

Solution (38.5): Copying the template equation, we have

$$\frac{dv}{dt} = -\frac{c}{m}v^2 + g,$$

where c is some constant. We see that the terminal velocity is

$$v_t = \sqrt{\frac{mg}{c}}.$$

Separating variables, we have

$$\begin{aligned}\frac{dv}{-\frac{c}{m}v^2 + g} &= dt \\ \frac{1}{g} \left(\frac{dv}{1 - \frac{c}{mg}v^2} \right) &= dt \\ \frac{1}{g} \left(\frac{dv}{1 - (v/v_t)^2} \right) &= dt.\end{aligned}$$

Using the substitution $u := v/v_t$, we have $du = \frac{1}{v_t} dv$, meaning that

$$v_t \int \frac{1}{1 - u^2} du = \int g dt.$$

The integral of $\frac{1}{1-u^2}$ is $\frac{1}{2} \ln \left(\frac{1+u}{1-u} \right) = \operatorname{arctanh}(u)$. Therefore, we have

$$\begin{aligned}\frac{v}{v_t} &= \tanh \left(\frac{g}{v_t} t \right) + K \\ v &= v_t \tanh \left(\frac{g}{v_t} t \right) + v_0 \\ &= \sqrt{\frac{mg}{c}} \tanh \left(\sqrt{\frac{c}{mg}} t \right) + v_0.\end{aligned}$$

Solution (38.6):

(a) Using the chain rule and letting $\frac{dm}{dt} = km^{2/3}$, we have

$$\begin{aligned}\frac{dv}{dt} &= km^{2/3} \frac{dv}{dm} \\ \frac{dv}{dm} + \frac{v}{m} &= -\frac{b}{km} v + \frac{g}{km^{2/3}}.\end{aligned}$$

With integrating factor $m^{1+\frac{b}{k}}$, we have

$$\begin{aligned}m^{1+\frac{b}{k}} v &= \frac{g}{k} \frac{m^{\frac{4}{3}+\frac{b}{k}}}{\frac{4}{3}+\frac{b}{k}} + C \\ v &= \frac{g}{k \left(\frac{4}{3} + \frac{b}{k} \right)} m^{\frac{1}{3}+\frac{b}{k}} + C m^{-1-\frac{b}{k}}.\end{aligned}$$

We let $v(m_0) = 0$, so that

$$C = -\frac{g}{k \left(\frac{4}{3} + \frac{b}{k} \right)} m_0^{\frac{4}{3}+\frac{b}{k}},$$

so

$$v = \frac{g}{\frac{4}{3}k + b} m^{\frac{1}{3}} \left(1 - \left(\frac{m_0}{m} \right)^{\frac{4}{3}+\frac{b}{k}} \right).$$

Thus,

$$\begin{aligned}\frac{dv}{dt} &= g - \frac{1}{m} \frac{dm}{dt} v \\ &= g - \frac{1}{m} \left(km^{2/3} \right) \left(\frac{g}{\frac{4}{3}k + b} m^{\frac{1}{3}} \left(1 - \left(\frac{m_0}{m} \right)^{\frac{4}{3} + \frac{b}{k}} \right) \right).\end{aligned}$$

| **Solution (38.7):**

| **Solution (39.5):**

| **Solution (39.7):**

| **Solution (39.8):**

| **Solution (39.13):**

| **Solution (39.17):**

| **Solution (39.18):**

| **Solution (39.21):**

| **Solution (39.22 (b)):**

| **Solution (39.28):**