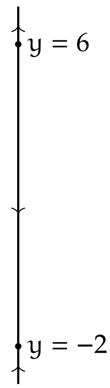
Part 1

1.6, Problem 2

$$\begin{aligned}\frac{dy}{dt} &= y^2 - 4y - 12\\ &= (y-6)(y+2).\end{aligned}$$

We can see that $\frac{dy}{dt} = 0$ at $y = 6$ and $y = -2$. Additionally, for $y > 6$, $\frac{dy}{dt} > 0$, for $y < -2$, $\frac{dy}{dt} > 0$, and for $y \in (6,2)$, $\frac{dy}{dt} < 0$. Thus, we get the following phase line.



The equilibrium point at $y = -2$ is a sink, while the equilibrium point at $y = 6$ is a source.

1.6, Problem 7

$$\begin{aligned}\frac{dv}{dt} &= -v^2 - 2v - 2\\ &= -\left(v^2 + 2v + 2\right)\\ &= -\left((v+1)^2 + 1\right).\end{aligned}$$

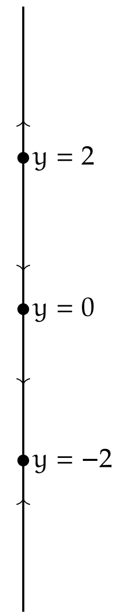
Thus, we can see that it is never the case that $v = 0$, and that $\frac{dv}{dt} < 0$ for all v. The analytical solution is as follows:

$$\begin{aligned}\frac{dv}{dt} - &= -\left((v+1)^2 + 1\right)\\ \int \frac{dv}{(v+1)^2 + 1} &= -\int dt\\ \arctan(v+1) &= -t + C\\ v + 1 &= \tan(-t + C)\\ v &= \tan(-t + C) - 1.\end{aligned}$$

1.6, Problem 8

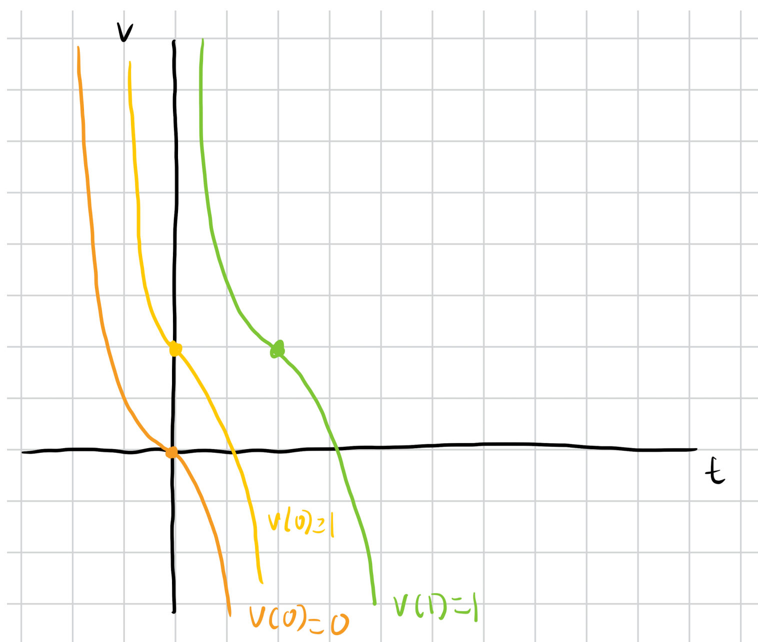
$$\begin{aligned}\frac{dw}{dt} &= 3w^3 - 12w^2\\ &= 3w^2(w-2)(w+2).\end{aligned}$$

We can see that $\frac{dw}{dt} = 0$ at $w = 0$, $w = 2$, and $w = -2$. Additionally, we can see that $\frac{dw}{dt} > 0$ for $w > 2$ and $w < -2$, and $\frac{dw}{dt} < 0$ for $w \in (-2, 2)$. Thus, the phase line is as follows.

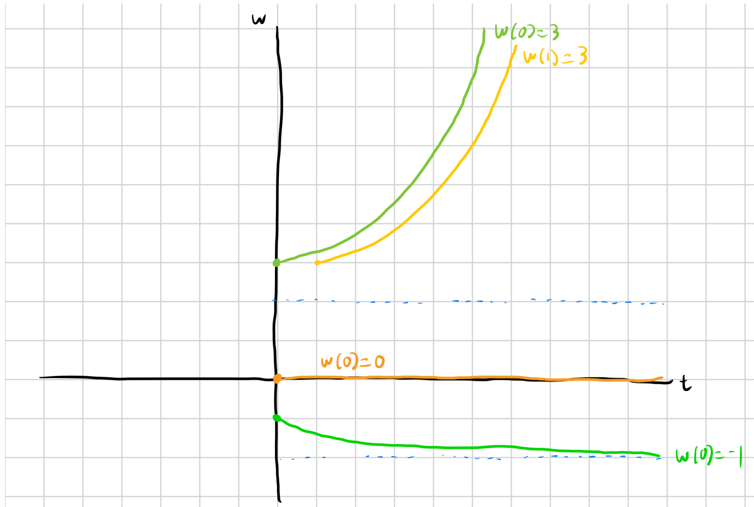


The equilibrium point at $y = -2$ is a sink, the equilibrium point at $y = 2$ is a source, and the equilibrium point at $y = 0$ is a node.

