Homework Section 1.1

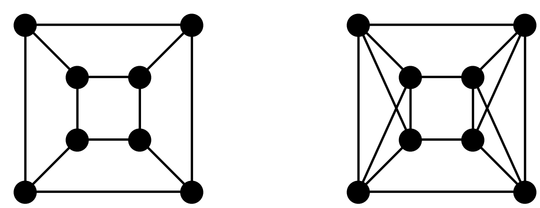
Individual:

1.1.1 Determine which complete bipartite graphs are complete graphs.

1.1.3 Using rectangular blocks (think block-matrix) whose entries are all equal, write down an adjacency matrix for $K_{m,n}$.

1.1.5 Prove or disprove: If every vertex of a simple graph G has degree 2, then G is a cycle.

1.1.8 Prove that the 8-vertex graph on the left below decomposes into copies of $K_{1,3}$ and also into copies of P_4.



1.1.9 Prove that the graph on the right above is isomorphic to the complement of the graph on the left.

1.1.10 Prove or disprove: The complement of a simple disconnected graph must be connected.

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Group:

"A" Group Problems:

1.1.13 Let G be the graph whose vertex set is the set of k-tuples with coordinates in $\{0,1\}$, with x adjacent to y when x and y differ in exactly one position. Determine whether G is bipartite.

1.1.26 Let G be a graph with girth 4 in which every vertex has degree k. Prove that G has at least $2k$ vertices. Determine all such graphs with exactly $2k$ vertices.

1.1.27 Let G be a graph with girth 5. Prove that if every vertex of G has degree at least k, then G has at least k^2+1 vertices. For $k=2$ and $k=3$, find one such graph with exactly k^2+1 vertices.

1.1.30 Let G be a simple graph with adjacency matrix A and incidence matrix M. Prove that the degree of v_i is the ith diagonal entry in A^2 and in MM^T. What do the entries in position (i,j) of A^2 and MM^T say about G?

1.1.34 Decompose the Petersen graph into three connected subgraphs that are pairwise isomorphic. Also decompose it into copies of P_4.

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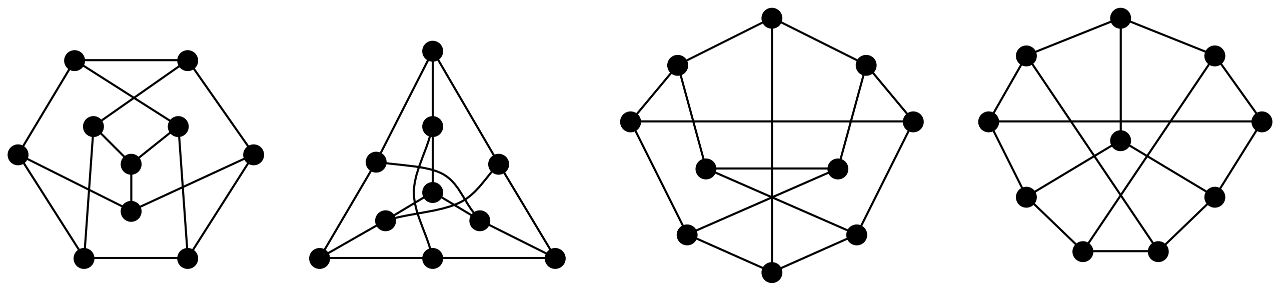
"B" Group Problems:

1.1.14 Prove that removing opposite corner squares from an 8-by-8 checkerboard leaves a subboard that cannot be partitioned in to 1-by-2 and 2-by-1 rectangles. Using the same argument, make a general statement about all bipartite graphs. (View the squares of the boards as vertices of a graph, join two squares with an edge if they can be tiled by a domino.)

1.1.24 Prove that the graphs in the middle of page 17 are all drawings of the Petersen graph (Definition 1.1.36) (Hint: Use the disjointness definition of adjacency.)



1.1.31 Prove that a self-complementary graph with n vertices exists if and only if n or $n-1$ is divisible by 4. (Hint: When n is divisible by 4, generalize the structure of P_4 by splitting the vertices into four groups. For $n \equiv 1 \pmod 4$, add one vertex to the graph constructed for $n-1$.)

1.1.35 Prove that K_n decomposes into three pairwise-isomorphic subgraphs if and only if $n+1$ is not divisible by 3. (Hint: For the case where n is divisible by 3, split the vertices into three sets of equal size.)

1.1.38 Let G be a simple graph in which every vertex has degree 3. Prove that G decomposes into claws if and only if G is bipartite.