Abstract

We show that if E is a module defined over a principal ideal domain R, then E is uniquely decomposable as $E \cong R^r \oplus R/\langle q_1 \rangle \oplus \cdots \oplus R/\langle q_n \rangle$, where R^r is a free module of rank r, and $q_1 | q_2 | \cdots | q_n$.

Definition. Let A be a ring. A *left A-module* M is an abelian group with an operation of A on M such that

$$
\begin{aligned}
(a+b)x &= ax + bx \\
a(x+y) &= ax + ay
\end{aligned}
$$

for all $a, b \in A$ and $x, y \in M$.

If M is an A-module, then $N \subseteq M$ is known as a *submodule* of N is a subgroup such that $AN \subseteq N$.