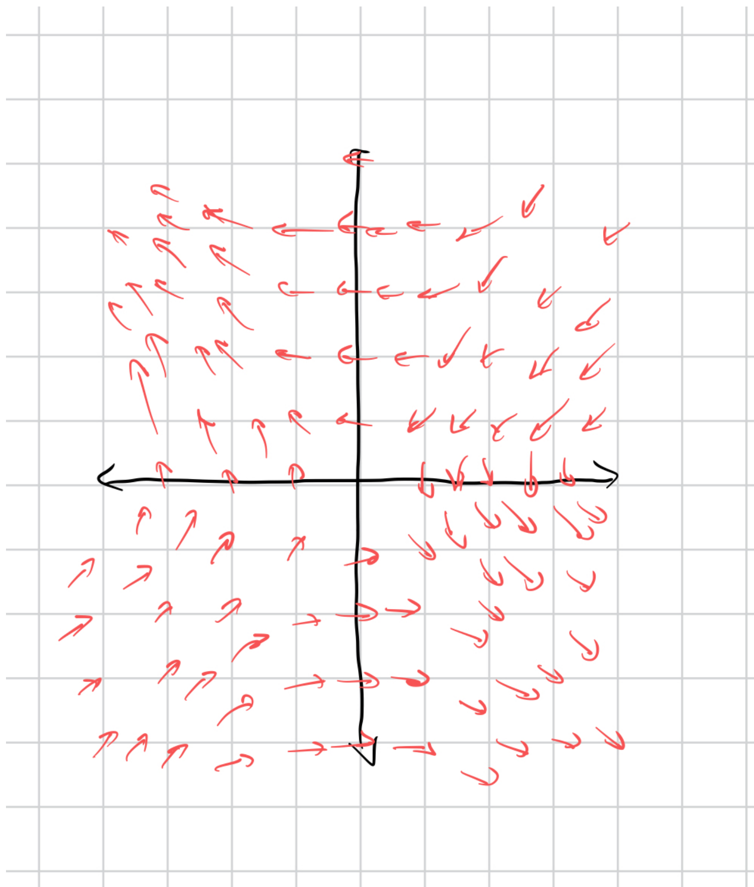
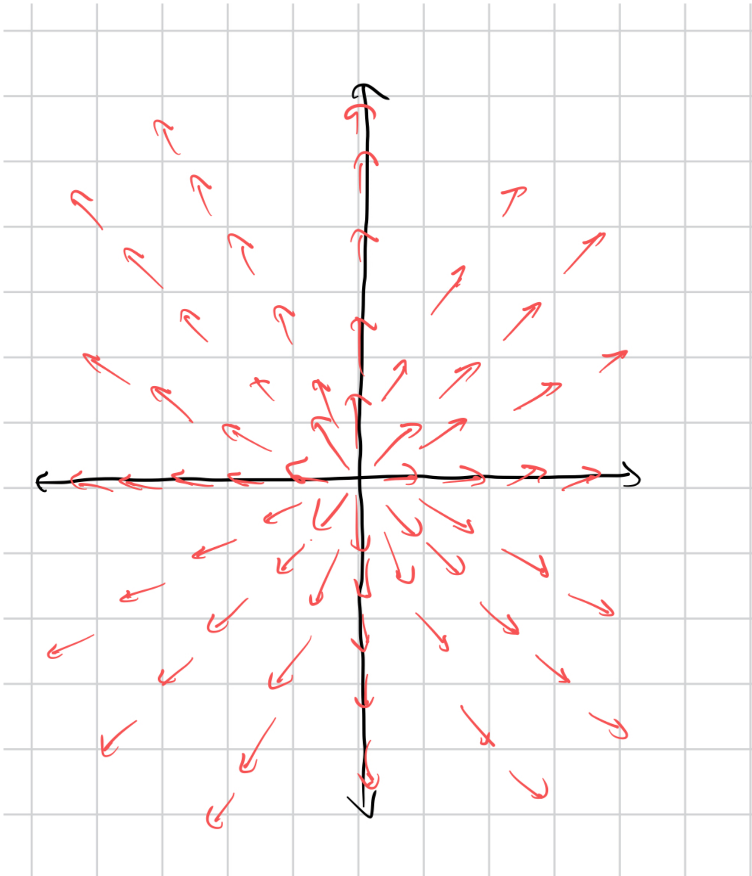
Chapter 11 Problems

Problem 1

(a) $\mathbf{F}(\mathbf{x}) = \frac{1}{\rho}\hat{\rho}$.



