Problem (Problem 1)**:** In this exercise, we prove another fundamental result in differential topology, called the tubular neighborhood theorem. Let M be a compact smooth manifold with orientable boundary N. For simplicity, assume that N is connected. The tubular neighborhood theorem asserts that N admits a neighborhood in M which is diffeomorphic to $N \times [0, 1)$.

(a) Choose a Riemannian metric on M, and show that N admits a nonvanishing vector field that is everywhere orthogonal to the tangent space of N. That is, a vector field X such that for all $p \in N$, $g(X_p, T_pN) = 0$.

(b) Use the flow generated by X to find the desired neighborhood.

Problem (Problem Set 7, Problem 5)**:** Suppose G is a finite group acting freely on a manifold M by diffeomorphisms.

(a) Show that M/G is a manifold.

(b) Show that the de Rham cohomology of M/G is isomorphic to the G-invariant cohomology of M.

Problem (Problem Set 8, Problem 3)**:** Compute the de Rham cohomology of $\mathbb{RP}^n$.