More Limit Pricing

An incumbent firm (player 1) is either a low-cost type $\theta_1 = \theta_L$ or a high cost type $\theta_1 = \theta_H$, each with equal probability. In period $t = 1$, the incumbent is a monopolist and sets one of two prices, p_L or p_H, and its profits in this period depend on its type and the price it chooses, given by the following table:

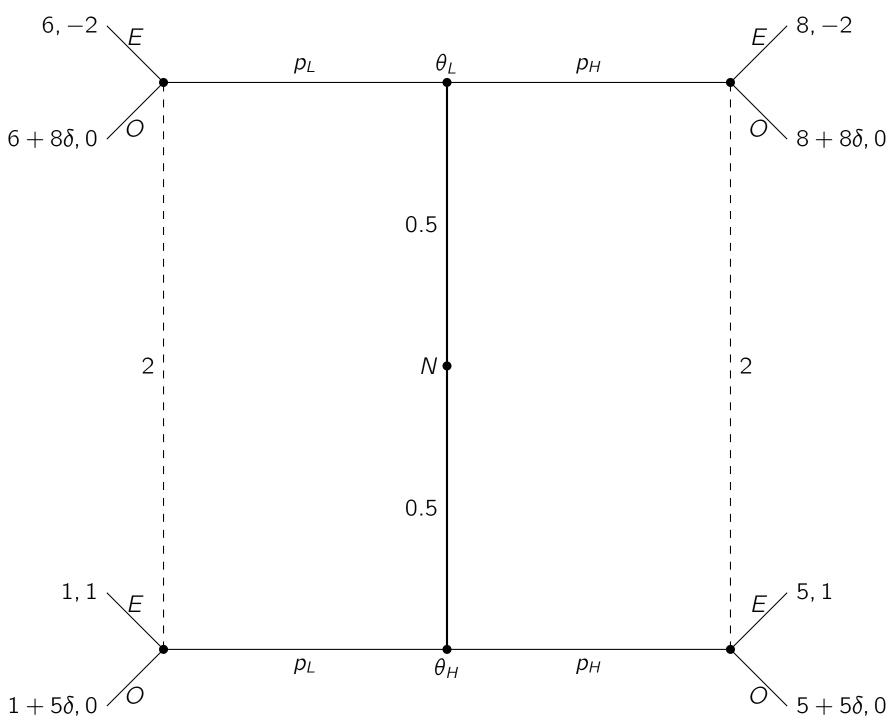
Type	Profit from p_L	Profit from p_H
θ_L	6	8
θ_H	1	5

After observing the period $t = 1$ price, a potential entrant (player 2), which does not know the incumbent's type but knows the distribution of types, can choose to either enter the market (E), or stay out (O) in period $t = 2$. The payoffs of both players in period 2 depend on the entrant's choice and on the incumbent's type and are given by the following table:

Incumbent's type	Entrant's choice	Incumbent's payoff	Entrant's payoff
θ_L	E	0	-2
θ_L	O	8	0
θ_H	E	0	1
θ_H	O	5	0

At the beginning of the game the incumbent discounts profits for period $t = 2$ using a discount factor $\delta \leq 1$.

(a)



(b)

Problem: For $\delta = 1$ find a pooling perfect Bayesian equilibrium of the game in which both types of player 1 choose p_L in period $t = 1$.

Strategy for Player 1:

$$s_1^*(\theta) = p_L \qquad \forall\theta$$

Belief System for Player 2:

$$\begin{aligned}
\mu_2(\theta_L|p_L) &= \frac{1}{2}\\
\mu_2(\theta_H|p_L) &= \frac{1}{2}\\
\mu_2(\theta_L|p_H) &= \lambda\\
\mu_2(\theta_H|p_H) &= 1-\lambda
\end{aligned}$$

Best Responses for Player 2:

After seeing p_L:

$$\begin{aligned}
Ev_2(E, p_L;\theta) &= \mu_2(\theta_L|p_L)v_2(E, p_L;\theta_L) + \mu_2(\theta_H|p_L)v_2(E, p_L;\theta_H)\\
&= -\frac{1}{2}\\
Ev_2(O, p_L;\theta) &= 0
\end{aligned}$$

