Abstract

We discuss the much celebrated Regular Value Theorem and Sard's Theorem, and discuss some of the consequences of these results.

A smooth map between manifolds $f\colon M \to N$ includes a certain family of local information; for instance, the derivative $D_p f\colon T_pM \to T_{f(p)}N$, which is a linear map between tangent spaces at p and q, is defined on a coordinate chart $U \subseteq M$ for p and a corresponding coordinate chart $V \subseteq N$ for $f(p)$. Yet, the properties of this linear map can give us information about the underlying map f.

To understand this, we need to dive into the world of regular and critical values.

Sard's Theorem

Definition: Let $f\colon M \to N$ be a smooth map, and let $p \in M$. We say p is a *critical point* for f if $D_p f$ does not have the same rank as the dimension of $T_{f(p)}N$. If $D_p f$ has the same rank as the dimension of $T_{f(p)}N$, then we say that p is a *regular point* of f.

We say $q \in N$ is a *critical value* for f if $f^{-1}(\{q\})$ contains a critical point for f. Else,we say that q is a *regular value*.