Studying and Modeling The Voltage Behavior In a First-Order RC Circuit

SP1 – Modeling & Analysis

By: Abbas Hussain, Mihiret Tolessa, Thomas, Ali Aladhab.

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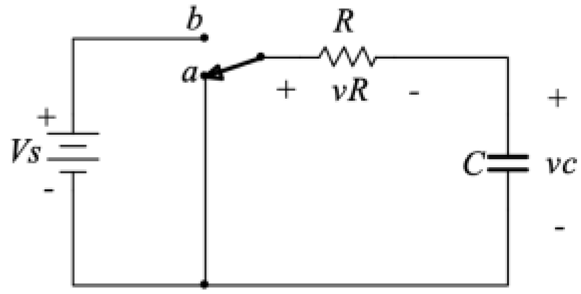
Prof. Zelalem Eshete

Introduction

In this project, we focused on studying and modeling the voltage behavior in a first-order RC circuit through conceptual/theoretical and simulation-based approaches. The main objective was to determine and graph the voltage across the capacitor under two specific pulse durations, $t_1 = 2\tau$ and $t_1 = 4\tau$. A key component of the project involved using Simscape simulations to confirm the conceptual/theoretical results and observe the circuit's dynamic performance. Ultimately, this project provided practical experience in applying differential equations and simulation software to realistic scenarios.



Discussion

Components of our project:

While doing the project we investigated how a RC circuit responds to changing input signals. The aim was to model and simulate the voltage across the capacitor, examining how different pulse durations impact the circuit's overall performance. Understanding the capacitor's charging and discharging behavior is crucial for grasping how energy is stored and dissipated within the system over time. By utilizing Simscape, the project showcases how theoretical models and simulations can be combined to verify the system's performance under specific conditions.

First, we went on to determine the vc response, which is the capacitor's voltage, with the help of the differential equations we derived during our theoretical analysis, which are also known as the charge and discharge equations for the RC circuit.

After that, we plotted the graph for vc vs t/τ in which we used the intervals that we mentioned before, $t_1 = 2\tau$ and $t_1 = 4\tau$. The charge and discharge equations that we had derived earlier were used to plot this graph.

Next, we made our own virtual RC circuit in Simscape as instructed by the guidelines. The simulation for the circuit we made in that software helped us further understand the concept of the given circuit and why it behaved the way it did. The Simscape simulation also provided us with a graph, which we used to compare to our own plotted graph in order to make sure that the solution was indeed correct.

Determining the response vc (voltage across the capacitor):

In order to analyze the change in voltage in the capacitor during its charging phase, we used the following equation: $V_c(t) = V_s(1 - e^{-\frac{t}{RC}})$

In the project instructions, we were told that V_s is 10 volts and the τ and RC are both equal to 10 seconds. Using that information and the two different scenarios that we previously discussed where in the first case $t_1 = 2\tau$ and in the second one $t_1 = 4\tau$. After substituting this in our $V_c(t)$ equation, we end up with $V_c(t) = V_s(1 - e^{-2})$ when $t_1 = 2\tau$ and, $V_c(t) = V_s(1 - e^{-4})$ when $t_1 = 4\tau$ respectively.

