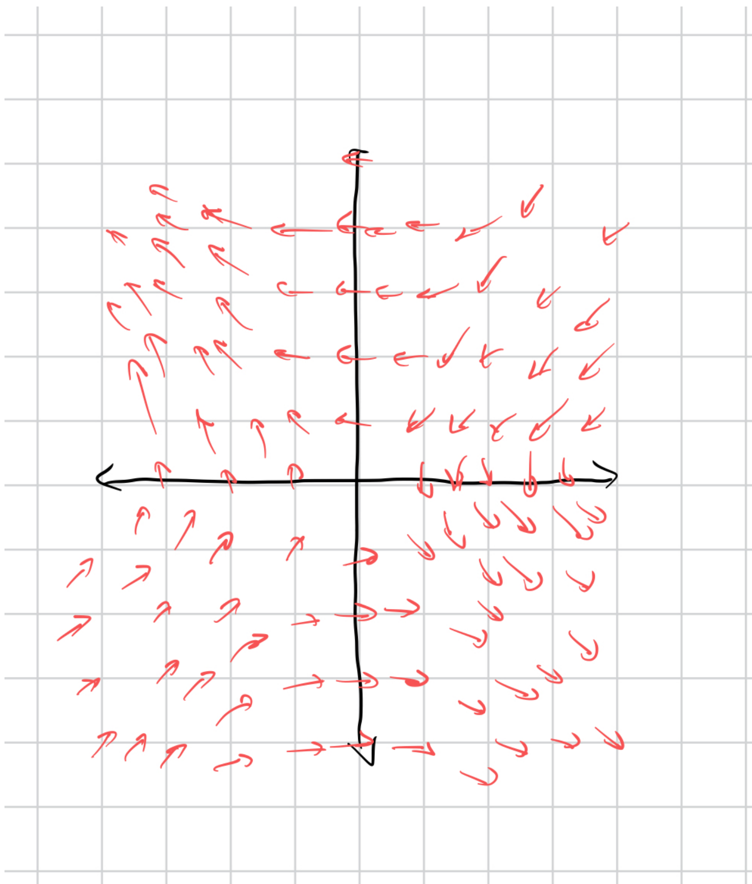
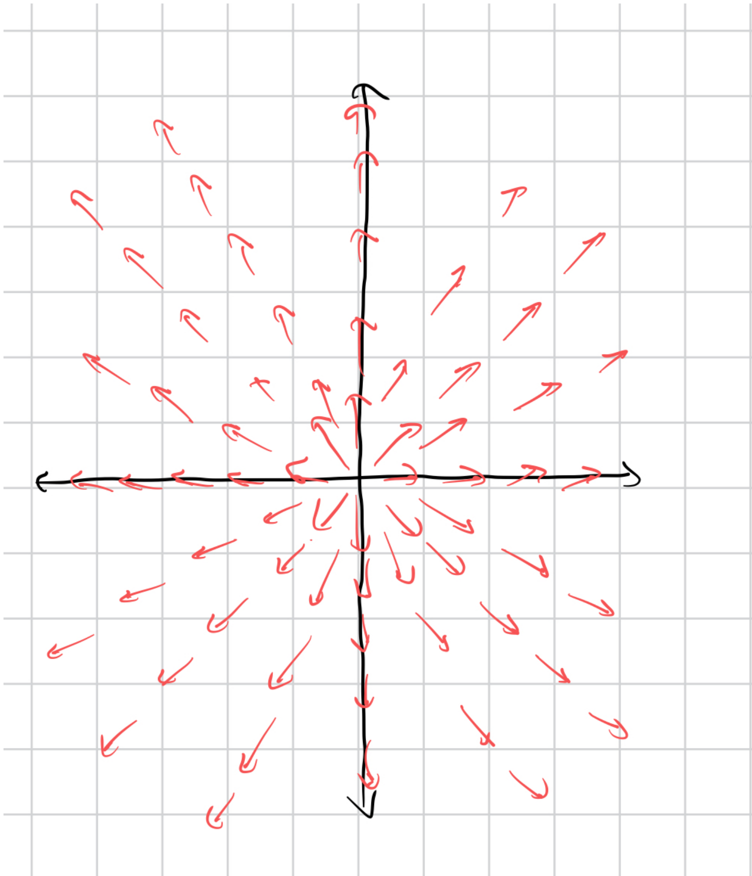
# Chapter 11 Problems

## Problem 1

(a) $\mathbf{F}(\mathbf{x}) = \frac{1}{\rho}\hat{\rho}$.



(b) $\mathbf{F}(\mathbf{x}) = y\hat{\imath} + x\hat{\jmath}$.



## Problem 2

The parametrized streamlines for $\mathbf{v} = (-y, x)$ are of the form $r\cos t\hat{\imath} + r\sin t\hat{\jmath}$.

