## 1.3.3

Let $u$ and $v$ be adjacent vertices in $G$. Prove that $uv$ belongs to at least $d(u) + d(v) - n(G)$ triangles in $G$.

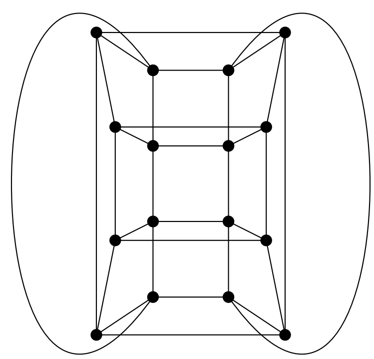
## Solution

Let $u \leftrightarrow v \in G$. In order for $uv$ to be in a triangle in $G$, $u$ and $v$ must share a common neighbor. By using the property of inclusion and exclusion, we can find the set as follows:

$$\begin{aligned}
|N(u) \cup N(v)| &= |N(u)| + |N(v)| - |N(u) \cap N(v)| \\
|N(u) \cap N(v)| &= |N(u)| + |N(v)| - |N(u) \cup N(v)| \\
&= d(u) + d(v) - n(G)
\end{aligned}$$

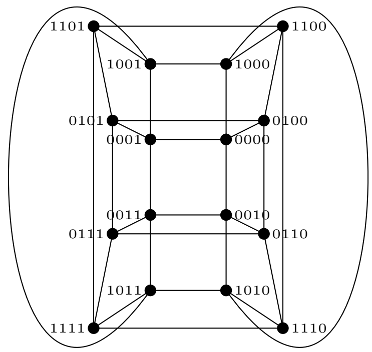
## 1.3.4

Prove that the graph below is isomorphic to $Q_4$.



## Solution

We can assign tuples to the graph as follows:




## 1.3.6

Given graphs $G$ and $H$, determine the number of components and maximum degree in $G+H$ in terms of the parameters for $G$ and $H$.

## 1.3.7

Determine the maximum number of edges in a bipartite subgraph of $P_n$, $C_n$, and $K_n$.

## 1.3.26 (a)

Count the 6-cycles in $Q_3$.