These are some definitions and ideas I will be using regularly throughout this presentation.

Definition (Groups). Let A be a set, and let $\star\colon A\times A\to A$ be such that

- $a\star(b\star c)=(a\star b)\star c$;
- there exists $e_A\in A$ such that $e_A\star a=a\star e_A$ for all $a\in A$;
- for each $a\in A$, there is a^{-1} such that $a\star a^{-1}=a^{-1}\star a=e_A$.

Then, we say the pair $(A,\star)$ is a *group*. We abbreviate $a\star b=ab$.

Definition (Subgroups and Quotient Groups). Let G be a group.

- If $H\subseteq G$ is a subset such that for all $a,b\in H$, $ab^{-1}\in H$, then we say H is a *subgroup*.
- If $N\subseteq G$ is a subgroup such that for all $g\in G$ and $h\in N$, $ghg^{-1}\in N$, then we say N is a *normal subgroup*.
- If N is a normal subgroup, we may define the equivalence relation $g\sim_N g'$ if $g^{-1}g'\in N$; the equivalence classes $gN :- [g]_{\sim_N}$ form the *quotient group*, G/N.
- If $H\subseteq G$ is a subgroup, then the *index* of H is the number of cosets, $gH :- \{gh\mid h\in H\}$, written $[G:H]$.

Definition (Group Actions). If G is a group, and X is a set, then $\rho\colon G\times X\to X$ is called an *action* of G onto X if ρ satisfies

- $\rho(e_G,x)=x$;
- $\rho(g,\rho(h,x))=\rho(gh,x)$.

We write $\rho(g,x)=g\cdot x$.

Every group is equipped with a family of canonical actions, $\sigma\colon G\times G\to G$, given by $(a,x)\mapsto ax$, known as *left-multiplication*.

Definition (Algebras, σ-Algebras of Subsets). If X is a set, then a collection $\mathcal{A}=\{A_i\}_{i\in I}\subseteq P(X)$ is known as an *algebra* of subsets of X if

(1) $\varnothing,X\in\mathcal{A}$;

(2) for all $A_i\in\mathcal{A}$, $A_i^c\in\mathcal{A}$;

(3) for all $A_i,A_j\in\mathcal{A}$, $A_i\cup A_j\in\mathcal{A}$.

If condition (3) holds for any countable subcollection $\{A_n\}_{n\geq 1}\subseteq\mathcal{A}$, then we say $\mathcal{A}$ is a *σ-algebra* of subsets.

Definition (Measures). If X is a set and $\mathcal{A}$ is a σ-algebra, then a map $\mu\colon\mathcal{A}\to[0,\infty]$ that satisfies

- $\mu(\varnothing)=0$;
- for disjoint $A,B\in\mathcal{A}$, $\mu(A\sqcup B)=\mu(A)+\mu(B)$,

then we say μ is a *finitely additive measure*. If, for any countable collection of disjoint sets, $\{A_n\}_{n\geq 1}$, we have

$$\mu\left(\bigsqcup_{n\geq 1}A_n\right)=\sum_{n\geq 1}\mu(A_n),$$

then we say μ is a *measure*. If $\mu(X)=1$, then we say μ is a probability measure.