## Introduction to Game Theory

Game Theory analyzes the *interaction* among a *group* of *rational* agents who *behave strategically*.

- A group consists of at least two individuals who are free to make decisions.
- An interaction means that the decisions of at least one member of the group must affect at least one other member of the group.
- In strategic behavior, members of the group account for the interaction in their decision making process.
- Rational agents act in their best decisions based on their knowledge.

Keynes's Beauty Contest: Choose the face that is the most chosen in a newspaper contest.

In many games, we are not asked to pick *our* favorite, we are asked to pick *everyone else's* favorite.

### Applications of Game Theory

- Labor Economics (compensation interactions, promotions)
- Industrial Organization (pricing, entry, exit, etc.)
- Public Finance (public goods games)
- Political Economy (strategic voting)
- Trade (tariff wars)
- Biology (hunting and mating)
- Linguistics

It's important to remember that game theory is a subfield of *mathematics*, not economics.

## Static Games of Complete Information

We will begin by covering *static games of complete information*.

- Static: Play happens at once and payoffs are realized. Decisions are not necessarily made at the same time.
- Complete information: the following four are all common knowledge in the game
  - (i) all possible actions of the players
  - (ii) all possible outcomes
  - (iii) how each combination of actions of all players affects which outcome will materialize
  - (iv) the preferences of each and every player over outcomes
- An event, $E$, is common knowledge if everyone knows $E$, everyone knows everyone knows $E$, *ad infinitum*.

### The Prisoner's Dilemma

- Two suspects are interrogated in separate rooms.
- There is enough evidence to convict each of them for a minor offense, but not enough to convict either of a major crime unless one finks (F).
- If they each stay quiet (Q), they only get 1 year in prison each.
- If only one finks, they are free, and the other gets 4 years in prison.
- If they both fink, they each will spend 3 years in prison.

We will try to write The Prisoner's Dilemma as a game. First, we can see this in a payoff matrix.

| | | Player Y: Q | Player Y: F |
|---|---|---|---|
| Player X | Q | $(2,2)$ | $(0,3)$ |
| | F | $(3,0)$ | $(1,1)$ |

### Normal-Form Game

The constituents of a *normal-form game* $G$ consist of the following:

- A finite set of players: $N = \{1, 2, \dots, n\}$.
- For each player $i$, a set $S_i$ denotes the *strategy space* of player $i$. We will let $S = S_1 \times S_2 \times \cdots S_n$ denote the strategy space of the entire game (i.e., the entire set of strategies possible).
  - Every element $s \in S$ is a *strategy profile*, where $s = (s_1, s_2, \dots, s_n)$.
  - We denote the strategy choices of all players except player $i$ as $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n)$.
- A payoff function: $v_i : S \to \mathbb{R}$. The payoff function depends on the strategies of *all players*.

#### Example

Let the following payoff matrix represent a game. Write the normal form.

| | X | Y |
|---|---|---|
| A | $(5,1)$ | $(2,6)$ |
| B | $(0,9)$ | $(3,2)$ |
| C | $(4,4)$ | $(4,7)$ |

- $n = 2$
- $S_1 = \{A, B, C\}$
  $S_2 = \{X, Y\}$

## Strategic Dominance

Recall the prisoner's dilemma.

| | | Player Y | |
|---|---|---|---|
| | | Q | F |
| Player X | Q | $(2,2)$ | $(0,3)$ |
| | F | $(3,0)$ | $(1,1)$ |

Suppose you were player 1. If player 2 stays quiet, it is more optimal for you to fink than to stay quiet. Similarly, if player 2 finks, then it is more optimal for you to fink than to stay quiet.

In a similar vein, for player 2, it is more optimal to fink in both cases. Therefore, the proper strategy is $(F, F)$.

### Dominated Strategy

A strategy $s_i'$ if *strictly dominated* for $i$ if there is one other strategy $s_i \in S_i$ such that $v_i(s_i, s_{-i}) > v_i(s_i', s_{-i})$ for all $s_{-i} \in S_{-i}$.

Essentially, a strategy is strictly dominated if there is another strategy that yields a strictly greater payoff regardless of the other strategies.

A rational player will *never* play a strictly dominated strategy.

