

4.1

Problem: Write a Turing machine that computes the constant function $C_0(n) = 0$.

Solution: The Turing machine with the instructions

$q_1 1 B q_2$
 $q_2 B R q_2$
 $q_2 1 B q_3$
 $q_3 B R q_4$
 $q_4 1 B q_3$

computes $C_0(n)$.

4.2

Problem: Write a Turing machine that computes the identity function $f(m) = m$ on \mathbb{N} . Then write a machine for the identity function on \mathbb{N}^n .

Solution: The Turing machine with instructions

$q_1 1 B q_2$

computes $f(m) = m$.

The Turing machine with instructions

$q_1 1 B q_2$
 $q_2 B R q_3$
 $q_3 1 R q_3$
 $q_3 B R q_4$
 $q_4 B B q_5$
 $q_4 1 B q_2$

computes $\text{id}_{\mathbb{N}^n}$.

4.3

Problem: Write a Turing machine that computes the function

$$Z(n) = \begin{cases} 1 & n = 0 \\ 0 & n \geq 1 \end{cases}.$$

Solution: The Turing machine with the instructions

$q_1 1 R q_2$
 $q_2 1 B q_3$
 $q_3 B L q_4$
 $q_4 1 B q_5$
 $q_5 B R q_5$
 $q_5 1 L q_6$
 $q_6 1 B q_7$
 $q_7 B R q_6$

computes $Z(n)$.

4.4

Problem: Write a Turing machine that computes $f(m, n) = |m - n|$.

Solution: The Turing machine with the instructions

$q_1 1 B q_2$
 $q_2 B R q_3$
 $q_3 1 R q_3$
 $q_3 B R q_4$
 $q_4 1 R q_5$
 $q_5 1 R q_5$
 $q_5 B L q_6$
 $q_6 1 B q_7$
 $q_7 B L q_8$
 $q_8 1 L q_8$
 $q_8 B L q_9$
 $q_9 1 L q_9$
 $q_9 B R q_1$

computes $f(m, n) = |m - n|$. In short, for an input tape

$$B, \underbrace{1, 1, \dots, 1}_{m+1}, B, \underbrace{1, 1, \dots, 1}_{n+1}, B$$

the machine successively deletes the left-most 1 on the tape denoting m and deletes the right-most 1 on the tape denoting n , until it reaches the end of one of the domain elements (since both q_1 and q_4 are not defined for blank inputs).