Problem: Construct infinitely many non-homotopic retractions $S^1 \vee S^1 \to S^1$.

Solution: We will let (A, a_0) be the first (pointed) copy of S^1 and (B, b_0) the second copy. By the universal property of the disjoint union, we observe that any pair of self-maps $f_a\colon A \to A$ and $f_b\colon B \to B$ induce a self-map on the disjoint union, defined by

$$f(x) = \begin{cases} f_a(x), & x \in A; \\ f_b(x), & x \in B. \end{cases}$$

If these maps preserve their respective basepoints, then we obtain a self-map on the wedge sum of A and B upon the identification $a_0 \sim b_0$.

Therefore, a self-map $A \vee B \to A \vee B$ is a retraction into S^1 if f_a is a retraction onto a_0 or f_b is a retraction onto b_0. Without loss of generality, we will assume that f_b is a retraction of B onto b_0. We may then define non-homotopic self-maps of A, which we will denote $\phi_n\colon A \to A$ to be representatives of the homotopy classes $[\omega_n]$ looping around S^1 a total of n times in a manner that preserves a_0.

Since we have established that, whenever $n \neq m$, we have that $[\omega_n] \neq [\omega_m]$, it follows that upon collapsing with the wedge sum, we have that $A \wedge B$ retracts onto A, with each map being non-homotopic to the next map.