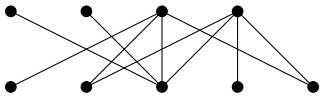
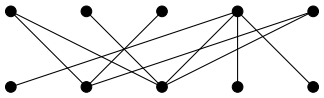
3.1.1

Find a maximum matching in each graph below. Prove that it is a maximum matching by exhibiting an optimal solution to the dual problem (minimum vertex cover). Explain why this proves that the matching is optimal.

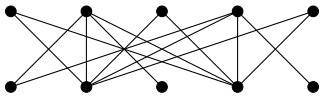
Graph 1:



Graph 2:



Graph 3:



3.1.2

Determine the minimum size of the maximal matching in C_n.

3.1.5

Prove that $\alpha(G) \geq \frac{n(G)}{\Delta(G)+1}$ for every graph G.

3.1.9

Prove that every maximal matching in a graph G has at least $\alpha'(G)/2$ edges.

3.1.28

Exhibit a perfect matching in the graph below or give a short proof that it has none.