Suppose we cut two disks out of a larger disk, let this equal to X. Pick $x_0 \in A$, and let $f, g : I \to X$ be loops centered at x_0. Then, $f \cdot g$ is equivalent to moving along $f(I)$ for the first half of the interval I, then moving along $g(I)$ for the second half of the interval I. Note here that $f \cdot g$ and $g \cdot f$ (analogous to $f \cdot g$) need not self-intersect at x_0 because they are homotopic.

Two paths are **path homotopic** if they start and end at the same point. In the example below, the points are not path homotopic.

> Let $\alpha : I \to X$, $\beta : I \to X$, $\alpha(0) = \beta(0) = x_0$, $\alpha(1) = \beta(1) = x_1$. Then, α is path homotopic to β if $\exists H : I \times I \to X$ where $H_0 = \alpha$, $H_1 = \beta$, $H_t(0) = x_0$ and $H_t(1) = x_1$.

In this respect, it $f \cdot g$ and $g \cdot f$ are not path homotopic to each other in the above example. If f_1 and f_2 are path homotopic, then we say that they are in the same equivalence class.

$$[f_1] = \{f : f \sim_{x_0} f_i\}$$

Let $[f] \cdot [g] = [f \cdot g]$. We will have to show that this is well defined.

Consider the following loops f and g on T^2. We know that f is not path homotopic to g. However, we can show that $f \cdot g$ and $g \cdot f$ are path homotopic to each other with the following set of diagrams. Because $f \cdot g = g \cdot f$, we have that the group created by the equivalence class of functions is abelian.

The fundamental group is the set of all equivalence classes of a loop under $\sim_p$. The product is defined as above. Because the fundamental group of $S^2 \times S^1 = \mathbb{Z}$ while the fundamental group of $S^3 = \{1\}$, we know that $S^2 \times S^1 \not\cong S^3$.