Problem 1

Let $X = \{0, 1\}^n$. Show that the Hamming distance:

$$\begin{aligned} d_H &: X \times X \to [0, \infty) \\ d_H\left((x_j)_{j=1}^n, (y_j)_{j=1}^n\right) &= \left|\{j \mid x_j \neq y_j\}\right| \end{aligned}$$

defines a metric on X.

Proof:

- Symmetry:

$$\begin{aligned} d_H\left((x_j)_{j=1}^n, (y_j)_{j=1}^n\right) &= \left|\{j \mid x_j \neq y_j\}\right| \\ &= \left|\{j \mid y_j \neq x_j\}\right| \\ &= d_H\left((y_j)_{j=1}^n, (x_j)_{j=1}^n\right) \end{aligned}$$

- Definiteness: it is only the case that $d_H(x_j, y_j) = 0$ if $x_j = y_j$ for all j, by the definition of the distance.
- Similarly, since $x_j = x_j$ for all j, $d_H(x_j, x_j) = 0$.