Chapter 4 Problems

4.7

Cylindrical Coordinates

In cylindrical coordinates, we have

$$d\mathbf{r} = \rho\cos\phi\hat{\imath} + \rho\sin\phi\hat{\jmath} + z\hat{k}.$$

We let $\hat{e}_1 = \hat{\rho}$, $\hat{e}_2 = \hat{\phi}$, and $\hat{e}_3 = \hat{z}$, with $u_1 = \rho$, $u_2 = \phi$, and $u_3 = z$. Thus, we get

- Line element:

$$\begin{aligned}(ds)^2 &= \sum_{i,j} \frac{\partial \mathbf{r}}{\partial u_i}\frac{\partial \mathbf{r}}{\partial u_j}\left(\hat{e}_i \cdot \hat{e}_j\right) du_i\, du_j \\ &= \sum_{i=1}\left(\frac{\partial \mathbf{r}}{\partial u_i}\right)(du_i)^2 && \text{The } \hat{\rho}, \hat{\phi}, \hat{z} \text{ basis is orthogonal} \\ &= (d\rho)^2 + \rho^2 (d\phi)^2 + (dz)^2.\end{aligned}$$

- Area element:

$$d\mathbf{a} = \left(\sum_k \epsilon_{ijk}\hat{e}_k\right)\frac{\partial \mathbf{r}}{\partial u_i}\cdot\frac{\partial \mathbf{r}}{\partial u_j} du_i\, du_j$$

Chapter 6 Problems

6.3

(a) Looking at the ratio test first, we find

- Ratio test:

$$\begin{aligned}\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty}\left|\sqrt{\frac{n}{n+1}}\right| \\ &= 1,\end{aligned}$$

which is an inconclusive result.

- Comparison test:

$$\frac{1}{\sqrt{n}} > \frac{1}{n} \qquad \forall n \geqslant 1.$$

Since $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, so too does $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$.

(b)
- Ratio test:

$$\begin{aligned}\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty}\left|\left(\frac{n}{n+1}\right)\left(\frac{1}{2}\right)\right| \\ &= \frac{1}{2} \\ &< 1,\end{aligned}$$

meaning the series converges by the ratio test.

- $$\frac{1}{n2^n} < \frac{1}{2^n} \qquad \text{for all } n \geqslant 1,$$

  and since $\sum_{n=1}^{\infty} \frac{1}{2^n}$ converges, it must be the case that $\sum_{n=1}^{\infty} \frac{1}{n2^n}$ converges.