

**Solution (32.20):** We start by taking the recurrence relation

$$(1 - x^2)P'_n = -n x P_n + n P_{n-1}.$$

Using the relation

$$(2n + 1)xP_n = (n + 1)P_{n+1} + nP_{n-1},$$

we get

$$(1 - x^2)P'_n = (n + 1)xP_n - (n + 1)P_{n+1}.$$

Differentiating, we then get

$$(1 - x^2)P''_n - 2xP'_n = (n + 1)P_n + (n + 1)xP'_n - (n + 1)P'_{n+1}$$

We want to evaluate

$$(n + 1)P_n + (n + 1)xP'_n - (n + 1)P'_{n+1} = -n(n + 1)P_n.$$

by using the generating function

$$P_n = \frac{1}{n!} \frac{\partial^n}{\partial t^n} \left( (1 - 2xt + t^2)^{-1/2} \right) \Big|_{t=0}$$

**Solution (32.21):**

**Solution (32.23):**

**Solution (35.4):**

**Solution (35.5):** Differentiating,

$$\begin{aligned} \frac{dJ_0}{dx} &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\partial}{\partial x} \left( e^{ix \sin(\gamma)} \right) d\gamma \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} (i \sin(\gamma)) e^{ix \sin(\gamma)} d\gamma \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} i \left( \frac{1}{2i} (e^{i\gamma} - e^{-i\gamma}) \right) d\gamma \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{2} e^{ix \sin(\gamma) + i\gamma} - \frac{1}{2} e^{ix \sin(\gamma) - i\gamma} d\gamma \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{2} (\cos(x \sin(\gamma) + i\gamma) + i \sin(x \sin(\gamma) + i\gamma) - (\cos(x \sin(\gamma) - i\gamma) + i \sin(x \sin(\gamma) - i\gamma))) d\gamma \end{aligned}$$

and with more tedious algebra, we obtain

$$\begin{aligned} &= -\frac{1}{\pi} \int_0^{\pi} \cos(x \sin(\gamma) - \gamma) d\gamma \\ &= -J_1(x). \end{aligned}$$

Evaluating

$$\frac{d}{dx}(xJ_1) = J_1 + x \frac{dJ_1}{dx},$$

we take

$$\begin{aligned} \frac{d}{dx}(xJ_1) &= \frac{1}{\pi} \int_0^{\pi} \cos(x \sin(\gamma) - \gamma) - x \sin(\gamma) \sin(x \sin(\gamma) - \gamma) d\gamma \\ &= \frac{1}{\pi} \int \cos(x \sin(\gamma)) \cos(\gamma) + \sin(x \sin(\gamma)) \sin(\gamma) - x \sin(\gamma) \sin(x \sin(\gamma) - \gamma) d\gamma \end{aligned}$$

| **Solution (35.7):**

| **Solution (35.8):**

| **Solution (35.10):**

| **Solution (35.11):**

| **Solution (35.12):**

| **Solution (35.16):**

| **Solution (35.17 (c)):**

| **Solution (35.21):**

| **Solution (35.25):**