

The basis of Multivariable Calculus

If a function is continuous and differentiable, on a small enough interval, the function will approximate a line (i.e., a function of x).

A similar intuition applies to functions of more than one variable (but with a plane, cube, hypercube, etc.). However, in multivariable functions, we will have to sacrifice the ability to visualize it.

For example, in multiple dimensions, it is possible for there to be a function that is both strictly decreasing (in one dimension) and strictly increasing (in another dimension).

Some Functions and Sets

$$f(x, y) = x^2 - y^2$$

DOMAIN: $\{(x, y) \mid \exists f(x, y)\}$

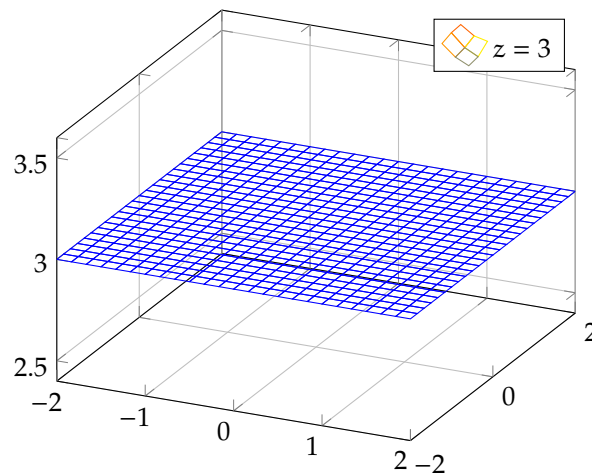
RANGE: $\{f(x, y) \mid (x, y) \in \text{Dom}(f)\} = \mathbb{R}$

GRAPH: $\text{Graph}(f) = \{x, y, f(x, y) \mid x, y \in \text{Dom}(f)\}$. For example, $(1, 3, 4) \notin \text{Graph}(f)$ since $1^2 - 3^2 \neq 4$.

Examples

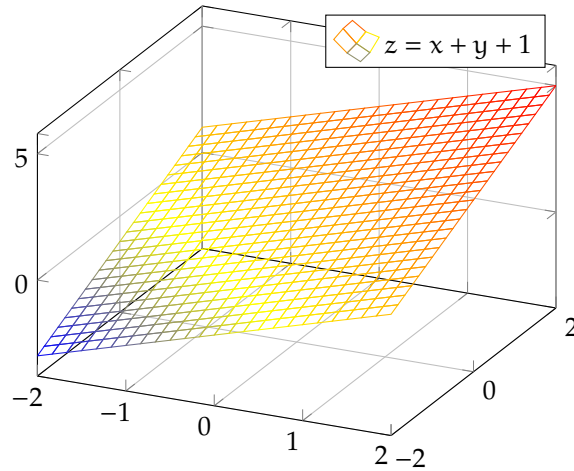
In \mathbb{R}^3 , in x, y, z coordinates, $z = 3$ is a plane defined as follows:

- Parallel to the xy plane.
- Passes through the point $(0, 0, 3)$.



Meanwhile, $y = 0$ would be a “wall” that passes through the origin that contains the line $y = 0$ in the xy plane.

Finally, $z = x + y + 1$ is a plane, as we can see below.



Visualizing a function of multiple variables

Consider the function $f(x, y) = x^2 - y^2$. We can try visualizing slices as follows:

- $f(-2, y) = 4 - y^2$
- $f(0, y) = -y^2$
- $f(2, y) = 4 - y^2$
- $f(x, -2) = x^2 + 4$
- $f(x, 0) = x^2$
- $f(x, 2) = x^2 + 4$

Alternatively, we can visualize via contour diagrams.