## Returns to Scale in Production

Do the following production functions exhibit increasing, constant, or decreasing returns to scale in inputs $K$ and $L$.

- $Y(K,L) = K^{1/2}L^{1/2}$
- $Y(K,L) = K^{1/3}L^{1/2}$
- $Y(K,L) = K + K^{1/3}L^{1/3}$
- $Y(K,L) = K^{1/3}L^{2/3} - \overline{A}$

- $Y(2K,2L) = (2K)^{1/2}(2L)^{1/2} = 2(K^{1/2}L^{1/2}) = 2Y(K,L)$ — **constant returns to scale**
- $Y(2K,2L) = (2K)^{1/3}(2L)^{1/2} < 2Y(K,L)$ — **decreasing returns to scale**
- $Y(2K,2L) = 2K = (2K)^{1/3}(2L)^{1/3} < 2Y(K,L)$ — **decreasing returns to scale**
- $Y(2K,2L) = (2K)^{1/3}(2L)^{2/3} - \overline{A} < 2Y(K,L)$ — **decreasing returns to scale**

## The Black Death

In the middle of the fourteenth century, an epidemic known as the Black Death killed about a third of Europe's population, about 34 million people. While this was an enormous tragedy, the macroeconomic consequences might surprise you: over the next century, wages are estimated to have been higher than before the Black Death.

- Use the production model to explain why wages might have been higher.
- Can you attach a number to your explanation? In the model, how much would wages rise if a third of the population died from disease?

The production model would explain why wages went up because, after a large level of dying, the marginal product of labor, expressed as $MP_L = \frac{2}{3}\frac{Y}{L}$ went up as capital stayed constant, workers "left," and output stayed constant.

In this model, if a third of the population died of disease, then wages would have gone up by 50% (which is equal to $1/(1-1/3)$), assuming output stayed constant.

## A Variant of the Production Model

Suppose the production function at the core of our model is $Y = \overline{A}K^{3/4}L^{1/4}$.

- Create a new version of Table 4.1 from the textbook for the new model. What are the five equations and five unknowns?
- Now solve these equations to get the solution to the model. Put your solutions in the same form as Table 4.2 from the textbook.
- What is the solution for the equilibrium level of output per person?

The Production Model, 5 Equations and 5 Unknowns

| Unknowns | $Y, K, L, r, w$ |
|---|---|
| Production Function | $Y = \overline{A}K^{3/4}L^{1/4}$ |
| Rule for hiring capital | $r = \frac{3}{4}\frac{Y}{K}$ |
| Rule for hiring labor | $w = \frac{1}{4}\frac{Y}{L}$ |
| Demand = Supply for Capital | $K = \overline{K}$ |
| Demand = Supply for Labor | $L = \overline{L}$ |
| Exogenous Variables | $\overline{A}, \overline{K}, \overline{L}$ |

| The Solution to the Production Model | |
|---|---|
| Capital | $K^* = \overline{K}$ |
| Labor | $L^* = \overline{L}$ |
| Rental rate | $r^* = \frac{3}{4}\frac{Y^*}{K^*} = \frac{3}{4}\overline{A}\left(\frac{\overline{L}}{\overline{K}}\right)^{1/4}$ |
| Wage rate | $w^* = \frac{1}{4}\frac{Y^*}{L^*} = \frac{1}{4}\overline{A}\left(\frac{\overline{K}}{\overline{L}}\right)^{3/4}$ |
| Output | $Y^* = \overline{AK}^{3/4}\overline{L}^{1/4}$ |

$$Y^*_{\text{per capita}} = \frac{\overline{AK}^{3/4}\overline{L}^{1/4}}{\overline{L}} = \overline{A}\left(\frac{\overline{K}}{\overline{L}}\right)^{3/4}$$

## Empirical Fit of the Production Model

The table below reports GDP per capita and capital per person in 2017 for ten countries. Fill out the missing data.

| TFP Calculations | In 2011 Dollars | | Relative to the US | | | |
|---|---|---|---|---|---|---|
| Country | Capital per person | GDP Per Capita | Capital per person | GDP Per Capita | Predicted $Y^*$ | Implied TFP |
| **United States** | 175075 | 54087 | 1 | 1 | 1 | 1 |
| **Canada** | 153390 | 42540 | 0.876 | 0.787 | 0.957 | 0.822 |
| **France** | 136004 | 38841 | 0.887 | 0.718 | 0.961 | 0.747 |
| **Hong Kong** | 154766 | 40603 | 1.138 | 0.751 | 1.044 | 0.719 |
| **South Korea** | 142891 | 36521 | 0.923 | 0.675 | 0.974 | 0.693 |
| **Indonesia** | 26620 | 10598 | 0.186 | 0.196 | 0.571 | 0.343 |
| **Argentina** | 31589 | 16469 | 1.187 | 0.304 | 1.059 | 0.287 |
| **Mexico** | 41866 | 17070 | 1.325 | 0.316 | 1.098 | 0.288 |
| **Kenya** | 4179 | 3069 | 0.1 | 0.057 | 0.464 | 0.123 |
| **Ethiopia** | 2938 | 1596 | 0.703 | 0.03 | 0.889 | 0.034 |

| TFP Calculations | Inverse Values (relative to the US) | | |
|---|---|---|---|
| Country | GDP per capita | Predicted Y* | Implied TFP |
| **United States** | 1 | 1 | 1 |
| **Canada** | 1.271 | 1.045 | 1.217 |
| **France** | 1.393 | 1.041 | 1.339 |
| **Hong Kong** | 1.332 | 0.958 | 1.391 |
| **South Korea** | 1.481 | 1.027 | 1.443 |
| **Indonesia** | 5.102 | 1.751 | 2.915 |
| **Argentina** | 3.289 | 0.944 | 3.484 |
| **Mexico** | 3.165 | 0.911 | 3.472 |
| **Kenya** | 17.544 | 2.155 | 8.13 |
| **Ethiopia** | 33.333 | 1.125 | 29.412 |

We find that roughly 20% of the difference between the United States and Kenya is due to differences in capital per person, and about 80% of the difference is due to differences in TFP. (i think i had a lot of errors propagated here)

## True/False

Decide whether the following statement is true, false, or uncertain, and why.

> Differences in the quality of institutions are the sole cause of differences in per capita income levels across countries.

This statement is **false**, because there are other major factors determining differences in per capita income, such as technological development, human capital, natural resources, and geography.