1 Returns to Scale

Part A

$$
\begin{aligned}
Q(L, K) &= L + L^{\frac{1}{3}} K^{\frac{2}{3}} + 2K + 5 \\
Q(2L, 2K) &= 2L + 2\left(L^{\frac{1}{3}} K^{\frac{2}{3}}\right) + 4K + 5 \\
&< 2(Q(L, K))
\end{aligned}
$$

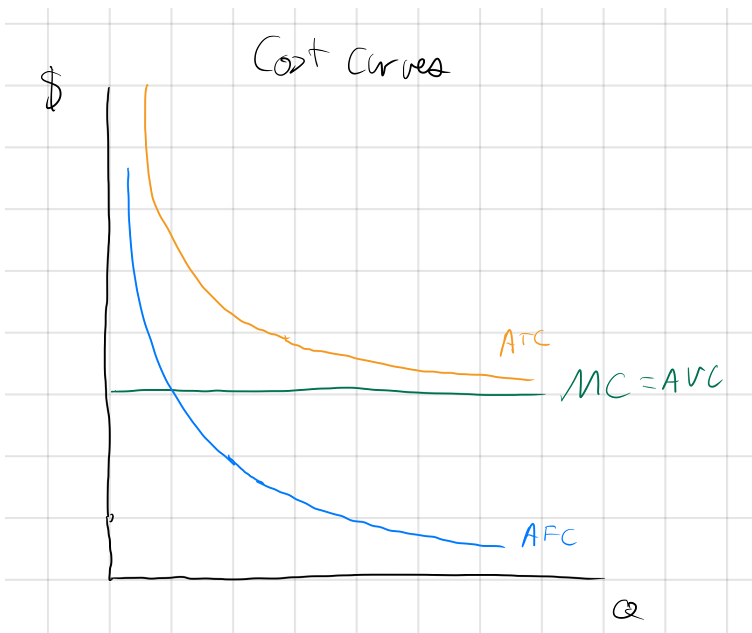
Therefore, production function exhibits **decreasing** returns to scale.

Part B

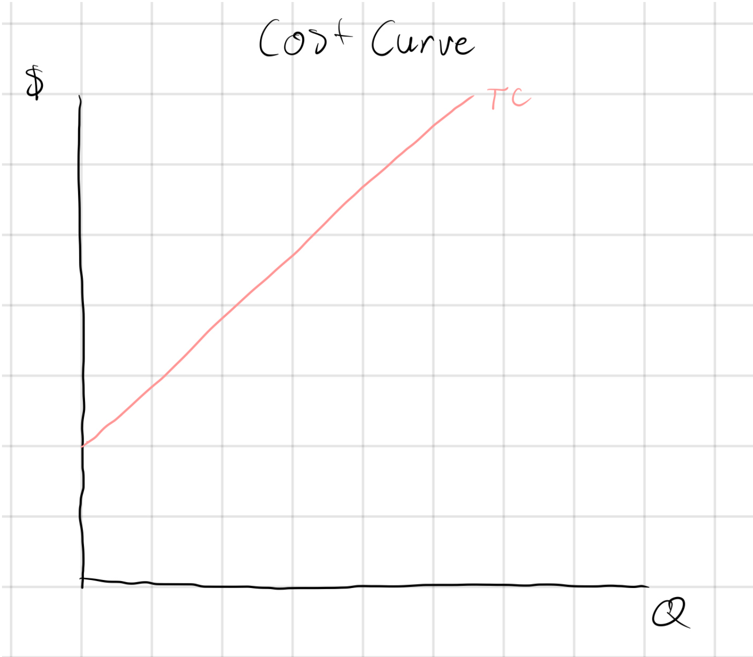
Consider the point on the $q = 70$ curve at $(20, 20)$. When increasing to $(30, 30)$, a factor of 1.5, the production only increases from 70 to 100, a factor of 1.43, so this function exhibits **decreasing** returns to scale.

