```
\begin{enumerate}
  \item If $\vec{u} = \begin{bmatrix}-1\\5\end{
     bmatrix}$ and if $\vec{v} = \begin{bmatrix
     }3\\2\end{bmatrix}$, and the vector $4\vec{u} +
      \vec{v}$ is drawn with its tail at the point $
     (10,-10)$, find the coordinates of the point at
      the head of $4\vec{u} + \vec{v}$.
  \item Find the general equation of the plane
     through the point $(1,1,1)$ that is
     perpendicular to the line with parametric
     equations
    \begin{align*}
      x &= 2-t\\
      y &= 3 + 2t\\
      z &= -1 + t
    \end{align*}
  \item Find the rank of the matrix: $\begin{bmatrix
     }1 & -2 & 0 & 3 & 2 \\ 3 & -1 & 1 & 3 & 4 \\ 3
     & 4 & 2 & -3 & 2 \\ 0 & -5 & -1 & 6 & 2\end{
     bmatrix}$
  \item Prove that $\vec{u} \cdot \vec v = \frac
     {1}{4}||\vec u + \vec v ||^2 - \frac{1}{4}||\
     vec u - \vec v||^2$ for all vectors $\vec u,\
     vec v$ in $\mathbb{R}^n$.
  \item Let $\{\vec{v}_1,\vec{v}_2,\dots,\vec{v}_k\}$
      be a linearly independent set of vectors in $\
     mathbb{R}^n$, and let $\vec{v}$ be a vector in
     $\mathbb{R}^n$. Suppose that $\vec{v} = c_1\vec
     {v}_1 + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k$,
     with $c_1\neq 0$. Prove that $\{\vec{v},\vec{v}
     _2,\dots,\vec{v}_k\}$ is linearly independent.
  \item Compute the following limit $\displaystyle\
     lim_{x\rightarrow \infty}\frac{x^3-3x^2+x}{x^4-
     x+2}$.
  \item Find the derivative of $f(x) = (5^{2x} + \sin
     ^{-1}(\pi x - 2))(x^2 + \log(x))^4$.
  \item Find the numbers at which the function $f$ is
      discontinuous. Explain clearly using the
     language of limits. Sketch the graph below,
     clearly marking points (with both coordinates)
     of all discontinuities.
    \[
      f(x) = \begin{cases}
        x + 2 & \text{if} ~ x < 0\\
        2x^2 & \text{if} ~ 0\leq x \leq 1\\
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          2-x & \text{if} ~ x>1
        \end{cases}
      \]
    \item True or False: Every solution of $\frac{dy}{
       dt} = y + sin(t)$ tends to $+\infty$ or $-\
       infty$ as $t\rightarrow \infty$.
    \item Consider the following special first-order
       differential equation
      \begin{equation}\label{diffeq}
        (3x^2 + 4xy) + (2x^2 + 3y^2)\frac{dy}{dx} = 0
      \end{equation}
      Use the change of variables $y=vx$ to show that
         Equation \eqref{diffeq} can be transformed
         into a separable differential equation in the
          variables $v$ and $x$. (\textbf{NOTE}: $v$
         is a function of $x$.)
  \end{enumerate}
```

1. If $\vec{u} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$ and if $\vec{v} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, and the vector $4\vec{u}+\vec{v}$ is drawn with its tail at the point $(10, -10)$, find the coordinates of the point at the head of $4\vec{u} + \vec{v}$.

2. Find the general equation of the plane through the point $(1, 1, 1)$ that is perpendicular to the line with parametric equations
$$\begin{aligned} x &= 2 - t \\ y &= 3 + 2t \\ z &= -1 + t \end{aligned}$$

3. Find the rank of the matrix: $\begin{bmatrix} 1 & -2 & 0 & 3 & 2 \\ 3 & -1 & 1 & 3 & 4 \\ 3 & 4 & 2 & -3 & 2 \\ 0 & -5 & -1 & 6 & 2 \end{bmatrix}$

4. Prove that $\vec{u} \cdot \vec{v} = \frac{1}{4}||\vec{u} + \vec{v}||^2 - \frac{1}{4}||\vec{u} - \vec{v}||^2$ for all vectors $\vec{u}, \vec{v}$ in $\mathbb{R}^n$.

5. Let $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k\}$ be a linearly independent set of vectors in $\mathbb{R}^n$, and let $\vec{v}$ be a vector in $\mathbb{R}^n$. Suppose that $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k$, with $c_1 \neq 0$. Prove that $\{\vec{v}, \vec{v}_2, \dots, \vec{v}_k\}$ is linearly independent.

6. Compute the following limit $\displaystyle\lim_{x\to\infty} \frac{x^3 - 3x^2 + x}{x^4 - x + 2}$.

7. Find the derivative of $f(x) = (5^{2x} + \sin^{-1}(\pi x - 2))(x^2 + \log(x))^4$.

8. Find the numbers at which the function f is discontinuous. Explain clearly using the language of limits. Sketch the graph below, clearly marking points (with both coordinates) of all discontinuities.

$$f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \leq x \leq 1 \\ 2-x & \text{if } x > 1 \end{cases}$$

9. True or False: Every solution of $\frac{dy}{dt} = y + sin(t)$ tends to $+\infty$ or $-\infty$ as $t \to \infty$.

10. Consider the following special first-order differential equation

$$(3x^2 + 4xy) + (2x^2 + 3y^2)\frac{dy}{dx} = 0 \tag{1}$$

Use the change of variables $y = vx$ to show that Equation (1) can be transformed into a separable differential equation in the variables v and x. (**NOTE**: v is a function of x.)