

Math 395
Homework 7
Due: 4/18/2024

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Problem 1

We say a field K/F is normal if K is the splitting field of a collection of polynomials. Equivalently, every polynomial in $F[x]$ that has a root in K splits into linear factors over K . Let $\alpha \in \mathbb{R}$ such that $\alpha^4 = 5$. We will show that $\mathbb{Q}(\alpha + i\alpha)$ is normal over $\mathbb{Q}(i\alpha^2)$, but $\mathbb{Q}(\alpha + i\alpha)$ is not normal over \mathbb{Q} .

Problem 3

For any prime p and any nonzero $a \in \mathbb{F}_p$, we will prove that $f(x) = x^p - x + a$ is irreducible and separable over \mathbb{F}_p .

First, we have that $D_x(f(x)) = px^{p-1} - 1 = -1$, meaning that $\gcd(f(x), D_x(f(x))) = 1$, so f is separable.

Let α be a root of f . Then, we have that $\alpha^p - \alpha + a = 0$. Notice that for $j \in \mathbb{F}_p$, $(\alpha + j)^p = \alpha^p + j^p = \alpha^p + j$, meaning that $(\alpha + j)^p - (\alpha + j) + a = 0$, so $\alpha + j$ is a root of f .

Problem 4

Let K be a finite extension of \mathbb{Q} . We will prove there are only a finite number of roots of unity in K .

Problem 6