# Activity: Justifying Mixed Strategy Nash Equilibria

Econ 305

Brandon Lehr

You can verify that the following game has a unique NE: $(3/4U + 1/4D, 1/2L + 1/2R)$

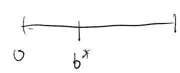
| | $L$ | $R$ |
|---|---|---|
| $U$ | 0,0 | 0,-1 |
| $D$ | 1,0 | -1,3 |

Consider introducing a little bit of incomplete information into the above game by changing the payoffs to

| | $L$ | $R$ |
|---|---|---|
| $U$ | $\varepsilon a, \varepsilon b$ | $\varepsilon a, -1$ |
| $D$ | $1, \varepsilon b$ | $-1, 3$ |

where $a$ and $b$ are known to players 1 and 2, respectively, and are drawn independently from the uniform distribution over $[0, 1]$

Determine the structure of the BNE:




$$s_1^*(a) = \begin{cases} U & \text{if } a \underline{\ \geq\ } a^* \\ D & \text{if } a \underline{\ <\ } a^* \end{cases}$$

$$s_2^*(b) = \begin{cases} L & \text{if } b \underline{\ \geq\ } b^* \\ R & \text{if } b \underline{\ <\ } b^* \end{cases}$$

Write down the local indifference equations:

$\underline{\varepsilon a^*} = Ev_1(U, s_2^*(b); a^*, b) = Ev_1(D, s_2^*(b); a^*, b) = \underline{(1)(1-b^*) + (-1)(b^*) \Rightarrow 1-2b^*}$

$\underline{\varepsilon b^*} = Ev_2(L, s_1^*(a); b^*, a) = Ev_2(R, s_1^*(a); b^*, a) = \underline{(-1)(1-a^*) + 3(a^*) \Rightarrow 4a^*-1}$

Solve these local indifference equations for $a^*$ and $b^*$:

$\varepsilon a^* = 1-2b^*$

$\varepsilon b^* = 4a^*-1$

$a^* = \frac{\varepsilon b^*+1}{4} \Rightarrow \varepsilon\left(\frac{\varepsilon b^*+1}{4}\right) = 1-2b^*$

$\varepsilon^2 b^* + \varepsilon = 4-8b^*$

$(8+\varepsilon^2)b^* = 4-\varepsilon$

$b^* = \frac{4-\varepsilon}{8+\varepsilon^2}$

$a^* = \frac{4\varepsilon-\varepsilon^2+8+\varepsilon^2}{4(8+\varepsilon^2)} = \frac{8+4\varepsilon}{32+4\varepsilon^2}$

What happens as private information becomes small ($\varepsilon \to 0$)?

$a^* \to \frac{1}{4}$

$b^* \to \frac{1}{2}$