

Problem (Problem 5): A smooth map $f: M \rightarrow N$ is called a submersion if it induces surjections on tangent spaces. Prove that if M and N are smooth manifolds and $A \subseteq N$ is a smooth submanifold, then f is transverse to A .

Solution: Let $p \in f^{-1}(A)$. By the definition of the submersion, we have $T_{f(p)}N = D_p f(T_p M)$, meaning that $D_p f(T_p M) + T_{f(p)}A = T_{f(p)}N$.