Machine Learning Prerequisites Diagnostic Test

Duration: 30–45 minutes

Instructions

- Answer all questions.
- Show intermediate steps for mathematical questions.
- For programming questions, write valid Python / NumPy code (it will not be executed).
- No machine learning knowledge is required.

Part A — Calculus

Question 1 — Derivatives

Let

$$f(x) = 3x^2 - 5x + 7.$$

(a) Compute $f'(x)$.

(b) Evaluate $f'(2)$.

Question 2 — Chain Rule

Let

$$f(x) = \sin(x^2).$$

Compute $f'(x)$.

Question 3 — Partial Derivatives and Gradient

Let

$$f(x, y) = x^2 y + 3y^2.$$

(a) Compute $\frac{\partial f}{\partial x}$.

(b) Compute $\frac{\partial f}{\partial y}$.

(c) Write the gradient $\nabla f(x, y)$.

Part B — Linear Algebra

Question 4 — Matrix Multiplication

Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}.$$

Compute AB.

Question 5 — Transpose

Let

$$C = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{pmatrix}.$$

(a) Compute $C^\top$.

(b) Give the dimensions of C and $C^\top$.

Question 6 — Solving a Linear System

Solve the system:

$$\begin{cases} x + y = 3, \\ 2x + y = 4. \end{cases}$$

Question 7 — Eigenvalues and Eigenvectors

Let

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

(a) Compute the eigenvalues of A.

(b) For each eigenvalue, give one associated eigenvector.

Question 8 — Conceptual Eigenvalue Questions

Answer briefly (one or two sentences):

(a) What does an eigenvector represent geometrically?

(b) Why are eigenvalues and eigenvectors important in data analysis or machine learning?

Question 9 — Invertibility (Conceptual)

Let $A \in \mathbb{R}^{n \times n}$.

(a) State a condition on the determinant of A for it to be invertible.

(b) Explain in one sentence what non-invertibility means for the system $Ax = b$.

Part C — Python Programming

Question 10 — Functions and Loops

Write a Python function that computes the sum of squares of the integers from 1 to n.

```
def sum_of_squares(n):
    ...
```

Question 11 — Conditionals

Write a function that takes a real number `x` and returns:

- `"positive"` if $x > 0$,
- `"negative"` if $x < 0$,
- `"zero"` otherwise.

Question 12 — Lists and Loops

Given the list:

```
x = [1, 4, 9, 16]
```

Write Python code to create a new list containing the square root of each element.

Part D — NumPy

Assume:

```
import numpy as np
```

Question 13 — Array Creation

Create a NumPy array containing the integers from 0 to 9.

Question 14 — Vectorized Operations

Given:

```
x = np.array([1, 2, 3, 4])
```

(a) Write code to compute the element-wise square of `x`.

(b) Write code to compute the mean of `x`.

Question 15 — Linear Algebra with NumPy

Given:

```
A = np.array([[1, 2],
              [3, 4]])
b = np.array([1, 0])
```

(a) Write NumPy code to compute the matrix–vector product Ab.

(b) Write NumPy code to solve the linear system $Ax = b$.

Question 16 — Shapes (Highly ML-Relevant)

Let:

```
X = np.random.randn(100, 5)
w = np.random.randn(5)
```

(a) What is the shape of `X`?

(b) What is the shape of `X @ w`?

(c) What does this operation represent intuitively?

Optional Bonus Question

Question 17 — Gradient Interpretation

Let

$$f(x, y) = x^2 + y^2.$$

(a) Compute $\nabla f(x, y)$.

(b) What does the gradient indicate about the direction of steepest ascent?