Computer Project 2: Design and Analysis of Aircraft Wing Spar

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Abstract

This document outlines the structural analysis and design process of a wing spar for a new aircraft model designed to comply with specified regulations. Emphasis is placed on the spar's ability to withstand the specified loading conditions while maintaining the required safety factor.

Contents

1 Introduction

The design of the main wing spar plays a critical role in the overall integrity and performance of aircraft structures. Given the maximum takeoff weight and load factors, this project involves the computation and validation of a wing spar capable of withstanding a distributed load while maintaining a required safety factor of 1.5.

2 Design Criteria

The spar is designed as a prismatic I-beam made from 2014-T6 aluminum, constrained by a maximum height of 20cm, a maximum flange width of 10cm and a maximum web thickness of 2mm. It was also given that the maximum allowed load on either wing is 24,560N. Given these constraints, the task that will be accomplished is finding the minimum flange thickness while being able to maintain a wing safety factor of 1.5.

3 Part 1. Prismatic Spar Design

3.1 a. Calculation of Distributed Load

The load per unit length w_0 is derived based on the equation $w(x) = w_0\sqrt{1 - \left(\frac{x}{L}\right)^2}$. This equation gives us the spanwise lift distribution of the wing. When this equation is integrated from 0 to L, the distributed load is found. Because the length of the wing is 4 meters, the value of L used in the integration will be 4 meters. $w(x)$ equals 24,560N because it is the maximum allowed load on either wing. After computing the integral and solving for w_0, it is found that w_0 is equal to 7,817.69N/m.

3.2 b. Maximum Internal Shear Force and Bending Moment

In order to find the maximum internal shear force and the maximum bending moment, a load function $w(x)$ was defined. The equation used for this was $w(x) = w_0\sqrt{1 - \left(\frac{x}{L}\right)^2}$. This equation describes how the load varies along the length of the wing—from maximum at the root to zero at the tip—according to a predetermined mathematical formula. The wing is split into N segments, and the load on each segment is approximated by averaging the loads at the start and end of each segment, then multiplying by the segment's width. For each segment, the code computes the resultant shear forces and bending moments. The shear force at any point along the spar is calculated by summing the loads from that point to the tip of the wing, reflecting how shear force accumulates in a cantilevered beam. Similarly, the bending moment at each point is computed by taking the sum of moments generated by each segment's load, multiplied by its distance from the point of interest to the center of the segment, thereby capturing how bending moments are influenced by the position and magnitude of loads. The maximum values for shear forces and bending moments are specifically extracted at the root of the wing, where they are expected to be greatest due to the cantilevered configuration. Doing this results in the maximum shear force being 24,560N and the maximum bending moment being 41,694Nm.

3.3 c. Selecting a Flange Thickness and Von-Mises Stress

In order to find the minimum flange thickness, an array was made to hold all of the possible values for the flange thickness from 0 to 10mm. Then a second array was made to hold all of the stress calculations for every possible value for flange thickness. The stress, σ, in a beam subject to a bending moment is given by the bending stress formula $\sigma = \frac{Mc}{I}$ where M is the maximum bending moment, c is the distance from the neutral axis to the outermost fiber, and I is the moment of inertia of the beam's cross-section. Once all of the stress values were found, the stress values were graphed with respect to all of the possible flange thickness values. Then the allowable stress was found through the equation $\sigma_{\text{allow}} = \frac{\sigma_{\text{y}}}{F.S.}$ where σ_{y} is equal to 414MPa for 2014-T6 aluminum and $F.S.$ is equal to the required safety factor of 1.5. This value was also graphed with respect to flange thickness which resulted in a graph that looked like this:

