Activity: Grim Trigger SPE

Econ 305

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Consider the stage game, G, between a manager (M) and worker (W):

- Simultaneously, the manager chooses a bonus payment $b \geq 0$ and the worker chooses an effort level $e \geq 0$.
- The stage-game payoffs for the manager and worker are:

$$v_M(b, e) = 4e - b \qquad \text{and} \qquad v_W(b, e) = b - e^2$$

One can confirm that in the stage game G:

- The efficient effort level is $e^* = 2$.
- The unique NE is $b = e = 0$.

Suppose the stage game G is infinitely repeated with discount factor $\delta < 1$ for both players. Assume that the players adopt the following grim trigger strategy: the worker supplies the efficient effort level e^* and the manager pays a bonus $b^* \in (4, 8)$ in every period until someone deviates from (e^*, b^*), in which case both players play the NE of G in every future period. Find the condition(s) on δ such that the grim trigger strategy is a SPE of $G(\infty, \delta)$.

Some deviation from $(2, b^*)$ in the past:

- no player can do better today by deviating due to playing NE

No deviation from $(2, b^*)$ in the past:

- Stay: avg. payoff
  Manager: $8 - b^* > 0$
  Employee: $b^* - 4 > 0$
- Deviate: avg. payoff:
  Manager: $b' = 0 \Rightarrow (1-\delta)(8 + 0 \cdots) = 8(1-\delta) \leq 8 - b^*$

  $8 - 8\delta \leq 8 - b^*$

  $\frac{b^*}{8} \leq \delta$

  Worker: $e = 0$

  $\Rightarrow (1-\delta)(b^* + 0 + \cdots)$

  $= b^* - b^*\delta \leq b^* - 4$

  $\delta \geq \frac{4}{b^*}$

$$\delta \geq \max\left(\frac{4}{b^*}, \frac{b^*}{8}\right)$$

Bonus: Is efficiency most likely to be sustained in an infinitely repeated interaction when the bonus on the equilibrium path, b^*, is low (just above 4), moderate (around 6), or high (just under 8)?

It should be equally likely in all cases, as there is no profitable deviation in any of the actions on the equilibrium path.