Cops and Robbers

Player 1 is a police officer who must decide whether to patrol the streets or to hang out at the coffee shop. His payoff from hanging out at the coffee shop is 10, while his payoff from patrolling the streets depends on whether he catches a robber, who is player 2. If the robber prowls the streets then the police officer will catch him and obtain a payoff of 20. If the robber stays in his hideaway then the officer's payoff is 0. The robber must choose between staying hidden or prowling the streets. If he stays hidden then his payoff is 0, while if he prowls the streets his payoff is -10 if the officer is patrolling the streets and 10 if the officer is at the coffee shop.

(a) Write down the matrix form of this game.

(b) Draw the best-response function of each player.

(c) Find the Nash equilibrium of this game.

(a)

Cop \ Robber	Stay	Prowl
Coffee	$10, 0$	$10, 10$
Patrol	$0, 0$	$20, -10$

(b)

In order to find the best response correspondences for each player, we assume that both are mixing strategies.

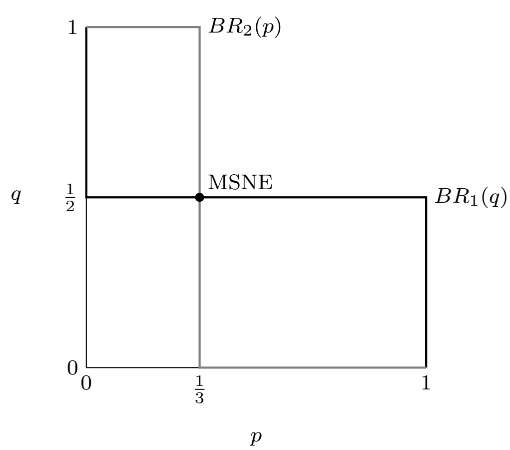
Cop's Indifference:

$$\begin{aligned} v_1(C, \sigma_2) &= 10 \\ v_1(P, \sigma_2) &= 0(q) + 20(1-q) \\ q &= \frac{1}{2} \end{aligned}$$

Robber's Indifference:

$$\begin{aligned} v_2(\sigma_1, S) &= 10p + 0(1-p) \\ v_2(\sigma_1, P) &= 10p - 10(1-p) \\ 10p &= 10p - 10(1-p) \\ p &= 1 \end{aligned}$$

The best response correspondences are plotted below:



(c)

The Nash equilibria are at $p^* = \frac{1}{3}$ and $q^* = \frac{1}{2}$.

Discrete All-Pay Auction

Each bidder submits a bid. The highest bidder gets the good, but *all players pay their bids*. Consider an auction in which player 1 values the item at 3 and player 2 values the item at 5. Each player can bid either 0 (Z), 1 (O), or 2 (T). If both players bid the same amount, a coin is flipped, but both players pay their bids nonetheless.

The bids correspond to the payoff matrix below.

Player 2

		Z	O	T
Player 1	Z	$1.5, 2.5$	$0, 4$	$0, 3$
	O	$2, 0$	$0.5, 1.5$	$-1, 3$
	T	$1, 0$	$1, -1$	$-0.5, 0.5$

(a)

Write down the reduced game that results from iteratively eliminating strictly dominated strategies.

- For Player 2, T strictly dominates Z.

Player 2

		Z	O	T
Player 1	Z	$\cancel{1.5, 2.5}$	$0, 4$	$0, 3$
	O	$\cancel{2, 0}$	$0.5, 1.5$	$-1, 3$
	T	$\cancel{1, 0}$	$1, -1$	$-0.5, 0.5$

- For Player 1, T strictly dominates O

Player 2

		Z	O	T
Player 1	Z	$\cancel{1.5, 2.5}$	$0, 4$	$0, 3$
	O	$\cancel{2, 0}$	$\cancel{0.5, 1.5}$	$\cancel{-1, 3}$
	T	$\cancel{1, 0}$	$1, -1$	$-0.5, 0.5$

(b)

Find the Nash equilibria for this game.

We will let p denote Player 1's chance of playing Z and let q denote Player 2's chance of playing O.

