Activity: BNE of Social Unrest Game

Econon 305

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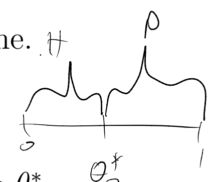
Consider a game of social unrest with incomplete information. Two people (players 1 and 2) have to simultaneously choose whether to protest (P) or stay home (H). A player who stays home gets a payoff of 0. Player i's payoff of protesting is determined by this player's protest value θ_i and whether the other player also protests. Each player knows her own protest value, but does not observe that of the other player. Assume that θ_1 and θ_2 are independently drawn from the uniform distribution on $[0, 1]$. The payoff matrix is:

	P ($\theta_2 \geq \theta_2^*$)	H ($\theta_2 < \theta_2^*$)
P	$\theta_1 - 1/3, \theta_2 - 1/3$	$\theta_1 - 2/3, 0$
H	$0, \theta_2 - 2/3$	$0, 0$

Find a pure-strategy Bayesian Nash equilibrium of this game.



First, guess that strategies take the following form:

$$s_i^*(\theta_i) = \begin{cases} \underline{\quad P \quad} & \text{if } \theta_i \geq \theta_i^* \\ \underline{\quad H \quad} & \text{if } \theta_i < \theta_i^* \end{cases}$$

Second, observe that when $\theta_i = \theta_i^*$, player i is indifferent between protesting and staying home. Use the indifference conditions to solve for the cutoffs θ_1^* and θ_2^*.

$$E\left(v\left(P, s^*(\theta_{-i}); \theta_i^*, \theta_{-i}\right)\right) = E\left(v_i\left(H, s_{-i}^*(\theta_{-i}); \theta_i^*, \theta_{-i}\right)\right)$$

$$\overbrace{\left(\theta_i^* - \tfrac{2}{3}\right) P(\theta_{-i} < \theta_{-i}^*)}^{\text{If } (-i) \text{ stays home}} + \underbrace{\left(\theta_i^* - \tfrac{1}{3}\right) \cdot P(\theta_{-i} \geq \theta_{-i}^*)}_{\text{if } (-i) \text{ protests}} = 0$$

$$\left(\theta_i^* - \tfrac{2}{3}\right)\left(\theta_{-i}^*\right) + \left(\theta_i^* - \tfrac{1}{3}\right)\left(1 - \theta_{-i}^*\right) = 0$$

$$\theta_i^* - \tfrac{1}{3} = \tfrac{1}{3}\theta_{-i}^*$$

Symmetric →

$$\theta_i^* - \tfrac{1}{3} = \tfrac{1}{3}\theta_i^*$$

$$\tfrac{2}{3}\theta_i^* = \tfrac{1}{3} \rightarrow \theta_i^* = \tfrac{1}{2}$$