2 Types of Cost Curves, Graphically

Part A



Part B



3 Types of Cost Curves, Mathematically

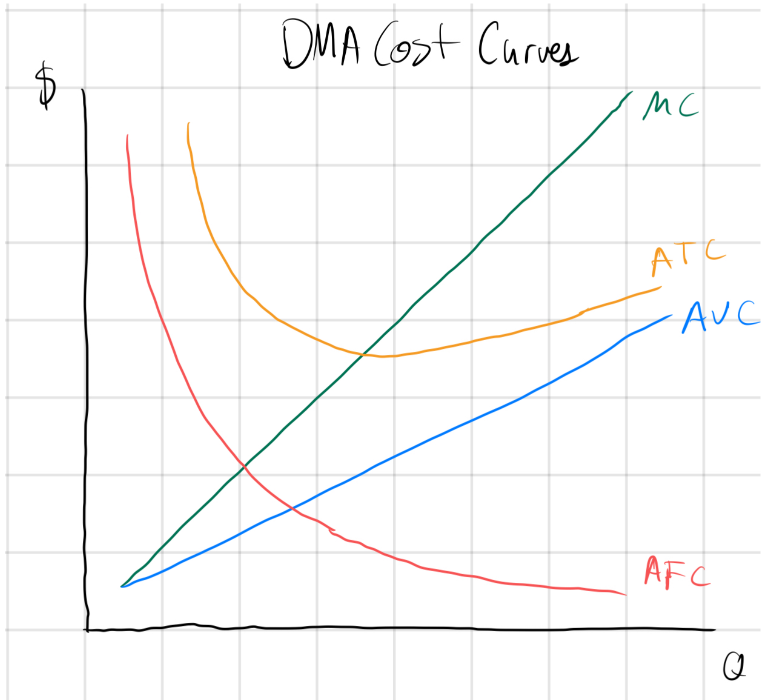
Part A

$$\begin{aligned} ATC(Q) &= \frac{TC(Q)}{Q} \\ &= \frac{288}{Q} + 3 + 2Q \\ AVC(Q) &= \frac{VC(Q)}{Q} \\ &= 3 + 2Q \\ MC(Q) &= \frac{dTC}{dQ} \\ &= 3 + 4Q \end{aligned}$$

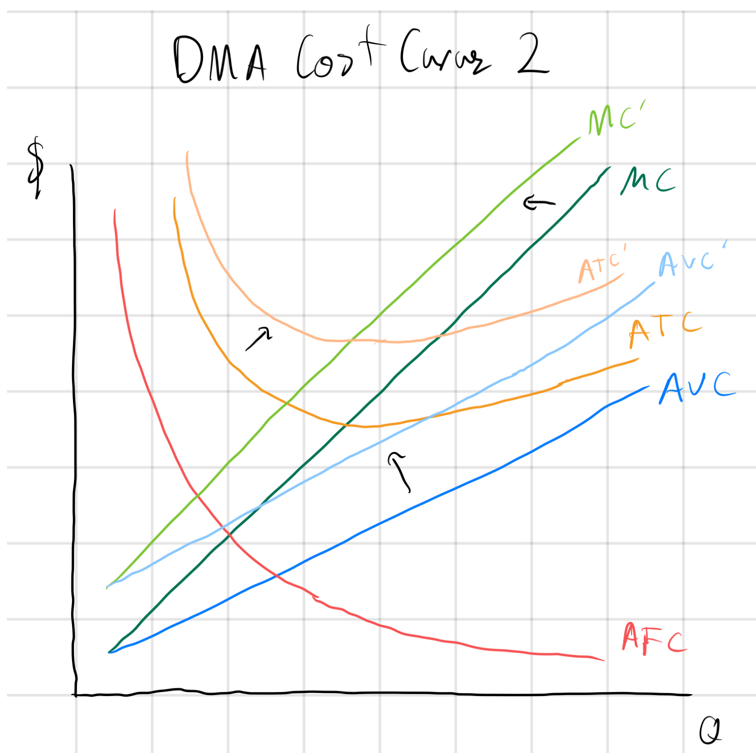
Part B

$$\begin{aligned} \frac{d(ATC)}{dQ} &= 0 \\ -\frac{288}{Q^2} + 2 &= 0 \\ Q &= 12 \\ ATC(Q) &= MC(Q) \\ \frac{288}{Q} + 3 + 2Q &= 3 + 4Q \\ \frac{288}{Q} &= 2Q \\ Q &= 12 \end{aligned}$$

