Theorem 1

- The theorem statement is incorrect: for example, if $a = 6, b = 3, c = 4$, then $a|(bc)$ but $a \not| b$ and $a \not| c$.
- The proof only looks at one case and generalizes to the entire integers.

Corrected Theorem and Proof

Theorem 1. *Let $a,\ b,\ c \in \mathbb{Z}$ such that $a < b < c$. If $a|(bc)$, then $a|b$ or $a|c$*

Proof. Suppose toward contradiction that for $a, b, c \in \mathbb{Z}$, $a|(bc)$, $a \not| b$, and $a \not| c$. Then $\forall x, y \in \mathbb{Z}$, $b \neq xa$ and $c \neq ya$. Then, $bc \neq (xy)a$. However, this means $a \not| bc$, as $xy \in \mathbb{Z}$. $\perp$ □

Theorem 2. *If $a \in \mathbb{R}$ and $a > 1$, then $0 < \frac{1}{a} < 1$.*

Proof. Assume that $1 \leq \frac{1}{1}$. Since $a > 1$, we can divide both sides by a (without reversing the inequality) to get $\frac{a}{a} > \frac{1}{a}$ so $1 > \frac{1}{a}$. This contradicts the assumption that $1 \leq \frac{1}{a}$. Thus it must be that $a > \frac{1}{a}$. □

Theorem 3. *If $absx < \epsilon$ for every real number $\epsilon > 0$, then $x = 0$.*

Proof. Suppose that $|x| < \epsilon$ for every positive number ϵ, but $x \neq 0$. Since $x \neq 0$, necessarily $\frac{|x|}{2} > 0$, so in particular $|x| < \epsilon$ for the positive number $\epsilon = \frac{|x|}{2} > 0$. This means

$$|x| < \frac{|x|}{2}.$$

But, $|x| \neq 0$ by assumption, so we can divide both sides by $|x|$ to conclude that $1 < \frac{1}{2}$, which is a contradiction. Thus, if $|x| < \epsilon$ for every real number $\epsilon > 0$, it must me the case that $x = 0$. □

Theorem 4. *Let $a, b \in \mathbb{Z}$ where $a \equiv 1 \mod 3$ and $b \equiv 2 \mod 3$. Then $(a + b) \equiv 0 \mod 3$.*

Proof. Since $a \equiv 1 \mod 3$ there is an integer k in $\mathbb{Z}$ such that $a = 3k + 1$. Since $b \equiv 2 \mod 3$, we can write $b = 3k + 2$. Thus, $a + b = (3k + 1) + (3k + 2) = 6k + 3 = 3(2k + 1)$, so $(a + b) \equiv 0 \mod 3$. □

Theorem 5. *There are no integers a, b for which $2a + 4b = 1$.*

Proof. Suppose the theorem is false, so that there are integers a, b for which $2a + 4b = 1$. Dividing both sides of this equation by 2, we conclude that $a + 2b = \frac{1}{2}$. Since $aandb$ are integers, $a + 2b$ is also an integer. But $\frac{1}{2}$ is not an integer, so this is impossible. Therefore, the theorem can not be false, so it must be true. □

Theorem 6. *Let n be an integer. If $n^2 + 5$ is odd, then n is even.*

Proof. Suppose, for the sake of contradiction, that $n^2 + 5$ is odd and n is also odd. By definition, then, there exists an integer k so that $n^2 + 5 = 2k + 1$ and $n = 2k + 1$. Hence we have

$$2k + 1 = n^2 + 5 = (2k + 1)^2 + 1 = 4k^2 + 4k + 1 + 5 = 2(2k^2 + 2k + 3)$$

. Therefore, $2k+1$ is even. This is clearly impossible, and hence we cannot have that n^2+5 is odd and n is also odd. Therefore, if that $n^2 + 5$ is odd, we must have n is even. □