(b) $\mathbf{F}(\mathbf{x}) = y\hat{\imath} + x\hat{\jmath}$.



Problem 2

The parametrized streamlines for $\mathbf{v} = (-y, x)$ are of the form $r\cos t\hat{\imath} + r\sin t\hat{\jmath}$.

Problem 3

We can see that $\mathbf{E}$ and $\mathbf{B}$ are mutually perpendicular by taking the standard inner product

$$\left\langle xy^2\hat{\imath} + x^2y\hat{\jmath}, x^2y\hat{\imath} - xy^2\hat{\jmath}\right\rangle = 0.$$

Additionally, for $\mathbf{E}$,

$$\begin{aligned}\frac{dy}{dt} &= x^2y\\ \frac{dx}{dt} &= xy^2\\ \frac{dy}{dx} &= \frac{x}{y}\\ y^2 &= x^2 + K,\end{aligned}$$

and for $\mathbf{B}$,

$$\frac{dy}{dt} = -xy^2$$

$$\frac{dx}{dt} = x^2 y$$
$$\frac{dy}{dx} = -\frac{y}{x}$$
$$y = \frac{K}{x}.$$

Problem 4

(a)

$$\int_V \mathbf{E}(\mathbf{r})\ d^3x = \int_0^{\pi/2}\int_0^{2\pi}\int_0^R \hat{r}\sin\theta\ dr d\phi d\theta$$
$$\int_V \mathbf{E}(\mathbf{r})\ d^3x = \int_{-R}^{R}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\int_0^{\sqrt{R^2-x^2-y^2}} \frac{x\hat{\imath} + y\hat{\jmath} + z\hat{k}}{(x^2+y^2+z^2)^{3/2}}\ dz dy dx$$

(b)

$$\int_V \mathbf{E}(\mathbf{r})\ d^3x = \int_0^{\pi/2}\int_0^{2\pi}\int_0^R \sin\theta\left(\cos\phi\sin\theta\hat{\imath} + \sin\phi\sin\theta\hat{\jmath} + \cos\theta\hat{k}\right)\ dr d\phi d\theta$$

This integral is more practical than the pure forms since the basis is position-independent and the integral is not a giant mess.

(c) Using symmetry, since $\cos\phi$ is integrated from 0 to 2π and $\sin\phi$ is integrated from 0 to 2π, both the $\hat{\imath}$ and $\hat{\jmath}$ components are 0.

$$\int_0^{\pi/2}\sin^2\theta\int_0^{2\pi}\cos\phi\int_0^R\ dr d\phi d\phi = 0$$
$$\int_0^{\pi/2}\sin^2\theta\int_0^{2\pi}\sin\phi\int_0^R\ dr d\phi d\phi = 0$$

(d) Evaluating the $\hat{k}$ component,

$$\int_0^{\pi/2}\sin\theta\cos\theta\int_0^{2\pi}\int_0^R\ dr d\phi d\theta = 2\pi R\int_0^{\pi/2}\sin\theta\cos\theta\ d\theta$$
$$= \pi R.$$

Problem 5

$$\mathbf{R}_{cm} = \frac{1}{M}\int_S \mathbf{r}\ dm$$
$$= \frac{\sigma}{M}\int_{-\ell/2}^{\ell/2}\int_0^{\pi}\left(R\cos\phi\hat{\imath} + R\sin\phi\hat{\jmath} + z\hat{k}\right) R\ d\phi dz$$
$$= \frac{\sigma}{M}\left(2R^2\right)\hat{\jmath}.$$

Chapter 12 Problems

Problem 1

Problem 2

Problem 3

Problem 6

Problem 7

Problem 9

Problem 15

Problem 19

Chapter 13 Problems

Problem 2