

**Solution (20.1):** We know that  $\sin(z)$  is conformal when  $\frac{d}{dz}(\sin(z)) \neq 0$ , meaning that we verify when  $\cos(z) \neq 0$ . This occurs at  $z = n\pi$ , where  $n \in \mathbb{Z}$ .

We know that  $\sin(z) = 0$  when  $z = \pi$ , with  $\cos(z) = -1 = e^{i\pi}$ . Therefore, the image of  $z = \pi$  is not stretched, and is rotated by an angle of  $\pi$ .

We know that the image of  $z = i\pi$  is stretched by a factor of  $|\cos(i\pi)| = |\cosh(\pi)|$ . Since  $\cosh(\pi) = |\cosh(\pi)|$ , the image is rotated by an angle of 0.

Evaluating  $\cos(\pi/2 + i\pi)$ , we get that it is equal to  $-\sin(\pi/2)\sin(i)$ , or  $-i\sinh(1) = \sinh(1)e^{-i\pi/2}$ . Therefore, the image of  $z = \pi/2 + i$  is stretched by a factor of  $\sinh(1)$  and rotated by an angle of  $-\pi/2$ .

**Solution (20.9):** Mapping  $|z - 1| < 1$  to the plane  $\operatorname{Re}(w) > 0$ , with  $w(0) = \infty$ , we have

$$w(z) = \frac{az + b}{cz}.$$

Now, we want  $z = 2$  to map to zero, giving

$$w(z) = \frac{a(z - 2)}{cz}.$$

Finally, an entirely arbitrary decision made by the problem solver has it such that  $z = 1$  maps to  $z = 1$ . Thus, we have

$$\frac{-a}{c} = 1.$$

Therefore, we get the Möbius transformation of

$$w(z) = \frac{2 - z}{z}.$$

**Problem Solver's Note:** It is not possible for  $|z - 1| < 0$ , as norms are always at least equal to zero. The problem solver has decided to interpret the question such that it becomes nontrivial.

**Solution (20.10):** The first map of  $e^z$  has it such that  $\operatorname{Re}(w)$  ranges from  $e^{\operatorname{Re}(z_1)}$  to  $e^{\operatorname{Re}(z_2)}$ , while  $\arg(w)$  ranges from 0 to  $\pi$ , which agrees with the map showing an annular strip in the  $w$ -plane.

The second map of  $e^z$  maps  $z_1, z_2$ , and  $z_3$  to  $e^{\operatorname{Re}(z_1)}, 1$ , and  $e^{\operatorname{Re}(z_3)}$ , eventually converging to 0 as  $z_3$  becomes more and more negative. Similarly,  $e^z$  maps  $z_4, z_5$ , and  $z_6$  to  $e^{i\pi \operatorname{Re}(z_4)}, -1$ , and  $e^{i\pi \operatorname{Re}(z_6)}$ , similarly converging to 0 as  $z_4$  becomes more and more negative.

**Solution (20.11):**

**Solution (20.12):**

**Solution (20.14):**

**Solution (20.15):**

**Solution (20.16):**

**Solution (20.17):**