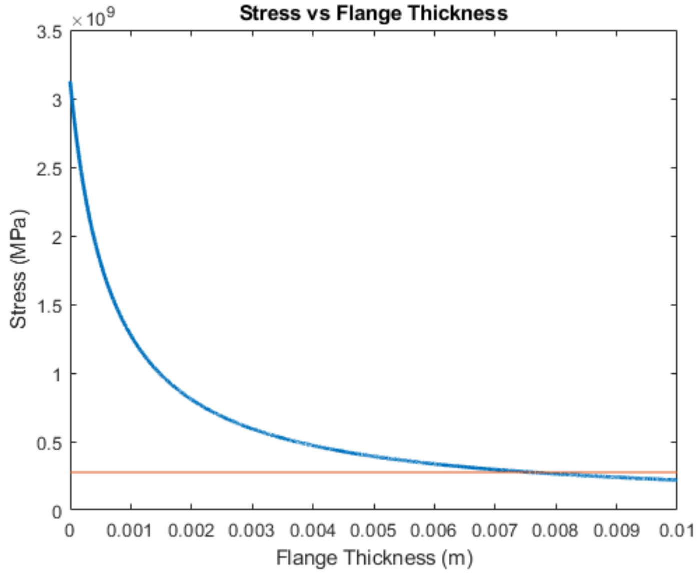

Figure 1: Stress vs. Flange Thickness

The intersection point of the two lines determines the minimum flange thickness that ensures the stress does not exceed what is allowed by the safety factor. This was found by searching the stress array for the first instance where the stress is less than or equal to the allowable stress. The flange thickness calculated from this method was 0.007589m.

In order to find the maximum von-Mises stress in the flange and the web, the equation $\sigma_{\text{v}} = \sqrt{\sigma_{\text{x}}^2 - \sigma_{\text{x}}\sigma_{\text{y}} + \sigma_{\text{y}}^2 + 3\tau_{\text{xy}}^2}$ was used. For the flanges, $\sigma_{\text{x}} = \frac{Mc}{I}$, $\sigma_{\text{y}} = 0$ and $\tau_{\text{xy}} = 0$. For the web, $\sigma_{\text{x}} = 0$, $\sigma_{\text{y}} = 0$ and $\tau_{\text{xy}} = \frac{VQ}{IT}$. After calculations, the maximum von-Mises stress for the flange was 275.99MPa and the maximum von-Mises stress for the web was 102.79MPa. The selected flange thickness is sufficient to prevent yielding of the spar with the required safety factor because 275.99MPa is less than 276MPa and 102.79MPa is less than 276MPa. The value of 276MPa was found through the equation $\sigma_{\text{allow}} = \frac{\sigma_{\text{y}}}{F.S.}$.

3.4 d. Stress Analysis Simulation

In order to run a stress simulation on the beam, it was first modeled by the use of Autodesk Inventor.

