Solution (21.1)**:**

(a) Doing a partial fraction decomposition, we find

$$\frac{1}{(z-1)(z+2)} = \frac{1}{3}\frac{1}{z-1} - \frac{1}{3}\frac{1}{z+2},$$

giving giving a residue of $\frac{1}{3}$ at $z = 1$ and a residue of $-\frac{1}{3}$ at $z = -2$.

(b) Evaluating the residue at $z = 1$, we may use the cover-up method to find

$$\text{Res}[f(z), 1] = \frac{e^{2i}}{27}.$$

To evaluate the residue at $z = -2$, we use the formula to calculate residues, giving

$$\begin{aligned}\text{Res}[f(z), -2] &= \frac{1}{2}\frac{d^2}{dz^2}\left(\frac{e^{2iz}}{z-1}\right)\Big|_{z=-2} \\ &= \frac{38}{27}e^{-4i}\end{aligned}$$

(c) Note that $\sin(z)$ is a simple zero at $z = n\pi$. Therefore, we evaluate

$$\text{Res}[f(z), n\pi] = (-1)^n e^{n\pi}.$$

(d) Using the Laurent series for $e^{1/z}$, we find that

$$e^{1/z} = 1 + \frac{1}{z} + \frac{1}{2z^2} + \cdots,$$

so that

$$\text{Res}[f(z), 0] = 1.$$

(e) Note that $e^{2z} + 1 = 0$ whenever $z = i(2n+1)\pi/2$. These are all simple zeros, so we may evaluate

$$\begin{aligned}\text{Res}[f(z), i(2n+1)\pi/2] &= \frac{-(2n+1)^2\pi^2(-1)}{4(-2)} \\ &= -\frac{(2n+1)^2\pi^2}{8}.\end{aligned}$$

Solution (21.2)**:**

Solution (21.6)**:**

Solution (21.8)**:**

Solution (21.10)**:**

Solution (21.12)**:**

Solution (21.16)**:**

Solution (21.17)**:**

Solution (21.22)**:**