

Solution (32.20): We start by taking the recurrence relation

$$(1 - x^2)P'_n = -nP_n + nP_{n-1}.$$

Using the relation

$$(2n + 1)xP_n = (n + 1)P_{n+1} + nP_{n-1},$$

we get

$$(1 - x^2)P'_n = (n + 1)xP_n - (n + 1)P_{n+1}.$$

Differentiating, we then get

$$(1 - x^2)P''_n - 2xP'_n = (n + 1)P_n + (n + 1)xP'_n - (n + 1)P'_{n+1}$$

We want to evaluate

$$(n + 1)P_n + (n + 1)xP'_n - (n + 1)P'_{n+1} = -n(n + 1)P_n.$$

by using the generating function

$$P_n = \frac{1}{n!} \frac{\partial^n}{\partial t^n} \left((1 - 2xt + t^2)^{-1/2} \right) \Big|_{t=0}$$

| **Solution (32.21):**

| **Solution (32.23):**

| **Solution (35.4):**

| **Solution (35.5):**

| **Solution (35.7):**

| **Solution (35.8):**

| **Solution (35.10):**

| **Solution (35.11):**

| **Solution (35.12):**

| **Solution (35.16):**

| **Solution (35.17 (c)):**

| **Solution (35.21):**

| **Solution (35.25):**