Activity: Auctions

Econ 305

1 First-Price Auction

Consider the $n-$player version of the private value first-price sealed bid auction where each player i has an independent valuation θ_i and believes that the valuations of the other players are each uniform on $[0, 1]$. Guess that every player uses the strategy $s_i^*(\theta_i) = k\theta_i + c$. Find k and c such that s^* is a BNE of the game. Is bidding higher or lower with more players?

Step One: Write the expected payoff for player i of type θ_i from choosing arbitrary bid b_i, given that all other players are choosing the strategy s_{-i}^*.

$$V_i = (\theta_i - b_i)\left(P_r\left(b_i > k\theta_{-i} + C\right)\right)^{n-1}$$

$$= (\theta_i - b_i)\left(\frac{b_i - c}{k}\right)^{n-1}$$

Step Two: Find the bid b_i that maximizes the payoff in Step One.

$$\underset{b_i}{\arg\max}\left((\theta_i - b_i)\left(\frac{b_i - c}{k}\right)^{n-1}\right) \Rightarrow 0 = -\left(\frac{b_i - c}{k}\right)^{n-1} + (\theta_i - b_i)\left(\frac{n-1}{k}\right)\left(\frac{b_i - c}{k}\right)^{n-2}$$

$$\left(\frac{b_i - c}{k}\right)^{n-1} = (\theta_i - b_i)\left(\frac{n-1}{k}\right)\left(\frac{b_i - c}{k}\right)^{n-2}$$

$$(b_i - c)^{n-1} = (\theta_i - b_i)(n-1)(b_i - c)^{n-2}$$

$$b_i - c = (\theta_i - b_i)(n-1)$$

$$(n)(b_i) - c = (\theta_i)(n-1) \Rightarrow b_i = \frac{(\theta_i)(n-1) + c}{n}$$

Step Three: In a BNE, it must be that the payoff-maximizing bid b_i in Step Two equals the guess that $s_i^*(\theta_i) = k\theta_i + c$ for all θ_i. Use this fact to solve for k and c.

$$k\theta_i + c = \frac{\theta_i(n-1)}{n} + \frac{c}{n}$$

$$\Rightarrow c = 0$$

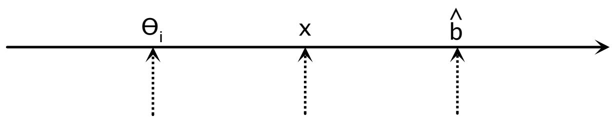
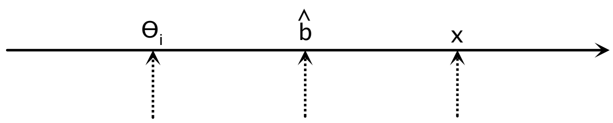
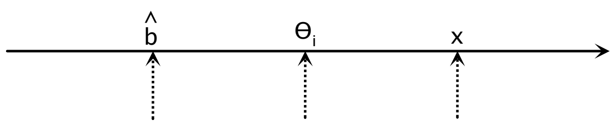
$$k = \frac{n-1}{n}$$

Step Four: Interpret the impact of n on the bidding strategy.

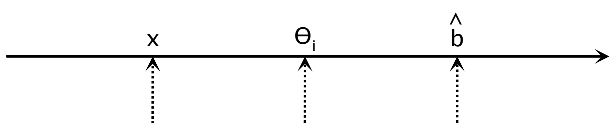
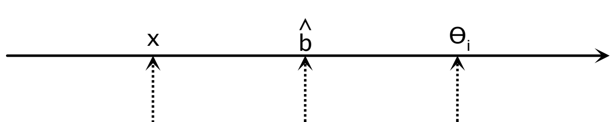
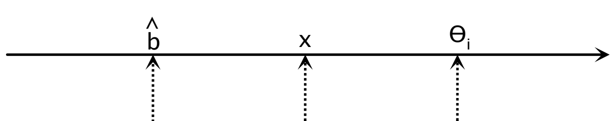
$n \to \infty \Rightarrow$ bidders bid closer to true valuation due to greater competition

2 Second-Price Auction

Consider a player with private value θ_i playing a private value second-price sealed bid auction. Let $\hat{b}$ denote the highest bid of the other players $j \neq i$. Compute the payoff to player i from bidding x vs. θ_i (where $x > \theta_i$) in the following three cases[1]:

Cases	x	θ_i
	0	0
	$\theta_i - \hat{b} < 0$	0
	$\theta_i - \hat{b} > 0$	$\theta_i - \hat{b} > 0$

Bonus: Compute the payoff to player i from bidding x vs. θ_i (where $x < \theta_i$) in the following three cases:

Cases	x	θ_i
	0	0
	0	$\theta_i - \hat{b} > 0$
	$\theta_i - \hat{b} > 0$	$\theta_i - \hat{b} > 0$

[1] I have not described what happens if there is a tie at the highest bid, but in fact, it will not matter what is specified at this contingency, so I will ignore it here.