Tadelis 3.7: Public Good Contribution

Three players live in a town, and each can choose to contribute to fund a streetlamp. The value of having the streetlamp is 3 for each player, and the value of not having it is 0. The mayor asks each player ot contribute either 1 or nothing. If at least two players contribute then the lamp will be erected. If one player or no players contribute then the lamp will not be erected, in which case any person who contributed will not get his money back. Write down the normal form of this game.

Players: $N = \{1, 2, 3\}$

Strategy Sets: $S_i = \{F, A\}$ for $i \in \{1, 2, 3\}$

Payoffs: Let $L = \{(F, F, F), (F, F, A), (F, A, F), (A, F, F)\}$ denote the set of possibilities that yields the streetlight being built, while $D = \{(A, A, A), (F, A, A), (A, F, A), (A, A, F)\}$ denotes the set of possibilities that yield the street light not being built. Then, the payoffs are as follows:

$$v_i = \begin{cases} 3 & \text{if } s_i = A, (s_1, s_2, s_3) \in L \\ 2 & \text{if } s_i = F, (s_1, s_2, s_3) \in L \\ -1 & \text{if } s_i = F, (s_1, s_2, s_3) \in D \\ 0 & \text{if } s_i = A, (s_1, s_2, s_3) \in D \end{cases}$$

Hermaphroditic Fish

Members of some species of hermaphroditic fish choose, in each mating encounter, whether to play the role of a male or a female. Each fish has a preferred role, which uses up fewer resources and hence allows more future mating. A fish obtains a payoff H if it mates in its preferred role and L if it mates in the other role, where $H > L$. Consider an encounter between two fish whose preferred roles are the same. Each fish has two possible strategies: mate in either role or insist on its preferred role. If both fish offer to mate in either role, the roles are assigned randomly, and each fish's payoff is $\frac{1}{2}(H + L)$. If each fish insists on its preferred role, the fish do not mate; each goes off in search of another partner, and obtains the payoff S. The higher the chance of meeting another partner, the larger S is. Formulate this situation as a normal-form game and determine the range of values of S, for any given values of H and L, for which the game differs from the Prisoner's Dilemma only in the names of the actions.

Stag Hunt

Jean-Jacques Rousseau in his *Discourse on the origin and foundations of inequality among men* discusses a group of hunters who wish to catch a stag. They will succeed if they all remain attentive, but each is tempted to desert her post and catch a hare.

We can model this as a group of n hunters, each of whom has two options: remain attentive to the stag (S) or catch a hare (H). There is only one stag, but more hares than hunters. Assume that at least m hunters (with $2 \leq m \leq n$) are needed to pursue the stag in order to catch it. A captured stag is only shared by the hunters who catch it. Find all the Nash equilibria of the strategic game under each of the following assumptions.

(a)

All hunters are needed to catch the stag: $m = n$. For each hunter, the strategy profile in which all hunters choose S is ranked highest, followed by any profile in which they choose H, followed by any profile in which they choose S and one or more of the other players chooses H.

$$v_i(s_i, s_{-i}) = \begin{cases} H & s_i = H \\ 0 & s_i = S, s_{-i} \neq (S, S, \dots, S) \\ S & s_i = S, s_{-i} = (S, S, \dots, S) \end{cases}$$

The cases are as follows:

Case 1: $s = (S, S, \dots, S)$

Follow: $v_i = S$

Deviate: $v_i = H$

Case 2: $s = (H, H, \dots, H)$

Follow: $v_i = H$

Deviate: $v_i = 0$

Case 3: $s \neq (S, S, \dots, S), (H, H, \dots, H),\ s_i = S$

Follow: $v_i = 0$

Deviate: $v_i = H$

Therefore, our two Pure Strategy Nash Equilibria are $s = (S, S, \dots, S)$ and $s = (H, H, \dots, H)$.