## Problem 3

We can see that $\mathbf{E}$ and $\mathbf{B}$ are mutually perpendicular by taking the standard inner product

$$\left\langle xy^2\hat{\imath} + x^2y\hat{\jmath}, x^2y\hat{\imath} - xy^2\hat{\jmath}\right\rangle = 0.$$

Additionally, for $\mathbf{E}$,

$$\begin{aligned}\frac{dy}{dt} &= x^2y\\ \frac{dx}{dt} &= xy^2\\ \frac{dy}{dx} &= \frac{x}{y}\\ y^2 &= x^2 + K,\end{aligned}$$

and for $\mathbf{B}$,

$$\frac{dy}{dt} = -xy^2$$

$$\begin{aligned}\frac{dx}{dt} &= x^2 y\\ \frac{dy}{dx} &= -\frac{y}{x}\\ y &= \frac{K}{x}.\end{aligned}$$

## Problem 4

(a)

$$\int_V \mathbf{E}(\mathbf{r})\ d^3x = \int_0^{\pi/2}\int_0^{2\pi}\int_0^R \hat{r}\sin\theta\ drd\phi d\theta$$

$$\int_V \mathbf{E}(\mathbf{r})\ d^3x = \int_{-R}^{R}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\int_0^{\sqrt{R^2-x^2-y^2}} \frac{x\hat{\imath}+y\hat{\jmath}+z\hat{k}}{(x^2+y^2+z^2)^{3/2}}\ dzdydx$$

(b)

$$\int_V \mathbf{E}(\mathbf{r})\ d^3x = \int_0^{\pi/2}\int_0^{2\pi}\int_0^R \sin\theta\left(\cos\phi\sin\theta\hat{\imath}+\sin\phi\sin\theta\hat{\jmath}+\cos\theta\hat{k}\right)\ drd\phi d\theta$$

This integral is more practical than the pure forms since the basis is position-independent and the integral is not a giant mess.

(c) Using symmetry, since $\cos\phi$ is integrated from 0 to $2\pi$ and $\sin\phi$ is integrated from 0 to $2\pi$, both the $\hat{\imath}$ and $\hat{\jmath}$ components are 0.

$$\int_0^{\pi/2}\sin^2\theta\int_0^{2\pi}\cos\phi\int_0^R\ drd\phi d\phi = 0$$

$$\int_0^{\pi/2}\sin^2\theta\int_0^{2\pi}\sin\phi\int_0^R\ drd\phi d\phi = 0$$

(d) Evaluating the $\hat{k}$ component,

$$\begin{aligned}\int_0^{\pi/2}\sin\theta\cos\theta\int_0^{2\pi}\int_0^R\ drd\phi d\theta &= 2\pi R\int_0^{\pi/2}\sin\theta\cos\theta\ d\theta\\ &= \pi R.\end{aligned}$$

## Problem 5

$$\begin{aligned}\mathbf{R}_{\text{cm}} &= \frac{1}{M}\int_S \mathbf{r}\,dm\\ &= \frac{\sigma}{M}\int_{-\ell/2}^{\ell/2}\int_0^{\pi}\left(R\cos\phi\hat{\imath}+R\sin\phi\hat{\jmath}+z\hat{k}\right)R\ d\phi dz\\ &= \frac{\sigma}{M}\left(2R^2\right)\hat{\jmath}.\end{aligned}$$

# Chapter 12 Problems

## Problem 1

(a) Letting $f(\mathbf{x}) = \rho$, we have

$$\nabla f = \hat{\rho}$$

in cylindrical coordinates, and

$$\nabla f = \frac{x}{\sqrt{x^2 + y^2}}\hat{\imath} + \frac{y}{\sqrt{x^2 + y^2}}\hat{\jmath}$$

in Cartesian coordinates. These results are equal to each other by the definition of $\hat{\rho}$.

(b) Letting $f(\mathbf{x}) = y$, we have

$$\nabla f = \hat{\jmath}$$

in Cartesian coordinates, and

$$\nabla f = \sin\phi\hat{\rho} + \cos\phi\hat{\phi},$$

which yields $\hat{\jmath}$ under the coordinate conversion.

(c) Letting $f(\mathbf{x}) = z\rho^2$, we have

$$\nabla f = 2\rho z\hat{\rho} + \rho^2\hat{k}$$

in cylindrical coordinates, and

$$\nabla f = 2xz\hat{\imath} + 2yz\hat{\jmath} + \left(x^2 + y^2\right)\hat{k},$$

which is equal under the coordinate conversion.

(d) Letting $f(\mathbf{x}) = \rho^2\tan\phi$, we have

$$\nabla f = 2\rho\tan\phi\hat{\rho} + \rho\sec^2\phi\hat{\phi}$$

and

$$\nabla f = \left(y - \frac{y^3}{x^2}\right)\hat{\imath} + \left(x + \frac{3y^2}{x}\right)\hat{\jmath},$$

which is equal under the coordinate conversion.

### Problem 2

### Problem 3

### Problem 6

### Problem 7

### Problem 9

### Problem 15

### Problem 19

## Chapter 13 Problems

### Problem 2