I have not shown most of the extraneous work because it is tedious to show.

Solution (12.1, Problem 2)**:** Separating with $u = X(x)Y(y)$, we have

$$Y\frac{dX}{dx} + 3X\frac{dY}{dy} = 0,$$

so that

$$\begin{aligned}\frac{dX}{dx} &= CX\\ \frac{dY}{dy} &= -\frac{C}{3}Y,\end{aligned}$$

meaning

$$u(x,y) = Ke^{Cx-\frac{C}{3}y}.$$

Solution (12.1, Problem 4)**:** Separating by taking $u(x,y) = X(x)Y(y)$, we have

$$\frac{1}{X}\left(\frac{dX}{dx}\right) = \frac{1}{Y}\left(\frac{dY}{dy}\right) + 1.$$

Therefore, this equation splits into

$$\begin{aligned}\frac{dX}{dx} &= CX\\ \frac{dY}{dy} &= (C-1)Y,\end{aligned}$$

yielding the solution of

$$u(x,y) = Ke^{Cx+(C-1)y}.$$

Solution (12.1, Problem 10)**:** Separating with $u(x,t) = X(x)T(t)$, we have

$$kT(t)\frac{d^2X}{dx^2} = X(t)\frac{dT}{dt},$$

so that

$$\frac{k}{X}\left(\frac{d^2X}{dx^2}\right) = \frac{1}{T}\left(\frac{dT}{dt}\right).$$

Setting these quantities equal to C, we have

$$u(x,t) = \begin{cases} e^{Ct}\left(A\cos\left(\sqrt{\frac{-C}{k}}x\right) + B\cos\left(\sqrt{\frac{-C}{k}}x\right)\right) & C < 0\\ e^{Ct}\left(Ae^{\sqrt{\frac{C}{k}}x} + Be^{-\sqrt{\frac{C}{k}}x}\right) & C > 0\\ Ax + B & C = 0.\end{cases}$$

Solution (12.1, Problem 12)**:** Separating with $u(x,t) = X(x)T(t)$, we get

$$\frac{a^2}{X}\left(\frac{d^2X}{dx^2}\right) = \frac{1}{T}\left(\frac{d^2T}{dt^2} + 2k\frac{dT}{dt}\right).$$

Setting equal to C and going through tedious algebra, we have the solution

$$u(x,t) = \begin{cases} \left(a_1 e^{\left(-k+\sqrt{k^2+C}\right)t} + a_2 e^{\left(-k+\sqrt{k^2+C}\right)t}\right)\left(b_1 e^{\frac{\sqrt{C}}{a}x} + b_2 e^{-\frac{\sqrt{C}}{a}x}\right) & c > 0 \\ \left(a_1 e^{\left(-k+\sqrt{k^2+C}\right)t} + a_2 e^{\left(-k+\sqrt{k^2+C}\right)t}\right)(Ax+B) & C = 0 \\ \left(a_1 e^{\left(-k+\sqrt{k^2+C}\right)t} + a_2 e^{\left(-k+\sqrt{k^2+C}\right)t}\right)\left(b_1 \cos\left(\sqrt{\frac{-C}{a}}x\right) + b_2 \sin\left(\sqrt{\frac{-C}{a}}x\right)\right) & -k^2 < C < 0 \\ \left(a_1 e^{-kt} + a_2 t e^{-kt}\right)\left(b_1 \cos\left(\sqrt{\frac{-C}{a}}x\right) + b_2 \sin\left(\sqrt{\frac{-C}{a}}x\right)\right) & C = -k^2 \\ e^{-kt}\left(a_1 \cos\left(\sqrt{\left|k^2+c\right|}x\right) + a_2 \sin\left(\sqrt{\left|k^2+c\right|}x\right)\right)\left(b_1 \cos\left(\sqrt{\frac{-C}{a}}x\right) + b_2 \sin\left(\sqrt{\frac{-C}{a}}x\right)\right) & C < -k^2 \end{cases}$$

Solution (12.1, Problem 18)**:** Since $B = 5$, $A = 3$, and $C = 1$, this is a hyperbolic PDE.

Solution (12.2, Problem 2)**:** The boundary value problem is

$$\begin{aligned} u(x,0) &= 0 \\ u(0,t) &= u_0 \\ u(L,t) &= u_1. \end{aligned}$$

Solution (12.2, Problem 4)**:** The boundary value problem is

$$\begin{aligned} \left.\frac{\partial u}{\partial x}\right|_{(0,t)} &= 0 \\ \left.\frac{\partial u}{\partial x}\right|_{(0,t)} &= 0 \\ u(x,0) &= 100 \\ \left.\frac{\partial u}{\partial t}\right|_{(x,t)} &= -50. \end{aligned}$$

Solution (12.2, Problem 6)**:**

Solution (11.1, Problem 2)**:**

Solution (11.1, Problem 4)**:**

Solution (11.1, Problem 10)**:**

Solution (11.1, Problem 12)**:**

Solution (Extra Problem)**:**