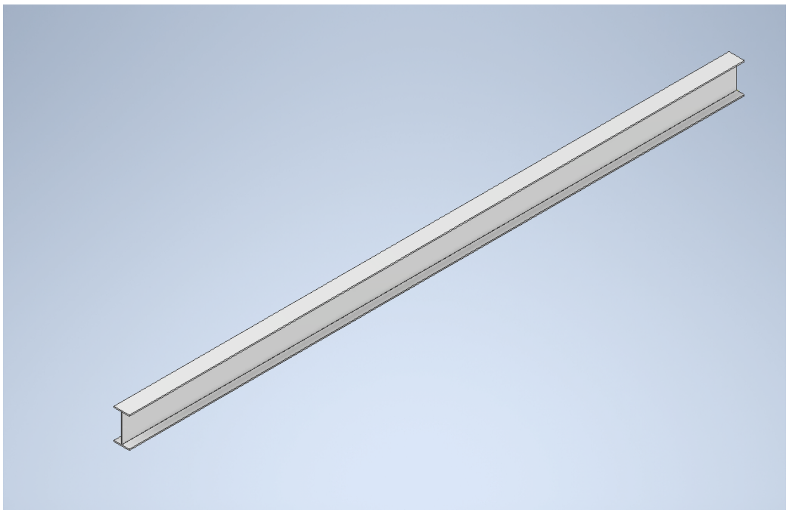

Figure 2: Designed Beam From Inventor

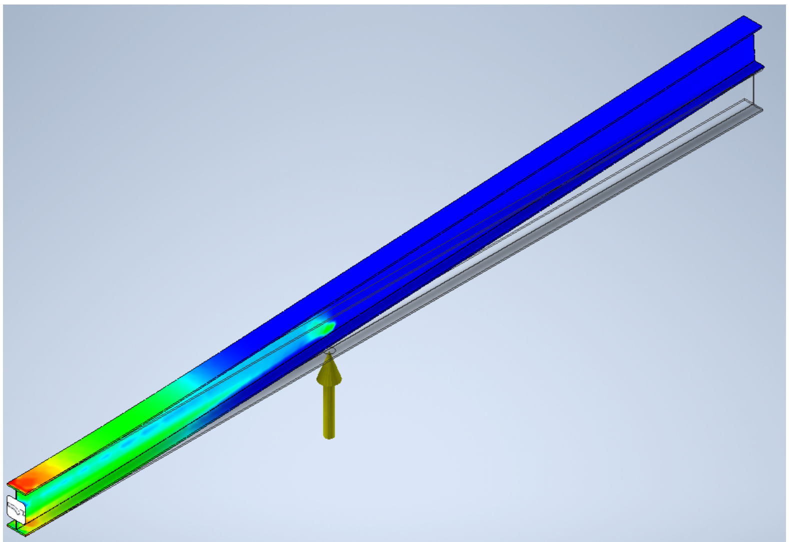

Figure 3: Designed Beam From Inventor with Applied Force

A force of 24560N was applied 1.6976 meters from the root of the wing and the simulation was run. The value of 1.6976 was found by using the equation $F_{\max} = M_{\max} x$, and solving for x. Probes were then added to display von Mises stress at the neutral axis and at the top of the beam flange at the wing root. The results of these are shown below:

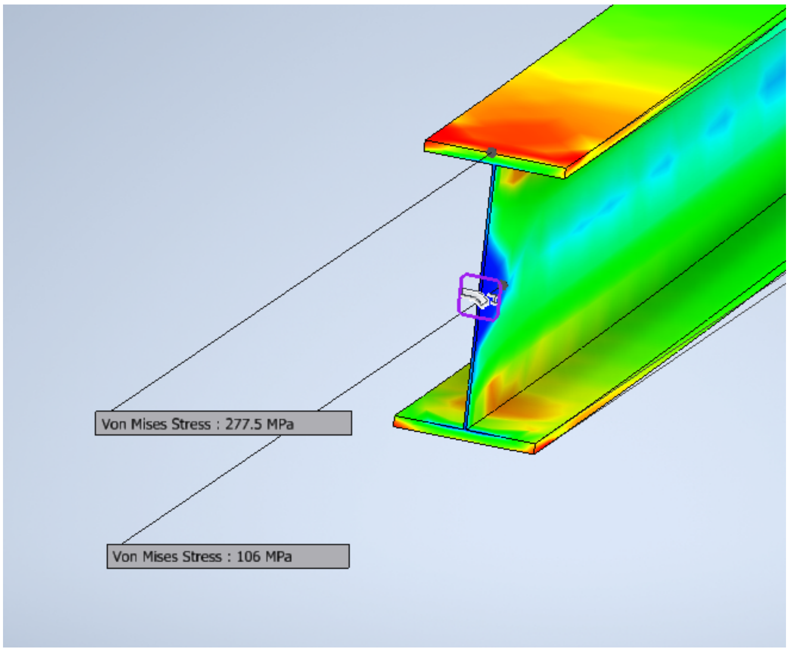

Figure 4: von-Mises From Inventor

The von-Mises stress found from Inventor was very close to the von-Mises stress that was calculated.

3.5 e. Weight of the Prismatic Spar

The weight of the spar was calculated through the use of the formula $D = \frac{m}{V}$. The density of 2014-T6 aluminum is known to be $2789 \frac{kg}{m^3}$. The volume of both the flange and the web was found using the formula $V = bhw$. Through the use of these formulas, it was found that the total weight of the prismatic spar was 21.06kg.

4 Part 2. Spar Safety Factor

4.1 a. Plotting Internal Shear and Bending Moments

The same arrays from part 1b were used to complete these plots. The same arrays that were used to find the maximum shear force and bending moment were plotted as a function of distance which resulted in these plots:

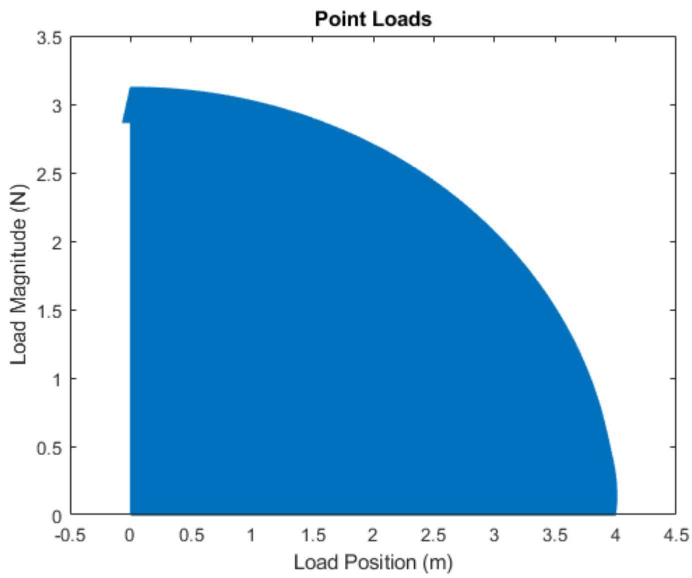

Figure 5: Point Loads along Beam

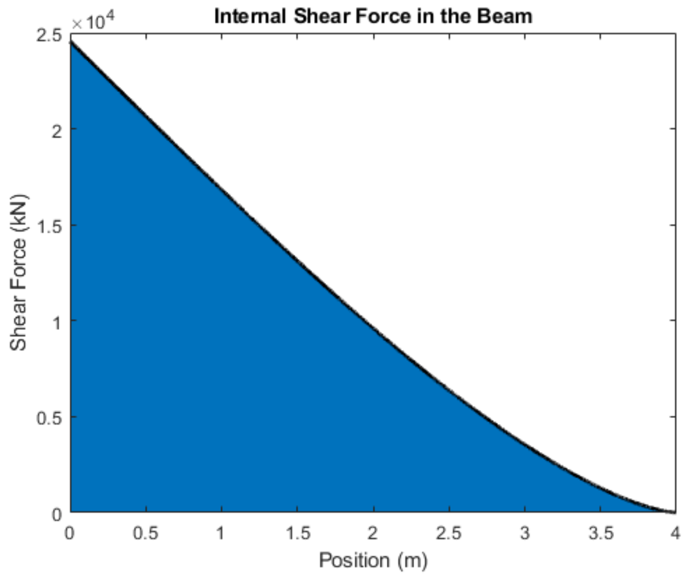

Figure 6: Internal Shear Force in Beam

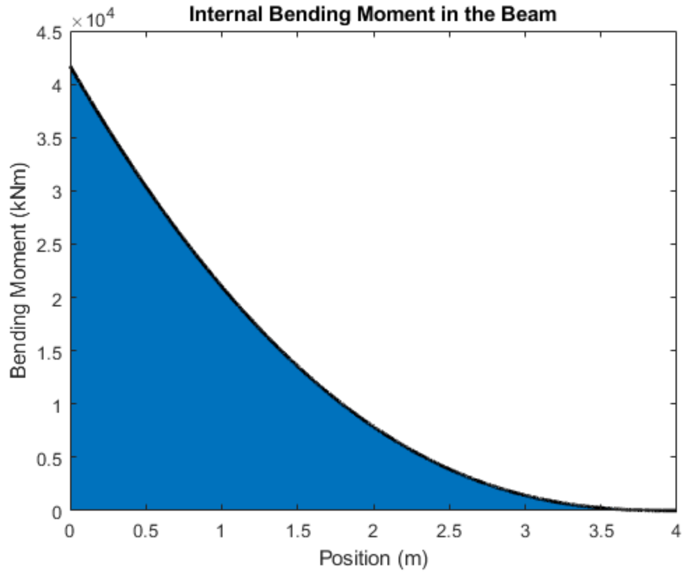

Figure 7: Internal Bending Moment in Beam

4.2 b. Safety Factor Analysis

In order to calculate the safety factor for bending stress relative to the allowable stress as a function of distance, the bending stress at every point along the beam was calculated. With the use of this value, the safety factor was calculated using the equation, $F.S. = \frac{\sigma_{\mathrm{y}}}{\sigma_{\mathrm{bending}}}$. The resulting graph has been shown below.

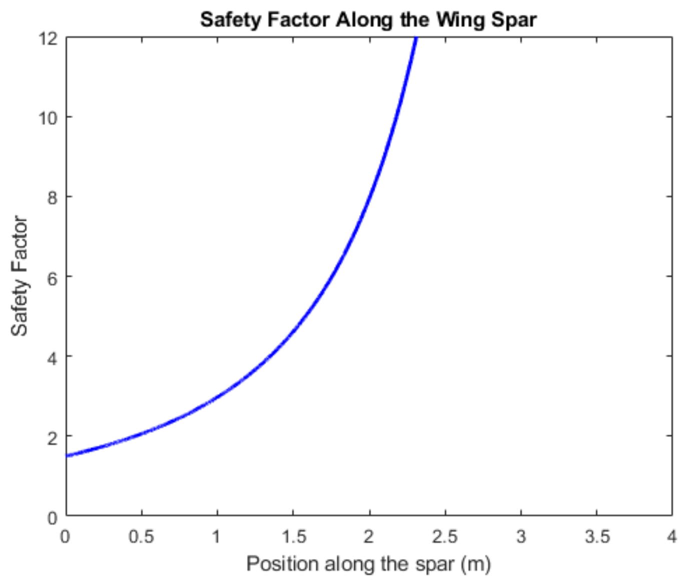

Figure 8: Safety Factor Along Wing Spar

5 Part 3. Extra Credit

5.1 a. Minimum Flange Width

A detailed examination of the flange width required to maintain uniform safety standards across the spar is presented. The methodology adjusts the flange width dynamically along the spar's length, optimizing material usage while ensuring safety. The flange width is initially set at 2mm because the flange width should be larger than the web width in order to rivet the beam to the wing, however in this project the flange width has been set to the web width for simplicity. The inertia of the beam was calculated with fixed values for the web thickness and web height which were in the instructions and a fixed value of flange thickness that was found in the previous part. These values were used to find the bending stress using the equation $\sigma_{\text{bending}} = \frac{Mc}{I}$. If this stress was less than the allowable stress then it was added to a list which was graphed with respect to distance. The resulting graph is shown below.

