

$$\begin{aligned}
 F(x) &= \int_{\ln(x)}^2 y^6 + 3^y \, dy \\
 &= - \int_2^{\ln(x)} y^6 + 3^y \, dy
 \end{aligned}$$

Flipping Bounds of Integration

letting $f(y) = y^6 + 3^y$ and $u(x) = \ln(x)$, we get

$$= - \int_2^{u(x)} f(y) \, dy.$$

Using chain rule,

$$\begin{aligned}
 F'(x) &= -f(u(x)) \frac{du}{dx} \\
 &= -f(\ln(x)) \frac{d}{dx}(\ln(x)) \\
 &= -\left((\ln(x))^6 + 3^{\ln x}\right) \left(\frac{1}{x}\right) \\
 &= -\frac{\left((\ln(x))^6 + 3^{\ln x}\right)}{x}.
 \end{aligned}$$