

Activity: MSNE in Larger Games

Econ 305

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1 MSNE in a 3-Pure Strategy Game

Find *all* of the Nash equilibria of the following game:

		U L	C	(1-p) R
	U	3,10	9,15	7,10
	p M	5,1	10,0	0,3
	(1-p) D	5,2	8,1	10,1

U strictly dominated by $0.8D + 0.2M$

C strictly dominated by L

$$v_1(M, \sigma_2) = 5q + 0(1-q)$$

$$v_1(D, \sigma_2) = 5q + 10(1-q)$$

$$5q = 5q + 10(1-q)$$

$$10q = 10$$

$$q = 1$$

Infinite NE: $q=1$ $0 \leq p \leq \frac{1}{3}$

$$p < \frac{7}{8}$$

$$p > \frac{2}{10}$$

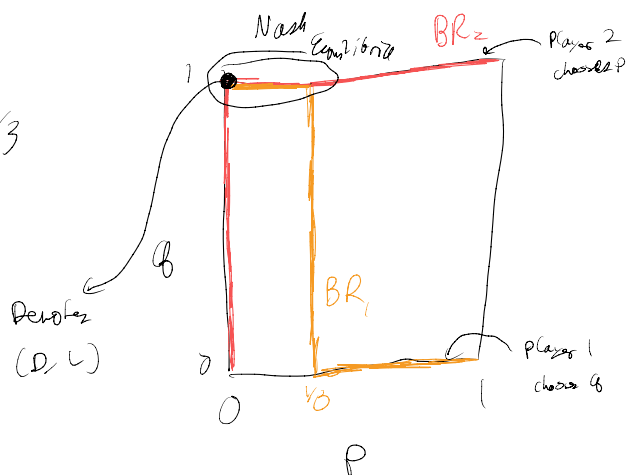
$$p = 0.8$$

$$v_2(\sigma_1, L) = p + 2(1-p)$$

$$v_2(\sigma_1, R) = 3p + (1-p)$$

$$-p + 2 = 3p + 1$$

$$p = \frac{1}{3}$$



2 Bonus: MSNE in a 3-Player Game

Consider the following three-player team production problem. Simultaneously and independently, each player chooses between exerting effort, E , or not exerting effort, N . Exerting effort imposes a cost of 2 on the player who exerts effort. If two or more players exert effort, each player receives a benefit of 4 regardless of whether she herself exerted effort. Otherwise, each player receives zero benefit. The payoff to each player is her realized benefit less the cost of her effort (if she exerted effort).

Find a symmetric mixed-strategy Nash equilibrium of this game. In particular, let the strategy for player i be denoted by $\sigma_i = pE + (1-p)N$ and determine p .

- Given NE, $\sigma_i^* = pE + (1-p)N \quad \forall i$

$$U_i(E, \sigma_{-i}) = U_i(N, \sigma_{-i})$$

$$U_i(N, \sigma_{-i}) = 4p^2 \quad (\text{benefit})$$

$$U_i(E, \sigma_{-i}) = (-2)(1) + 4 \quad (\text{at least one does effort})$$

$$= -2 + 4(1 - (1-p)^2) \quad (1 - \text{probability no one does effort})$$

$$4p^2 = -2 + 4(2p - p^2)$$

$$8p^2 - 8p + 2 = 0$$

$$\boxed{p = \frac{1}{2}}$$