After seeing p_H:

$$\begin{aligned}
Ev_2(E, p_H;\theta) &= \mu_2(\theta_L|p_H)v_2(E, p_H;\theta_L) + \mu_2(\theta_H|p_H)v_2(E, p_H;\theta_H)\\
&= 1-3\lambda\\
Ev_2(O, p_H;\theta) &= 0
\end{aligned}$$

Condition to play entry upon p_H:

$$\begin{aligned}
1-3\lambda &> 0\\
\lambda &< \frac{1}{3}.
\end{aligned}$$

So,

$$s_2^*(a_1) = \begin{cases} O, & a_1 = p_L, \lambda \in [0,1]\\ E, & a_1 = p_H, \lambda < 1/3\\ O, & a_1 = p_H, \lambda > 1/3\\ qE + (1-q)O, & a_1 = p_H, \lambda = 1/3 \end{cases}$$

Best Responses for Player 1:

Follow:

$$\begin{aligned}
v_1(p_L, s_2^*(a_1);\theta_L) &= 14\\
v_1(p_L, s_2^*(a_1);\theta_H) &= 6
\end{aligned}$$

Deviate:

$$\begin{aligned}
v_1(p_H, s_2^*(a_1);\theta_L) &= 8\\
v_1(p_H, s_2^*(a_1);\theta_H) &= 5
\end{aligned}$$

(c)

Problem: Find the range of discount factors for which a separating perfect Bayesian equilibrium exists in which type θ_L chooses p_L and type θ_H chooses p_H in period $t = 1$.

Strategy for Player 1:

$$s_1^*(\theta) = \begin{cases} p_L, & \theta = \theta_L \\ p_H, & \theta = \theta_H \end{cases}$$

Best Response for Player 2:

$$s_2^*(a_1) = \begin{cases} E, & a_1 = p_H \\ O, & a_1 = p_L \end{cases}$$

Player 1 Equilibrium Conditions:

Low Cost:

$$\begin{aligned} v_1(p_L, s_2^*(a_1); \theta_L) &= 6 + 8\delta \\ v_1(p_H, s_2^*(a_1); \theta_L) &= 8 \\ 6 + 8\delta &\geq 8 \\ \delta &\geq \frac{3}{4} \end{aligned}$$

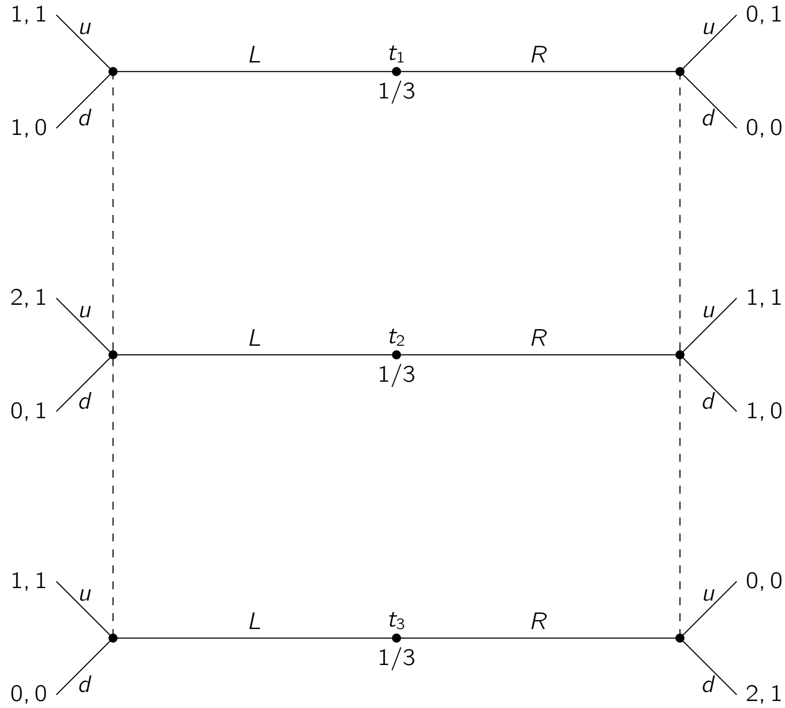
High Cost:

$$\begin{aligned} v_1(p_H, s_2^*(a_1); \theta_H) &= 5 \\ v_1(p_L, s_2^*(a_1); \theta_L) &= 1 + 5\delta \\ 1 + 5\delta &\leq 5 \\ \delta &\leq \frac{4}{5} \end{aligned}$$

Therefore, $\delta \in [3/4, 4/5]$.

