Activity: Cournot Competition

Econ 305

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Consider the standard duopoly ($n = 2$) Cournot competition game in which $c_1(q_1) = 10q_1$ and

$$P(Q) = \begin{cases} 100 - Q & \text{if } Q \leq 100 \\ 0 & \text{if } Q > 100 \end{cases}$$

In class I showed that the best response function for firm 1 is:

$$BR_1(q_2) = \begin{cases} \frac{1}{2}(90 - q_2) & \text{if } q_2 \leq 90 \\ 0 & \text{if } q_2 > 90 \end{cases}$$

Determine the Nash equilibrium of the following Cournot games.

Example 1: Asymmetric Costs where $c_2(q_2) = 40q_2$, but all else is unchanged.

$BR_1(q_2) = \frac{1}{2}(90 - q_2)$

$BR_2(q_1) = \max_{q_2} q_2(100 - q_2 - q_1) - 40q_2$

$= \max_{q_2} q_2(60 - q_2 - q_1)$

$0 = 60 - 2q_2 - q_1$

$q_2 = \frac{1}{2}(60 - q_1)$

$q_1^* = BR_1(BR_2(q_1^*))$

$= \dfrac{90 - \frac{60 - q_1^*}{2}}{2}$

$q_1^* = 30 + \dfrac{q_1^*}{4}$

$\boxed{\begin{aligned} q_1^* &= 40 \\ q_2^* &= 10 \end{aligned}}$

Example 2: n **Identical Firms, each with** $c_i(q_i) = 10q_i$ **for all** $i = 1, 2, ..., n$.

Hint: After finding the best response functions, you can assume that every firm will choose the same quantity in the Nash equilibrium.

$$BR_1(q_{-i}) = \max_{q_1} q_1(100 - \Sigma q_j) - 10q_1$$

$$= \max_{q_1} q_1(90 - \Sigma q_j)$$

$$0 = 90 - 2q_1 - \Sigma q_{-i}$$

$$q_i = \frac{1}{2}(90 - \Sigma q_{-i})$$

$$BR_i(q_i) = \frac{1}{2}(90 - \underbrace{\Sigma q_{-i}}_{(n-1)q_i^*})$$

$$q_i^* = \frac{1}{2}(90 - (n-1)(q_i^*))$$

$$2q_i^* = 90 - nq_i^* + q_i^*$$

$$q_i^* = \frac{90}{1+n}$$