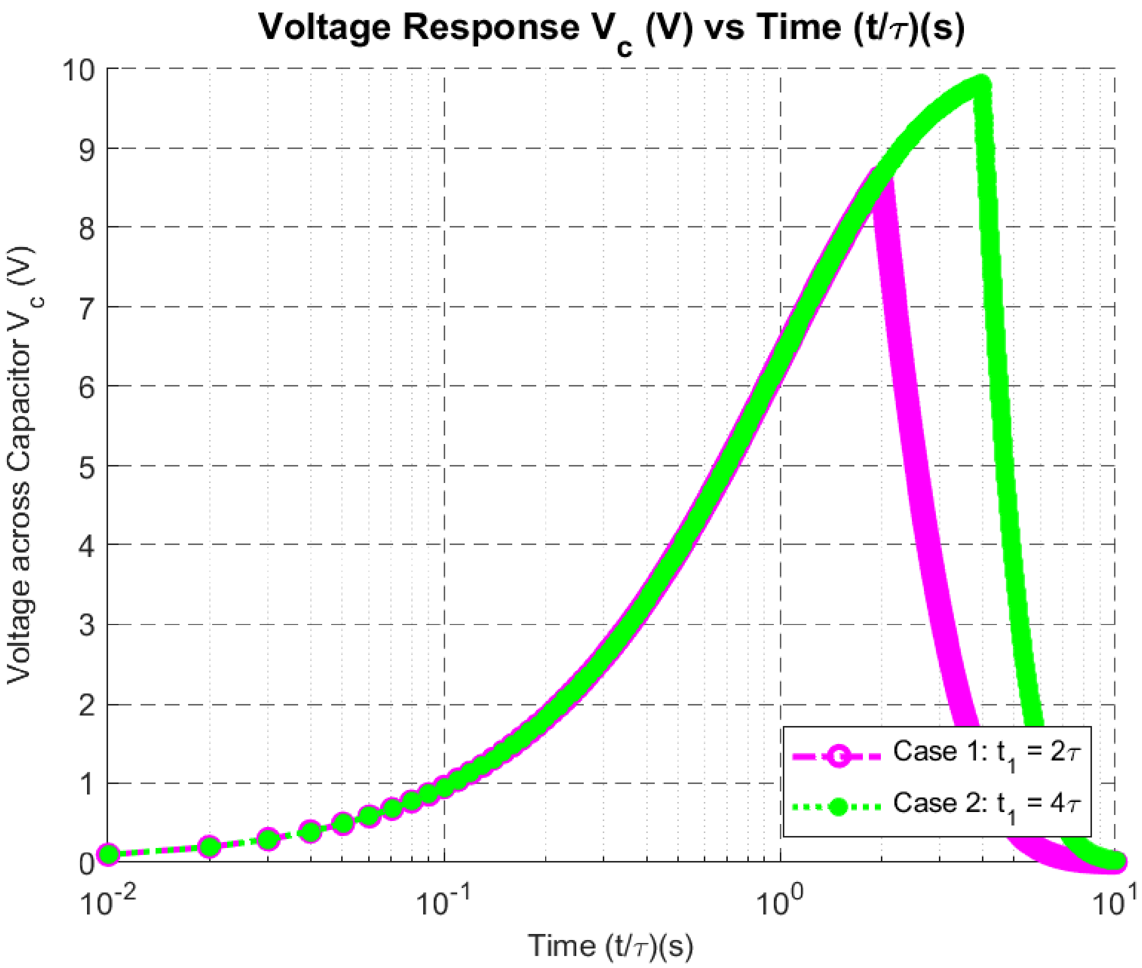
As for during the discharging phase of the capacitor, we used the following equation: $V_c(t) = V_c(t_1)e^{-\frac{(t-t_1)}{RC}}$

Through which we found out that $V_c(t) = V_s(1 - e^{-2})e^{-\frac{(t-20)}{10}}$ when $t_1 = 2\tau$ and, $V_c(t) = V_s(1 - e^{-4})e^{-\frac{(t-40)}{10}}$ when $t_1 = 4\tau$.

Afterwards, we plotted the graph with the help of these charging and discharging equations and MATLAB to further visualize the concept of the RC circuit we were dealing with.

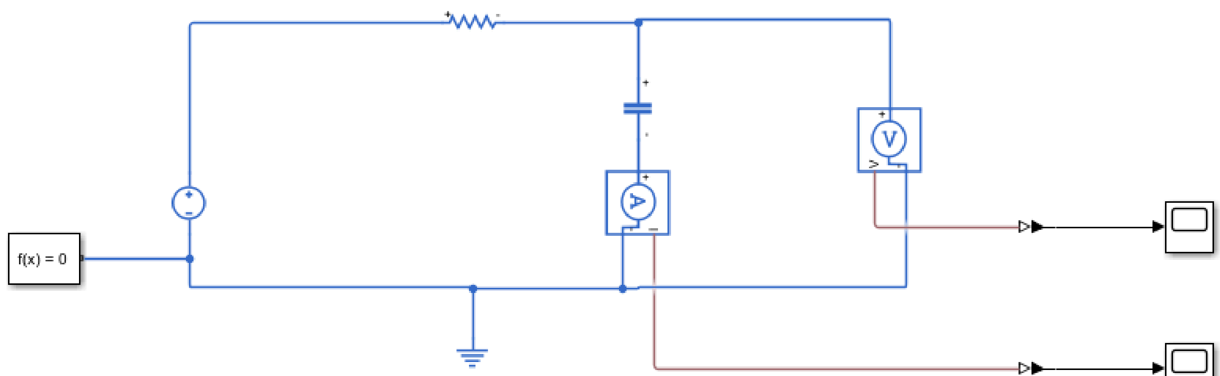
Plotting the graph of vc vs t/τ using the equations:

