Consideration of multiple (infrared) optical components between a camera and the target object

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In making infrared measurements, it is usually the case that objects, including air, exist between the camera and the target that affect the infrared radiation between the two. In addition, all objects reflect some amount of radiation from the environment. A method is detailed here of using summary properties of the objects involved to estimate the actual radiance emitted by the target. Self-emission, transmittances, and reflectances of all objects are considered, also allowing for reflections between objects.

The System

When using infrared imaging to measure object temperatures, there are often objects along the imaging path that may alter the radiation through absorption, reflection, or addition of their own emission. These should all be taken into account when using a measured radiance to estimate that of a target. In this work, the properties of each object are assumed to be single values, and should therefore be calculated with the spectral response of the detector included.

An example system is shown in Figure 1, indicating that each object n out of N objects can have its own self-emission M_n, transmittance τ_n, and reflectance ρ_n. Also defined are the emittance ϵ_T and self-emission M_T of the target, self-emission of the background around the target M_B, the emission measured by the camera M_M, and F_n and B_n, being the forward and backward travelling radiation, respectively, between objects n and $n+1$. The self-emission of each object, with the exception of the target, should already have the effect of any reflected background taken into account.

From this arrangement, one can begin to define the quantities F_n and B_n in a recursive manner based on the properties of the objects. The quantity F_{n-1} must be the result of the interaction of object n with B_{n-1} and F_n, leading to Equations (1) and (2).

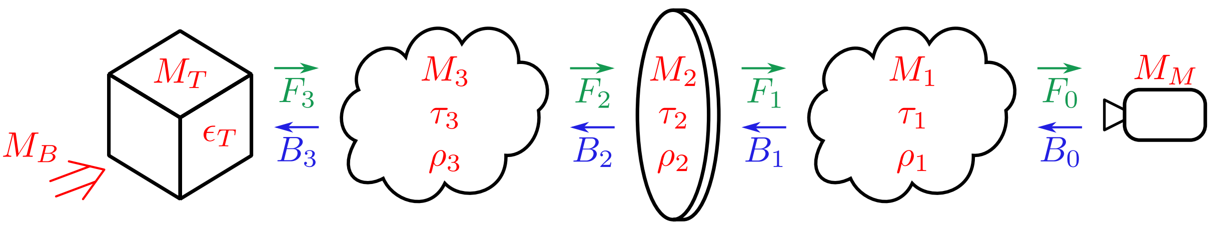

Figure 1: An example system with a window between the camera and the target, showing the atmosphere between each object should also be considered.

$$F_{n-1} = \rho_n B_{n-1} + \tau_n F_n + M_n \tag{1}$$

$$F_n = \frac{F_{n-1} - \rho_n B_{n-1} - M_n}{\tau_n} \tag{2}$$

Similarly, B_n also results from the same interaction, but in the other direction, as given in Equation (3).

$$\begin{aligned} B_n &= \rho_n F_n + \tau_n B_{n-1} + M_n \\ &= \rho_n \frac{F_{n-1} - \rho_n B_{n-1} - M_n}{\tau_n} + \tau_n B_{n-1} + M_n \\ &= \frac{\rho_n}{\tau_n} F_{n-1} + \left(\tau_n - \frac{\rho_n^2}{\tau_n}\right) B_{n-1} + \left(1 - \frac{\rho_n}{\tau_n}\right) M_n \end{aligned} \tag{3}$$

The pair of coupled recursive equations Equations (2) and (3) are linear in F_{n-1} and B_{n-1}, and can be turned into a single recursive matrix equation, Equation (4), by defining the matrices that follow.

$$\boldsymbol{X}_n = \begin{bmatrix} F_n \\ B_n \end{bmatrix} \qquad \boldsymbol{A}_n = \begin{bmatrix} \frac{1}{\tau_n} & -\frac{\rho_n}{\tau_n} \\ \frac{\rho_n}{\tau_n} & \tau_n - \frac{\rho_n^2}{\tau_n} \end{bmatrix} \qquad \boldsymbol{b}_n = \begin{bmatrix} -\frac{M_n}{\tau_n} \\ \left(1 - \frac{\rho_n}{\tau_n}\right) M_n \end{bmatrix}$$

$$\boldsymbol{X}_n = \boldsymbol{A}_n \boldsymbol{X}_{n-1} + \boldsymbol{b}_n \tag{4}$$

With the relatively light assumption that $F_1 = M_M$ and that $B_1 = 0$ (i.e. negligible radiation is travelling from the camera back toward the target), $\boldsymbol{X}_1 = [M_M \;\; 0]^T$, and the system can be solved iteratively if the properties of all objects (apart from the target) are known.

To avoid iterative solutions, the recursion can be unravelled into a single formula, given in Equation (5). In this and following expressions, "Big-Pi" notation is used to mean matrix multiplication, with subsequent terms being ordered left to right.

$$\boldsymbol{X}_n = \left(\prod_{i=1}^{n} \boldsymbol{A}_{n-i}\right) \boldsymbol{X}_0 + \sum_{i=0}^{n-2} \left(\prod_{j=1}^{n-i-1} \boldsymbol{A}_{n-j}\right) \boldsymbol{b}_i + \boldsymbol{b}_n \tag{5}$$

By isolating the two constant terms can be taken out of the expression and evaluated individually to give a single, non-recursive, expression for $\boldsymbol{X}_N$ in Equation (6), from which the value of F_N can be extracted.

$$\boldsymbol{D}_n = \left(\prod_{i=1}^{n} \boldsymbol{A}_{n-i}\right) \qquad \boldsymbol{C}_n = \sum_{i=0}^{n-2} \left(\prod_{j=1}^{n-i-1} \boldsymbol{A}_{n-j}\right) \boldsymbol{b}_i + \boldsymbol{b}_n$$

$$\boldsymbol{X}_N = \boldsymbol{D}_N \boldsymbol{X}_0 + \boldsymbol{C}_N \tag{6}$$

The value of F_N does not immediately give us the estimated object radiance however, as it is also reflecting some radiance from the background. The quantity M_T is the radiance after correcting for this, and is given in Equation (7).

$$M_T = \frac{F_N - (1 - \epsilon_T)\, M_B}{\epsilon_T} \tag{7}$$

By extracting F_N as the first element of $\boldsymbol{X}_N$ and applying the correction in Equation (7), one arrives at a linear relationship between the measured radiance M_M and the estimated radiance of the target object M_T, with coefficients that are fully determined by the properties of the N objects and the radiance of the background. This is given in Equation (8), where d_{11} and c_{11} are the first elements of $\boldsymbol{D}_N$ and $\boldsymbol{C}_N$ respectively.

$$M_T = \frac{d_{11}}{\epsilon_T} M_M + \frac{c_{11} - (1 - \epsilon_T)\, M_B}{\epsilon_T} \tag{8}$$

The coefficients of the linear expression in Equation (8) are independent of the measured radiance, and therefore only need to be calculated once for each system, which can improve efficiency.