14.6

4:

$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}\\ &= \frac{2\frac{1}{t}}{\frac{1}{t^2}+t}\left(-\frac{1}{t^2}\right) + \frac{2\sqrt{t}}{t+\frac{1}{t^2}}\end{aligned}$$

6:

$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}\\ &= 2e^{1-t^2} - 2\left((2-t^2)e^{1-t^2}\right)\end{aligned}$$

8:

$$\begin{aligned}\frac{\partial z}{\partial u} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}\\ &= \frac{\left(u^3+v^3\right)^2\left(4u^3+4v^2u\right)}{\left(\left(u^2+v^2\right)\left(u^2+v^3\right)\right)^2} + \frac{\left(u^2+v^2\right)^2\left(6u^5+6u^2v^3\right)}{\left(\left(u^2+v^2\right)\left(u^2+v^3\right)\right)^2}\\ \frac{\partial z}{\partial v} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}\\ &= \frac{\left(u^3+v^3\right)^2\left(4v^3+4u^2v\right)}{\left(\left(u^2+v^2\right)\left(u^2+v^3\right)\right)^2} + \frac{\left(u^2+v^2\right)^2\left(6v^5+6v^2u^3\right)}{\left(\left(u^2+v^2\right)\left(u^2+v^3\right)\right)^2}\end{aligned}$$

14:

$$\begin{aligned}\frac{\partial z}{\partial u} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}\\ &= -2\left(\cos(v)\sin(u^2) + \sin(v)\sin(u^2)\right)\\ \frac{\partial z}{\partial v} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}\\ &= 2u\sin(v)\sin(u^2) - 2u\cos(v)\sin(u^2)\end{aligned}$$

16:

$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}\\ &= 3t^{10}(3t^2) + 2t^{11}(2t)\\ &= 13t^{12}\\ z &= t^{13}\\ \frac{dz}{dt} &= 13t^{12}\end{aligned}$$

38: I don't know how to do this problem.

14.7

2:

$$\begin{aligned}f_{xx} &= 2\\ f_{xy} &= 2\\ f_{yx} &= 2\\ f_{yy} &= 2\end{aligned}$$

6:

$$\begin{aligned} f_{xx} &= 0 \\ f_{xy} &= e^y \\ f_{yx} &= e^y \\ f_{yy} &= xe^y \end{aligned}$$

12:

$$\begin{aligned} \ell(x,y) &= -1 + (1)x + (-1)y \\ q(x,y) &= \ell(x,y) + (-1)x^2 + (1)(xy) \\ &= -1 + x - y - x^2 + xy \end{aligned}$$

14:

$$\begin{aligned} \ell(x,y) &= 1 \\ q(x,y) &= 1 - 2x^2 - y^2 \end{aligned}$$

42:

$$\begin{aligned} \ell(x,y) &= 1 + \frac{1}{2}x + y \\ q(x,y) &= \ell(x,y) - \frac{1}{8}x^2 - xy - \frac{1}{4}y^2 \\ f(0.9, 0.2) &= 1.14 \\ \ell(0.9, 0.2) &= 1.65 \\ q(0.9, 0.2) &= 1.36 \end{aligned}$$

44:

$$\begin{aligned} \ell(x,y) &= 1 + x - y \\ q(x,y) &= \ell(x,y) - xy + y^2 \\ f(0.9, 0.2) &= 0.737 \\ \ell(0.9, 0.2) &= 1.7 \\ q(0.9, 0.2) &= 1.56 \end{aligned}$$

48:

$$\begin{aligned} \frac{\partial^2 f}{\partial x^2} &= \frac{2xy}{(x^2+y^2)^2} \\ \frac{\partial^2 f}{\partial y^2} &= \frac{-2xy}{(x^2+y^2)^2} \\ 0 &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \end{aligned}$$

50 (a):

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{e^{-\frac{x^2}{4t}}(x^2 - 2t)}{8t^2\sqrt{\pi t}} \\ \frac{\partial^2 u}{\partial x^2} &= \frac{e^{-\frac{x^2}{4t}}(x^2 - 2t)}{8t^2\sqrt{\pi t}} \end{aligned}$$

52: We can assume that $z_{yy} = 0$, as y is only of degree 1 in z.

15.1

2:
- D: Saddle Point
- B: Local Maximum
- C: Saddle Point
- G: Local Minimum
- F: Saddle Point
- E: Local Minimum

4: Because neither f_x nor f_y are equal to zero, $(1,2)$ cannot be a critical point.

