Activity: Spence's Job Market Signaling Game

Econ 305

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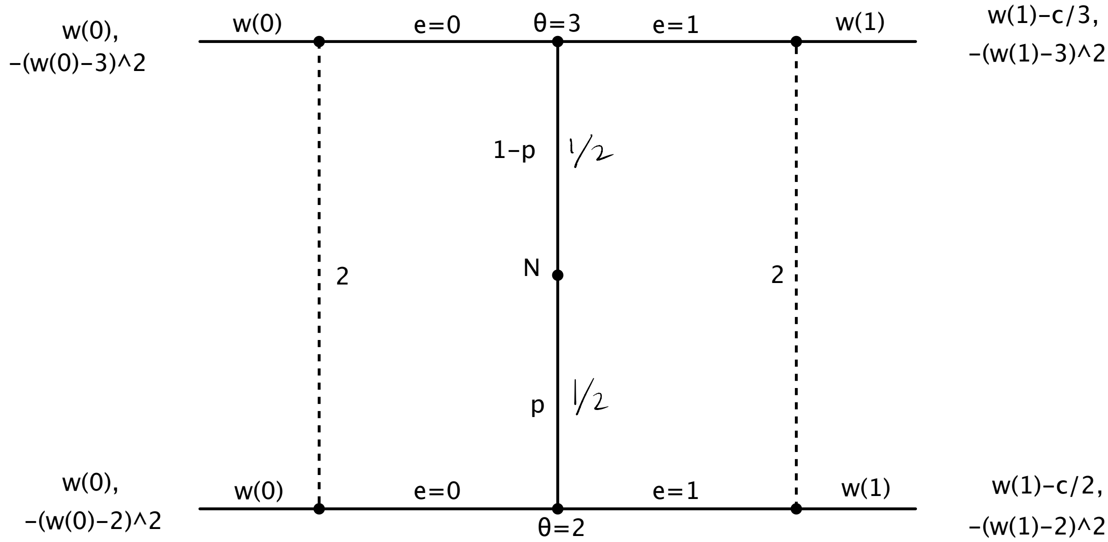

Consider the above signaling game with $p = 1/2$ and various costs, c, of education.

3 pieces of PBE:

- $s_1^*(\theta_1)$ Best response
- $\mu_2(\theta_1 | a_1)$ Best response
- $s_2^*(a_1)$

Best Response

1 Separating: $2 \le c \le 3$

1. Guess that there is a separating PBE in which only the high ability types of player 1 get an education:

$$s_1^*(\theta) = \begin{cases} \underline{e=0} & \text{if } \theta = 2 \\ \underline{e=1} & \text{if } \theta = 3 \end{cases}$$

2. Use Bayes' rule to obtain a consistent belief system, given strategy of player 1:

$$\mu_2(3|a_1 = 0) = \underline{0}$$

$$\mu_2(3|a_1 = 1) = \underline{1}$$

3. Player 2's best response is to set the wage equal to the expected ability/productivity of the worker, given their beliefs:

$$w^*(a_1) = \begin{cases} 2 & \text{if } \underline{a_1} = 0 \\ 3 & \text{if } \underline{a_1} = 1 \end{cases}$$

4. Finally, we need to verify that every type of player 1 is best responding to the strategy of player 2:

$$v_1(a_1, w^*(a_1); \theta = 2) = \begin{cases} \underline{2} & \text{if } a_1 = 0 \\ \underline{3 - \frac{c}{2}} & \text{if } a_1 = 1 \end{cases}$$

$$v_1(a_1, w^*(a_1); \theta = 3) = \begin{cases} \underline{2} & \text{if } a_1 = 0 \\ \underline{3 - \frac{c}{3}} & \text{if } a_1 = 1 \end{cases}$$

strategy is best response

NOTE: There is no separating PBE where the low type gets an education and the high type does not.

2 Pooling: $c \leq 1$

1. Guess that there is a pooling PBE in which everyone gets an education:

$$s_1^*(\theta) = 1 \text{ for all } \theta$$

2. Use Bayes' rule, when possible, to obtain a consistent belief system, given strategy of player 1:

$$\mu_2(3|a_1 = 0) = \underline{\lambda} \quad \leftarrow \text{free-ish Parameter}$$
$$\mu_2(3|a_1 = 1) = \underline{\tfrac{1}{2}}$$

3. Player 2's best response is to set the wage equal to the expected ability/productivity of the worker, given their beliefs:

$$w^*(a_1) = \begin{cases} 3\lambda + 2(1-\lambda) = 2+\lambda & \text{if } a_1 = 0 \\ \frac{1}{2}(2)+\frac{1}{2}(3) = 2.5 & \text{if } a_1 = 1 \end{cases}$$

4. Finally, we need to verify that every type of player 1 is best responding to the strategy of player 2:

$$v_1(a_1, w^*(a_1); \theta_1 = 2) = \begin{cases} 2+\lambda & \text{if } a_1 = 0 \\ 2.5 - \frac{c}{2} & \text{if } a_1 = 1 \end{cases}$$

$$v_1(a_1, w^*(a_1); \theta_1 = 3) = \begin{cases} 2+\lambda & \text{if } a_1 = 0 \\ 2.5 - \frac{c}{3} & \text{if } a_1 = 1 \end{cases}$$

Condition for best response: $2+\lambda \leq 2.5 - \frac{c}{2} \quad \forall c \in [0,1]$

$\rightarrow \lambda = 0$

NOTE: There is also always a pooling PBE where no one gets an education and the firm thinks that any worker who gets an education is of the low type.

3 Semi-Separating: $1 < c < 2$

1. Guess that there is a semi-separating PBE in which the high ability worker gets an education and the low ability worker gets an education with probability q:

$$\frac{100 + (100)(q)}{200} = \frac{100(1+q)}{200}$$

$$s_1^*(\theta) = \begin{cases} q(e=1) + (1-q)(e=0) & \text{if } \theta = 2 \\ e=1 & \text{if } \theta = 3 \end{cases}$$

2. Use Bayes' rule to obtain a consistent belief system, given strategy of player 1:

$$\mu_2(3|a_1 = 0) = 0$$

$$\mu_2(3|a_1 = 1) = \frac{\left(\frac{1}{2}\right)}{\frac{1}{1+q}} = \frac{1}{2}(1+q)$$

3. Player 2's best response is to set the wage equal to the expected ability/productivity of the worker, given their beliefs:

$$w^*(a_1) = \begin{cases} 2 & \text{if } a_1 = 0 \\ 3\left(\frac{1}{2}(1+q)\right) + 2\left(1 - \frac{1}{2}(1+q)\right) & \text{if } a_1 = 1 \end{cases}$$

4. Finally, we need to verify that every type of player 1 is best responding to the strategy of player 2:

$$v_1(a_1, w^*(a_1); \theta_1 = 2) = \begin{cases} 2 & \text{if } a_1 = 0 \\ 2 + \frac{1}{2}(1+q) - \frac{c}{2} & \text{if } a_1 = 1 \end{cases}$$

$$v_1(a_1, w^*(a_1); \theta_1 = 3) = \begin{cases} 2 & \text{if } a_1 = 0 \\ 2 + \frac{1}{2}(1+q) - \frac{c}{3} & \text{if } a_1 = 1 \end{cases}$$

NOTE: In the limiting cases where $c \to 2$, then $q \to 0$ and we have the separating equilibrium, while for $c \to 1$, then $q \to 1$ and we have the pooling equilibrium.