Useful Results

These will almost certainly not be given to you on the final exam, but it is important to have these ones memorized to the point where if someone woke you up at 3am you would be able to recite these.

- Pythagorean Identities:

$$\begin{aligned} \sin^2(x) + \cos^2(x) &= 1 \\ 1 + \tan^2(x) &= \sec^2(x) \\ 1 + \cot^2(x) &= \csc^2(x). \end{aligned}$$

- Double-Angle identities:

$$\begin{aligned} \sin(2x) &= 2\sin(x)\cos(x) \\ \cos(2x) &= \cos^2(x) - \sin^2(x) \\ &= 1 - 2\sin^2(x) \\ &= 2\cos^2(x) - 1. \end{aligned}$$

- Limits:

$$\begin{aligned} \lim_{x \to 0} \frac{\sin(x)}{x} &= 1 \\ \lim_{n \to 0} \frac{1}{n} &= 0. \end{aligned}$$

- Derivatives:

$$\begin{aligned} \frac{d}{dx}(\sin(x)) &= \cos(x) \\ \frac{d}{dx}(\cos(x)) &= -\sin(x) \\ \frac{d}{dx}(\tan(x)) &= \sec^2(x) \\ \frac{d}{dx}(\cot(x)) &= -\csc^2(x) \\ \frac{d}{dx}(\sec(x)) &= \sec(x)\tan(x) \\ \frac{d}{dx}(\csc(x)) &= -\csc(x)\cot(x) \\ \frac{d}{dx}(x^n) &= nx^{n-1} \\ \frac{d}{dx}(e^x) &= e^x \\ \frac{d}{dx}(\ln(x)) &= \frac{1}{x} \\ \frac{d}{dx}(\arctan(x)) &= \frac{1}{1+x^2}. \end{aligned}$$

Practice Problems

These are some relatively involved multi-step practice problems that are similar (but likely more difficult) than the ones you may encounter on the final exam.

Practice Problem 1

(a) Consider the solid defined by rotating the region bounded by $x = 0$, $x = \pi/2$, $y = 0$, and $y = \cos(x)$. Set up integrals I_1 and I_2 for the volume and surface area of this solid respectively.

(b) Find the volume of the solid by resolving the integral.

(c) Similarly, find an expression for the surface area of this solid. To find the antiderivative, you may find the following steps useful.

(i) Use a substitution to express the integral entirely in terms of square roots and polynomial expressions.

(ii) Take $u = \tan(\theta)$ and use trigonometric identities to express the integral solely in terms of $\sec(\theta)$.

(iii) Extract a factor of $\sec^2(\theta)$ and use integration by parts and a trigonometric identity to reduce this integral to that of $\sec(\theta)$.

(iv) To evaluate the integral of $\sec(\theta)$, multiply top and bottom by $\sec(\theta) + \tan(\theta)$, then use a substitution.

Practice Problem 2

Evaluate the convergence of the following series, determining whether a series is absolutely convergent, conditionally convergent, or divergent. Explicitly indicate which test(s) you are using to arrive at your conclusion, and carefully verify the hypotheses.

(a)

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}.$$

(b)

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}.$$

(c)

$$\sum_{n=0}^{\infty} \frac{\sin\left(\frac{1}{n}\right)}{n^2}.$$

Practice Problem 3

Find the *interval* of convergence for the following power series.

(a)

$$\sum_{n=1}^{\infty} \left(n^5 + 3n^2 + 2n + 3\right) \frac{(x-3)^n}{4^n}.$$

(b)

$$\sum_{n=1}^{\infty} \frac{2^{3n}}{n^2 5^{2n}} (x-2)^{n^2}.$$

(c)

$$\sum_{n=1}^{\infty} \frac{n^2}{2^{n/2}} x^{4n}.$$