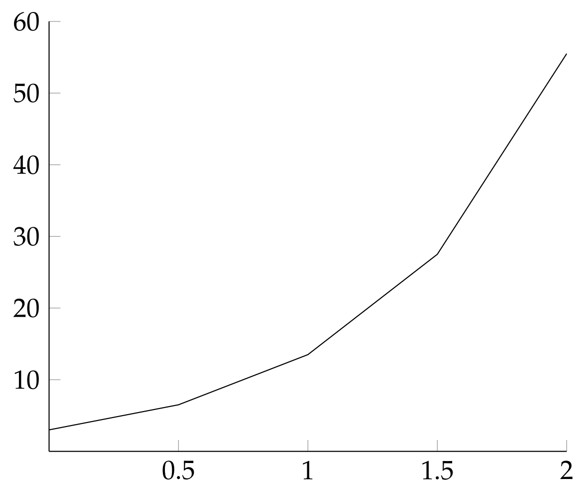
Part 1

1.4, Problem 2

$$\begin{aligned}\frac{dy}{dt} &= 2y + 1\\ y(0) &= 3\\ 0 \leqslant t &\leqslant 2\\ \Delta t &= 0.5\end{aligned}$$

k	t	y	f
0	0	3	7
1	0.5	6.5	14
2	1	13.5	28
3	1.5	27.5	56
4	2	55.5	—

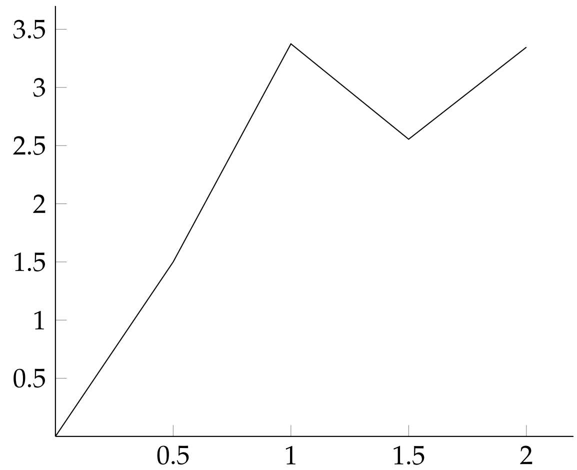


Excel was used for calculation and TikZ/PGF was used to graph the coordinate outcomes from Euler's Method.

1.4, Problem 6

$$\begin{aligned}\frac{dw}{dt} &= (3 - w)(w + 1)\\ w(0) &= 0\\ 0 \leqslant t &\leqslant 2\\ \Delta t &= 0.5\end{aligned}$$

k	t	w	f
1	0	3	3
2	0.5	1.5	3.75
3	1	3.375	−1.641
4	1.5	2.555	1.583
5	2.0	3.346	—

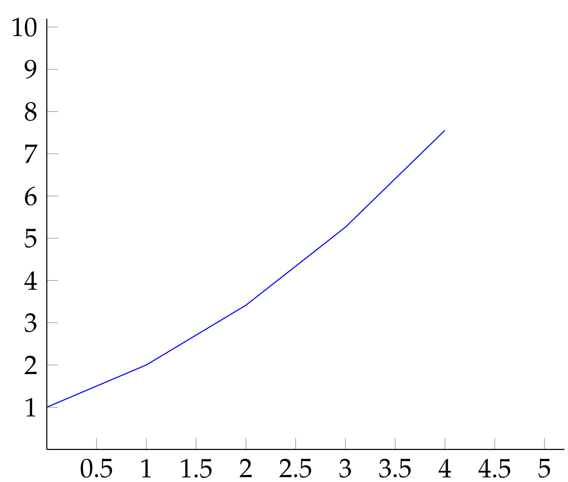


1.4, Problem 11

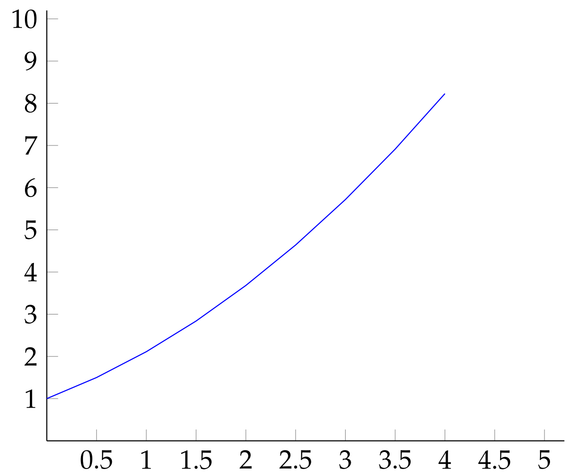
The equilibrium solutions to $\frac{dw}{dt} = (3 - w)(w + 1)$ occur for $w(t) = 3$; however, we had our initial condition at $w(0) = 0$ and yet the solution seemed to oscillate around the equilibrium point (rather than approaching it from below, as we would expect for a solution that started below the equilibrium value).

