

The primary text for Algebra II is Dummit and Foote's *Abstract Algebra*, and will cover the following topics:

- modules and advanced linear algebra;
- representation theory of finite groups;
- field theory and Galois theory.

Contents

Modules and Advanced Linear Algebra

Tensor Products of Modules

To motivate tensor products, we recall a basic fact from linear algebra. If we assume that R is a field, and M, N are finite-dimensional R -vector spaces, then the following equation necessarily holds:

$$\dim(M \oplus N) = \dim(M) + \dim(N).$$

We want to construct a similar operation on vector spaces, $M \otimes N$, that satisfies

$$\dim(M \otimes N) = \dim(M) \dim(N).$$

For now, we will label this by $M \bar{\otimes} N$, where we use the $\bar{\otimes}$ to refer to the fact that this is a temporary definition. Naively, we might seek to define $M \bar{\otimes} N$ as follows. If we let $\{x_1, \dots, x_k\}$ be a basis for M and $\{y_1, \dots, y_\ell\}$ a basis for N , then we will define $M \bar{\otimes} N$ to be all the formal R -linear combinations over the basis

$$B = \{x_i \otimes y_j \mid 1 \leq i \leq k, 1 \leq j \leq \ell\}.$$