

Problem (Problem 1): Find simply connected covering spaces of \mathbb{RP}^2 and $S^2 \vee S^1$.

Solution: We observe that $\mathbb{RP}^2 = S^2/(x \sim -x)$ can be identified as a quotient space. Therefore, if we let $q: S^2 \rightarrow \mathbb{RP}^2$ be the quotient map, then q is a covering map as any open subset of \mathbb{RP}^2 , upon taking preimages, maps to two antipodal homeomorphic open subsets of S^2 . Thus, this is our desired (simply connected) covering space.

In the case of $S^2 \vee S^1$, we observe that from Homework 12, a covering space is given by $\mathbb{R} \sqcup S^2$ attached at $(1, 0, 0) \sim 0$. Since their intersection is a single point, and both $\pi_1(S^2)$ and $\pi_1(\mathbb{R})$ are the trivial group, it follows from a van Kampen argument that $\pi_1(\mathbb{R} \vee S^2)$ is the trivial group, so this is a simply connected cover for $S^2 \vee S^1$.