Part 1

1.8, Problem 4

To solve

$$\frac{dy}{dt} = 2y + \sin 2t,$$

we start by solving the homogeneous equation

$$\frac{dy}{dt} = 2y,$$

which yields $y_h = ke^{2t}$.

For the particular solution, we guess that y is of the form

$$y_p(t) = A\cos 2t + B\sin 2t.$$

Plugging this into our equation, we get

$$\begin{aligned} -2A\sin 2t + 2B\cos 2t + 2\left(A\cos 2t + B\sin 2t\right) &= \sin 2t \\ (2B - 2A)\sin 2t + (2B + 2A)\cos 2t &= \sin 2t, \end{aligned}$$

meaning $A = -\frac{1}{4}$ and $B = \frac{1}{4}$. Thus, our general solution is

$$y(t) = -\frac{1}{4}\cos 2t + \frac{1}{4}\sin 2t + ke^{2t}.$$

1.8, Problem 8

To solve

$$\frac{dy}{dt} - 2y = 3e^{-2t},$$

with the initial condition of $y(0) = 10$, we start by solving the homogeneous equation, which yields $y_h(t) = ke^{2t}$.

For the particular solution, we guess that y is of the form

$$y_p(t) = Ae^{-2t}.$$

Substituting into our equation, we have

$$\begin{aligned} -2Ae^{-2t} - 2\left(Ae^{-2t}\right) &= 3e^{-2t} \\ -4Ae^{-2t} &= 3e^{-2t}, \end{aligned}$$

which yields $A = -\frac{3}{4}$. Thus, our general solution is of the form

$$y(t) = -\frac{3}{4}e^{-2t} + ke^{2t}.$$

The initial condition yields $k = \frac{43}{4}$.

1.8, Problem 9

To solve

$$\frac{dy}{dt} + y = \cos 2t,$$

with the initial condition of $y(0) = 5$, we start by solving the homogeneous equation, which yields $y_h(t) = ke^{-t}$.

For the particular solution, we guess that y is of the form

$$y_p(t) = A\cos 2t + B\sin 2t.$$

Substituting into our equation, we get

$$\begin{aligned} -2A\sin 2t + 2B\cos 2t + (A\cos 2t + B\sin 2t) &= \cos 2t \\ (2B + A)\cos 2t + (B - 2A)\sin 2t &= \cos 2t, \end{aligned}$$

meaning $A = \frac{1}{3}$ and $B = \frac{2}{3}$. Thus, our general solution is

$$y(t) = \frac{1}{3}\cos 2t + \frac{2}{3}\sin 2t + ke^{-t}.$$

Solving the initial condition yields $k = \frac{14}{3}$.

1.8, Problem 17

1.8, Problem 18

1.8, Problem 20

1.8, Problem 31

1.9, Problem 4

1.9, Problem 5

1.9, Problem 9

1.9, Problem 12

1.9, Problem 19

1.9, Problem 21