

MACROECONOMICS

PROBLEM SET 2: MODEL OF PRODUCTION

Due by the start of class on Fri Feb 3

Instructions: For all questions, be sure to explain your answers and to use graphs whenever appropriate. You should work on each question first, without the aid of the solutions (on the honor system). Once you have either finished solving the question or have gotten as far into solving it as possible, in a different color pen or pencil, you should correct your own work, or finish solving the problem, with the aid of the solutions. If your answer was correct, indicate that you still checked your answer against the solutions by placing a “check mark” in a different color pen or pencil. Please staple your answers before submitting.

1. *Returns to Scale in Production.* Do the following production functions exhibit increasing, constant, or decreasing returns to scale in inputs, K and L? Show mathematically. Assume that \bar{A} is some fixed positive number. Hint 1: Recall that a production function displays increasing returns to scale if doubling inputs more than doubles output, decreasing returns to scale if doubling inputs less than doubles output, and constant returns to scale if doubling inputs exactly doubles output. Hint 2: For each function, please show what happens to output when inputs, K and L, are doubled, i.e. $Y(2K, 2L)$. Then, compare this amount to doubling output, i.e. $2Y(K, L)$, to determine whether the function displays increasing, decreasing, or constant returns to scale.

- a. $Y(K, L) = K^{1/2}L^{1/2}$
- b. $Y(K, L) = K^{1/3}L^{1/2}$
- c. $Y(K, L) = K + K^{1/3}L^{1/3}$
- d. $Y(K, L) = K^{1/3}L^{2/3} - \bar{A}$

2. *The Black Death.* In the middle of the fourteenth century, an epidemic known as the Black Death killed about a third of Europe’s population, about 34 million people. While this was an enormous tragedy, the macroeconomic consequences might surprise you: over the next century, wages are estimated to have been higher than before the Black Death.

- a. Use the production model to explain why wages might have been higher.
- b. Can you attach a number to your explanation? In the model, by how much would wages rise if a third of the population died from disease?

3. *A Variant of the Production Model.* Suppose the production function at the core of our model is given by $Y = \bar{A}K^{3/4}L^{1/4}$ (that is, assume the exponents on capital and labor are 3/4 and 1/4, rather than 1/3 and 2/3).

- a. Create a new version of Table 4.1 (from the textbook) for the new version of the model. What are the five equations and five unknowns?
- b. Now solve these equations to get the solution to the model. Put your solution in the same form as Table 4.2 (from the textbook).
- c. What is the solution for the equilibrium level of output per person?

4. *The Empirical Fit of the Production Model.* The table below reports per capita GDP and capital per person in the year 2017 for ten countries. Your task, in parts a-c, is to fill in the missing columns.

I have uploaded the table below as an Excel spreadsheet to Moodle under the category “Problem Sets.” You may use the Excel spreadsheet for your calculations. If you prefer, you may do the calculations with a calculator. You need not show your work for all of the calculations. Instead, showing the calculations for one representative country, and then filling in the calculations in the table for the other countries, will suffice.

- Given the values in columns 1 and 2, fill in columns 3 and 4. That is, compute per capita GDP and capital per person relative to the U.S. values.
- In column 5, use the production model (with a capital exponent of 1/3) to compute predicted per capita GDP for each country relative to the United States, assuming no TFP differences.
- In column 6, compute the level of TFP for each country that is needed to match up the model and the data. Comment on the general results you find.

	In 2011 dollars		Relative to the U.S. values (U.S. = 1)			
	(1)	(2)	(3)	(4)	(5)	
Country	Capital per person	Per capita GDP	Capital per person	Per capita GDP	Predicted y^*	Implied TFP to match data
United States	175,075	54,807				
Canada	153,390	42,540				
France	136,004	38,841				
Hong Kong	154,766	40,603				
South Korea	142,891	36,521				
Indonesia	26,620	10,598				
Argentina	31,589	16,469				
Mexico	41,866	17,070				
Kenya	4,179	3,069				
Ethiopia	2,938	1,596				

- Now, create a new table that contains only the last three columns of the previous table, except this time, instead of reporting the numbers relative to the U.S. value, report the inverse of those numbers. In other words, in the new table, the three columns should report the inverse of columns (4), (5), and (6) from the previous table. (For example, you should have found that per capita GDP in Kenya relative to that in the United States was 0.055996 (or, if rounded to three decimal points, 0.056). Now express this number as the ratio of U.S. to Kenya per capita GDP, i.e. the inverse: $1/0.055996 = 17.858$. Fill in all three columns for the remaining countries.)

Note: If you have done the calculations in Excel, try to arrange your printer settings so that everything is on one page (but only if it is easy for you to do so). On my apple computer, I simply click “Scaling: Fit.”

- The interpretation of the three columns in this new table are as follows:
 - In column 1, we say that the U.S. is X times richer than that particular country.
 - In column 2, we say that capital differences alone make the United States Y times richer than that particular country.
 - In column 3, we say that TFP differences alone make the United States Z times richer than that particular country.

Explain intuitively why the three columns in the new table reflect these interpretations.

- f. In class, we found that about one-fourth of the difference in per capita GDP across countries were due to differences in capital per person and about three-fourths were due to differences in TFP. Carry out this calculation for Kenya. The United States is roughly 18 times richer than Kenya; what fraction is due to differences in capital per person and what fraction is due to differences in TFP?

5. *True/False*. Decide whether the following statement is true, false, or uncertain and explain why. Your explanation is the important part of the answer.

Statement to evaluate: Differences in the quality of institutions are the sole cause of differences in per capita income levels across countries.

Hint: The article, “What Makes Countries Rich or Poor? A Review of *Why Nations Fail*” by Jared Diamond, assigned for class (see Moodle), will be helpful.

Note: Because this question is about a reading, I will not provide solutions to this question until after you turn in the problem set. Please answer this question on your own.