19.4

2:

- Direct Calculation:

$$\begin{aligned}\int_S \vec{F}\cdot d\vec{A} &= \int_0^{2\pi}\int_{-1}^{1} \sin\theta \; dz \; d\theta \\ &= 0\end{aligned}$$

- Divergence Theorem:

$$\begin{aligned}\int_S \vec{F}\cdot d\vec{A} &= \int_{-1}^{1}\int_0^{2\pi}\int_0^{1} \; dr \; d\theta \; dz \\ &= 0\end{aligned}$$

4:

6:

$$\begin{aligned}\int_S \vec{F}\cdot d\vec{A} &= \int_V 10 dV \\ &= 240.\end{aligned}$$

8:

$$\begin{aligned}\int_S \vec{H}\cdot d\vec{A} &= \int_0^4\int_0^3\int_0^2 (y) \; dx \; dy \; dz \\ &= \int_0^4\int_0^3 2y \; dy \; dz \\ &= \int_0^4 9 \; dz \\ &= 36.\end{aligned}$$

10:

$$\begin{aligned}\int_S \vec{N}\cdot d\vec{A} &= \int_V \nabla\cdot\vec{N} \; dV \\ &= 0.\end{aligned}$$

14:

$$\begin{aligned}\int_S \vec{F}\cdot d\vec{A} &= \int_V \nabla\cdot\vec{F} \; dV \\ &= \int_V x+y+z \; dV \\ &= \int_0^{\pi}\int_0^{2\pi}\int_0^{1} \rho(\sin\phi\cos\theta+\sin\phi\sin\theta+\cos\phi) \; \rho^2\sin\phi \; d\rho \; d\theta \; d\phi \\ &= 0\end{aligned}$$

16:

$$\begin{aligned}\int_S \vec{F}\cdot d\vec{A} &= \int_V \nabla\cdot\vec{F} d\vec{V} \\ &= \int_0^{\pi/4}\int_0^{2\pi}\int_2^{3} 3\rho^4\sin\phi \; d\rho \; d\theta \; d\phi \\ &= \frac{633(2-\sqrt{2})\pi}{5}\end{aligned}$$

22:

20.1

6:

$$\nabla \times \vec{F} = \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$$

8:

$$\nabla \times \vec{F} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

10:

$$\nabla \times \vec{F} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

12:

$$\nabla \times \vec{F} = \begin{pmatrix} 2zx^3y + 6y^5x^7 - xy \\ y - 7x^6y^6 \\ zy - z \end{pmatrix}$$

22:

$$\vec{F} = \begin{pmatrix} a(x) \\ b(y) \\ c(z) \end{pmatrix}$$

24:

(a) Counterclockwise.

(b) Clockwise.

(c)

$$\nabla \times \vec{F} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

20.2

2: Zero.

4:

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= \int_0^1 \begin{pmatrix} 3(\cos(\pi t) + \sin(\pi t)) \\ 3\cos(\pi t) \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -3\pi \sin(\pi t) \\ 0 \\ 3\pi \cos(\pi t) \end{pmatrix} dt + \int_0^1 \begin{pmatrix} 6t - 3 \\ 6t - 3 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} dt \\ &= \frac{9\pi}{2} \\ \int_S \nabla \times \vec{F} \cdot d\vec{A} &= \frac{9\pi}{2} \int_S \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} dA \\ &= \frac{9\pi}{2} \end{aligned}$$

10:

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= \int_S \nabla \times \vec{F} \cdot d\vec{A} \\ &= -5\pi \end{aligned}$$

12:

$$\begin{aligned}\int_C \vec{F}\cdot d\vec{r} &= \int_S \nabla\times\vec{F}\cdot d\vec{A}\\ &= 0\end{aligned}$$

28:

(a) $\vec{C} = \begin{pmatrix}\sin t\\ \cos t\\ 2\end{pmatrix}$

(b)

$$\begin{aligned}\int_S \nabla\times\vec{F}\cdot d\vec{A} &= \int_C \vec{F}\cdot d\vec{r}\\ &= \int_C \begin{pmatrix}-\cos t\\ \sin t\\ 2\end{pmatrix}\cdot\begin{pmatrix}\cos t\\ -\sin t\\ 0\end{pmatrix}\ dt\\ &= -2\pi\end{aligned}$$

34:

$$\begin{aligned}\int_{C_1} \vec{F}\cdot d\vec{r} &= \int_S \nabla\times\vec{F}\cdot d\vec{A}\\ &= \int_0^5\int_0^{2\pi} 12\ d\theta\ dz\\ &= 120\pi\\ &= \int_{C_2} \vec{F}\cdot d\vec{r}\\ \int_{C_1}\vec{F}\cdot d\vec{r} + \int_{C_2}\vec{F}\cdot d\vec{r} &= 240\pi\end{aligned}$$

20.3

4:

$$\begin{aligned}\nabla\times\vec{F} &= \begin{pmatrix}0\\ -4\\ 0\end{pmatrix}\\ &\neq \vec{0}\end{aligned}$$

6:

$$\nabla\times\vec{F} = \vec{0}$$

8:

$$\nabla\cdot\vec{F} = 0$$

Thus, the vector field is a curl field.

24: There does exist a vector potential for the vector field since the divergence is zero. I don't know how to find it.

28:

$$\begin{aligned}\int_C \begin{pmatrix}u\\ v\end{pmatrix}\cdot d\vec{r} &= \int_S \nabla\times\begin{pmatrix}u\\ v\\ 0\end{pmatrix}\cdot\begin{pmatrix}0\\ 0\\ 1\end{pmatrix} dA\\ &= \int_R \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\ dx\ dy\end{aligned}$$