# Section 4.1

**Solution** (Problem 4)**:** Evaluating with the initial conditions, we get

$$\begin{aligned} c_1 - c_2 &= 0 \\ -c_3 &= 2 \\ c_2 &= -1. \end{aligned}$$

We see that $c_1 = -1$, $c_2 = -1$, and $c_3 = -2$. This yields the particular solution of

$$y = -1 - \cos x - 2\sin x.$$

**Solution** (Problem 10)**:** The interval $(-\pi, \pi)$ contains a unique solution to the initial value problem.

**Solution** (Problem 14)**:**

(a) We have

$$\begin{aligned} c_1 + c_2 + 3 &= 0 \\ c_1 + c_2 + 3 &= 4, \end{aligned}$$

which is not possible.

(b) We have

$$\begin{aligned} 3 &= 0 \\ c_1 + c_2 + 3 &= 2, \end{aligned}$$

which is yet again not possible.

(c) We have

$$\begin{aligned} 3 &= 3 \\ c_1 + c_2 + 3 &= 0, \end{aligned}$$

meaning that the solution set is all pairs $(c_1, c_2)$ such that $c_1 + c_2 = -3$.

(d) We have

$$\begin{aligned} c_1 + c_2 + 3 &= 3 \\ 4c_1 + 16c_2 + 3 &= 15, \end{aligned}$$

or

$$\begin{aligned} c_1 + c_2 &= 0 \\ 4c_1 + 16c_2 &= 12 \end{aligned}$$

meaning

$$\begin{aligned} c_1 &= -1 \\ c_2 &= 1. \end{aligned}$$

**Solution** (Problem 22)**:** Since

$$\sinh(x) = \frac{1}{2}(e^x + e^{-x}),$$

the functions are not linearly independent anywhere on $(-\infty, \infty)$.

**Solution** (Problem 28)**:** First, we verify that both solutions work.

$$\begin{aligned} x^2\frac{d^2}{dx^2}(\cos(\ln(x))) + x\frac{d}{dx}(\cos(\ln(x))) + \cos(\ln(x)) &= x^2\left(-\frac{\cos(\ln(x))}{x^2} + \frac{\sin(\ln(x))}{x^2}\right) + x\left(-\frac{\sin(\ln(x))}{x}\right) + \cos(\ln(x)) \\ &= 0 \end{aligned}$$

$$
\begin{aligned}
x^2\frac{d^2}{dx^2}(\sin(\ln(x))) + x\frac{d}{dx}(\sin(\ln(x))) + \sin(\ln(x)) &= x^2\left(-\frac{\cos(\ln(x))}{x^2} - \frac{\sin(\ln(x))}{x^2}\right) + x\left(\frac{\cos(\ln(x))}{x}\right) + \sin(\ln(x)) \\
&= 0.
\end{aligned}
$$

Additionally, we find that

$$
\begin{aligned}
\det\begin{pmatrix} \cos(\ln(x)) & \sin(\ln(x)) \\ \frac{-\sin(\ln(x))}{x} & \frac{\cos(\ln(x))}{x} \end{pmatrix} &= \frac{1}{x} \\
&\neq 0,
\end{aligned}
$$

so the solutions are linearly independent. Since the differential equation $x^2y'' + xy' + y' = 0$ is a second order equation, there are no other linearly independent solutions. Thus, we have the general solution of

$$
y = \alpha\cos(\ln(x)) + \beta\sin(\ln(x)).
$$

**Solution** (Problem 30)**:**

**Solution** (Problem 36)**:**

# Section 4.2

**Solution** (Problem 2)**:**

**Solution** (Problem 8)**:**

**Solution** (Problem 16)**:**

**Solution** (Problem 20)**:**

**Solution** (Problem 22)**:**

# Section 4.3

**Solution** (Problem 4)**:**

**Solution** (Problem 6)**:**

**Solution** (Problem 12)**:**

**Solution** (Problem 16)**:**

**Solution** (Problem 22)**:**

**Solution** (Problem 36)**:**

**Solution** (Problem 38)**:**

**Solution** (Problem 50)**:**