Avinash
Iyer

Homework 1:

(1): $Y_1, \dots, Y_{10}$, normal distribution μ, σ unknown.

$$P\left(|\bar{Y}-\mu| < \frac{2S}{\sqrt{10}}\right) = P\left(-2 \leq \sqrt{n}\left(\frac{\bar{Y}-\mu}{S}\right) \leq 2\right) \Rightarrow DF = 9$$

$$= P(-2 \leq T \leq 2)$$

$$= 1 - 2P(T > 2)$$

$$\approx 0.93 \text{ (using table)}$$

$$= 0.923 \text{ (using R)}$$

7.25:

a) Table: $t_{.10} = 1.476$

R: $t_{.10} = 1.475884$

b) $t_{.10}$ corresponds to the 1.476 quantile, or the 90^{th} percentile.

c) $DF = 30$: $t_{.10} = 1.311$

$DF = 60$: $t_{.10} = 1.297$

$DF = 120$: $t_{.10} = 1.289$

d) The variance of the t distribution is larger than the variance of the normal distribution.

e) I would guess that $t_{.10}$ converges to $z_{.10}$ as t gets large.

7.30. a) $E(Z^2) = V(Z) + (E(Z))^2$

$= 1 + (0)^2$

$= 1$

b)

i) $E\left(\frac{Z}{\sqrt{Y/r}}\right)$

$= E(Z)\, E\left(\sqrt{\frac{r}{Y}}\right)$

$= 0$

ii) $V(T) = E(T^2) - (E(T))^2$

$= E(T^2)$

$= E\left(\frac{Z^2 r}{Y}\right)$

$= r\,E(Z^2) \cdot E(Y^{-1})$

$= r \cdot \frac{\Gamma(r/2 - 1)}{\Gamma(r/2)} \cdot 2^{-1}$

$= r \cdot \frac{\Gamma(r/2 - 1)}{(r/2 - 1)(2)\Gamma(r/2 - 1)}$

$= \frac{r}{r-2}.$