Derivation

Output:

$$Y_t = C_t + I_t + G_t + EX_t - IM_t$$
$$\frac{Y_t}{\overline{Y}_t} = \frac{C_t + I_t + G_t + EX_t - IM_t}{\overline{Y}_t}$$

Assumptions:

$$\frac{C_t}{\overline{Y}_t} = \overline{a}_c \qquad (\overline{a}_c = 0.67)$$
$$\frac{G_t}{\overline{Y}_t} = \overline{a}_g$$
$$\frac{EX_t}{\overline{Y}_t} = \overline{a}_{ex}$$
$$\frac{IM_t}{\overline{Y}_t} = \overline{a}_{im}$$

Investment:

$$\frac{I_t}{\overline{Y}_t} = \overline{a}_i - \overline{b}(R_t - \overline{r})$$

where R_t represents the interest rate and $\overline{r}$ represents the marginal product of capital, and $\overline{b}$ represents the sensitivity of investment to interest rates

$$\frac{Y_t}{\overline{Y}_t} = \overline{a}_c + \left(\overline{a}_i - \overline{b}(R_t - \overline{r})\right) + \overline{a}_g + \overline{a}_{ex} - \overline{a}_{im}$$

$$\frac{Y_t}{\overline{Y}_t} - 1 = \overline{a}_c + \left(\overline{a}_i - \overline{b}(R_t - \overline{r})\right) + \overline{a}_g + \overline{a}_{ex} - \overline{a}_{im} - 1$$
$$\tilde{Y}_t = \frac{Y_t - \overline{Y}_t}{\overline{Y}_t}$$
$$= \frac{Y_t}{\overline{Y}_t} - 1$$

$$\tilde{Y}_t = \left(\overline{a}_c + \overline{a}_i + \overline{a}_g + \overline{a}_{ex} - \overline{a}_{im} - 1\right) - \overline{b}(R_t - \overline{r})$$
$$= \boxed{\overline{a} - \overline{b}(R_t - \overline{r})}$$

where $\overline{a} = \overline{a}_c + \overline{a}_i + \overline{a}_g + \overline{a}_{ex} - \overline{a}_{im} - 1$

We can see from this derivation that the IS curve is downward sloping — if R_t increases, then $\tilde{Y}_t$ decreases. In long run equilibrium, we have $\tilde{Y}_t = 0$, meaning $\overline{a} = 0$ and $R_t = \overline{r}$.