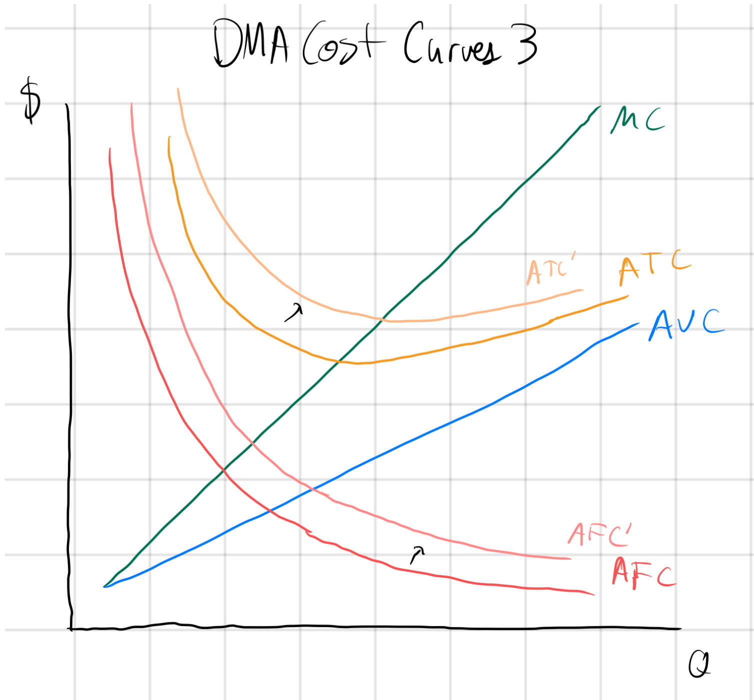
4 Cost Curves, Graphically



Part A



Part B



5 Cost Curves, Verbal Description

Part A

$$
\begin{aligned}
C_{\text{direct}} &= 200 + 30Q \\
Q &= 5T \\
C_{\text{direct}} &= 200 + 150T \\
C_{\text{opp}} &= 50(T^2 - 3T) \\
C_{\text{tot}} &= C_{\text{direct}} + C_{\text{opp}} \\
&= 200 + 150T + 50T^2 - 150T \\
&= 20 + 50T^2 \\
&= \boxed{20 + 2Q^2}
\end{aligned}
$$

Part B

Marjorie's business exhibits **diseconomies of scale**, as doubling production yields greater than a doubling in costs.

6 Conceptual Questions

1. Casey did **not** make a profit, as the opportunity cost is equal to $4 \times 35 = 140$ and the direct cost was 15, which is greater than the 150 he received at the end of the tournament.

2. Philo should **wait**—the \$20K spent on planting the corn is sunk costs, and the marginal revenue from harvesting the corn is only \$2 in September, while it is \$10 in May. The cost of harvesting the corn is fixed, too, so marginal revenue is higher than marginal cost in May.

7 Short Run Total Cost Curves

$$
\begin{aligned}
Q &= \min(0.5K, L) \\
Q &= \min(0.5(8), L) \\
4 &= \min(4, L) \\
SRTC(Q) &= 64 + 4Q, Q \leq 4
\end{aligned}
$$

8 Short Run Cost Functions

$$\begin{aligned}
Q(S,L) &= 0.1S^{\frac{1}{2}}L^{\frac{3}{4}} \\
Q(\overline{S},L) &= 0.1(10)L^{\frac{3}{4}} \\
&= L^{\frac{3}{4}} \\
L &= Q^{\frac{4}{3}} \\
L(Q) &= Q^{\frac{3}{4}} \\
SRTC(Q) &=
\end{aligned}$$

9 Short Run Total Cost Function

Part A

$$\begin{aligned}
Q(K,L) &= K + 2L \\
Q(\overline{K},L) &= 20 + 2L \\
L &= \frac{Q-20}{2} \\
SRTC(Q) &= 800 + 10Q, Q \geq 20
\end{aligned}$$

