8.1

(a)

$$\begin{aligned}
\int_0^1 2^x \, dx &= \int_0^1 e^{x(\ln 2)} \, dx \\
&= \frac{1}{\ln 2}\left(e^{x(\ln 2)}\Big|_0^1\right) && u = x(\ln 2) \\
&= \frac{1}{\ln 2}\left(2^x\Big|_0^1\right) \\
&= \frac{1}{\ln 2}(2-1) \\
&= \frac{1}{\ln 2}.
\end{aligned}$$

(b)

$$\begin{aligned}
\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} e^x \, dx &= \int_{-\infty}^{\infty} e^{\left(-\frac{x^2}{2}+x-\frac{1}{2}\right)+\frac{1}{2}} \, dx && \text{Completing the square.} \\
&= e^{\frac{1}{2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x-1)^2} \, dx \\
&= \sqrt{2\pi e}
\end{aligned}$$

(c)

(d)

$$\int_{-a}^{a} \sin x e^{-\alpha x^2} \, dx = 0 \qquad \text{Even/odd.}$$

(e)

$$\begin{aligned}
\int_0^1 e^{\sqrt{x}} \, dx &= x e^{\sqrt{x}}\Big|_0^1 - \frac{1}{2}\int_0^1 \sqrt{x} e^{\sqrt{x}} \, dx \\
&= e - \int_0^1 u^3 e^u \, du && u = \sqrt{x} \\
&= e - \left(u^3 e^u\Big|_0^1 - 3u^2 e^u\Big|_0^1 + 6u\Big|_0^1 - 6e^u\Big|_0^1\right) && \text{Repeated integration by parts.} \\
&= 3e - 6.
\end{aligned}$$

To evaluate $\int_0^1 u^3 e^u \, du$, we used tabular integration as follows:

Sign	Differentiate	Integrate
+	u^3	e^u
-	$3u^2$	e^u
+	$6u$	e^u
-	6	e^u
+	0	e^u

Taking the boundary integrals, we obtain

$$u^3 e^u\Big|_0^1 - 3u^2 e^u\Big|_0^1 + 6ue^u\Big|_0^1 - 6e^u\Big|_0^1 = 6 - 2e$$

(f)

$$\begin{aligned}
\int \frac{1}{\sqrt{1+x^2}}\,dx &= \int \frac{1}{\cosh(u)}\cosh(u)\,du && x = \sinh(u)\\
&= u + C\\
&= \sinh^{-1}(x) + C.
\end{aligned}$$

(g)

$$\begin{aligned}
\int \tanh x\,dx &= \int \frac{\sinh x}{\cosh x}\,dx\\
&= \int \frac{1}{u}\,du && u = \cosh x\\
&= \ln|u| + C\\
&= \ln|\cosh x| + C.
\end{aligned}$$

(h)

$$\begin{aligned}
\int \tan^{-1} x\,dx &= x\tan^{-1} x - \int \frac{x}{1+x^2}\,dx && \text{integration by parts}\\
&= x\tan^{-1} x - \frac{1}{2}\ln|1+x^2| + C. && u\text{-substitution implicit}
\end{aligned}$$