1.4, Problem 15

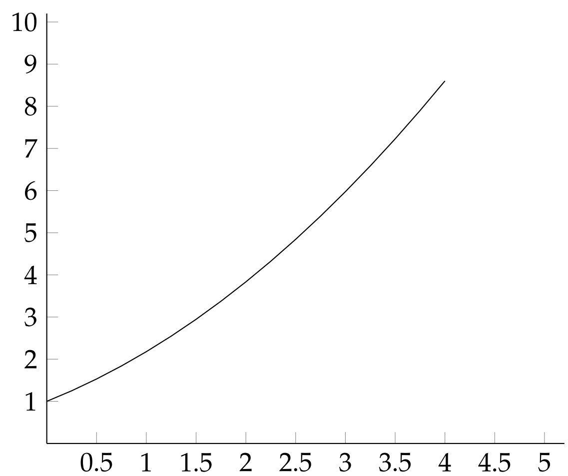
- $\Delta t = 1$:



- $\Delta t = 0.5$:



- $\Delta t = 0.25$:



The actual solution to the initial value problem should be some quadratic function.

Part 2

1.5, Problem 2

Since $0 < y(0) < 2$ and $y_2(t) = 2$, $y_3(t) = 0$ are equilibrium solutions for $\frac{dy}{dt} = f(y)$, it is the case that $y(t)$ that solves the initial value problem with $y(0) = 1$ will be restricted between 0 and 2 for all t.

1.5, Problem 3

For the initial condition $y(0) = 1$, we can see that it is, in a sense, trapped between $y_1(t)$ and $y_2(t)$, meaning that $y(t)$ that solves the initial value problem will be restricted between $y_1(t) = t + 2$ and $y_2(t) = -t^2$.

1.5, Problem 12

(a)

$$\frac{dy_1}{dt} = -\frac{1}{(t-1)^2}$$

$$
\begin{aligned}
&= -y_1^2 \\
\frac{dy_2}{dt} &= -\frac{1}{(t-2)^2} \\
&= -y_2^2
\end{aligned}
$$

(b) If $-1 < y(0) < -1/2$, we know that y will be of the form $\frac{1}{t+y(0)}$, as this satisfies the initial value problem, and $f(y)$, $\frac{\partial f}{\partial y}$ are continuous in a region about $(0, y(0))$ (satisfying the uniqueness condition).

1.5, Problem 14

$$
\begin{aligned}
\frac{dy}{dt} &= \frac{1}{(y+1)(t-2)} \\
\int (y+1)\ dy &= \int \frac{1}{t-2}\ dt \\
\frac{1}{2}(y+1)^2 &= \ln|t-2| + C \\
y &= \sqrt{2\ln|t-2| + K} - 1.
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
0 &= \sqrt{2\ln|(0)-2| + K} - 1 \\
2\ln|-2| + K &= 1 \\
K &= \frac{1}{2} - \ln 2.
\end{aligned}
$$

The solution is defined for all $t \neq 2$ and $y \neq -1$, implying that the solution's domain is $-2 < t < 2$.. The solution's slope blows up (to negative infinity) as it approaches the edge of its domain.

1.5, Problem 15

$$
\begin{aligned}
\frac{dy}{dt} &= \frac{1}{(y+2)^2} \\
\int (y+2)^2\ dy &= \int\ dt \\
\frac{1}{3}(y+2)^3 &= t + C \\
y &= \sqrt[3]{3t + K} - 2.
\end{aligned}
$$

Evaluating the initial condition, we have

$$
\begin{aligned}
1 &= \sqrt[3]{3(0) + K} - 2 \\
3 &= \sqrt[3]{K} \\
K &= 27.
\end{aligned}
$$

In particular, since $y(0) > -2$, the allowed values for y are $-2 < y < 2$, meaning $-9 < t < 9$.