6:

$$\begin{aligned} f_{xx} &= 2 \\ f_{yy} &= \cos y \\ f_{xy} &= 0 \\ D(0,0) &= 4 > 0 \end{aligned}$$

Therefore, the point is a local minimum.

8:

$$\begin{aligned} f_{xx} &= 12x^2 \\ f_{yy} &= 6y \\ D(0,0) &= 0 \end{aligned}$$

The critical point is indeterminate.

10:

$$\begin{aligned} f_{xx} &= 0 \\ f_{yy} &= 0 \\ f_{xy} &= 1 \\ D(0,0) &= -1 < 0 \end{aligned}$$

The critical point is a saddle point.

14:

$$\begin{aligned} \frac{\partial f}{\partial x} &= 2x - 2y \\ \frac{\partial f}{\partial y} &= 6y - 8 - 2x \\ \frac{\partial f}{\partial x} &= 0 \\ y &= 2 \\ \frac{\partial f}{\partial y} &= 0 \\ x &= 2 \\ \frac{\partial^2 f}{\partial x^2} &= 2 \\ \frac{\partial^2 f}{\partial y^2} &= 6 \\ \frac{\partial^2 f}{\partial x \partial y} &= -2 \\ D &= 8 > 0 \end{aligned}$$

Therefore, the point $f(2,2)$ is a local minimum.

20:

$$\begin{aligned}
\frac{\partial f}{\partial x} &= 6x^2 - 6xy + 12x \\
\frac{\partial f}{\partial y} &= -3x^2 - 12y \\
\frac{\partial f}{\partial x} &= 0 \\
\frac{\partial f}{\partial y} &= 0 \\
y &= -\frac{1}{4}x^2 \\
0 &= 6x^2 + \frac{3}{2}x^3 + 12x \\
0 &= \frac{3}{2}x\left(x^2 + 4x + 8\right) \\
(x, y) &= \{(0, 0)\} \\
\left.\frac{\partial^2 f}{\partial x^2}\right|_{(0,0)} &= 12 \\
\left.\frac{\partial^2 f}{\partial y^2}\right|_{(0,0)} &= -12 \\
\frac{\partial^2 f}{\partial x \partial y} &= 0 \\
D(0, 0) &= -144 < 0
\end{aligned}$$

Therefore, the point $f(0, 0)$ is a saddle point

24:

$$\begin{aligned}
\frac{\partial f}{\partial x} &= 2xy + 1 \\
\frac{\partial f}{\partial y} &= 2xy + 1 \\
y &= -\frac{1}{2x} \\
(x, y) &= \left\{\left(t, -\frac{1}{2t}\right) \mid t \neq 0\right\} \\
\frac{\partial^2 f}{\partial x^2} &= 2y \\
\frac{\partial^2 f}{\partial y^2} &= 2x \\
\frac{\partial^2 f}{\partial x \partial y} &= 2y \\
D &= 4xy - 4y^2 \\
&= -2 - \frac{1}{4t^2} \\
&< 0
\end{aligned}$$

Therefore, the points $f(t, -1/2t)$ are saddle points.

28:

$$\begin{aligned}
f_{xx} &= 6x \\
f_{yy} &= 2k \\
f_{xy} &= 9 \\
D &= 12tk - 81
\end{aligned}$$

If $k > 81/12$, then the point is a local minimum, and if $k < 81/12$, then the point is a saddle. It is not possible for the point to be a local maximum.

32: (a)

$$\begin{aligned}
\frac{\partial f}{\partial x} &= -2(x-a)e^{-(x-a)^2-(y-b)^2} \\
\frac{\partial f}{\partial y} &= -2(y-b)e^{-(x-a)^2-(y-b)^2} \\
\frac{\partial^2 f}{\partial x^2} &= 2\left(2x^2-4ax+(2a^2-1)\right)e^{-(x-a)^2-(y-b)^2} \\
\frac{\partial^2 f}{\partial y^2} &= 2\left(2y^2-4by+(2b^2-1)\right)e^{-(x-a)^2-(y-b)^2} \\
\frac{\partial^2 f}{\partial x \partial y} &= -4(x-a)(y-b)e^{-(x-a)^2-(y-b)^2} \\
D(a,b) &> 0
\end{aligned}$$

Therefore, the critical point is at (a,b).

(b) $a=-1$, $b=5$

(c) The point at (a,b) is a local maximum.

34: (a) Local Maximum.

(b) Saddle Point

(c) Local Minimum

(d) None.

38:

42: