Problem 1

(a)

$$\begin{aligned}\int_{C_1} \left(x^2 + y^2\right)\, d\ell &= \int_0^1 x^2\, dx + \int_0^1 y^2 + 1\, dy\\ &= \frac{5}{3}.\end{aligned}$$

(b)

$$\begin{aligned}\int_{C_2} \left(x^2 + y^2\right)\, d\ell &= \int_0^1 2x^2\, dx\\ &= \frac{2}{3}.\end{aligned}$$

(c)

$$\begin{aligned}\int_{C_3} \left(x^2 + y^2\right)\, d\ell &= \int_0^1 x^2 + x^4\, dx\\ &= \frac{8}{15}.\end{aligned}$$

Problem 2

(a) Since $\oint_C \, d\ell$ "adds up" the infinitesimal lengths along C, this integral gives the length of C.

(b) Since $\oint_C \, d\vec{\ell}$ is a vector-valued integral along C, and since C is closed, this integral gives 0.

Problem 3

(a) We have $d\ell = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$, and $\frac{dy}{dx} = \frac{-x}{\sqrt{a^2 - x^2}}$, so

$$\begin{aligned}\int_C \, d\ell &= \int \sqrt{1 + \frac{x^2}{a^2 - x^2}}\, dx\\ &= \int \frac{1}{\sqrt{a^2 - x^2}}\, dx\\ &= a \arcsin\left(\frac{x}{a}\right).\end{aligned}$$

Evaluated from $x = -a$ to $x = a$, we get that $\int_C \, d\ell = \pi a$.

(b) We have $d\ell = \sqrt{dr^2 + r^2 d\theta^2}$, so

$$\begin{aligned}\int \, d\ell &= \int_0^\pi a\, d\theta\\ &= \pi a\end{aligned}$$

Problem 7

(a) $\oint_S \, dA$ gives the area of the sphere, as we do not have to integrate with respect to a direction.

(b) $\oint_S \, d\mathbf{A}$ yields zero, as $\hat{n}$ is symmetrical with respect to S.

Problem 11

$$\begin{aligned}
\int_S \mathbf{r}\cdot d\mathbf{A} &= \int (R\hat{r})\cdot\hat{r}\left(R^2 d\Omega\right)\\
&= R^3\int_0^{2\pi}\int_0^{\pi/2}\sin^2\theta\ d\phi d\theta\\
&= 2\pi R^3\frac{\pi}{4}\\
&= \frac{\pi^2}{2}R^3.
\end{aligned}$$

Problem 18

Problem 19

Problem 20

Problem 21

Problem 22

Problem 26

Problem 28