18.4

2: $f(x,y) = x^2y$

6: $f(x,y) = x^2y^3 + xy$

10: There is no function that serves this purpose.

12:

$$\begin{aligned}\oint_C \vec{F} \cdot d\vec{r} &= \int_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\ dx\ dy\\ &= \int_0^1 \int_0^1 -x\ dx\ dy\\ &= \frac{1}{2}\end{aligned}$$

14:

$$\begin{aligned}\oint_C \vec{F} \cdot d\vec{r} &= \int_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\ dx\ dy\\ &= \int_R (x-3)\ dx\ dy\\ &= \int_0^{2\pi} \int_0^1 (r\cos\theta - 3)r\ dr\ d\theta\\ &= -3\pi\end{aligned}$$

24:

$$\begin{aligned}\oint_C \vec{F} \cdot d\vec{r} &= \int_0^1 \int_{x^3}^{x^2} \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\ dy\ dx\\ &= \int_0^1 \int_{x^3}^{x^2} (-y)\ dy\ dx\\ &= \frac{1}{2}\int_0^1 x^6 - x^4\ dx\\ &= \frac{1}{35}\end{aligned}$$

28: (a) $\int_C \vec{F} \cdot d\vec{r} = f(2,4) - f(0,0) = 4e^4$

(b) $\int_C \vec{G} \cdot d\vec{r} = 4$

34:

$$\int_C \vec{F} \cdot d\vec{r} = \int_R 1\ dx\ dy$$

19.1

2:

$$\vec{A} = \begin{pmatrix} 0 \\ 0 \\ 25\pi \end{pmatrix}$$

4:

$$\vec{A} = \begin{pmatrix} 0 \\ 15 \\ 0 \end{pmatrix}$$

8: (a) Negative
(b) Positive
(c) Negative
(d) Negative
(e) Zero

12: (a) -32π
(b) 32π

16:

$$\begin{aligned} \vec{A} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 0 \\ -2 & 0 & 3 \end{vmatrix} \\ &= \begin{pmatrix} 6 \\ 0 \\ 4 \end{pmatrix} \\ \vec{v} \cdot \vec{A} &= -6 \end{aligned}$$

20:

$$\begin{aligned} \vec{n} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} \\ &= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\ \hat{n} &= \frac{1}{\sqrt{3}}\vec{n} \\ \vec{A} &= \frac{\sqrt{3}}{2}\frac{1}{\sqrt{3}}\vec{n} \\ &= \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \end{pmatrix} \\ \vec{v} \cdot \vec{A} &= \frac{3}{2} \end{aligned}$$

26:

$$\int_S \vec{H} \cdot d\vec{A} = 3$$

30:

$$\int_S \vec{F} \cdot d\vec{A} = 45\pi$$

40:

$$\int_S \vec{F} \cdot d\vec{A} = 28\pi$$

50:

$$\begin{aligned}\int_S \vec{F} \cdot d\vec{A} &= \int_0^2 \int_0^3 (2z+4)\ dz\ dy \\ &= 42\end{aligned}$$

19.2

2:

$$\vec{A} = \begin{pmatrix} -8 \\ -7 \\ 1 \end{pmatrix}$$

4:

$$\vec{A} = \begin{pmatrix} -y \\ -x-2y \\ 1 \end{pmatrix}$$

6:

$$\int_S \vec{F} \cdot d\vec{A} = \int_0^8 \int_0^4 ((-4)(50-4x+10y)+10x+y)\ dx\ dy$$

12:

$$\begin{aligned}\int_S \vec{F} \cdot d\vec{A} &= \int_0^1 \int_{-1}^1 (2x+y+2)\ dx\ dy \\ &= \frac{5}{2}\end{aligned}$$

16:

18:

20:

22:

$$\int_S \vec{F} \cdot d\vec{A} = \int_0^{2\pi} \int_0^{\pi} \left(25\sin\phi(\sin^2\phi\cos^2\theta + 2\sin^2\phi\sin^2\theta + 3\cos^2\theta)\right)\ d\phi\ d\theta$$

24:

$$\int_S \vec{F} \cdot d\vec{A} = \int_0^{\pi} \int_0^{\pi/2} 9\sin\phi\cos\phi e^{\cos\theta\sin\phi}\ d\phi\ d\theta$$

19.3

2: Scalar:

$$\nabla \cdot \begin{pmatrix} 2\sin(xy) + \tan(z) \\ \tan(y) \\ e^{x^2+y^2} \end{pmatrix} = 2y\cos(xy) + \sec^2(y)$$

4:

$$\nabla \cdot \vec{F} = 0$$

6:

$$\nabla \cdot \vec{F} = -1$$

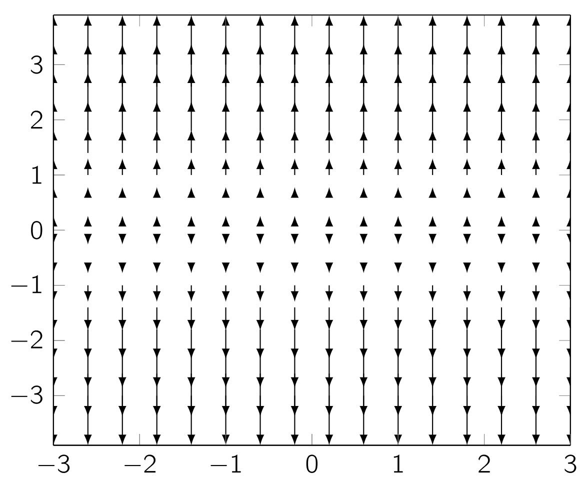
24:

$$\begin{aligned} \nabla \cdot \nabla F &= \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} \cdot \begin{pmatrix} ay + 2axy \\ ax + ax^2 + 3y^2 \end{pmatrix} \\ &= 2ay + 6y \\ a &= \boxed{-3} \end{aligned}$$

28:

$$\nabla \cdot \vec{F} = 1$$

Viewed facing the yz plane, we can see the following field, which indicates that the vector field does have zero divergence.



38: