Solution (4.4, Problem 2)**:** By the method of inspection, we get the general solution of

$$y(x) = c_1 \cos\left(\frac{3}{2}x\right) + c_2 \sin\left(\frac{3}{2}x\right) + \frac{5}{3}.$$

Solution (4.4, Problem 4)**:** By the method of inspection (basically just undetermined coefficients without actually going through the full steps), we get the general solution of

$$y(x) = c_1 e^{3x} + c_2 e^{-2x} - \frac{1}{3}x + \frac{1}{8}.$$

Solution (4.4, Problem 12)**:** Using the power of inspection, we have the homogeneous solution of $k_1 e^{4x} + k_2 e^{-4x}$. For the particular solution, we guess

$$y_p(x) = (ax + b)e^{4x},$$

and use the method of computation through Sage's `desolve` command to obtain the general solution of

$$y(x) = k_1 e^{4x} + k_2 e^{-4x} + \frac{1}{4} x e^{4x}$$

This can be independently verified by using undetermined coefficients on $y_p(x) = (ax + b)e^{4x}$, giving

$$\begin{aligned} y'' - 16y &= 8ae^{4x} \\ &= 2e^{4x}. \end{aligned}$$

Solution (4.6, Problem 2)**:** We guess $y_p(x) = u_1 y_1(x) + u_2 y_2(x)$, and use the variation of parameters derivation to find

$$\begin{pmatrix} u_1' \\ u_2' \end{pmatrix} = \begin{pmatrix} -\frac{\sin^2 x}{\cos(x)\cos(2x)} \\ \frac{\sin(x)}{\cos(2x)} \end{pmatrix}.$$

After many tedious, error-prone calculations that are better performed in Mathematica, we get the particular solution of

$$y_p(x) = -\frac{1}{2}\cos(x)\ln(\sin(x) + 1) + \frac{1}{2}\cos(x)\ln(\sin(x) - 1).$$

This adds to the homogeneous solution of $k_1 \cos(x) + k_2 \sin(x)$.

Solution (4.6, Problem 8)**:** We find homogeneous solutions of $y_1 = e^x$ and $y_2 = e^x$. We use variation of parameters to find

$$y_p(x) = \frac{1}{3}\sinh(2x),$$

giving the general solution of

$$y(x) = k_1 e^x + k_2 e^{-x} + \frac{1}{3}\sinh(2x).$$

Solution (4.6, Problem 14)**:**

Solution (4.6, Problem 31)**:**

Solution (4.7, Problem 4)**:**

Solution (4.7, Problem 10)**:**

Solution (4.7, Problem 12)**:**

Solution (4.7, Problem 14)**:**

Solution (4.7, Problem 16)**:**

Solution (4.7, Problem 18)**:**

Solution (4.7, Problem 32)**:**