As aforementioned, we plotted the graphs for the final equations using MATLAB for the two different cases. We started off by setting the parameters for the graph so that it had constant set values overall. We decided to plot both cases on the same graph in order to better present the difference observed in both cases. We plotted t/τ on the x-axis and $V_c(t)$ on the y-axis. We used different colors for each case to better visualize the results. Following is the final graph we obtained after plotting everything accordingly:

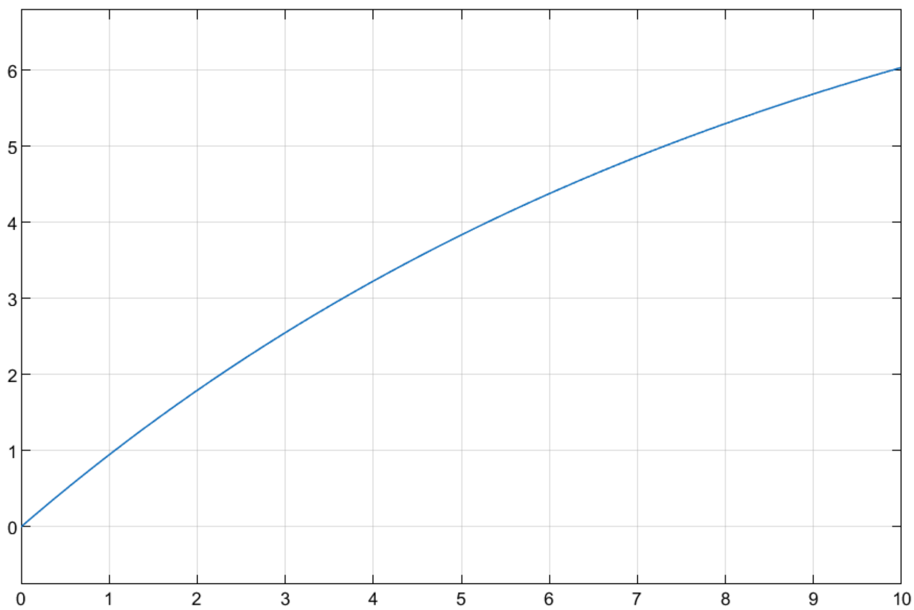


Simscape Simulation Experimentation:

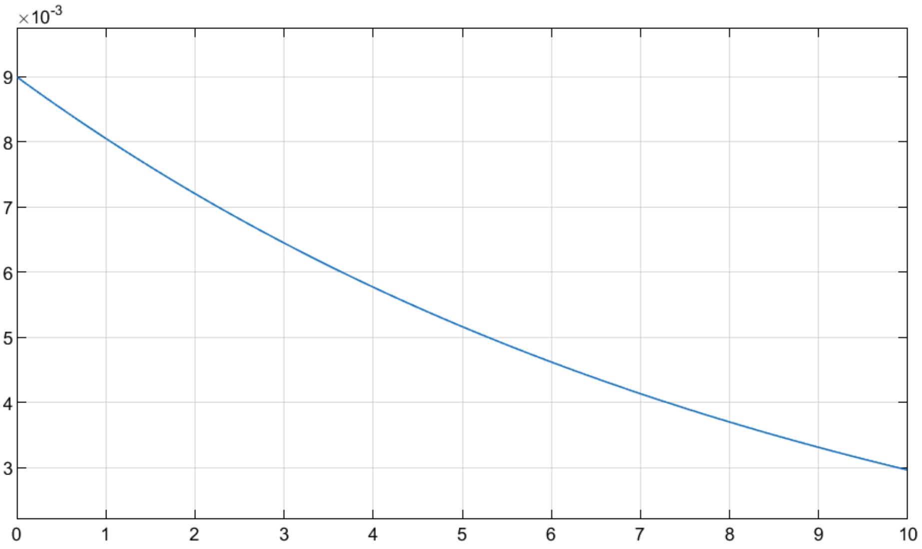
We recreated the given RC circuit in Simscape to make it run and test it. The circuit was powered by a D.C voltage source which provided voltage to the components present in the circuit, which includes the capacitor, resistor, ground, voltmeter and ammeter. We also added scopes to the sensor so that a graph could be produced based on the voltage received. We edited the voltage value for the D.C source to 10 volts as per the instructions and the RC/ τ value to 10 seconds too. Following is the RC circuit we formed with the help of the Simscape software:



