Part 1

1.1, Problem 2

The equilibrium solution occurs when $\frac{dy}{dt} = 0$, meaning

$$\begin{aligned} 0 &= \frac{\left(t^2-1\right)\left(y^2-2\right)}{y^2-4} \\ y^2 - 2 &= 0 \\ y(t) &= \sqrt{2} \\ y(t) &= -\sqrt{2}. \end{aligned}$$

1.1, Problem 3

(a) When $P = 0$ or $P = 230$, the population is in equilibrium.

(b) If P is between 0 and 230, the population is increasing.

(c) If P is greater than 230 or less than 0, the population is decreasing.

1.1, Problem 13

Learning occurs most rapidly when $L = 0$.

1.1, Problem 14

(a) The student who knows one half of the list at $t = 0$ learns at a slower rate than the student who knows none of the list.

(b) The student who starts out knowing none of the list will never catch up to the student who starts out knowing one half of the list because solutions to initial value problems cannot intersect.

Part 2

1.2, Problem 1

(a) Glen and Bob are correct; taking $y(t) = 2t + 1$, we see that $\frac{dy}{dt} = 2 = \frac{2t+2}{t+1}$, and similarly, taking $y(t) = t$, we see $\frac{dy}{dt} = 1 = \frac{t+1}{t+1}$.

(b) The solution of $y(t) = t$ should be immediately obvious from separation of variables.

1.2, Problem 2

Substituting $y = e^{2t}$, we find that $t = \frac{1}{2}\ln y$, meaning we have $y(t) = e^{2t}$ is a solution to $\frac{dy}{dt} = 2y - t + \frac{1}{2}\ln y$.

1.2, Problem 3

The derivative $\frac{dy}{dt}$ for $y(t) = e^{t^3}$ is $3t^2e^{t^3}$. Substituting $y = e^{t^3}$, we find that $y(t) = e^{t^3}$ is a solution to the equation $\frac{dy}{dt} = 3t^2y$.

1.2, Problem 27

Since $y(t) = 0$ is an equilibrium solution for $\frac{dy}{dt} = -y^2$, and $y(0) = 0$, we have that $y(t) = 0$ solves the initial value problem.

1.2, Problem 32

$$\begin{aligned}
\frac{dy}{dt} &= ty^2 + 2y^2\\
\frac{dy}{dt} &= y^2(t+2)\\
\frac{1}{y^2}\,dy &= \frac{1}{t+2}\,dt\\
\int \frac{1}{y^2}\,dy &= \int \frac{1}{t+2}\,dt\\
-\frac{1}{y} &= \ln|t+2| + C_1\\
\frac{1}{y} &= C - \ln|t+2|\\
y &= \frac{1}{C - \ln|t+2|}.
\end{aligned}$$

Including our initial value $y(0) = 1$, we find that

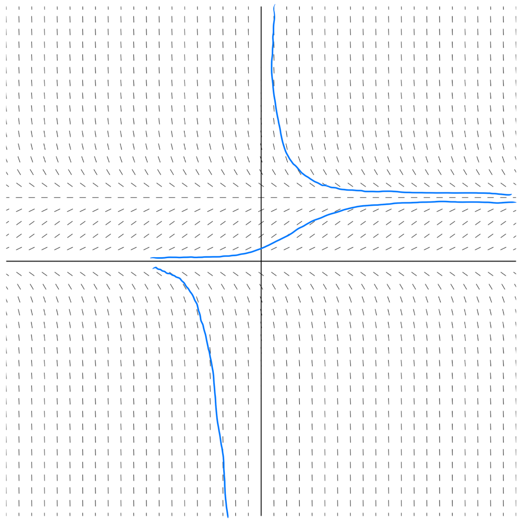
$$C = 1 + \ln 2.$$

Thus, the initial value problem has a solution of

$$y(t) = \frac{1}{(1+\ln 2) - \ln|t+2|}.$$

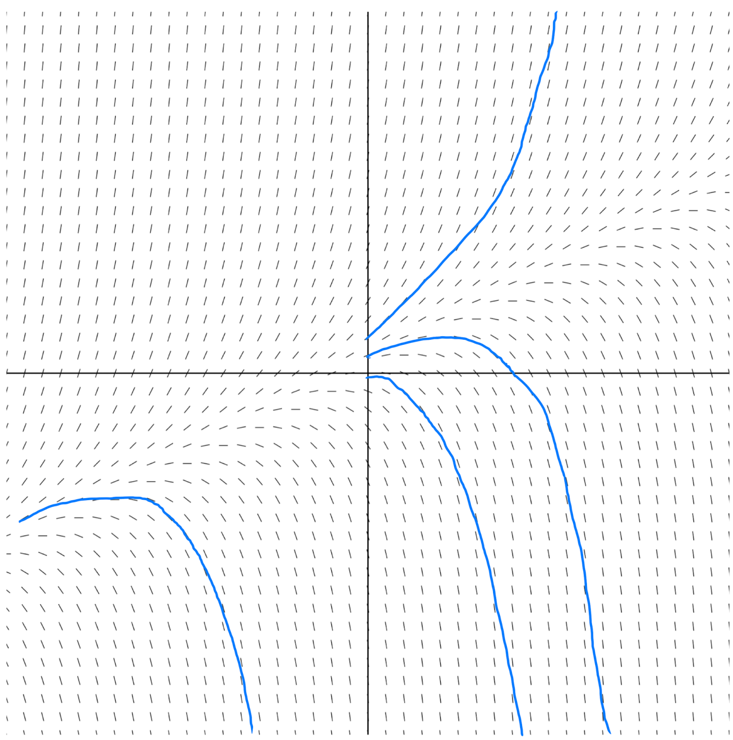
Part 3

1.3, Problem 7



For the solution with initial condition of $y(0) = 1/2$, the slope field suggests that, as t increases, $y(t)$ approaches the equilibrium solution of $y(t) = -1$.