In the prisoner's dilemma, $Q$ is strictly dominated by $F$ in both cases. Oddly, this yields the worst outcome from a social perspective (i.e., it has the lowest aggregate welfare).

| | L | M | R |
|---|---|---|---|
| T | 2,2 | 1,1 | 4,0 |
| B | 1,2 | 4,1 | 3,5 |

Through *iterated elimination of strictly dominated strategies*, we start by removing $M$ from the strategy profile of player 2 as playing $L$ is strictly better. Then, Player 1 realizes that player 2 is rational, and thus does not play $B$ (as $B$ is strictly dominated by $T$ once $M$ is removed from the strategy space of player 2). Finally, Player 2 does not play $R$, as $R$ is strictly dominated by $L$ given that player 1 will play $T$. Thus, we get our answer of **T**, **L**.

A game is *dominance solvable* if it can be solved via iterated elimination of strictly dominated strategies. However, only a small number of games are not dominance solvable.

Strategic Dominance and Normal-Form Activity

# Activity: Strategic Games and Dominance
Econ 305

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## 1 Strategic Games

For each of the games described below, determine the normal form of the game: number of players $n$, strategy space for each player $S_i$, and payoffs (as a matrix or function).

a. Matching Pennies (a zero-sum game). Two players simultaneously place a penny on a table. If the pennies match (e.g., both placed heads up), player 2 pays player 1 a dollar. If the pennies do not match, player 1 pays player 2 a dollar.

Players: $N = \{1,2\}$

Strategy Space: $S_i = \{H, T\}$

Payoff Functions: $u_1 = \begin{cases} 1, & s_1 = s_2 \\ -1, & s_1 \neq s_2 \end{cases}$

$u_2 = \begin{cases} 1, & s_1 \neq s_2 \\ -1, & s_1 = s_2 \end{cases}$

| | | Player 2 | |
|---|---|---|---|
| | | H | T |
| Player 1 | H | 1,-1 | -1,1 |
| | T | -1,1 | 1,-1 |

b. Bach or Stravinsky / Battle of the Sexes (a coordination game with some conflict). A couple wants to be together on their date night rather than alone, but they have different preferences over which type of concert they attend. They simultaneously — and without communication — choose to go to either the Bach or Stravinsky concert. Conditional on being together, player 1 prefers Bach and player 2 prefers Stravinsky.

Players: $N = \{1,2\}$

Strategy Sets: $S_i = \{B, S\}$

Payoffs:

| | | Player 2 | |
|---|---|---|---|
| | | B | S |
| Player 1 | B | 5,3 | 2,2 |
| | S | 0,0 | 3,5 |

c. Hawk vs. Dove / Chicken (an anti-coordination game). Two teenagers ride their bikes at high speed towards each other along a narrow ride. Neither of them wants to "chicken out" and lose their pride, but even worse is getting hurt by crashing into the oncoming biker.

Players: $n=2$

Strategy Space: $\{D, H\}$

Payoffs:

| | | Player 2 | |
|---|---|---|---|
| | | D | H |
| Player 1 | D | 1,1 | 1,2 |
| | H | 2,1 | 0,0 |

d. Cournot Competition (an industrial organization game). Two firms compete by simultaneously choosing how much to produce of a homogenous good (e.g., oil, soybeans) for a market.

Players: $n=2$

Strategy Space: $S_i = [0, \infty)$

$P = d^{-1}(q_1 + q_2)$

Payoff: $\pi_i = d^{-1}(q_1 + q_2)\, q_i - C_i(q_i)$

## 2 Strict Dominance

Are the following games dominance solvable? Justify your answers.

a. A 4 × 4 game:

Yes, (B, X) is the result from IESDS on this game

| | $W$ | $X$ | $Y$ | $Z$ |
|---|---|---|---|---|
| $A$ | 5,2 | 2,6 | 1,4 | 0,4 |
| $B$ | 0,0 | 3,2 | 2,1 | 1,1 |
| $C$ | 7,0 | 2,2 | 1,5 | 5,1 |
| $D$ | 9,5 | 1,3 | 0,2 | 4,8 |

b. The beauty contest game, i.e., to win, come closest to guessing two-thirds the average of numbers between 0 and 100 selected by players.

Yes, every strategy is strictly dominated by 0 the game is dominance solvable