Player 1's Indifference:

$$\begin{aligned} v_1(Z, \sigma_2) &= 0 \\ v_1(T, \sigma_2) &= q - 0.5(1-q) \\ 0 &= q - 0.5(1-q) \\ q &= \frac{2}{3} \end{aligned}$$

Player 2's Indifference:

$$\begin{aligned} v_2(\sigma_1, O) &= 4p - (1-p) \\ v_2(\sigma_1, T) &= 3p + 0.5(1-p) \\ 5p - 1 &= 2.5p + 0.5 \\ p &= \frac{3}{5} \end{aligned}$$

Therefore, the Nash equilibrium is $p^* = \frac{3}{5}, q^* = \frac{2}{3}$.

Mixed Up

In the following normal-form games, find all the Nash equilibria.

(a)

	L	R
T	$0,0$	$10,12$
B	$4,4$	$6,0$

Player 1's Indifference:

$$\begin{aligned} v_1(T, \sigma_2) &= 10(1-q) \\ v_1(B, \sigma_2) &= 4q + 6(1-q) \\ 10 - 10q &= 6 - 2q \\ q &= \frac{1}{2} \end{aligned}$$

Player 2's Indifference:

$$\begin{aligned} v_2(\sigma_1, L) &= 4(1-p) \\ v_2(\sigma_1, R) &= 12p \\ 12p &= 4 - 4p \\ p &= \frac{1}{2} \end{aligned}$$

Therefore, the Nash equilibrium is $p^* = \frac{1}{2}, q^* = \frac{1}{2}$

(b)

	L	R
T	$2, 4$	$2, 6$
B	$6, 6$	$2, 4$

Player 1's Indifference:

$$\begin{aligned} v_1(T, \sigma_2) &= 2 \\ v_1(B, \sigma_2) &= 6q + 2(1-q) \\ 2 &= 6q + 2(1-q) \\ q &= 0 \end{aligned}$$

Player 2's Indifference:

$$\begin{aligned} v_2(\sigma_1, L) &= 4p + 6(1-p) \\ v_2(\sigma_1, R) &= 6p + 4(1-p) \\ 6 - 2p &= 4 + 2p \\ p &= \frac{1}{2} \end{aligned}$$

So the Nash equilibria are (T, R) and (B, R), each with equal frequency.

(c)

	W	X	Y	Z
A	$3,1$	$1,6$	$3,1$	$6,5$
B	$6,6$	$2,3$	$1,1$	$1,1$
C	$1,1$	$2,3$	$6,6$	$5,2$
D	$2,3$	$7,2$	$2,3$	$2,1$

- Z is strictly dominated by X

	W	X	Y	Z
A	$3,1$	$1,6$	$3,1$	$\cancel{6,5}$
B	$6,6$	$2,3$	$1,1$	$\cancel{1,1}$
C	$1,1$	$2,3$	$6,6$	$\cancel{5,2}$
D	$2,3$	$7,2$	$2,3$	$\cancel{2,1}$

- A is strictly dominated by $\frac{1}{2}B + \frac{1}{2}C$

	W	X	Y	Z
A	$\cancel{3,1}$	$\cancel{1,6}$	$\cancel{3,1}$	$\cancel{6,5}$
B	$6,6$	$2,3$	$1,1$	$\cancel{1,1}$
C	$1,1$	$2,3$	$6,6$	$\cancel{5,2}$
D	$2,3$	$7,2$	$2,3$	$\cancel{2,1}$

- X is strictly dominated by $\frac{1}{2}W + \frac{1}{2}Y$

	W	X	Y	Z
A	$\cancel{3,1}$	$\cancel{1,6}$	$\cancel{3,1}$	$\cancel{6,5}$
B	$6,6$	$\cancel{2,3}$	$1,1$	$\cancel{1,1}$
C	$1,1$	$\cancel{2,3}$	$6,6$	$\cancel{5,2}$
D	$2,3$	$\cancel{7,2}$	$2,3$	$\cancel{2,1}$

- D is strictly dominated by $\frac{1}{2}B + \frac{1}{2}C$

	W	X	Y	Z
A	$\cancel{3,1}$	$\cancel{1,6}$	$\cancel{3,1}$	$\cancel{6,5}$
B	$6,6$	$\cancel{2,3}$	$1,1$	$\cancel{1,1}$
C	$1,1$	$\cancel{2,3}$	$6,6$	$\cancel{5,2}$
D	$\cancel{2,3}$	$\cancel{7,2}$	$\cancel{2,3}$	$\cancel{2,1}$

Thus, the two Nash equilibria are (B, W) and (C, Y), occurring with equal frequency.