Problem 1

Let $(x_k)_k$ be a sequence of strictly positive numbers such that

$$(kx_k)_k \to L > 0.$$

Show that $\sum_k x_k$ diverges.

Since $(kx_k)_k \to L$, every subsequence of $(kx_k)_k$ converges to L. Let $n_k = 2^k$. Then,

$$(2^k x_{2^k})_k \to L > 0,$$

implying that

$$\sum_k 2^k x_{2^k} = \infty.$$

By the Cauchy Condensation test, this implies that $\sum_k x_k$ diverges.