- 5 questions:
  (1) (Discrete) game tree: count information sets, pure strategies, subgames, etc.
  (2) (Discrete) game tree: find SPE and Pure Strategy Nash equilibria.
  (3) SPE in Stackelberg-style game: player 1 chooses $a_1 \in \mathbb{R}$, player 2 sees a_1, then chooses $a_2 \in \mathbb{R}$.
    - Strategy for Player 1: a_1^*
    - Strategy for Player 2: $a_2(a_1^*)$ (best response function)
  (4) Repeated games: given a stage game G, find
    - SPE in finitely repeated game $G(T, \delta)$.
    - In $G(\infty, \delta)$, find δ for which grim trigger is a SPE.
    - Conceptual: limited punishment, Folk Theorem.
  (5) Bargaining: given a strategy, and a given player, examine
    - Proposer node:
      * Find payoff from following
      * Find payoff from deviating (offer less to other)
    - Responder to z:
      * Find payoff from accepting (z)
      * Find payoff from rejecting — accept any z at least as large as this value

Positive Externality Repeated Game: Consider a game with effort x_i and payoff

$$v_i = \left(16 - x_i + \frac{1}{2}x_j\right)x_i - 4x_i$$

The Unique Nash Equilibrium for this game is $x_1 = x_2 = 8$, where $v_1 = v_2 = 64$. However, the social optimum is $x_1 = x_2 = 12$, where $v_1 = v_2 = 72$.

(a) $G(T, \delta)$, where T is finite: there must be only one SPE — playing the Nash equilibrium in every stage game. Therefore, it is not possible to play $(12, 12)$ in any period of a SPE.

(b) $G(\infty, \delta)$ with grim trigger:
- Deviation in past $\Rightarrow$ play Nash equilibrium $\Rightarrow$ no incentive to deviate.
- No deviation in past:
  - Discounted Average Payoff from follow: 72
  - Discounted Average Payoff from deviation:

$$\begin{aligned} BR_i(12) &\Rightarrow \frac{\partial v_i}{\partial x_i} = 0 \\ 18 - 2x_1 &= 0 \\ x_1 &= 9 \\ v_1 &= 81 \end{aligned}$$

Payoff from Deviation:

$$\begin{aligned} \overline{v}_1 &= (1-\delta)\left(81 + \overbrace{64\delta + 64\delta^2 + \cdots}^{\text{grim trigger payoffs}}\right) \\ &= 81(1-\delta) + 64\delta \\ 72 &\geq 81(1-\delta) + 64\delta \\ 17\delta &\geq 9 \\ \delta &\geq \frac{9}{17} \end{aligned}$$

(c) If the punishment period was shorter, would need future punishment to be weighted more, meaning δ would need to be higher.