Third Type's a Charm

The following three-type signaling game begins with a move by Nature (not shown in the tree) that yields one of three types with equal probability.



(a)

Problem: Find the pure strategy pooling perfect Bayesian equilibrium.

Strategy for Player 1:

$$s_1^*(\theta) = L \qquad \forall\theta$$

Belief System for Player 2:

$$\mu_2(t_1|L) = \frac{1}{3}$$
$$\mu_2(t_2|L) = \frac{1}{3}$$
$$\mu_2(t_3|L) = \frac{1}{3}$$
$$\mu_2(t_1|R) = \lambda_1$$
$$\mu_2(t_2|R) = \lambda_2$$
$$\mu_2(t_3|R) = \lambda_3$$

Best Response for Player 2:

$$\begin{aligned}
Ev_2(u, L; \theta) &= \sum_i \mu_2(t_i|L)v_2(u, L; t_i)\\
&= 1\\
Ev_2(d, L; \theta) &= 0\\
Ev_2(u, R; \theta) &= \sum_i \mu_2(t_i|R)v_2(u, R; t_i)\\
&= \frac{\lambda_1 + \lambda_2}{3}\\
&= 1 - \frac{\lambda_3}{3}\\
Ev_2(d, R; \theta) &= \sum_i \mu_2(t_i|R)v_2(d, R; t_i)\\
&= \frac{\lambda_3}{3}
\end{aligned}$$

Therefore,

$$s_2^*(a_1) = u \qquad \forall \lambda_1, \lambda_2, \lambda_3$$

Best Responses for Player 1:

Follow:

$$\begin{aligned}
v_1(L, s_2^*(a_1); t_1) &= 1\\
v_1(L, s_2^*(a_1); t_2) &= 2\\
v_1(L, s_2^*(a_1); t_3) &= 1
\end{aligned}$$

Deviate:

$$\begin{aligned}
v_1(R, s_2^*(a_1); t_1) &= 0\\
v_1(R, s_2^*(a_1); t_2) &= 1\\
v_1(R, s_2^*(a_1); t_3) &= 0
\end{aligned}$$

Therefore, $(s_1^*(\theta), s_2^*(a_1), \mu_2(\theta|a_1))$ is a pooling perfect Bayesian equilibrium.

(b)

Problem: Find the pure strategy separating perfect Bayesian equilibrium.

Strategy for Player 1:

$$s_1^*(\theta) = \begin{cases} L, & \theta = t_1, t_2\\ R, & \theta = t_3 \end{cases}$$

Belief System for Player 2:

$$\begin{aligned}
\mu_2(t_1|L) &= \frac{1}{2}\\
\mu_2(t_2|L) &= \frac{1}{2}\\
\mu_2(t_3|L) &= 0\\
\mu_2(t_1|R) &= 0\\
\mu_2(t_2|R) &= 0\\
\mu_2(t_3|R) &= 1
\end{aligned}$$

Best Response for Player 2:

$$\begin{aligned}
Ev_2(u, L; \theta) &= \sum_i \mu_2(t_i|L)v_2(u, L; t_i)\\
&= 1\\
Ev_2(d, L; \theta) &= \sum_i \mu_2(t_i|L)v_2(d, L; t_i)\\
&= 0\\
Ev_2(u, R; \theta) &= \sum_i \mu_2(t_i|R)v_2(u, R; t_i)\\
&= 2\\
Ev_2(d, R; \theta) &= \sum_i \mu_2(t_i|R)v_2(d, R; t_i)\\
&= 1
\end{aligned}$$

Therefore,

$$s_2^*(a_1) = \begin{cases} u, & a_1 = L\\ d, & a_1 = R \end{cases}$$

Best Responses for Player 1:

Follow:

$$\begin{aligned}
v_1(L, s_2^*(a_1); t_1) &= 1\\
v_1(L, s_2^*(a_1); t_2) &= 2\\
v_1(R, s_2^*(a_1); t_3) &= 2
\end{aligned}$$

