

## 19.4

2:

- Direct Calculation:

$$\begin{aligned}\int_S \vec{F} \cdot d\vec{A} &= \int_0^{2\pi} \int_{-1}^1 \sin \theta \, dz \, d\theta \\ &= 0\end{aligned}$$

- Divergence Theorem:

$$\begin{aligned}\int_S \vec{F} \cdot d\vec{A} &= \int_{-1}^1 \int_0^{2\pi} \int_0^1 dr \, d\theta \, dz \\ &= 0\end{aligned}$$

4:

6:

$$\begin{aligned}\int_S \vec{F} \cdot d\vec{A} &= \int_V 10 \, dV \\ &= 240.\end{aligned}$$

8:

$$\begin{aligned}\int_S \vec{H} \cdot d\vec{A} &= \int_0^4 \int_0^3 \int_0^2 (y) \, dx \, dy \, dz \\ &= \int_0^4 \int_0^3 2y \, dy \, dz \\ &= \int_0^4 9 \, dz \\ &= 36.\end{aligned}$$

10:

$$\begin{aligned}\int_S \vec{N} \cdot d\vec{A} &= \int_V \nabla \cdot \vec{N} \, dV \\ &= 0.\end{aligned}$$

14:

$$\begin{aligned}\int_S \vec{F} \cdot d\vec{A} &= \int_V \nabla \cdot \vec{F} \, dV \\ &= \int_V x + y + z \, dV \\ &= \int_0^\pi \int_0^{2\pi} \int_0^1 \rho(\sin \phi \cos \theta + \sin \phi \sin \theta + \cos \phi) \, \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi \\ &= 0\end{aligned}$$

16:

$$\begin{aligned}\int_S \vec{F} \cdot d\vec{A} &= \int_V \nabla \cdot \vec{F} \, dV \\ &= \int_0^{\pi/4} \int_0^{2\pi} \int_2^3 3\rho^4 \sin \phi \, d\rho \, d\theta \, d\phi \\ &= \frac{633(2 - \sqrt{2})\pi}{5}\end{aligned}$$

22:

## 20.1

6:

$$\nabla \times \vec{F} = \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$$

8:

$$\nabla \times \vec{F} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

10:

$$\nabla \times \vec{F} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

12:

$$\nabla \times \vec{F} = \begin{pmatrix} 2zx^3y + 6y^5x^7 - xy \\ y - 7x^6y^6 \\ zy - z \end{pmatrix}$$

22:

$$\vec{F} = \begin{pmatrix} a(x) \\ b(y) \\ c(z) \end{pmatrix}$$

24:

- (a) Counterclockwise.
- (b) Clockwise.
- (c)

$$\nabla \times \vec{F} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

## 20.2

2: Zero.

4:

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= \int_0^1 \begin{pmatrix} 3(\cos(\pi t) + \sin(\pi t)) \\ 3\cos(\pi t) \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -3\pi \sin(\pi t) \\ 0 \\ 3\pi \cos(\pi t) \end{pmatrix} dt + \int_0^1 \begin{pmatrix} 6t-3 \\ 6t-3 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} dt \\ &= \frac{9\pi}{2} \\ \int_S \nabla \times \vec{F} \cdot d\vec{A} &= \frac{9\pi}{2} \int_S \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} dA \\ &= \frac{9\pi}{2} \end{aligned}$$

10:

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= \int_S \nabla \times \vec{F} \cdot d\vec{A} \\ &= -5\pi \end{aligned}$$

12:

$$\int_C \vec{F} \cdot d\vec{r} = \int_S \nabla \times \vec{F} \cdot d\vec{A} \\ = 0$$

28:

$$(a) \vec{C} = \begin{pmatrix} \sin t \\ \cos t \\ 2 \end{pmatrix}$$

34:

**20.3**

4:

6:

8:

24:

28: