I am using $\overline{z}$ to denote the conjugate of a complex number and T^* to denote the adjoint of an operator.

Chapter 25 Problems

Problem 1

(a)

$$\begin{aligned}
\||1\rangle\|^2 &= \langle 1|1\rangle\\
&= (1)\overline{(1)} + (i)\overline{(i)}\\
&= 2\\
\||2\rangle\|^2 &= \langle 2|2\rangle\\
&= (-i)\overline{(-i)} + (2i)\overline{(2i)}\\
&= 5\\
\||3\rangle\|^2 &= \langle 3|3\rangle\\
&= \left(e^{i\phi}\right)\overline{(e^{i\phi})} + (-1)\overline{(-1)}\\
&= 2\\
\||4\rangle\|^2 &= \langle 4|4\rangle\\
&= (1)\overline{(1)} + (-2i)\overline{(-2i)} + (1)\overline{(1)}\\
&= 6\\
\||5\rangle\|^2 &= \langle 5|5\rangle\\
&= (i)\overline{(i)} + (1)\overline{(1)} + (i)\overline{(i)}\\
&= 3.
\end{aligned}$$

(b)

$$\begin{aligned}
\langle 2|1\rangle &= (1)\overline{(-i)} + (i)\overline{(2i)}\\
&= \overline{-i\overline{(1)} + 2i\overline{(i)}}\\
&= 2 + i\\
&= \overline{\langle 1|2\rangle}\\
\langle 3|1\rangle &= (1)\overline{(e^{i\phi})} + (i)\overline{(-1)}\\
&= \overline{e^{i\phi}\overline{(1)} + (-1)\overline{(i)}}\\
&= e^{-i\phi} - i\\
&= \overline{\langle 1|3\rangle}\\
\langle 3|2\rangle &= (-i)\overline{(e^{i\phi})} + (2i)\overline{(-1)}\\
&= \overline{(e^{i\phi})\overline{(-i)} + (-1)\overline{(2i)}}\\
&= -ie^{-i\phi} - 2i\\
&= \overline{\langle 2|3\rangle}.\\
\langle 5|4\rangle &= (1)\overline{(i)} + (-2i)\overline{(1)} + (1)\overline{(i)}\\
&= \overline{i\overline{(1)} + (1)\overline{(-2i)} + (i)\overline{(1)}}\\
&= -4i
\end{aligned}$$

$$= \overline{\langle 4 \,|5 \rangle}$$

Problem 4

(a)

$$\begin{aligned}
|u\rangle^* &= (M\,|v\rangle)^* \\
&= \left(\begin{pmatrix} 1 & i \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix}\right)^* \\
&= \left(\begin{pmatrix} 2 \\ 2-i \end{pmatrix}\right)^* \\
&= \begin{pmatrix} 2 & 2+i \end{pmatrix} \\
&= \begin{pmatrix} 1 & i \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -i & 1 \end{pmatrix} \\
&= \langle u| \,.
\end{aligned}$$

(b)

$$\begin{aligned}
\langle w\,|v\,\rangle &= \langle w\,|Mv\,\rangle \\
&= \langle w\,|u\,\rangle \\
&= \begin{pmatrix} -1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 2-i \end{pmatrix} \\
&= -i \\
&= \overline{\langle u\,|w\,\rangle} \\
&= \overline{\begin{pmatrix} 2 & 2+i \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix}} \\
&= \overline{(i)} \\
&= -i.
\end{aligned}$$

Problem 5

$$\begin{aligned}
\langle v\,|Lw\,\rangle &= \langle v|\,L\,|w\rangle \\
&= \langle L^* v\,|w\rangle \\
&= \overline{\langle w\,|L^* v\rangle} \\
&= \overline{\langle w|\,L^*\,|v\rangle}.
\end{aligned}$$

Problem 6

(a)

$$\begin{aligned}
\overline{\overline{\langle v|\,T\,|w\rangle}} &= \overline{\langle w|\,T^*\,|v\rangle} \\
&= \langle v|\,T^{**}\,|w\rangle \,.
\end{aligned}$$

(b)

$$\begin{aligned}
\langle v|\,(ST)^*\,|w\rangle &= \overline{\langle w|\,S\,(T\,|v\rangle)} \\
&= \overline{\langle w|\,S\,|u\rangle} && |u\rangle = T\,|v\rangle
\end{aligned}$$

$$
\begin{aligned}
&= \langle u| S^* |w\rangle \\
&= \langle Tv| S^* |w\rangle \\
&= \langle v| T^*S^* |w\rangle .
\end{aligned}
$$

Alternatively,

$$
\begin{aligned}
\langle v| (ST)^* |w\rangle &= \langle (ST)\, v\, |w\rangle \\
&= \langle Tv| S^* |w\rangle \\
&= \langle v| T^*S^* |w\rangle .
\end{aligned}
$$

Problem 8

Problem 9

Problem 13

Problem 17

Problem 18

Problem 19

Problem 26

Problem 29

Problem 30