Problem (Problem 1)**:** Every square S in an infinite chessboard contains a positive integer equal to the average of the four non-diagonal neighbors of S. Are the integers necessarily equal?

Problem (Problem 2)**:** Alice has $n+1$ binary strings of length $n \geq 2$. Bob knows this but knows nothing about the bits in the strings. Bob wants to specify a binary string of length n that is not in Alice's set. Bob can ask Alice questions of the form: "What is the jth bit of the kth string?" What is the minimal number of questions that Bob needs to ask to guarantee success?

Problem (Problem 3)**:** Show that among $n+1$ integers not greater than $2n$ that there are two such that one divides the other.

Problem (Problem 4)**:** Show that for every graph there is an orientation of the edges such that for every vertex the out-degree and in-degree differ by at most 1.

Solution: Without loss of generality, we assume G is connected. If G is disconnected, we apply the following procedure to each of the connected components.

If all vertices in G are of even degree, we may apply an orientation on G by following an Eulerian circuit. Every vertex in this directed Eulerian circuit has degree zero.

If there are any vertices in G of odd degree, we know that there must be an even number of these vertices of odd degree; call them $\{v_1, \dots, v_{2n}\}$. We add vertices $\{w_1, \dots, w_n\}$ and edges such that w_i has an edge to v_{2i-1} and v_{2i}. This new graph, call it G', has an Eulerian circuit; we follow this Eulerian circuit to place an orientation on all edges. Since each v_i has only one edge out to a w_i (or one edge in from a w_i), we may delete $\{w_1, \dots, w_n\}$ to yield G with an orientation of all edges that has difference at most one between in-degree and out-degree.

Problem (Problem 5)**:** A convex board is surrounded by some nails hammered into a table; the nails make it impossible to slide the board in any direction, but if any of them is missing then this is no longer true. What is the maximal number of nails?