Time-warped PCA: simultaneous alignment and dimensionality reduction of neural data

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Summary. Analysis of multi-trial neural data often relies on a rigid alignment of neural activity to stimulus triggers or behavioral events. However, the onset and rate of neural dynamics may vary across trials for a variety of reasons including differences in attentional state, biophysical kinetics (CITE), and the phase of stimulus delivery with respect to internal brain rhythms. This variability can inflate the apparent dimensionality of the data and obscure analysis of otherwise simple, low-dimensional dynamics. We demonstrate this phenomenon in spike-triggered covariance analysis of retinal ganglion cells and in primate motor cortical neurons during a reaching task. In both cases, the principal eigenvector of the data covariance matrix matches the temporal derivative of the trial-averaged signal, suggesting that differences in timing are a substantial source of trial-to-trial variability in these systems. Critically, this means that variability in neural timing could overshadow other signals, preventing their discovery and examination. To disentangle the contributions of timing from intrinsic variability in neural dynamics, we developed a new technique, *time-warped PCA*, which flexibly aligns (warps?) trials while finding a low-dimensional representation of population dynamics. (ADD REACTION TIME RESULT.)

...Need to flesh out the technique and results...

Additional Detail.

The synthetic dataset in Fig 1 illustrates how variability in neural timing produces artifacts in data analysis, including an apparent increase in dimensionality. We considered a single neuron over multiple trials. On trial k the activity pattern of the neuron was given by $y_k(t) = a_k z(t + \tau_k)$, allowing for changes in amplitude, a_k, and timing, τ_k. Inspection of the Taylor expansion,

$$y_k(t) \approx a_k z(t) + a_k \tau_i z'(t) + a_k \tau_i^2 z''(t) + ...$$

shows that, as τ_i increases, an approximation to the full dataset requires a greater number of temporal basis functions — i.e., $z(t)$ and its temporal derivatives.

This analysis is important and relevant in many contexts since we expect neural timing vary across trials, even after alignment. We provide evidence for this in two very different datasets: (a) in motor cortical neurons during point-to-point reaching in a Rhesus monkey (Fig. 2A), and in spike-triggered covariance analysis of Salamander retinal ganglion cells (Fig. 2B). In both cases, the top principal component qualitatively matches the first temporal derivative of the trial-averaged signal. The presence of derivatives in the principal components of neural data can also be found by careful inspection of existing literature [**1**, **2**]. However, the source of this phenomenon is rarely discussed, and, more importantly, its consequences for neural data analysis have not been systematically studied.

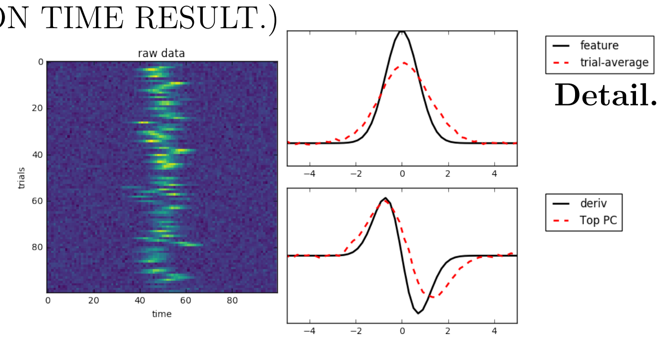

Figure 1: Variability in trial-to-trial timing of a synthetic signal (panel A) causes the trial-averaged signal to over-estimate the width (panel B) and the eigenvector of the covariance matrix to resemble the temporal derivative (panel C).

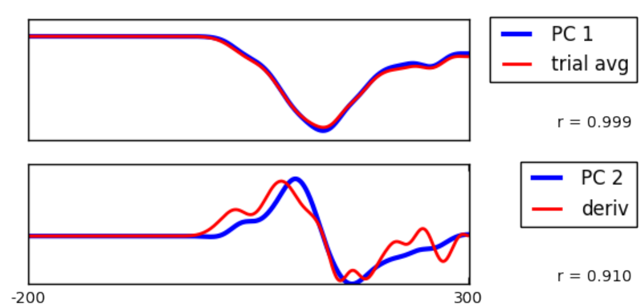

Figure 2: An example neuron from the primate reaching dataset. Similar to Fig 1, the top PC matches the trial-averaged firing rate, while the second PC matches the temporal derivative of the first PC. could show a histogram of the correlation coefficient for each neuron?

Method. Given a dataset of N neurons over M trials
with each trial having length T. For each trial, i, we let $X^{(i)} \in \mathbb{R}^{T \times N}$ be a matrix with each column containing the activity of an individual neuron over time. Our objective is then:

$$\underset{U,V,\tau^{(1)},\cdots,\tau^{(K)}}{\text{minimize}} \sum_{i=1}^{M} \left\| X^{(i)} - \tau^{(i)} \left(UV^T \right) \right\|_F^2 \qquad (1)$$

where $U \in \mathbb{R}^{T \times K}$ and $V \in \mathbb{R}^{N \times K}$ are the matrices that make up the low-rank approximation of the aligned data, and $\tau^{(i)}$ is a transformation from the data space to t

Representaiton of the transformation τ. I believe this

Curvature regularizer on tau, interpolates between no warping and arbitrary warping.

Note differentiability, mention how we optimize it, etc.

Application to monkey data: data preprocessing, aligned to go cue, used one trial to predict RT

[**1**] Fairhall. *Journal of Neurophysiology* 96.5 (2006). [**2**] Kobak. *eLife* 5 (Apr. 2016).

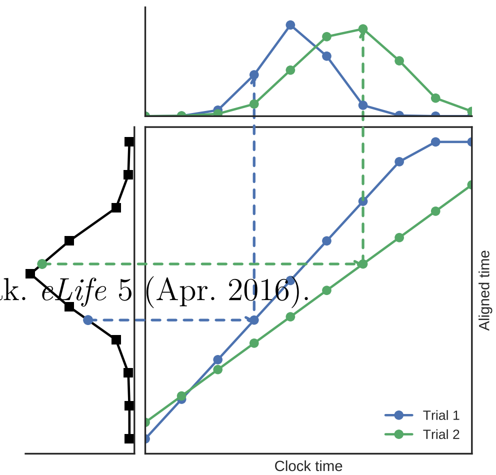

Figure 3: Schematic of time warping method.

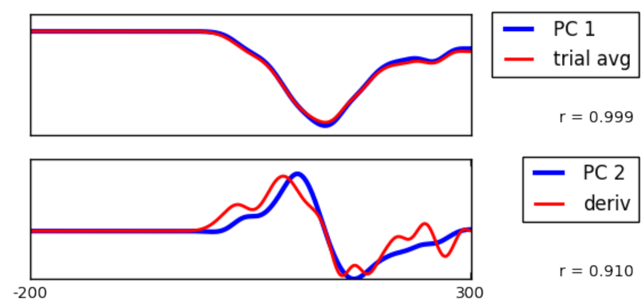

Figure 4: monkey data