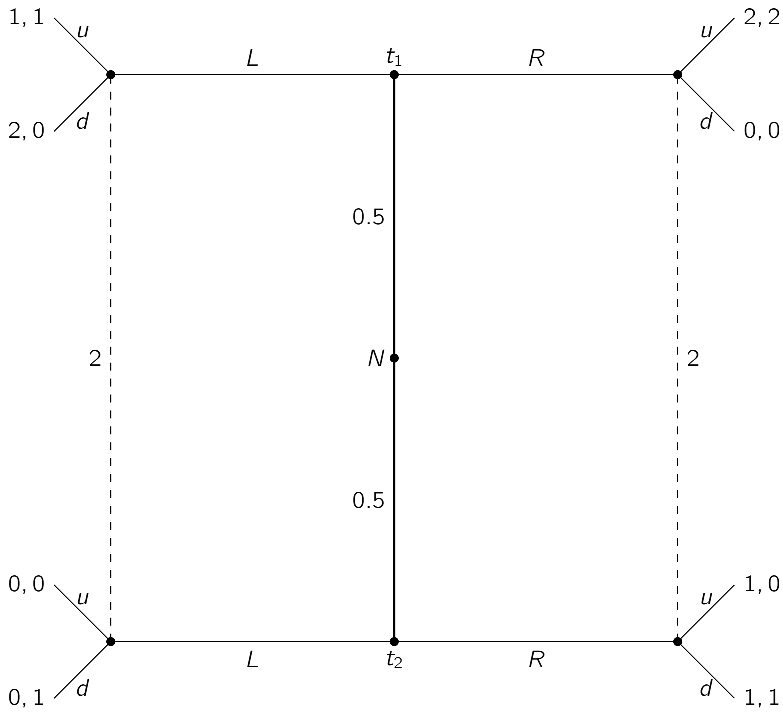
Deviate:

$$\begin{aligned}
v_1(R, s_2^*(a_1); t_1) &= 0\\
v_1(R, s_2^*(a_1); t_2) &= 1\\
v_1(L, s_2^*(a_1); t_3) &= 1
\end{aligned}$$

Therefore, $(s_1^*(\theta), s_2^*(a_1), \mu_2(\theta|a_1))$ is a separating perfect Bayesian equilibrium.

Semi-Separating Signals

Problem: Find a semi-separating PBE in the below signaling game.



Strategy for Player 1:

$$s_1^*(\theta) = \begin{cases} qR + (1-q)L, & \theta = t_1 \\ R, & \theta = t_2 \end{cases}$$

Belief System for Player 2:

$$\begin{aligned} \mu_2(t_1|L) &= 1 \\ \mu_2(t_2|L) &= 0 \\ \mu_2(t_1|R) &= \frac{q}{1+q} \\ \mu_2(t_2|R) &= \frac{1}{1+q} \end{aligned}$$

Best Response for Player 2:

$$\begin{aligned}
Ev_2(u, L; \theta) &= \sum_i \mu_2(t_i|L)v_2(u, L; t_i) \\
&= 1 \\
Ev_2(d, L; \theta) &= \sum_i \mu_2(t_i|L)v_2(d, L; t_i) \\
&= 0 \\
Ev_2(u, R; \theta) &= \sum_i \mu_2(t_i|R)v_2(u, R; t_i) \\
&= \frac{2q}{1+q} \\
Ev_2(d, R; \theta) &= \sum_i \mu_2(t_i|R)v_2(d, R; t_i) \\
&= \frac{1}{1+q}
\end{aligned}$$

Indifference Condition:

$$\begin{aligned}
\frac{2q}{1+q} &= \frac{1}{1+q} \\
q &= \frac{1}{2}
\end{aligned}$$

therefore,

$$s_2^*(a_1) = \begin{cases} u, & a_1 = L \\ u, & a_1 = R, q > \frac{1}{2} \\ d, & a_1 = R, q < \frac{1}{2} \\ pu + (1-p)d, & a_1 = R, q = \frac{1}{2} \end{cases}$$

Best Response for Player 1:

$$\begin{aligned}
v_1(L, s_2^*(a_1); t_1) &= 1 \\
v_1(R, s_2^*(a_1); t_1) &= 2
\end{aligned}$$

Indifference Condition:

$$\begin{aligned}
q(v_1(R, s_2^*(a_1); t_1)) &= (1-q)(v_1(L, s_2^*(a_1); t_1)) \\
2q &= 1 - q \\
q &= \frac{1}{3}
\end{aligned}$$

Type t_2 will always play R as R dominates L for t_2.

Therefore, $(s_1^*(\theta), s_2^*(a_1), \mu_2(\theta|a_1))$ is a PBE.