Graph received from scope connected to voltmeter:



Graph received from scope connected to ammeter:



Summary

Examining the voltage behavior of an initial RC circuit by combining intellectual and analytical approaches with simulation-based methodologies. This report summarizes on determining the voltage across the capacitor for the two distinct pulse lengths, t1=2 and t1=4. Depending on the length of the pulse (t1), the voltage recorded in a capacitor's response to an RC circuit's rectangular pulse input is allowed. The capacitor charges to a greater voltage before discharging when the pulse width is longer. In both cases, the circuit's resistive element is what drives the capacitor voltage to drop down to zero. The rate of charge or discharge of the capacitor is shown by the time constant (τ). Utilizing Simscape simulations to verify hypothetical conclusions and monitor the circuit's changing efficiency was a crucial aspect of the project. Closing, this project gave a real-world experience using simulation tools and differential equations to create plausible situations.

Appendix

Charging Phase: when $t_1 = 2\tau$, $V_c(t) = V_s(1-e^{-\frac{t}{RC}})$

$$\downarrow$$

$$V_c(t) = V_s\left(1-e^{-\frac{2\tau}{\tau}}\right)$$

$$\downarrow$$

$$\boxed{V_c(t) = V_s(1-e^{-2})}$$

when $t_1 = 4\tau$, $V_c(t) = V_s(1-e^{-\frac{t}{RC}})$

$$\downarrow$$

$$V_c(t) = V_s\left(1-e^{-\frac{4\tau}{\tau}}\right)$$

$$\downarrow$$

$$\boxed{V_c(t) = V_s(1-e^{-4})}$$

Discharging Phase: when $t_1 = 2\tau$, $V_c(t) = V_s(t_1)e^{-\frac{(t-t_1)}{\tau}}$

$$\downarrow$$

$$V_c(t) = V_s(1-e^{-2})e^{\frac{-(t-2\tau)}{\tau}}$$

$$\boxed{V_c(t) = V_s(1-e^{-2})e^{-\frac{t}{10}+2}}$$

when $t_1 = 4\tau$, $V_c(t) = V_s(t_1)e^{-\frac{(t-t_1)}{\tau}}$

$$V_c(t) = V_s(1-e^{-4})e^{-\frac{(t-4\tau)}{\tau}}$$

$$\boxed{V_c(t) = V_s(1-e^{-4})e^{-\frac{t}{10}+4}}$$

Assumptions

1. Ideal Components Assumption

- **Assumption:** The resistor and capacitor in the RC circuit are assumed to be ideal components, meaning they exhibit no parasitic properties like inductance in the resistor or leakage in the capacitor.
- **Reason:** Ideal components simplify the analysis, making it easier to derive the theoretical equations for charging and discharging. Real-world components have imperfections, but in educational or initial project phases, assuming ideal behavior provides a baseline to compare against practical results.

2. Steady-State Input Assumption

- **Assumption:** The voltage source ($V_s = 10V$) is assumed to remain constant during the duration of the simulation.
- **Reason:** This assumption ensures a predictable input for analyzing the capacitor's charging and discharging behavior. Real-world power supplies might have fluctuations, but a steady-state assumption is crucial for isolating the effects of the time constant and pulse durations in a controlled environment.

3. Negligible Wiring Resistance and Capacitance

- **Assumption:** The connecting wires in the RC circuit are assumed to have negligible resistance and capacitance.
- **Reason:** In most basic circuit simulations, the wires are assumed to be perfect conductors to focus purely on the performance of the key components (resistor and capacitor). Including wire effects would complicate the model unnecessarily for this project's objectives.

4. Initial Voltage Across Capacitor is Zero

- **Assumption:** At the start of the simulation, the initial voltage across the capacitor is assumed to be zero ($V_c(0) = 0V$).
- **Reason:** This is a common assumption in RC circuit analysis to simplify the charging equations. It allows for a more straightforward application of the exponential growth formula for voltage across the capacitor, ensuring a clear understanding of its charging behavior from an uncharged state.