Math 395

Homework 7

Due: 4/18/2024

Name: Avinash Iyer

Collaborators: Antonio Cabello, Timothy Rainone

Problem 1

We say a field K/F is normal if K is the splitting field of a collection of polynomials. Equivalently, every polynomial in $F[x]$ that has a root in K splits into linear factors over K. Let $\alpha \in \mathbb{R}$ such that $\alpha^4 = 5$. We will show that $\mathbb{Q}(\alpha + i\alpha)$ is normal over $\mathbb{Q}(i\alpha^2)$, but $\mathbb{Q}(\alpha + i\alpha)$ is not normal over $\mathbb{Q}$.

Problem 3

For any prime p and any nonzero $a \in \mathbb{F}_p$, we will prove that $f(x) = x^p - x + a$ is irreducible and separable over $\mathbb{F}_p$.

First, we have that $D_x(f(x)) = px^{p-1} - 1 = -1$, meaning that $\gcd(f(x), D_x(f(x))) = 1$, so f is separable. Additionally, we have that $x^p - x + a = (x^p - x) + a = 0 + a = a$, so f is irreducible over $\mathbb{F}_p$.

Problem 4

Let K be a finite extension of $\mathbb{Q}$. We will prove there are only a finite number of roots of unity in K.

Problem 6