## 15.3

2:

$$
\begin{aligned}
\nabla f &= \lambda \nabla g \\
\begin{pmatrix} 1 \\ 3 \end{pmatrix} &= \lambda \begin{pmatrix} 2x \\ 2y \end{pmatrix} \\
2\lambda x &= 1 \\
2\lambda y &= 3 \\
x &= \frac{1}{2\lambda} \\
y &= \frac{3}{2\lambda} \\
x^2 + y^2 &= 10 \\
\frac{10}{4\lambda^2} &= 10 \\
\lambda &= \pm\frac{1}{2} \\
x &= \pm 1 \\
y &= \pm 3 \\
f &= 12, -8
\end{aligned}
$$

Therefore, $f$ is maximized subject to the constraint at $(1, 3, 12)$ and minimized at $(-1, -3, -8)$.

4:

$$
\begin{aligned}
\nabla f &= \lambda \nabla g \\
\begin{pmatrix} 3x^2 \\ 1 \end{pmatrix} &= \lambda \begin{pmatrix} 6x \\ 2y \end{pmatrix} \\
6\lambda x &= 3x^2 \\
3x\left(x - 6\lambda\right) &= 0 \\
x &= 0, 6\lambda \\
2\lambda y &= 1 \\
y &= \frac{1}{2\lambda} \\
3x^2 + y^2 &= 4 \\
\frac{1}{4\lambda^2} &= 4 && x = 0 \\
\lambda &= \pm\frac{1}{4} \\
(x,y) &= \left(0, \pm\frac{1}{4}\right) \\
f(x,y) &= \pm\frac{1}{4} \\
\frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} &= 4 && x = 6\lambda \\
\lambda &= \pm\frac{1}{2\sqrt{2}} \\
x &= \pm\frac{3}{\sqrt{2}} \\
y &= \pm\frac{1}{4\sqrt{2}} \\
(x,y) &= \left(\pm\frac{3}{\sqrt{2}}, \pm\frac{1}{4\sqrt{2}}\right) \\
f(x,y) &= \pm\frac{55}{2\sqrt{2}}
\end{aligned}
$$

Therefore, $f$ is maximized subject to the constraint at $\left(\frac{3}{\sqrt{2}}, \frac{1}{4\sqrt{2}}, \frac{55}{2\sqrt{2}}\right)$ and minimized at $\left(-\frac{3}{\sqrt{2}}, -\frac{1}{4\sqrt{2}}, -\frac{55}{2\sqrt{2}}\right)$.

10:

$$
\begin{aligned}
\nabla f &= \lambda \nabla g\\
\begin{pmatrix}1\\3\\5\end{pmatrix} &= \lambda\begin{pmatrix}2x\\2y\\2z\end{pmatrix}\\
2\lambda x &= 1\\
2\lambda y &= 3\\
2\lambda z &= 5\\
x^2+y^2+z^2 &= 1\\
\frac{35}{4\lambda^2} &= 1\\
\lambda &= \frac{\pm\sqrt{35}}{2}\\
(x,y,z) &= \left(\pm\frac{2}{\sqrt{35}}, \pm\frac{6}{\sqrt{35}}, \pm\frac{10}{\sqrt{35}}\right)\\
f(x,y,z) &= \pm 2\sqrt{35}
\end{aligned}
$$

Therefore, $f$ is maximized at $\left(\frac{2}{\sqrt{35}}, \frac{6}{\sqrt{35}}, \frac{10}{\sqrt{35}}, 2\sqrt{35}\right)$, and minimized at $\left(-\frac{2}{\sqrt{35}}, -\frac{6}{\sqrt{35}}, -\frac{10}{\sqrt{35}}, -2\sqrt{35}\right)$

12:

$$\begin{aligned}
\nabla f &= \lambda \nabla g\\
\begin{pmatrix} yz \\ xz \\ xy \end{pmatrix} &= \lambda \begin{pmatrix} 2x \\ 2y \\ 8z \end{pmatrix}\\
2\lambda x &= yz\\
2\lambda y &= xz\\
8\lambda z &= xy\\
z &= \frac{xy}{8\lambda}\\
2\lambda x &= \frac{y^2 x}{8\lambda}\\
16\lambda^2 &= y^2 && x \neq 0\\
16\lambda^2 &= x^2 && y \neq 0\\
z^2 &= 32\lambda^2\\
x^2 + y^2 + 4z^2 &= 12\\
32\lambda^2 + 128\lambda^2 &= 12\\
\lambda^2 &= \frac{3}{40}\\
\lambda &= \pm\sqrt{\frac{3}{40}}\\
x &= \pm\sqrt{\frac{6}{5}}\\
y &= \pm\sqrt{\frac{6}{5}}\\
z &= \pm\sqrt{\frac{12}{5}}\\
f(x,y,z) &= \pm\sqrt{\frac{6\sqrt{12}}{5\sqrt{5}}}
\end{aligned}$$

Therefore, $f$ is maximized when $x, y, z$ are positive at $\frac{6\sqrt{12}}{5}$ and minimized when $x, y, z$ are negative at $-\frac{6\sqrt{12}}{5}$.

36:

$$
\begin{aligned}
f &= 2\pi r^2 + 2\pi rh \\
\pi r^2 h &= 100 \\
\nabla f &= \lambda \nabla g \\
\begin{pmatrix} 4\pi r + 2\pi h \\ 2\pi r \end{pmatrix} &= \lambda \begin{pmatrix} 2\pi rh \\ \pi r^2 \end{pmatrix} \\
2\pi\lambda rh &= 4\pi r + 2\pi h \\
\pi\lambda r^2 &= 2\pi r \\
r &= \frac{2}{\lambda} \\
4\pi h &= \frac{8\pi}{\lambda} + 2\pi h \\
h &= \frac{4}{\lambda} \\
\pi r^2 h &= 100 \\
\pi \frac{16}{\lambda^3} &= 100 \\
\lambda &= \sqrt[3]{\frac{16\pi}{100}} \\
r &= \frac{2\sqrt[3]{100}}{\sqrt[3]{16\pi}} \\
h &= \frac{4\sqrt[3]{100}}{\sqrt[3]{16\pi}}
\end{aligned}
$$

## 16.1

2:

Lower Estimate:

$$
\begin{aligned}
\int_R f(x,y)dA &\approx (4)(0.1)(0.2) + (6)(0.1)(0.2) + (3)(0.1)(0.2) + (5)(0.1)(0.2) \\
&= 0.36
\end{aligned}
$$

Upper Estimate:

$$
\begin{aligned}
\int_R f(x,y)dA &\approx (7)(0.1)(0.2) + (10)(0.1)(0.2) + (6)(0.1)(0.2) + (8)(0.1)(0.2) \\
&= 62
\end{aligned}
$$

4:

Lower Estimate:

$$
\begin{aligned}
\int_R f(x,y)dA &\approx (50)\left(2+4+8+4+6+8+6+8+10\right) \\
&= 2800
\end{aligned}
$$

Upper Estimate:

$$\begin{aligned}\int_R f(x,y)dA &\approx (50)\left(4+6+8+6+8+8+8+10+10\right)\\ &= 3400\end{aligned}$$

6: The integral represents total bacteria population.

8: The integral is positive.

14: The integral is negative.

20: I don't know how to do this problem.