2.1

Problem: Recall that an ordered pair (a, b) can be defined as the set $\{\{a\}, \{a, b\}\}$. Show that $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$

Solution. Let $(a, b) = (c, d)$. Then, $\{\{a\}, \{a, b\}\} = \{\{c\}, \{c, d\}\}$. Since $\{\{a\}, \{a, b\}\} \subseteq \{\{c\}, \{c, d\}\}$, it is the case that $\{a\} \in \{\{c\}, \{c, d\}\}$, meaning $\{a\} = \{c\}$ or $\{a\} = \{c, d\}$. Since it cannot be the case that $\{a\} = \{c, d\}$, as the latter contains two elements, it is the case that $\{a\} = \{c\}$. Since singleton sets are equal if and only if their respective elements are equal, this means $a = c$. Similarly, since by elimination, $\{a, b\} = \{c, d\}$, and since $a = c$, we have $\{c, b\} = \{c, d\}$; thus, $b \in \{c, d\}$ and $c \in \{c, d\}$, with $d \in \{c, b\}$ and $c \in \{c, b\}$; thus, $b = d$.

Let $a = b$ and $c = d$. Then, by the replacement schema, we have $\{\{a\}, \{a, b\}\} = \{\{c\}, \{c, d\}\}$ (under the map $a \mapsto c$ and $b \mapsto d$), implying $(a, b) = (c, d)$.