Part B

$$STRTC(Q) = \boxed{1000}$$

10 Long Run Total Costs, Standard Case

Part A

$$\begin{aligned}
Q &= K^{\frac{1}{2}}L^{\frac{1}{2}} \\
MRTS_{KL} &= \frac{L}{K} \\
\frac{L}{K} &= \frac{10}{5} \\
L &= 2K \\
Q &= K^{\frac{1}{2}}\left(2K^{\frac{1}{2}}\right) \\
Q &= K\sqrt{2} \\
K &= \frac{Q\sqrt{2}}{2} \\
L &= Q\sqrt{2} \\
LRTC(Q) &= 10\frac{Q\sqrt{2}}{2} + 5Q\sqrt{2} \\
&= 10Q\sqrt{2}
\end{aligned}$$

Part B

The production function exhibits constant economies of scale as the $LRTC$ function is linear in Q.

11 Deriving Total Cost Curves, Special Case

$$
\begin{aligned}
Q &= K + 10\sqrt{L} \\
MRTS_{KL} &= \frac{1}{10\frac{1}{2\sqrt{L}}} \\
\frac{\sqrt{L}}{5} &= 1 \\
\sqrt{L} &= 5 \\
L &= 25 \\
K &= Q - 50 \\
LRTC(Q) &= \begin{cases} Q - 25, & Q \geq 25 \\ 0, & Q < 25 \end{cases}
\end{aligned}
$$

12 Operation Decisions

Part A

Since the value of P is below the value of ATC at 11 units, the owner will shut down in the long run.

Part B

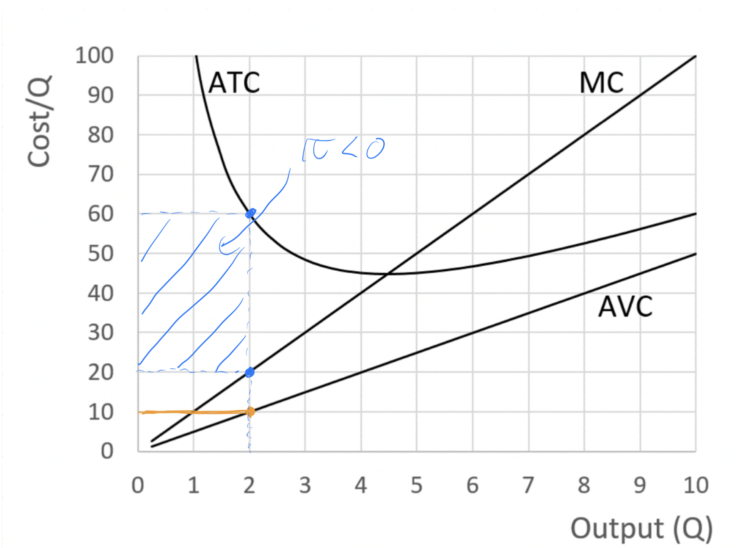
The short run loss will be equal to $(AFC)(Q^*) = (10.25 - 7.50)(11) = 30.25$

Part C

If the price is 7, it is better for the firm to produce zero units in the short run than to produce units, as the price is below the average variable cost. If the price is 9, then it is fine for the firm to produce units in the short run as the producer surplus is positive, but in the long run the firm will shut down as 9 is still below the average total cost curve.

13 Shutdown Decisions, Graphically

Part A



$$
\begin{aligned}
\pi &= (P - ATC)(Q^*) \\
&= (20 - 60)(2) \\
&= -80
\end{aligned}
$$

Part B

Lindsey **should** operate in the short run as her producer surplus $(P - AVC)$ is positive.

14 Shutdown Decisions, Math

$$
\begin{aligned}
TC(Q) &= Q^3 - 3Q^2 + 10Q \\
ATC(Q) &= \frac{TC(Q)}{Q} \\
&= Q^2 - 3Q + 10 \\
MC(Q) &= \frac{dTC}{dQ} \\
&= 3Q^2 - 6Q + 10 \\
MC &= MR \\
MR &= 10 \\
10 &= 3Q^2 - 6Q + 10 \\
Q(3Q - 6) &= 0 \\
Q &= 2 \\
ATC(2) &= (4) - (6) + 10 \\
&= 8
\end{aligned}
$$

Since $MR = 10$ is greater than $ATC = 8$, The York **should** operate in the long run.