} \boldsymbol{A}_{n-j} \right) \boldsymbol{b}_i + \boldsymbol{b}_n
$$

$$
\boldsymbol{X}_N = \boldsymbol{D}_N \boldsymbol{X}_0 + \boldsymbol{C}_N \tag{6}
$$

The value of F_N does not immediately give us the estimated object radiance however, as it is also reflecting some radiance from the background and from B_N. The quantity M_T is the radiance after correcting for this, and is given in Equation (7).

$$
\begin{aligned}
M_T &= \frac{F_N - \sigma_T M_B - \rho_T B_N}{\epsilon_T} \\
M_T &= \frac{1}{\epsilon_T} F_N - \frac{\rho_T}{\epsilon_T} B_N - \frac{\sigma_T}{\epsilon_T} M_B \\
M_T &= \begin{bmatrix} \frac{1}{\epsilon_T} & -\frac{\rho_T}{\epsilon_T} \end{bmatrix} \boldsymbol{X}_N - \frac{\sigma_T}{\epsilon_T} M_B
\end{aligned}
\tag{7}
$$

$$\boldsymbol{P}_T = \begin{bmatrix} \frac{1}{\epsilon_T} & -\frac{\rho_T}{\epsilon_T} \end{bmatrix} \qquad \boldsymbol{K} = \boldsymbol{P}_T \boldsymbol{D}_N \qquad l = \boldsymbol{P}_T \boldsymbol{C}_N - \frac{\sigma_T}{\epsilon_T} M_B$$

$$M_T = \boldsymbol{K}\boldsymbol{X}_0 + l \tag{8}$$

By collecting terms into Equation (8), and extracting M_M as the first element of $\boldsymbol{X}_0$, one arrives at a linear relationship between the measured radiance M_M and the estimated radiance of the target object M_T, with coefficients that are fully determined by the properties of the N objects and the radiance of the background. This is given in Equation (9), where k_1 is the first element of $\boldsymbol{K}$.

$$M_T = k_1 M_M + l \tag{9}$$

The coefficients of the linear expression in Equation (9) are independent of the measured radiance, and therefore only need to be calculated once for each system, which can improve efficiency.