Solution (19.1)**:**

(a) There is a simple pole at $z = 0$. The residue at this pole is 0.
(b) There is a pole of order 4 at $z = 0$. The residue at this pole is 0.
(c) There is a pole of order 4 at $z = 0$. The residue at this pole is $\frac{1}{120}$.
(d) There is an essential singularity at $z = 0$.
(e) There is a removable singularity at $z = 0$.

Solution (19.2)**:** The poles of $\frac{e^z}{\sin z}$ occur when $\sin z = 0$, which happens when $z = n\pi$.

Solution (19.4)**:** There are no residues within $|z| < 1$.

For $1 < |z| < 2$, evaluating the a_{-1} term, we have the residue of $\frac{1}{3}$.

For $|z| > 2$, evaluating the a_{-1} term, we have a residue of $\frac{1}{3}$.

Solution (19.5)**:**

(a) There is a pole of order 2 at $z = 1$ and a pole of order 1 at $z = 0$.
(b) Around $z = 0$, we have the expansion

$$\begin{aligned}\frac{1}{z(z-1)^2} &= \frac{1}{z(1-z)^2}\\ &= \frac{1}{z}\left(\sum_{k=1}^{\infty} kz^{k-1}\right)\\ &= \sum_{k=1}^{\infty} kz^{k-2},\end{aligned}$$

which converges for all $0|z| < 1$. Around $z = 1$, we have the expansion

$$\begin{aligned}\frac{1}{(z-1)^2 z} &= \frac{1}{(z-1)^2(1+z-1)}\\ &= \frac{1}{(z-1)^2}\left(\sum_{k=0}^{\infty}(-1)^k(z-1)^k\right)\\ &= \sum_{k=0}^{\infty}(-1)^k(z-1)^{k-2}.\end{aligned}$$

This series converges for all $0 < |z-1| < 1$.
(c) The residue at $z = 0$ is 1, and the residue at $z = 1$ is -1.

Solution (19.9)**:** If a is not a singularity of $w(z)$, the Laurent expansion collapses into the Taylor expansion.

Solution (19.11)**:**

Solution (19.13)**:**

Solution (19.18)**:**

Solution (19.24)**:** We must move $e^{2\pi i}$ back into the principal branch to evaluate the square root.

Solution (19.28)**:**

(a) We have

$$\begin{aligned}e^{iz} &= \cos(z) + i\sin(z)\\ &= \left(1 - \sin^2(z)\right)^{1/2} + i\sin(z).\end{aligned}$$

Thus, defining $w = \sin(z)$, we have

$$\begin{aligned} iz &= \ln\left(iw + \left(1 - w^2\right)^{1/2}\right) \\ z &= -i\ln\left(iw + \left(1 - w^2\right)^{1/2}\right). \end{aligned}$$

Similarly, defining $w = \cos(z)$, we have

$$\begin{aligned} e^{iz} &= \cos(z) + i\left(1 - \cos^2(z)\right)^{1/2} \\ iz &= \ln\left(w + i\left(1 - w^2\right)^{1/2}\right) \\ &= \ln\left(w + i\left((-1)\left(w^2 - 1\right)\right)^{1/2}\right) \\ &= \ln\left(w + i(-i)\left(w^2 - 1\right)^{1/2}\right) \\ &= \ln\left(w + \left(w^2 - 1\right)^{1/2}\right). \end{aligned}$$