Chapter 27 Problems

Problem 11

(a)

$$\begin{aligned}
\lambda \langle v \mid v \rangle &= \langle v | \lambda | v \rangle && \text{(Moving } \lambda \text{ into braket.)} \\
&= \langle v | H | v \rangle && \text{(Definition of } \lambda\text{.)} \\
&= \overline{\langle v | H^* | v \rangle} && \text{(Definition of adjoint operator.)} \\
&= \overline{\langle v | H | v \rangle} && \text{(Definition of Hermitian operator.)} \\
&= \overline{\langle v | \lambda | v \rangle} && \text{(Definition of } \lambda\text{.)} \\
&= \overline{\lambda} \langle v \mid v \rangle && \text{(Moving } \lambda \text{ out of braket.)}
\end{aligned}$$

(b) It is the case that $\langle Hv_1 \mid v_2 \rangle = \overline{\lambda_1} \langle v_1 \mid v_2 \rangle$ for any operator — since our operator is Hermitian, it must be the case that $\lambda_1 = \overline{\lambda_1}$, else it would be possible for there to be $\lambda_2 - \overline{\lambda_1} = 0$ with λ_1, λ_2 distinct in (27.52b).

Problem 22

$$\begin{aligned}
M &= \sum_i \lambda_i \left|\hat{v}_i\right\rangle \left\langle\hat{v}_i\right| \\
&= (2)\left(\frac{1}{6}\right)\begin{pmatrix}1\\2\\1\end{pmatrix}\begin{pmatrix}1 & 2 & 1\end{pmatrix} \\
&+ (-1)\left(\frac{1}{2}\right)\begin{pmatrix}1\\0\\-1\end{pmatrix}\begin{pmatrix}1 & 0 & -1\end{pmatrix} \\
&+ (1)\left(\frac{1}{3}\right)\begin{pmatrix}1\\-1\\1\end{pmatrix}\begin{pmatrix}1 & -1 & 1\end{pmatrix} \\
&= \begin{pmatrix}-1 & 1 & -1\\ 1 & 1 & 1\\ -1 & 1 & -1\end{pmatrix}.
\end{aligned}$$

Problem 26

(a) Let M be a normal matrix. Then, there exists a unitary operator U such that

$$U\Lambda U^* = M,$$

where Λ is the diagonal matrix of eigenvalues. Since Λ and M are in the same similarity class, they have the same trace, so

$$\begin{aligned}
\operatorname{tr}(M) &= \operatorname{tr}(\Lambda) \\
&= \sum_i \lambda_i.
\end{aligned}$$

(b) Let M be a normal matrix. Then, there exists a unitary operator U such that

$$U\Lambda U^* = M,$$

where Λ is the diagonal matrix of eigenvalues. Since Λ and M are in the same similarity class, they have the same determinant, so

$$\begin{aligned}\det(M) &= \det(\Lambda)\\ &= \prod_i \lambda_i.\end{aligned}$$

Problem 27

I don't know what you can say about their eigenvalues.

Chapter 28 Problems

Problem 1

$$\begin{aligned}M\left|\ddot{Q}\right\rangle &= -K\left|Q\right\rangle\\ m\ddot{q}_1 &= -2kq_1 + kq_2\\ m\ddot{q}_2 &= -2kq_2 + kq_1\\ \\ m\ddot{q}_1 &= k\left(-2q_1 + q_2\right)\\ m\ddot{q}_2 &= k\left(-2q_2 + q_1\right)\end{aligned}$$

We have

$$\begin{aligned}m\left(\ddot{q}_1 + \ddot{q}_2\right) &= -k\left(q_1 + q_2\right)\\ m\left(\ddot{q}_1 - \ddot{q}_2\right) &= -3k\left(q_1 - q_2\right).\end{aligned}$$

Thus, we have

$$\begin{aligned}\frac{d^2}{dt^2}\left(q_1 - q_2\right) &= -\frac{3k}{m}\left(q_1 - q_2\right)\\ \frac{d^2}{dt^2}\left(q_1 + q_2\right) &= -\frac{k}{m}\left(q_1 + q_2\right),\end{aligned}$$

so

$$\begin{aligned}q_1 + q_2 &= A_1\cos\left(\omega_1 t + \delta_1\right)\\ q_1 - q_2 &= A_2\cos\left(\omega_2 t + \delta_2\right)\\ q_1 &= a_1\cos\left(\omega_1 t + \delta_1\right) + a_2\cos\left(\omega_2 t + \delta_2\right)\\ q_2 &= a_1\cos\left(\omega_1 t + \delta_1\right) - a_2\cos\left(\omega_2 t + \delta_2\right).\end{aligned}$$

Problem 2

Problem 3

Problem 6

Problem 7

Problem 10

Problem 15