Show that if X_1 and X_2 are simply connected, then $X_1 \times X_2$ is simply connected.

To start, we will show that $X_1 \times X_2$ is path connected. Let $p = (p_1, p_2), q = (q_1, q_2) \subseteq X_1 \times X_2$. By the definition of path connected, $\exists f_i : I \to X_i$ from p_i to q_i for $i = 1, 2$. Let $f : I \to X_1 \times X_2$ be given by $f(t) = (f_1(t), f_2(t))$. Then, f is continuous since its "component functions" are continuous, and $f(0) = (p_1, p_2)$ and $f(1) = (q_1, q_2)$. Therefore, $\exists$ a continuous map from I to $X_1 \times X_2$, so $X_1 \times X_2$ is continuous.

Let $\updownarrow = S^1 \to X_1 \times X_2$. Then, $\pi_1 \circ \updownarrow = S^1 \to X_1$, and similarly for π_2. Since X_1 is simply connected, $\pi_1 \circ \updownarrow$ is null homotopic. So $\exists H : S^1 \times I \to X_1$ such that $H_0 = \pi_1 \circ \updownarrow$ and $H_1 = a$. Similarly, $\exists G : S^1 \times I \to X_2$ such that $G_0 = \pi_2 \circ \updownarrow$ and $G_1 = b$ for constants a, b. Define $F : S^1 \times I \to X_1 \times X_2$. Let $F(x,t) = (H(x,t), G(x,t))$. Then, $F_0 = (H(x,0), G(x,0)) = (\pi_1 \circ l(x), \pi_2 \circ l(x)) = l(x)$ and $F_1 = (H(x,1), G(x,1)) = (a, b)$, so F is a homotopy between $\updownarrow$ and the constant map.