Activity: Bertrand Duopoly

Econon 305

Brandon Lehr

We can write profits in the symmetric Bertrand duopoly game ($n = 2$) with marginal costs, c, and demand function, $D(p)$, as follows:

$$v_i(p_i, p_j) = \begin{cases} (p_i - c)D(p_i) & \text{if } p_i < p_j \\ (p_i - c)D(p_i)/2 & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

where $p_1, p_2 \in \mathbb{R}^+$.

Claim: The unique NE of the Bertrand game is $(p_1^*, p_2^*) = (c, c)$.

Proof: There are two parts we have to prove. First, we must show that the proposed action profile is in fact a NE. Next, we have to show that there is no other NE.

1. *Profile is NE*

   - change $\uparrow \to v = 0 = NE$
   - change $\downarrow \to v < 0 < NE$

   no better than current situation

2. *Uniqueness*

   Consider all other possible action profiles:

   (a) If $p_i < c$ for either $i = 1$ or $i = 2$:

   $\exists i$ s.t. $v(p_i, p_j) < 0 \to$ profitable to change marginal cost for $v(p_i', p_j) = 0$

   (b) If $p_i = c$ and $p_j > c$:

   $p_i \uparrow$ by $\varepsilon < (p_j - c) \to v(p_i + \varepsilon, p_j) > 0$

   (c) If $p_i > c$ and $p_j > c$ (without loss of generality let $p_i \geq p_j$):

   $$v(p_i, p_j) = \begin{cases} 0, & p_i > p_j \\ \frac{1}{2}(p_i - c)D(p_i), & p_i = p_j \end{cases}$$

   profitable to go just below p_j, payoff: $0 \to > 0$

Bonus: Suppose we modify the game so that firms can only charge discrete prices measured to the precision of a cent (as opposed to all non-negative real numbers). Argue that $(c+1, c+1)$ is also a Nash equilibrium (where c is given in cents). Assume that $D(c+1) > 0$.

Hold: $v_i = \frac{1}{2} D(c+1)$

Down by 1: $v_i = 0$

Down by >1: $v_i < 0$

Up: $v_i = 0$

all worse than staying at $c+1$