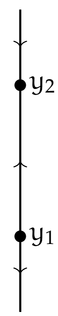
1.6, Problem 19



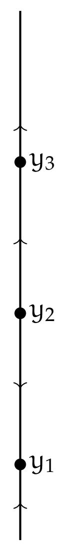
1.6, Problem 20



1.6, Problem 30



1.6, Problem 31



1.6, Problem 41

(a) The phase line is is qualitatively similar for $a > 0$ and $a < 0$; in the former case the phase line has zero equilibrium solutions, while in the latter case, the phase line has two equilibrium solutions.

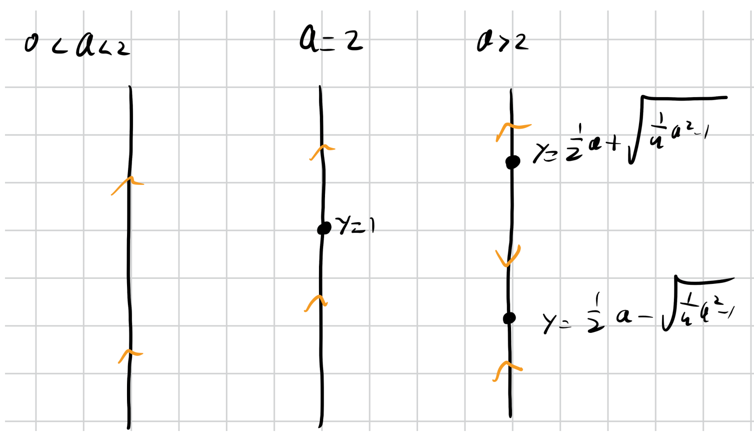
(b) The phase line shifts when $a = 0$, as it has only one equilibrium solution for $a = 0$.

Part 2

1.7, Problem 3

$$\begin{aligned}
\frac{dy}{dt} &= y^2 - ay + 1\\
y^2 - ay + 1 &= 0\\
y^2 - ay + \frac{1}{4}a^2 &= -1 + \frac{1}{4}a^2\\
y &= \frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 - 1}.
\end{aligned}$$

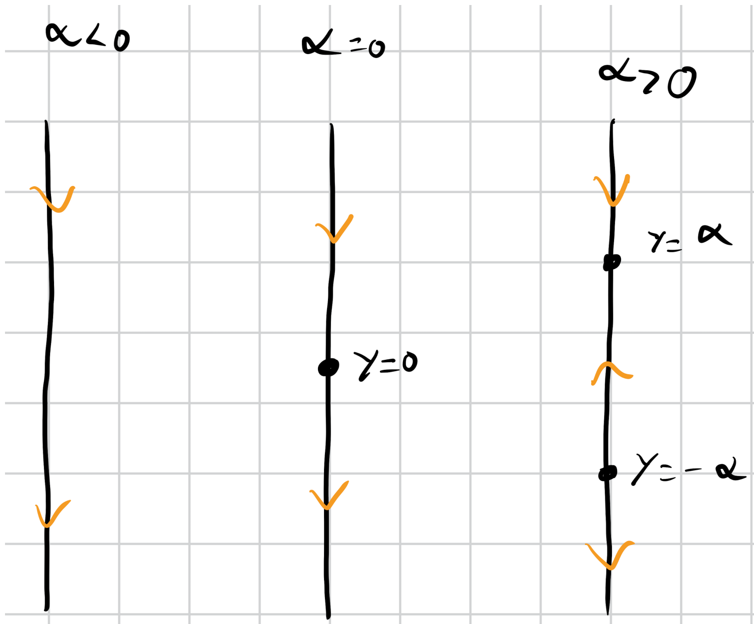
For $a > 2$, there are two equilibrium solutions, while for $a = 2$, there is one equilibrium solution, and for $a < 2$, there are no equilibrium solutions.



1.7, Problem 6

$$\begin{aligned}
\frac{dy}{dt} &= \alpha - |y|\\
\alpha - |y| &= 0\\
y &= \pm\alpha.
\end{aligned}$$

For $\alpha > 0$, there are two equilibrium solutions, while for $\alpha = 0$, there is one equilibrium solution, an for $\alpha < 0$, there are no equilibrium solutions.



1.7, Problem 18

(a) There are no bifurcations as C varies; the sole equilibrium value occurs at $P = \frac{C}{k}$. In particular, for $C > k$, the equilibrium population is zero.

(b) The population $P(t)$ approaches the equilibrium population whenever $P(0) > 0$.