1.3, Problem 8



For the solution with initial condition of $y(0) = 1/2$, the slope field suggests that the solution will increase without bound as t increases.

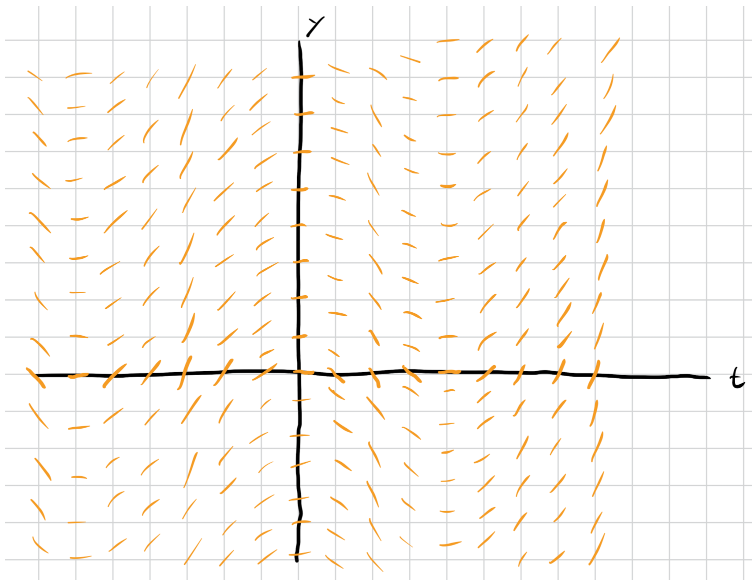
1.3, Problem 12

(a) $f(t, y)$ is equal to zero when $y = 2$.

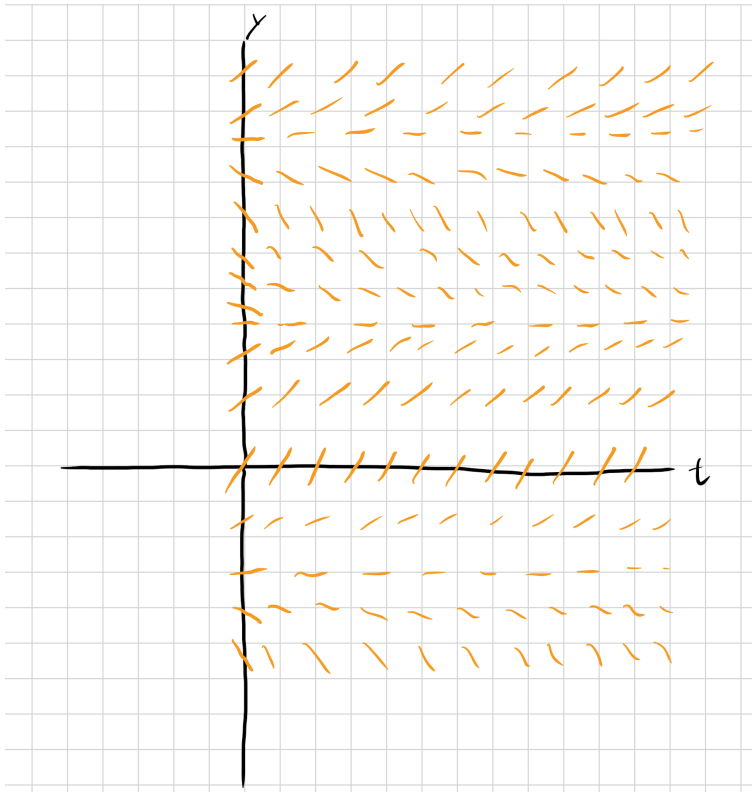
(b) We are able to sketch the equilibrium solution of $y(t) = 2$.

(c) This does not tell us any other information for when $y(0) \neq 2$.

1.3, Problem 13



1.3, Problem 14



1.3, Problem 16

(a) This slope field is for equation, (iii), as the slopes are purely dependent on y and equal to zero at $y = 0, y = -1$. Additionally, the slope is negative for all $y < -1$, which implies that it cannot be of the form y^2.

(b) This slope field is for equation (viii), as it reaches slop 0 at $\pm\sqrt{2}$, and for $t > \sqrt{2}$, the slope is positive.

(c) This slope field is for equation (v), as it has slope zero at $y = 0$ and $y = -1$ and has t-dependence that changes with sign.

(d) This slope field is for equation (vi), as it has slope zero at $y = -1$ and t-dependence with positive sign (i.e., always negative when $y + 1 < 0$).