Problem (Problem 1)**:** Every square S in an infinite chessboard contains a positive integer equal to the average of the four non-diagonal neighbors of S. Are the integers necessarily equal?

Problem (Problem 2)**:** Alice has $n+1$ binary strings of length $n \geq 2$. Bob knows this but knows nothing about the bits in the strings. Bob wants to specify a binary string of length n that is not in Alice's set. Bob can ask Alice questions of the form: "What is the jth bit of the kth string?" What is the minimal number of questions that Bob needs to ask to guarantee success?

Problem (Problem 3)**:** Show that among $n+1$ integers not greater than $2n$ that there are two such that one divides the other.

Problem (Problem 4)**:** Show that for every graph there is an orientation of the edges such that for every vertex the out-degree and in-degree differ by at most 1.

Solution: Without loss of generality, we assume G is connected. If G is disconnected, we apply the following procedure to each of the connected components.

We define the "net degree" of a vertex in a directed graph to be equal to the difference between its out-degree and its in-degree. Additionally, we say the directed graph G is strongly oriented if, for any v and w in G, there is a directed path from v to w, and a directed path from w to v. Note that if G is strongly oriented, then every vertex in G has net degree zero, as any pair of vertices is contained in a directed circuit.

By Robbins's theorem, if G does not contain a cut-edge (or bridge), then G always admits a strong orientation. Otherwise, we let G admit a bridge, e, and suppose that $G-e$ contains two components that are not single vertices. We may delete e and place a strong orientation on the connected components; then, any orientation on e will result in the endpoints of e having degree $+1$ and -1 respectively.

If G contains multiple bridges $e_1, \ldots, e_n$, then by alternating the choice of out-degree vs. in-degree, we can ensure that all vertices are either $+1$ or -1 in net degree.

Problem (Problem 5)**:** A convex board is surrounded by some nails hammered into a table; the nails make it impossible to slide the board in any direction, but if any of them is missing then this is no longer true. What is the maximal number of nails?