Math 395

Homework 3

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Problem 1

Let $\varphi : R \to S$ be a ring homomorphism. Let $\mathfrak{p} \in \text{Spec}(S)$. We will prove that $\varphi^{-1}(\mathfrak{p}) \subset R$ is an element of $\text{Spec}(R)$.

Let $\mathfrak{p} \in \text{Spec}(S)$. Let $ab \in \varphi^{-1}(\mathfrak{p})$. Then, $\varphi(ab) \in \mathfrak{p}$. So, $\varphi(a)\varphi(b) \in \mathfrak{p}$, meaning either $\varphi(a) \in \mathfrak{p}$ or $\varphi(b) \in \mathfrak{p}$. Therefore, $a \in \varphi^{-1}(\mathfrak{p})$ or $b \in \varphi^{-1}(\mathfrak{p})$. Therefore, $\varphi^{-1}(\mathfrak{p})$.