Solution (30.1)**:** (a) This is a legal expression.

(b) This is a legal expression.

(c) This is not a legal expression; we should obtain a dual vector upon acting with B_{ij} on the vector C^i.

(d) This is not a legal expression assuming summation convention; we cannot have a repeated index on the same tensor.

(e) This is not a legal expression assuming summation convention; we cannot have a repeated index on the same tensor.

(f) This is a legal expression.

(g) This is not a legal expression; we should have a $(0, 2)$ tensor in the dual space, A_{ij}, rather than a $(1, 1)$ tensor of the form A_i^j.

(h) This is a legal expression.

Solution (30.3)**:** Using the chain rule, we obtain

$$\begin{aligned}A^{j'}B_{j'} &= \frac{\partial u^{j'}}{\partial u^j}A^j\frac{\partial u^j}{\partial u^{j'}}B_j\\ &= \delta_j^j A^j B_j\\ &= A^j B_j.\end{aligned}$$

Meanwhile,

$$\begin{aligned}A^{j'}B^{j'} &= \frac{\partial u^{j'}}{\partial u^j}A^j\frac{\partial u^{j'}}{\partial u^j}B^j\\ &= \frac{\partial u^{j'}}{\partial u^j}\frac{\partial u^{j'}}{\partial u^j}A^j B^j,\end{aligned}$$

which means $A^{j'}B^{j'}$ is a rank $(2, 0)$ tensor.

Solution (30.5)**:**

Solution (30.6)**:**

Solution (30.16)**:**

Solution (30.20)**:**

Solution (30.21)**:**

Solution (30.22)**:**

Solution (30.28)**:**