ECE 657 Assignment 1

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June 13, 2021

QUESTION 2

$o_1 = neuron3.$
$o_2 = neuron4.$
$o_3 = neuron5.$

Ⓐ $W_{13}x + W_{23}y + b = o_1$
Ⓑ $W_{14}x + W_{24}y + b = o_2$
Ⓒ $W_{15}x + W_{25}y + b = o_3$

h1 = activation function at neuron 3
h2 = activation function at neuron 4
h3 = activation function at neuron 5

$w_{36}h_1$ + $w_{46}h_2$ + $w_{56}h_3$+ b= 2

Class = 1 if point falls within the triangle
Class = 0 if point falls outside the triangle
At (0,0) (1,3) (3,1):

x	y	z
0	0	1
1	3	1
3	1	1

$b + h_1w_{36} + h_2w_{46} + h_3w_{56} > 0$ – Ⓓ
$b + h_1w_{36} + h_2w_{46} + h_3w_{56} > 0$ – Ⓔ
$b + h_1w_{36} + h_2w_{46} + h_3w_{56} > 0$ – Ⓕ

At (0,0)
Ⓐ becomes $b = o_1$
Ⓑ becomes $b = o_2$
Ⓒ becomes $b = o_3$
if b=1 then o_1,o_2,o_3 = 1

At (1,3)
Ⓐ becomes $w_{13} + 3w_{23} + 1 = o_1 > 0$
Ⓑ becomes $w_{14} + 3w_{24} + 1 = o_2 > 0$
Ⓒ becomes $w_{15} + 3w_{25} + 1 = o_3 > 0$

At (3,1)
Ⓐ becomes $3w_{13} + w_{23} + 1 = o_1 > 0$

Ⓑ becomes $3w_{14} + w_{24} + 1 = o_2 > 0$
Ⓒ becomes $3w_{15} + w_{25} + 1 = o_3 > 0$

Using activation function as sign function ; Sign $o_1, o_2, o_3 = -1, 0, 1$
if
$o_1, o_2, o_3 < 0$
$o_1, o_2, o_3 = 0$
$o_1, o_2, o_3 > 0$
At (0,0), o_1, o_2, o_3 are all greater than zero. Thus, h_1, h_2, h_3 are all equal to 1

(0,0):
b + $h_1 w_{36}$ + $h_2 w_{46}$ + $h_3 w_{56}. > 0$
b + w_{36} + w_{46} + $w_{56}. > 0$
At (3,1) and (1,3) o_1, o_2, o_3 are all greater than 0.
Thus, h1, h2 , h3 =1

Ⓔ and Ⓕ becomes
b + w_{36} + w_{46} + $w_{56}. > 0$
b $> -w_{36}$ - w_{46} - w_{56}
if w_{36} = w_{46} = $w_{56} = 1$ and $b = 1$ then Ⓖ and Ⓗ are satisfied

From Ⓐ. $w_{13}x + w_{23}y + b > 0$
From Ⓑ. $w_{14}x + w_{24}y + b > 0$
From Ⓒ. $w_{15}x + w_{25}y + b > 0$ if b=1 then o_1,o_2,o_3 = 1

b $> -w_{13}x$ - $w_{23}y$
b $> -w_{14}x$ - $w_{24}y$
b $> -w_{15}x$ - $w_{25}y$

At (0,0): $b > 0, b = 1$

At (1,3):
$b > -w_{14} - 3w_{24}$
$b > -w_{13} - 3w_{23}$
$b > -w_{15} - 3w_{25}$

if b=1 and $w_{14} = w_{24} = w_{13} = w_{23} = w_{15} = w_{25} = 1$ then the above holds

At (3,1):
$b > -3w_{13} - w_{23}$
$b > -3w_{14} - w_{24}$

$b > -3w_{15} - w_{25}$

The equality holds as well with b = 1 and all weight values as 1

b=1
w_{13} = 1
w_{23}. = 1
w_{14}. = 1
w_{24}. = 1
w_{15}. = 1
w_{25}. = 1
w_{36}. = 1
w_{46} = 1
w_{56}. = 1

QUESTION 3

Widrow Hoff is mathematically represented as:
$\Delta w^k. = \eta(t^k - w^k x^k) \frac{x^k}{||x^k||^2}$ —— (1)

$\Delta w^k. = w^{(k+1)} - w^k$ —— (2)

Show that if the same input vector x^k is presented at iteration (k + 1) then:
$\Delta w^{k+1}. = (1-\eta)\Delta w^{(k)}$ —— (3)

With x^k being present at iteration $(k+1)$, equation 3 is valid because if x^k is present, t^k is present as well

$x^{k+1} = x^k$ ——(4)

$t^{k+1} = t^k$ —— (5)

Then we derive:
$\Delta w^{k+1}. = \eta(t^k - w^{k+1} x^k) \frac{x^k}{||x^k||^2}$ —— (6)

From equation 2, we make $w^{(k+1)}$ the subject of the formula
$w^{(k+1)} = \Delta w^k + w^k$ —— (7)

Sub (7) in (6)
$\Delta w^{k+1}. = \eta(t^k -(\Delta w^k + w^k)\, x^k) \frac{x^k}{||x^k||^2}$ —— (8)

$\Delta w^{k+1}. = \eta t^k [\frac{x^k}{||x^k||^2}] - \eta \Delta w^k x^k [\frac{x^k}{||x^k||^2}] + \eta w^k x^k [\frac{x^k}{||x^k||^2}] - - - - - (9)$

$\Delta w^{k+1}. = \eta(t^k - w^k x^k)[\, x^k \frac{}{||x^k||^2}] - \eta \Delta w^k [\frac{x^k x^k}{||x^k||^2}] - - - - - (10)$

$\Delta w^{k+1}. = \Delta w^k - \eta \Delta w^k [\frac{x^k x^k}{||x^k||^2}] - - - - - (11)$