

Solution (18.1):

- (a) The function $f(z) = z^n$ is analytic on $\mathbb{C} \cup \{\infty\}$.
- (b) The functions $f(z) = \sin(z)$ is analytic on \mathbb{C} , $f(z) = \cos(z)$ is analytic on $\mathbb{C} \cup \{\infty\}$, while $f(z) = \tan(z)$ is analytic everywhere except for singularities at $n\pi/2$.
- (c) The function $f(z) = |z|$ is analytic nowhere.
- (d) The function $f(z) = \frac{z-i}{z+1}$ is analytic everywhere except for $z = -1$.
- (e) The function $f(z) = \frac{z^2+1}{z}$ is analytic everywhere except for $z = 0$.
- (f) The function $f(z) = \frac{p_n(z)}{q_m(z)}$ is analytic everywhere except for the roots of q .
- (g) The function $x^2 + y^2$ is analytic nowhere.
- (h) The function e^z is analytic on \mathbb{C} .
- (i) The function e^{-iy} is analytic nowhere.
- (j) The function $\ln(z)$ is analytic everywhere except for $(-\infty, 0]$.

Solution (18.2): Let $w(z) = u(x, y) + iv(x, y)$. Then,

$$\begin{aligned} i \frac{\partial}{\partial x} (w(x + iy)) - \frac{\partial}{\partial y} (w(x + iy)) &= i \frac{\partial}{\partial x} (u(x, y) + iv(x, y)) - \frac{\partial}{\partial y} (u(x, y) + iv(x, y)) \\ &= i \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) - \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right). \end{aligned}$$

Solution (18.4):

Solution (18.5):

Solution (18.6):

Solution (18.7):

Solution (18.11):

Solution (18.14):

Solution (18.15):

Solution (18.18):