Theorem 1

- The theorem statement is incorrect: for example, if $a = 6, b = 3, c = 4$, then $a|(bc)$ but $a \not| b$ and $a \not| c$.
- The proof only looks at one case and generalizes to the entire integers.

Corrected Theorem and Proof

Theorem 1. *Let $a,\ b,\ c \in \mathbb{Z}$ such that $a < b < c$. If $a|(bc)$, then $a|b$ or $a|c$*

Proof. Suppose toward contradiction that for $a, b, c \in \mathbb{Z}$, $a|(bc)$, $a \not| b$, and $a \not| c$. Then $\forall x, y \in \mathbb{Z}$, $b \neq xa$ and $c \neq ya$. Then, $bc \neq (xy)a$. However, this means $a \not| bc$, as $xy \in \mathbb{Z}$. $\perp$ □

Theorem 2

- The proof states the wrong assumption.
- The proof states the wrong conclusion, it is supposed to be that $1 > \frac{1}{a}$, not $a > \frac{1}{a}$.

Corrected Theorem and Proof

Theorem 2. *If $a \in \mathbb{R}$ and $a > 1$, then $0 < \frac{1}{a} < 1$.*

Proof. Let $a \in \mathbb{R},\ a > 1$. Then, $a > 0$, so $\frac{1}{a} > 0$.

By the order properties of $\mathbb{R}$, multiplication by $\frac{1}{a}$ must preserve the sign in the inequality $a > 1$. So, $\frac{a}{a} > \frac{1}{a}$.

Thus, we have $0 < \frac{1}{a} < 1$. □

Theorem 3

- Instead of `\abs{x}`, the command for absolute value is $|x|$.
- Instead of `\epsilon`, the proof writer should have used `\varepsilon`.
- The proof does not state that it is toward contradiction.

Corrected Theorem and Proof

Theorem 3. *If $|x| < \varepsilon$ for every real number $\varepsilon > 0$, then $x = 0$.*

Proof. Suppose toward contradiction that $\exists x \neq 00$ such that $|x| < \varepsilon$ for every $\varepsilon > 0$.
Then, $\frac{|x|}{2} > 0$, as $|x| \neq 0$. This means $|x| < \frac{|x|}{2}$ by the theorem hypothesis.

Dividing by $|x|$, we get $1 < \frac{1}{2}$. $\perp$ □

Theorem 4

- The theorem's proof uses k to denote the values of both a and b.

Corrected Theorem and Proof

Theorem 4. *Let $a, b \in \mathbb{Z}$ where $a \equiv 1 \mod 3$ and $b \equiv 2 \mod 3$. Then, $(a + b) \equiv 0 \mod 3$.*

Proof. Let $a \equiv 1 \mod 3$ and $b \equiv 2 \mod 3$. Then, for some $k, \ell \in \mathbb{Z}$, $a = 3k + 1$ and $b = 3\ell + 2$.

Then, $a + b = (3k + 1) + (3\ell + 2) = 3(k + \ell + 1)$, so $a + b \equiv 0 \mod 3$. □

Theorem 5

- The proof of the theorem is often imprecise, using words such as "impossible," and does not state that it is toward contradiction.
- In the third sentence of the proof, math mode is used even though the word "and" should not be in math mode.

Corrected Theorem and Proof

Theorem 5. *There are no integers a, b for which $2a + 4b = 1$.*

Proof. Suppose toward contradiction that $\exists a, b \in \mathbb{Z}$ such that $2a+4b = 1$. Then, $a+2b = \frac{1}{2}$. However, since $a, b \in \mathbb{Z}$, and $a + 2b$ are all operations □

Theorem 6

- The proof of the theorem uses k in reference to both n and $n^2 + 5$.

Corrected Theorem and Proof

Theorem 6. *Let n be an integer. If $n^2 + 5$ is odd, then n is even.*

Proof. Let n be odd. Then, $n = 2k+1$ for some $k \in \mathbb{Z}$. So, $n^2+5 = (2k+1)^2+5$, or $(4k^2+4k+1)+5$. So, $n^2 + 5 = 2(2k^2 + 2k + 3)$. □

Theorem 7

- The proof states that it is "to the contrary," rather than by contradiction.
- n^2 cannot be less than n by the ordering properties of $\mathbb{N}$.

Theorem 8

- The series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ is conditionally convergent, meaning that its terms can be rearranged to satisfy any condition, implying that this proof cannot hold.
- On the first line of the second `align` section, there are two equal signs.
- On the third line, a $-$ is put in place of a 2.