Problem (Problem 1)**:** Prove that if X is a path-connected and locally-path connected space with $\pi_1(X)$ a finite group, then every map $X \to S^1$ is null-homotopic.

Solution: We know that if $f\colon X \to S^1$ is any map, then f induces a homomorphism $f_*\colon \pi_1(X) \to \pi_1(S^1) \cong \mathbb{Z}$. In particular, since $\pi_1(X)$ is finite, and $\mathbb{Z}$ is infinite with the only element of finite order being 0, it follows that f_* is the zero map.

Thus, f admits a lift to $\mathbb{R}$, which we call $\widetilde{f}$ as X is both path-connected and locally path-connected. Since X is path-connected, so too is $\widetilde{f}(X) \subseteq \mathbb{R}$, but this means that $\widetilde{f}(X)$ is an interval, hence contractible. Thus, $\widetilde{f}$ is null-homotopic, so upon applying the covering map, we get that f is null-homotopic.