

# Cse 105 Hw1

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January 11, 2016

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1.  $A(B+C) = AB + AC$

operations on left side of the equation to match the right and prove the distributive law

$$A \cup (B+C) = AB + AC$$

$$\exists xy: x \in A \cup ((y \in B) + (y \in C)) =$$

$$\exists xy: ((x \in A) \cup (y \in B)) + ((x \in A) \cup (y \in C)) =$$

$$(A \cup B) + (A \cup C) =$$

$$AB + AC = AB + AC$$

2.  $A(B+C) = AB + AC$

operations on left side of the equation to match the right and prove the distributive law

$$A \cap (B+C) = AB + AC$$

$$\exists x: x \in A \cap ((x \in B) + (x \in C)) =$$

$$\exists x: ((x \in A) \cap (x \in B)) + ((x \in A) \cap (x \in C)) =$$

$$(A \cap B) + (A \cap C) =$$

$$AB + AC = AB + AC$$

3.  $A^* = \text{empty} + AA^*$

By definition,  $A^*$  includes all combinations of the language  $A$  including the empty set. This makes the first part of the statement true where  $A^* = \text{empty}$ . The second half of the statement where  $A^* = AA^*$  is true because  $AA^*$  only includes strings that exist in the language  $A$  and  $A^*$  is all combinations possible in the language  $A$ .

4.  $A^* = A^{**}$

Since  $A^*$  is all combinations of the symbols in the language  $A$ , and  $A^{**} = A_0^* + A_1^* + A_2^* + \dots$ . All of the symbols in  $A^{**}$  are from the language  $A$ , and both  $A^*$  and  $A^{**}$  contain all combinations of the symbols in that language,  $A^* = A^{**}$ .

5. Yes both of these cases can be true. Given a finite alphabet from which a language is constructed from, all other combinations of strings that are possible using this alphabet make the complement of this language, so both are finite. If there is an infinite language constructed from an alphabet then the complement can construct another infinite language.

6. Since  $A$  is closed under concatenation, all the concatenations of strings in  $A$  are a part of  $A$ . This means that there can be infinite concatenations made using each of the newly used concatenations.

7. Since the sets in  $X$  are closed under union, the union of each set exists within the other sets. The complement of this also exists in each set since  $X$  is closed under complement ie: every other combination of symbols including and ex-

cluding union exists within each set.  $X$  is closed under intersection because the sets in  $X$  are infinite and intersecting (if an element exists in one set it exists in the other).