14.2

2:

$$\nabla Q = \begin{pmatrix} 50 \\ 100 \end{pmatrix}$$

8:

$$\nabla f = \begin{pmatrix} 0.3\left(\frac{L}{K}\right)^{0.7} \\ 0.7\left(\frac{K}{L}\right)^{0.3} \end{pmatrix}$$

14:

$$\nabla z = \begin{pmatrix} \frac{e^y}{x+y} - \frac{xe^y}{(x+y)^2} \\ \frac{xe^y}{x+y} - \frac{xe^y}{(x+y)^2} \end{pmatrix}$$

22:

$$\nabla f(0,1) = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

32: Approximately zero.

38: $\hat{i}$

42: $\hat{i} - \hat{j}$

44: $\frac{5}{2\sqrt{2}}$

48:

$$\nabla f(1,2) \cdot \vec{u} = \begin{pmatrix} \frac{6}{5} \\ -\frac{4}{5} \end{pmatrix}$$

50: $\partial f = y\partial x + x\partial y$

14.3

2:

$$\nabla f = \begin{pmatrix} 2x \\ 0 \\ 0 \end{pmatrix}$$

8:

$$\nabla f = \begin{pmatrix} e^y \sin z \\ xe^y \sin z \\ xe^y \cos z \end{pmatrix}$$

10:

$$\nabla f = \begin{pmatrix} 2x_1 x_2^3 x_3^4 \\ 3x_1^2 x_2^2 x_3^4 \\ 4x_1^2 x_2^3 x_3^3 \end{pmatrix}$$

14:

$$\nabla f(1,1,1) = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

16:

$$\nabla f(1,1,1) = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$$

18:

$$\nabla f(2,1,e) = \begin{pmatrix} 1 \\ \frac{2}{e} \\ \frac{2}{e} \end{pmatrix}$$

22:

$$\nabla f \cdot \vec{u} = 1$$

28:

Verifying Point on Level Surface:

$$(-1)^2 - (-1)(1)(2) = 3$$

Finding Gradient:

$$f(x,y,z) = x^2 - xyz$$

$$\nabla f = \begin{pmatrix} 2x - yz \\ -xz \\ -xy \end{pmatrix}$$

$$\nabla f(-1,1,2) = \begin{pmatrix} -4 \\ 2 \\ 1 \end{pmatrix}$$

Tangent Plane:

$$0 = -4(x+1) + 2(y-1) + 1(z-2)$$

30:

Verifying Point on Level Surface:

$$1 = \frac{4}{2(-1)+3(2)}$$

Finding Gradient:

$$f(x,y,z) = \frac{4}{y(2x+3z)}$$

$$\nabla f = \begin{pmatrix} -\frac{8}{y(2x+3z)^2} \\ -\frac{4}{y^2(2x+3z)} \\ -\frac{12}{y(2x+3z)^2} \end{pmatrix}$$

$$\nabla f(-1,1,2) = \begin{pmatrix} -\frac{1}{2} \\ -1 \\ -\frac{3}{4} \end{pmatrix}$$

Tangent Plane:

$$0 = -\frac{1}{2}(x+1) - (y-1) - \frac{3}{4}(z-2)$$

14.4

2:

$$\frac{dz}{dt} = \sin^2 t + 2\sin t e^{-t}$$

10:

$$\frac{\partial z}{\partial u} = \frac{e^v}{u}$$
$$\frac{\partial z}{\partial v} = 2e^v + \frac{e^v}{u}$$

16:

$$\begin{aligned} \frac{dz}{dt} &= \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} \\ &= 3t^{10} + 2t^{11} \end{aligned}$$

18:

$$\begin{aligned} \left.\frac{dz}{dt}\right|_{t=1} &= \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} \\ &= 802.2 \end{aligned}$$

20:

$$\begin{aligned}\frac{dV}{dt} &= \frac{\partial V}{\partial R}\frac{dR}{dt} + \frac{\partial V}{\partial I}\frac{dI}{dt}\\ &= \frac{\partial V}{\partial R}\left(\frac{\partial R}{\partial R_1}\frac{dR_1}{dt} + \frac{\partial R}{\partial R_2}\frac{dR_2}{dt}\right) + \frac{\partial V}{\partial I}\frac{dI}{dt}\\ &= \frac{61}{160}\end{aligned}$$

30:

$$\begin{aligned}\frac{dh}{dt} &= \frac{\partial h}{\partial f}\frac{df}{dt} + \frac{\partial h}{\partial g}\frac{dg}{dt}\\ &= f'(t)g(t) + g'(t)f(t)\end{aligned}$$

32: I don't know how to do this problem.

34:

$$\begin{aligned}\frac{\partial z}{\partial v} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}\\ &= bk + dq\end{aligned}$$

14.5

6:

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= 0\\ \frac{\partial^2 f}{\partial y^2} &= xe^y\\ \frac{\partial^2 f}{\partial x\partial y} &= e^y\\ \frac{\partial^2 f}{\partial y\partial x} &= e^y\end{aligned}$$

10:

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= -2\sin(x^2+y^2) + 2\cos(x^2+y^2)\\ \frac{\partial^2 f}{\partial y^2} &= -2\sin(x^2+y^2) + 2\cos(x^2+y^2)\\ \frac{\partial^2 f}{\partial x\partial y} &= -4xy\sin(x^2+y^2)\\ \frac{\partial^2 f}{\partial x\partial y} &= -4xy\sin(x^2+y^2)\end{aligned}$$

16:

$$f(x,y) \approx 1 - 2x + y + 8x^2 + 2y^2 - 4xy$$

20:

$$g(x,y) \approx x - 2y - x^2 - 4y^2 + 2xy$$

22: (a) $f_x(P) < 0$

(b) $f_y(P) = 0$

(c) $f_{xx}(P) > 0$

(d) $f_{yy}(P) = 0$

(e) $f_{xy}(P) = 0$

24: (a) $f_x(P) > 0$

(b) $f_y(P) = 0$

(c) $f_{xx}(P) < 0$

(d) $f_{yy}(P) = 0$

(e) $f_{xy}(P) = 0$