3 SPE in Self-Control Problem Game

Boris has \$700 to allocate between three periods of consumption: x_1, x_2, and x_3. He does not discount future utility exponentially, which generates self-control problems, i.e., he will consume more in period 2 than he originally planned to consume when in period 1. It is helpful to think of this as a game between different versions of himself: player 1 is his first period self and player 2 is his second period self. Let's find the SPE of this game.

a. We reason backwards, starting in period 2. Boris' second period self has the following utility function over the final two periods of consumption:

$$v_2(x_2, 700 - x_1 - x_2) = \ln(x_2) + \frac{2}{3}\ln(700 - x_1 - x_2),$$

where consumption in period 3 is $x_3 = 700 - x_1 - x_2$ because it is whatever remains of the \$700 after the first two periods. Find the period-2 consumption $x_2(x_1)$, which is a function of the first period self's consumption choice x_1.

$$\frac{1}{x_2} - \frac{2}{3(700 - x_1 - x_2)} = 0$$

$$3(700 - x_1 - x_2) = 2x_2$$

$$2100 - x_1 = 5x_2$$

$$x_2 = 420 - \frac{x_1}{5}$$

b. In the first period Boris' first period self has the following utility function over all three periods of consumption:

$$v_1(x_1, x_2, 700 - x_1 - x_2) = \ln(x_1) + \frac{2}{3}\ln(x_2) + \frac{2}{3}\ln(700 - x_1 - x_2),$$

Find the optimal choice of x_1, taking into account that Boris' first period self correctly anticipates how he will behave when he arrives in period 2.

$$v_1 = \ln(x_1) + \frac{2}{3}\left(420 - \frac{x_1}{5}\right) + \frac{2}{3}\left(280 - \frac{6x_1}{5}\right)$$

$$BR_1 = \frac{1}{x_1} - \frac{2}{15(420 - x_1/5)} - \frac{12}{15(280 - 6x_1/5)}$$