Solution (20.1)**:** We know that $\sin(z)$ is conformal when $\frac{d}{dz}(\sin(z)) \neq 0$, meaning that we verify when $\cos(z) \neq 0$. This occurs at $z = n\pi$, where $n \in \mathbb{Z}$.

We know that $\sin(z) = 0$ when $z = \pi$, with $\cos(z) = -1 = e^{i\pi}$. Therefore, the image of $z = \pi$ is not stretched, and is rotated by an angle of π.

We know that the image of $z = i\pi$ is stretched by a factor of $|\cos(i\pi)| = |\cosh(\pi)|$. Since $\cosh(\pi) = |\cosh(\pi)|$, the image is rotated by an angle of 0.

Evaluating $\cos(\pi/2 + i\pi)$, we get that it is equal to $-\sin(\pi/2)\sin(i)$, or $-i\sinh(1) = \sinh(1)e^{-i\pi/2}$. Therefore, the image of $z = \pi/2 + i$ is stretched by a factor of $\sinh(1)$ and rotated by an angle of $-\pi/2$.

Solution (20.9)**:** From Table 20.1, we find that

$$w = \frac{z+1}{1-z}$$

maps the *unit* circle to the right half plane. Therefore, scaling everything by $\sqrt{2}$, we have

$$w = \frac{\sqrt{2}z+1}{1-\sqrt{2}z}.$$

Problem Solver's Note: It is not possible for $|z-1| < 0$, as norms are always at least equal to zero. The problem solver has decided to interpret the question such that it becomes nontrivial.

Solution (20.10)**:** The first map of e^z has it such that $\mathrm{Re}(w)$ ranges from $e^{\mathrm{Re}(z_1)}$ to $e^{\mathrm{Re}(z_2)}$, while $\arg(w)$ ranges from 0 to π, which agrees with the map showing an annular strip in the w-plane.

The second map of e^z maps z_1, z_2, and z_3 to $e^{\mathrm{Re}(z_1)}$, 1, and $e^{\mathrm{Re}(z_3)}$, eventually converging to 0 as z_3 becomes more and more negative. Similarly, e^z maps z_4, z_5, and z_6 to $e^{i\pi\,\mathrm{Re}(z_4)}$, -1, and $e^{i\pi\,\mathrm{Re}(z_6)}$, similarly converging to 0 as z_4 becomes more and more negative.

Solution (20.11)**:**

(a) Since w is a composition of conformal maps (a Möbius transformation and the principal branch ln function), $w(z)$ is conformal.

(b) The cut line occurs when the argument, $\frac{z-1}{z+1}$, is less than or equal to zero, meaning that the cut line is along the real axis with $z \leqslant 1$.

(c) To start, we know that in the UHP, $\arg(w)$ ranges from 0 to π. Now, the Möbius transformation $\frac{z-1}{z+1}$ maps $\infty \to 1$, $0 \to -1$, and $1 \to 0$. Now, we have

$$\ln\left(\frac{(x-1)+iy}{(x+1)+iy}\right) = \ln(r) + i\arctan\left(\frac{2y}{(x^2-1)+y^2}\right),$$

where r is somewhat immaterial.

Solution (20.12)**:** We know that the strip $0 < y < \pi$ maps to the UHP under $w_1 = e^z$. Then, using either the cross ratio or the result in Example 20.3, we use the ratio $\frac{z-i}{z+i}$ to map the UHP into the unit disk. Thus, we have the final conformal map of

$$w(z) = \frac{e^z - i}{e^z + i}.$$

Solution (20.14)**:** Note that if $z = e^{i\varphi}$ for $0 \leqslant \varphi \leqslant \pi$, then

$$z + \frac{1}{z} = e^{i\varphi} + e^{-i\varphi},$$

which ranges from -2 to 2. Now, if $|x| > 1$, then

$$w(z) = x + \frac{x}{x^2+y^2} + i\left(y - \frac{y}{x^2+y^2}\right),$$

and, setting $y = 0$, we have

$$w(x) = x + \frac{1}{x}.$$

This gives a map from the x axis to the x axis.

Solution (20.15)**:**

Solution (20.16)**:**

Solution (20.17)**:**