Rectangular Coordinates

- Integrating over surface defined in rectangular coordinates.
- Primarily applies to non-closed surfaces defined in rectangular coordinates.

$$z = f(x, y)$$
$$\vec{F} = \vec{F}(x, y, f(x, y))$$
$$\int_S \vec{F} \cdot d\vec{A} = \int_S \vec{F} \cdot \begin{pmatrix} -\frac{\partial f}{\partial x} \\ -\frac{\partial f}{\partial y} \\ 1 \end{pmatrix} \, dx \, dy$$

Cylindrical Coordinates

- Integrating over side of non-closed cylinder with defined radius R.

$$\vec{F} = \vec{F}(R, \theta, z)$$
$$x = R\cos\theta$$
$$y = R\sin\theta$$
$$z = z$$
$$\int_S \vec{F} \cdot d\vec{A} = \int_{z_1}^{z_2} \int_{\theta_1}^{\theta_2} \vec{F} \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} R \, d\theta \, dz$$

Spherical Coordinates

- Integrating over shell of non-closed sphere with defined radius ρ.

$$\vec{F} = \vec{F}(\rho, \theta, \varphi)$$
$$x = \rho\cos\theta\sin\varphi$$
$$y = \rho\sin\theta\sin\varphi$$
$$z = \rho\cos\varphi$$
$$\int_S \vec{F} \cdot d\vec{A} = \int_{\varphi_1}^{\varphi_2} \int_{\theta_1}^{\theta_2} \vec{F} \cdot \begin{pmatrix} \cos\theta\sin\varphi \\ \sin\theta\sin\varphi \\ \cos\varphi \end{pmatrix} \rho^2 \sin\varphi \, d\theta \, d\varphi$$

Divergence Theorem

- Flux integral over closed surface S, defined by W in any coordinate system.

$$\oiint_S \vec{F} \cdot d\vec{A} = \iiint_W \nabla \cdot \vec{F}\ dV$$

Stokes Theorem

- Flux integral of curl of field over open surface S (evaluate using above techniques), with border C in any coordinate system.

$$\int_S \nabla \times \vec{F} \cdot d\vec{A} = \oint_C \vec{F} \cdot d\vec{r}$$