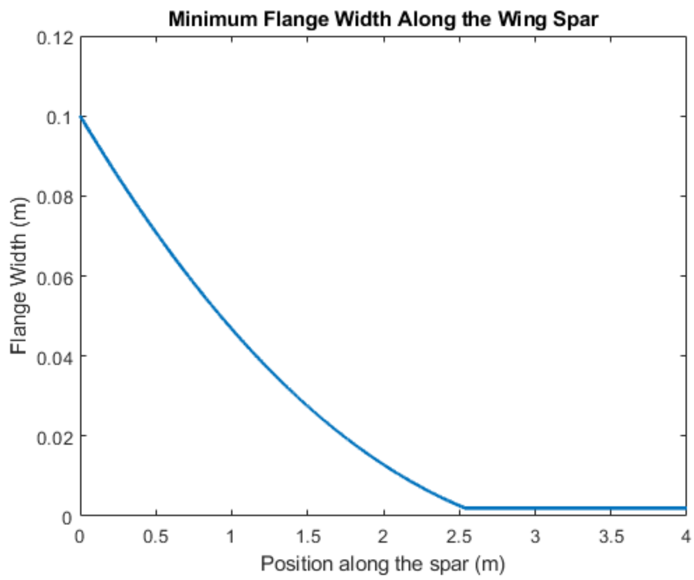

Figure 9: Minimum Flange Width Along the Wing Spar

5.2 b. Autodesk Inventor Simulation

The spar was designed by taking the width of the flange every 0.4 meters from the root and removing the portion of the wing that was not needed. The width of the flange every 0.4 meters was found using the method discussed in part 3a. The spar design is modeled and simulated in Autodesk Inventor. There were 10 point forces added on the beam. The point force values were found by going through the point loads array and summing up the forces in every 0.4 meters interval. The results of this are shown below.

Table 1: Flange Width Along Beam

$L(m)$	Flange Width(m)
0.40	0.0763
0.80	0.0558
1.20	0.0385
1.60	0.0242
2.00	0.0129
2.40	0.0043
2.80	0.0020
3.20	0.0020
3.60	0.0020
4.00	0.0020

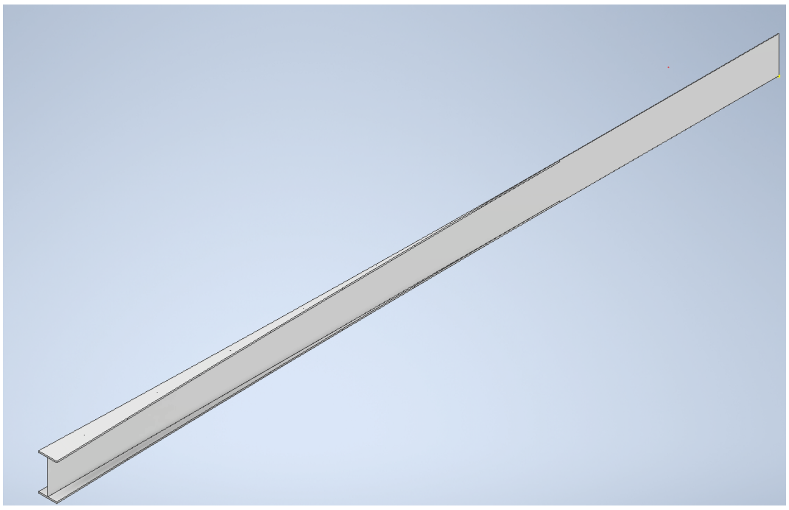

Figure 10: Designed Beam From Inventor with Changing Width

Table 2: Summation of Forces 0.4m Intervals Along Beam

$\Delta L(m)$	$\Sigma F(N)$
0.0-0.4	3124.968066
0.4-0.8	3090.300275
0.8-1.2	3026.261595
1.2-1.6	2927.584503
1.6-2.0	2790.579085
2.0-2.4	2609.176574
2.4-2.8	2373.125770
2.8-3.2	2063.477904
3.2-3.6	1637.686947
3.6-4.0	916.830087

