Jan 30:

4 key facts about Economic growth

- Enormous variation - poorest places have < 5% of per capita income of rich countries

- rate of growth vary substantially
    - Some countries grow fast, others not so

- growth rates not constant over time
    - for individual countries, growth rates change drastically over time
    - until about 1800, no growth
    - Example: Philippines and SK
        - in 1960, PC GDP was similar
        - both had similar geo/social makeup & population
        - Over decades, performance diverged drastically
            - Korea: 5.1%
            - Philippines: 1.3%

- A country can change distribution in ranking
    - Argentina was rich, now it's not
    - China and India have grown rapidly

Why?
- Models:
    - What is it leaving out?
    - Does it fit the data?
        - A problem: past performance is no guarantee of future success
    - institutions?

A model of production:

Notation:

- $\bar{A}, \bar{K}, \bar{L}$ → constants / exogenous variables

Production function: Output given a set of inputs

Cobb-Douglas production function:

$$Y = F(K, L) = \bar{A} K^{\alpha} L^{\beta}$$; in this case, we have $\alpha + \beta = 1$

K: capital — non-consumed inputs not labor
L: labor — workers

$$Y = \bar{A} K^{1/3} L^{2/3}$$

$\bar{A}$: Total factor productivity

Exponents sum to one — constant returns to scale
- double input → double output

IRS: double input → > double outputs

Behavior of firm:

- firms seek maximal profits: $\pi_{max}(K,L) = F(K,L) - rK - wL$

r,w in normalized price

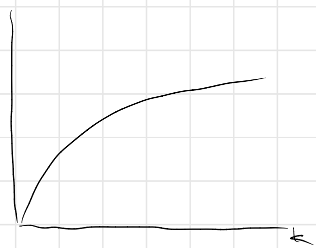
Marginal Product of Capital MPK: extra amount of output when we add an additional unit of capital, holding all else constant

- MPK decreases as $K\uparrow$ → Diminishing returns to capital
  - we reach "saturation"

$Y = \bar{A} K^{1/3} \bar{L}^{2/3}$; $Y = (K^{1/3})(\text{constant})$

↓

Diminishing returns



Solve for MPK: $\frac{dY}{dK}$

$$Y = \bar{A} K^{1/3} \bar{L}^{2/3}$$

$$\frac{dY}{dK} = \frac{1}{3}\bar{A}\bar{L}^{2/3} K^{-2/3}$$

$$= \frac{\bar{A}\bar{L}^{2/3}}{3K^{2/3}}$$

$$= \frac{1}{3}\bar{A}\left(\frac{L}{K}\right)^{2/3} = \frac{1}{3}\frac{Y}{K}$$

MPL = certain unit of labor → extra output

Diminishing returns to labor → workers become redundant

$$MPL = \frac{2}{3}\frac{Y}{L} = \frac{2}{3}\bar{A}\left(\frac{K}{L}\right)^{1/3}$$

- Exogenous variable set outside model
- Endogenous variable determined inside the model
- Profit max condition:
  - $MP_K = \frac{1}{3}\frac{Y}{K}$
  - $MP_L = \frac{2}{3}\frac{Y}{L}$

$MP_K > r \Rightarrow$ hire a machine $\rightarrow$ until $MP_K = r$
$MP_L > w \rightarrow$ hire a worker $\rightarrow$ until $MP_L = w$