Problem 1.1.1

Determine which bipartite graphs are complete graphs

The graph $K_{1,1}$ is the only bipartite graph that is complete.

Problem 1.1.3

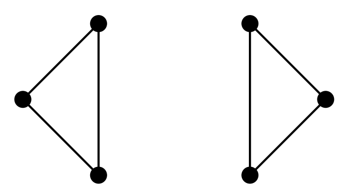
Using rectangular blocks whose entries are equal, write down an adjacency matrix for $K_{m,n}$

$$K_{m,n} = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_m \\ b_1 \\ b_2 \\ \vdots \\ b_n \end{array} \begin{array}{c} \begin{array}{cccccccc} a_1 & a_2 & \cdots & a_m & b_1 & b_2 & \cdots & b_n \end{array} \\ \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \\ 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \end{array}$$

Problem 1.1.5

Prove or disprove: If every vertex of a simple graph G has degree 2, then G is a cycle.

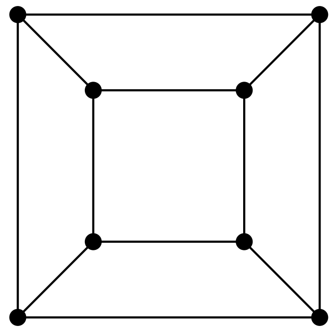
Let G be the following graph:

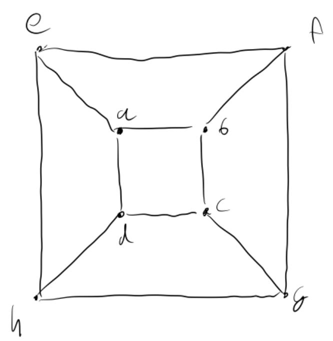


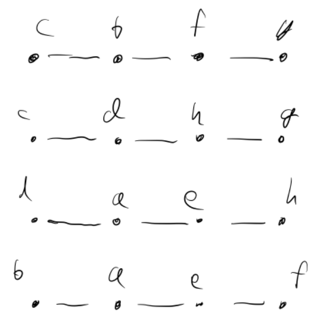
Every vertex in G has a degree 2, yet G is not a cycle.

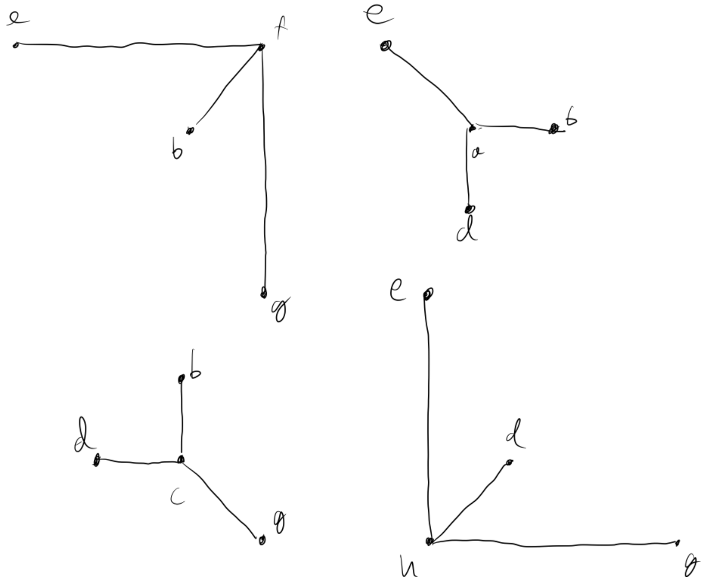
Problem 1.1.8

Prove that the 8 vertex graph below decomposes into copies of $K_{1,3}$ and also into copies of P_4



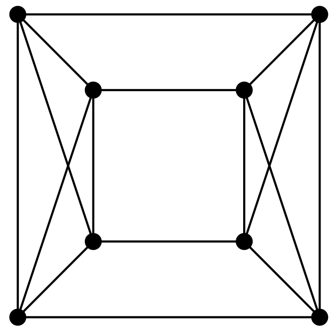


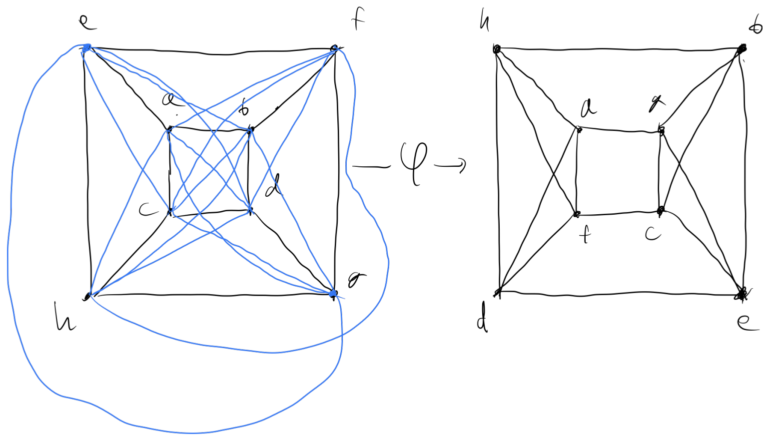




Problem 1.1.9

Prove that the graph below is isomorphic to the complement of the previous graph





Problem 1.1.10

Prove or disprove: the complement of a simple disconnected graph must be connected.

Let G be a graph that is disconnected. We want to show that $\forall x, y \in V(G), \exists x, z$ path. We can split into two cases.

- Suppose $x \not\leftrightarrow y$ in G. Then, in $\overline{G}$, $x \leftrightarrow y$ by the definition of a graph complement.
- Suppose $x \leftrightarrow y$ in G. Then, since G is disconnected, we know that there must be some $z \in V(G)$ such that there is no x, z path. Since there is no x, z path, then there is no y, z path. In particular, this means $x \not\leftrightarrow z$ and $y \not\leftrightarrow z$ in G. Therefore, in $\overline{G}$, we have that $x \leftrightarrow z$ and $y \leftrightarrow z$, meaning there is a path between x and y.