Homework Section 1.2

Most problems you that will be assigned in this class will require proofs. Words like "***construct***," "***show***," "***obtain***," "***determine***," etc., explicitly state that **proof** is required. Disproof by providing a counterexample requires *confirming* that it is a counterexample.

Individual:

1.2.1 *Determine* whether the statements below are true or false.

(a) Every disconnected graph has an isolated vertex.

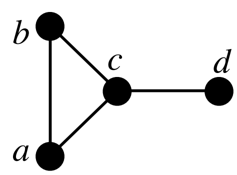
(b) A graph is connected if and only if some vertex is *connected*[1] to all other vertices.

(c) The edge set of every close trail can be partitioned into edge sets of cycles.

(d) (optional) If a maximal trail in a graph is not closed, then its endpoints have odd degree.

1.2.5 Let v be a vertex of a connected simple graph G. Prove that v has a neighbor in every component of $G - v$. Explain why this allows us to conclude that no graph has a cut-vertex of degree 1.

1.2.6 In the graph below (the paw), find all the maximal paths, maximal cliques, and maximal independent sets. Also, find all the maximum paths, maximum cliques, and maximum independent sets.



1.2.8 *Determine* the values of m and n such that $K_{m,n}$ is Eulerian. (There will be an infinite number of values, so describe/give a formula for the values of m and n.)

1.2.10 Prove or disprove:

(a) Every Eulerian bipartite graph has an even number of edges.

(b) Every Eulerian simple graph with an even number of vertices has an even number of edges.

(OVER)

[1] Notice that it says "connected," not joined, not adjacent.

Group:

"A" Group Problems:

1.2.20 Let v be a cut-vertex of a simple graph G. Prove that $\overline{G} - v$ is connected.

1.2.22 Prove that a graph is connected if and only if for every partition of its vertices into two nonempty sets, there is an edge with endpoints in both sets.

1.2.26 Prove that a graph G is bipartite if and only if every subgraph H of G has an independent set consisting of at least half of $V(H)$.

1.2.38 Prove that every n-vertex graph with at least n edges contains a cycle.

"B" Group Problems:

1.2.18 Let G be the graph whose vertex set is the set of k-tuples with elements in $\{0, 1\}$, with x adjacent to y if x and y differ in exactly two positions. *Determine* the number of components of G.

1.2.25 Use ordinary induction on the number of edges or vertices to prove that absence of odd cycles is a sufficient condition for a graph to be bipartite.

1.2.29 Let G be a connected simple graph not having P_4 or C_3 as an induced subgraph. Prove that G is a biclique (complete bipartite graph).

1.2.39 Suppose that every vertex of a loopless graph G has degree at least 3. Prove that G has a cycle of even length. (Hint: Consider a maximal path.) (P. Kwok)