Homework 1

1.1:

2c: $\left(1+\frac{3}{1+i}\right)^2$

$$= \left(1+\frac{(3-3i)}{2}\right)^2$$

$$= \frac{(5-3i)^2}{4}$$

$$= 4 - \frac{15}{2}i$$

7: No — $\mathrm{Re}(zw) = \frac{zw+\overline{zw}}{2}$

$$\mathrm{Re}(z)\,\mathrm{Re}(w) = \left(\frac{z+\bar{z}}{2}\right)\left(\frac{w+\bar{w}}{2}\right)$$

$$= \frac{zw + \overline{zw} + \bar{z}w + z\bar{w}}{2} \neq \frac{zw+\overline{zw}}{2}.$$

8: $\mathrm{Re}(az) = \frac{az+\overline{az}}{2}$

$$= \frac{az + a\bar{z}}{2}$$

$$= a\left(\frac{z+\bar{z}}{2}\right) = a\,\mathrm{Re}(z)$$

$\mathrm{Im}(az) = \frac{az-\overline{az}}{2i}$

$$= \frac{az - a\bar{z}}{2i}$$

$$= a\left(\frac{z-\bar{z}}{2i}\right) = a\,\mathrm{Im}(z)$$

$\mathrm{Re}(az+bw) = \frac{(az+bw)+\overline{(az+bw)}}{2}$

$$= \frac{(az+bw)+(\overline{az}+\overline{bw})}{2}$$

$$= \frac{az+\overline{az}}{2} + \frac{bw+\overline{bw}}{2}$$

$$= \frac{az+a\bar{z}}{2} + \frac{bw+b\bar{w}}{2}$$

$$= a\,\mathrm{Re}(z) + b\,\mathrm{Re}(w)$$

10: a: $w = a + bi$

$\Rightarrow zw = (ax - by) + (xb + ya)i$

$= \begin{pmatrix} ax - by \\ bx + ay \end{pmatrix}$

equivalent

$\begin{pmatrix} x & -y \\ y & x \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ax - by \\ ay + bx \end{pmatrix}$

b: $z_1 = x_1 + y_1 i$, $\phi_{z_1} = \begin{pmatrix} x_1 & -y_1 \\ y_1 & x_1 \end{pmatrix}$

$z_2 = x_2 + y_2 i$ $\phi_{z_2} = \begin{pmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{pmatrix}$

$z_1 z_2 = (x_1 x_2 - y_1 y_2) + (x_1 y_2 + y_1 x_2)i$, $\phi_{z_1 z_2} = \begin{pmatrix} x_1 x_2 - y_1 y_2 & -(x_1 y_2 + y_1 x_2) \\ x_1 y_2 + y_1 x_2 & x_1 x_2 - y_1 y_2 \end{pmatrix}$

$\phi_{z_1} \phi_{z_2}$

$= \begin{pmatrix} x_1 & -y_1 \\ y_1 & x_1 \end{pmatrix} \begin{pmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{pmatrix}$

equivalent

$= \begin{pmatrix} x_1 x_2 - y_1 y_2 & -(x_1 y_2 + y_1 x_2) \\ x_2 y_1 + y_2 x_1 & x_1 x_2 - y_1 y_2 \end{pmatrix}$

1.2:

3: $\overline{\left(\frac{(3+8i)^4}{(1+i)^{10}} \right)} = \frac{(3-8i)^4}{(1-i)^{10}}$

7: $\left| \frac{i(2+3i)(5-2i)}{-2-i} \right| = \sqrt{\frac{13 \cdot 29}{5}}$

$= \sqrt{\frac{377}{5}}$

9: $w \neq 1$, $w^n - 1 = 0$

$= (w-1)(1 + w + \cdots + w^{n-1})$

$\therefore 1 + w + \cdots + w^{n-1} = 0$

14: Wlog, let $|z|=1$. Then, $z=a+bi$, $\sqrt{a^2+b^2}=1$, $w=c+di$, $|zw|=|w|$

$=\sqrt{c^2+d^2}$

$$\left|\frac{z-w}{1-\bar{z}w}\right| = \frac{|z-w|}{|1-\bar{z}w|}$$

$(a-bi)(c+di)$

$(ac+bd)+(ad-bc)i$

$$= \frac{|(a-c)+(b-d)i|}{|(1-ac+bd)+(bc-ad)i|}$$

$$= \frac{\sqrt{(a-c)^2+(b-d)^2}}{\sqrt{(ac+bd-1)^2+(ad-bc)^2}}$$

$$= \sqrt{\frac{a^2-2ac+c^2+b^2-2bd+d^2}{(ac-bd)^2-2(ac+bd)+1+(ad-bc)^2}}$$

$$= \sqrt{\frac{c^2+d^2+1-2(ac+bd)}{c^2+d^2+1-2(ac+bd)}}$$

$= 1$

16: I don't know how to do this problem.