# Activity: Bargaining with Different Discount Factors

## Econ 305

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Consider the Rubinstein bargaining game of alternating offers in which the players bargain over a pie of size 1. We will specify that the payoffs if $(z, 1-z)$ is accepted at date $t$ are

$$(\delta_1^{t-1} z, \delta_2^{t-1}(1-z))$$

where $0 < \delta_i < 1$ is player $i'$s discount factor.

a. Verify that the following strategy profile is an SPE of the above game:

- Player 1 always proposes $\left(\boxed{\frac{1-\delta_2}{1-\delta_1\delta_2}}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right)$ and accepts a proposal $y$ if and only if $y_1 \geq \frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}$
- Player 2 always proposes $\left(\frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}, \frac{1-\delta_1}{1-\delta_1\delta_2}\right)$ and accepts a proposal $x$ if and only if $x_2 \geq \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}$

*First consider a subgame in which player 1 is the proposer.*

- *Follow:* $v_1 = \frac{1-\delta_2}{1-\delta_1\delta_2}$
- *Best Deviation (offer less to player 2):* $v_1 = \delta_1 \frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}$

↳ deviation ⇒ worse off

*Now consider a subgame in which player 1 is responding to some offer $z$.*

- *Accept:* $v_1 = z$
- *Rejects:* $v_1 = \delta_1 \frac{(1-\delta_2)}{1-\delta_1\delta_2}$

accept if $z \geq \frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}$

*Note that symmetric arguments apply for player 2.*

b. What happens when $\delta_1 \to 1$ for fixed $\delta_2$ and when $\delta_2 \to 1$ for fixed $\delta_1$? Explain why there is a difference in the equilibrium shares of the pie.

$\delta_1 \to 1$ holding fixed $\delta_2$ ⇒ Player 1 takes higher share (reduces to $\sim \frac{1-\delta_2}{1-\delta_2}$)

$\delta_2 \to 1$ holding fixed $\delta_1$ ⇒ Player 2 takes higher share.