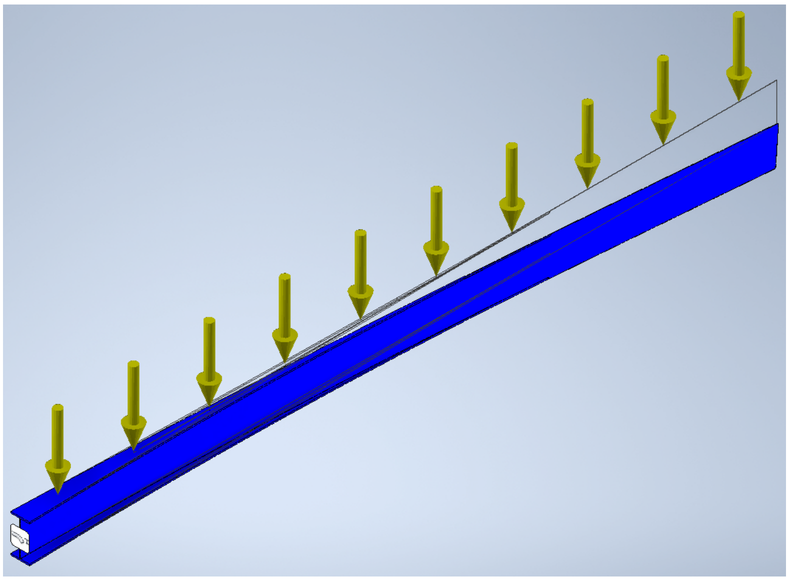

Figure 11: Designed Beam From Inventor with Changing Width: Point Loads

The maximum Von Mises stress is below the yield stress for the material throughout the designed beam, however the safety factor of this beam does not match the FAA's required safety factor of 1.5. A reason for this is the line for approximation of the beam width in between the 0.4 meter intervals were inside the actual curve.

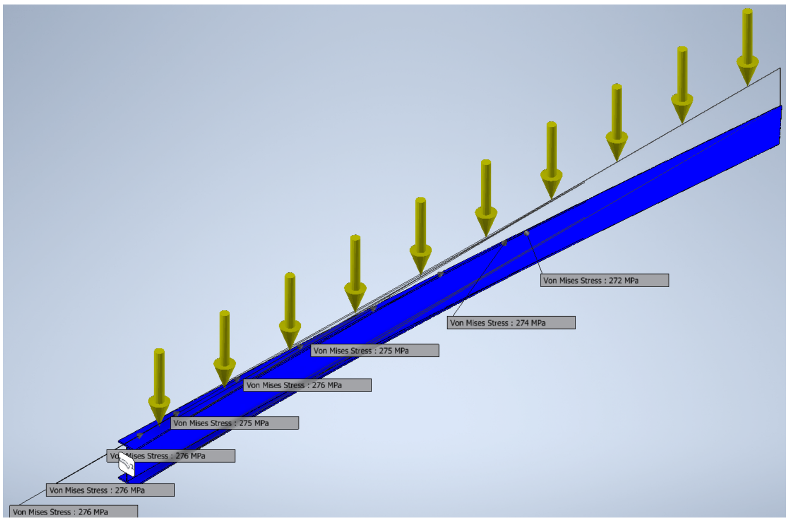

Figure 12: Designed Beam From Inventor with Changing Width: von-Mises Stress

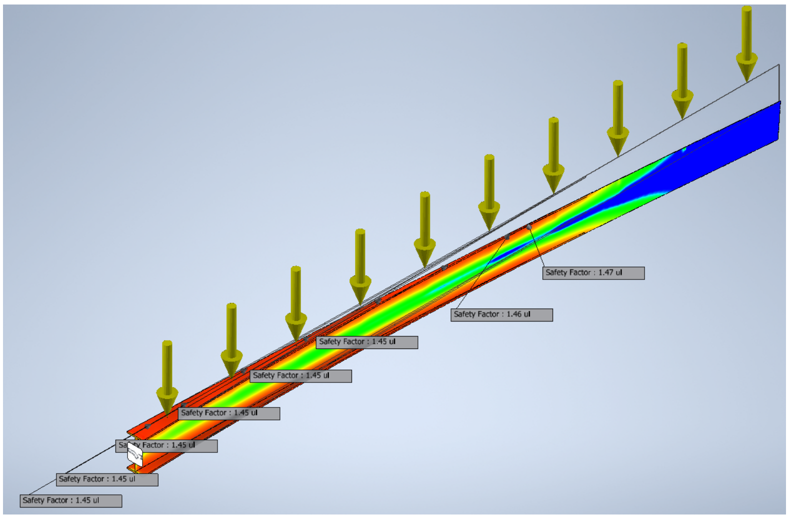

Figure 13: Designed Beam From Inventor with Changing Width: Safety Factor

The calculated weight for this beam was equal to 8.64kg. This was found using the same method from part 1e, resulting in a 58.9 percent weight reduction from the prismatic beam.

6 Conclusion

The project successfully demonstrates the feasibility of designing a wing spar that meets the required specifications and safety standards. The integration of theoretical calculations with practical simulations ensures that the design is both efficient and reliable.