

## 8.1

(a)

$$\begin{aligned}
 \int_0^1 2^x dx &= \int_0^1 e^{x(\ln 2)} dx \\
 &= \frac{1}{\ln 2} \left( e^{x(\ln 2)} \Big|_0^1 \right) && u = x(\ln 2) \\
 &= \frac{1}{\ln 2} \left( 2^x \Big|_0^1 \right) \\
 &= \frac{1}{\ln 2} (2 - 1) \\
 &= \frac{1}{\ln 2}.
 \end{aligned}$$

(b)

$$\begin{aligned}
 \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} e^x dx &= \int_{-\infty}^{\infty} e^{\left(-\frac{x^2}{2} + x - \frac{1}{2}\right) + \frac{1}{2}} dx && \text{Completing the square.} \\
 &= e^{\frac{1}{2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x-1)^2} dx \\
 &= \sqrt{2\pi} e && \text{Gaussian Integral}
 \end{aligned}$$

(c)

(d)

$$\int_{-a}^a \sin x e^{-\alpha x^2} dx = 0 \quad \text{Even/odd.}$$

(e)

$$\begin{aligned}
 \int_0^1 e^{\sqrt{x}} dx &= x e^{\sqrt{x}} \Big|_0^1 - \frac{1}{2} \int_0^1 x e^{\sqrt{x}} dx \\
 &= e - \int_0^1 u^3 e^u du && u = \sqrt{x} \\
 &= e - \left( u^3 e^u \Big|_0^1 - 3u^2 e^u \Big|_0^1 + 6u e^u \Big|_0^1 - 6e^u \Big|_0^1 \right) && \text{Repeated integration by parts.} \\
 &= 3e - 6.
 \end{aligned}$$

To evaluate  $\int_0^1 u^3 e^u du$ , we used tabular integration as follows:

Sign	Differentiate	Integrate
+	$u^3$	$e^u$
-	$3u^2$	$e^u$
+	$6u$	$e^u$
-	$6$	$e^u$
+	$0$	$e^u$

Taking the boundary integrals, we obtain

$$u^3 e^u \Big|_0^1 - 3u^2 e^u \Big|_0^1 + 6u e^u \Big|_0^1 - 6e^u \Big|_0^1 = 6 - 2e$$

(f)

$$\begin{aligned}
 \int \frac{1}{\sqrt{1+x^2}} dx &= \int \frac{1}{\cosh(u)} \cosh(u) du & x &= \sinh(u) \\
 &= u + C \\
 &= \sinh^{-1}(x) + C.
 \end{aligned}$$

(g)

$$\begin{aligned}
 \int \tanh x \, dx &= \int \frac{\sinh x}{\cosh x} \, dx \\
 &= \int \frac{1}{u} \, du & u &= \cosh x \\
 &= \ln |u| + C \\
 &= \ln |\cosh x| + C.
 \end{aligned}$$

(h)

$$\begin{aligned}
 \int \tan^{-1} x \, dx &= x \tan^{-1} x - \int \frac{x}{1+x^2} \, dx && \text{integration by parts} \\
 &= x \tan^{-1} x - \frac{1}{2} \ln |1+x^2| + C. && \text{u-substitution implicit}
 \end{aligned}$$

(i)

$$\begin{aligned}
 \int_S z^2 \, d\mathbf{a} &= \int_0^{\pi/2} \int_0^{\pi/2} \cos^2 \theta \sin \theta \, d\phi d\theta \\
 &= \frac{\pi}{2} \int_0^{\pi/2} \cos^2 \theta \sin \theta \, d\theta \\
 &= -\frac{\pi}{2} \int_0^{-1} t^2 \, dt && t = \cos \theta \\
 &= \frac{\pi}{2} \left( \frac{t^3}{3} \Big|_{-1}^0 \right) \\
 &= \frac{\pi}{6}
 \end{aligned}$$