



## Federated Learning with Only Positive Labels

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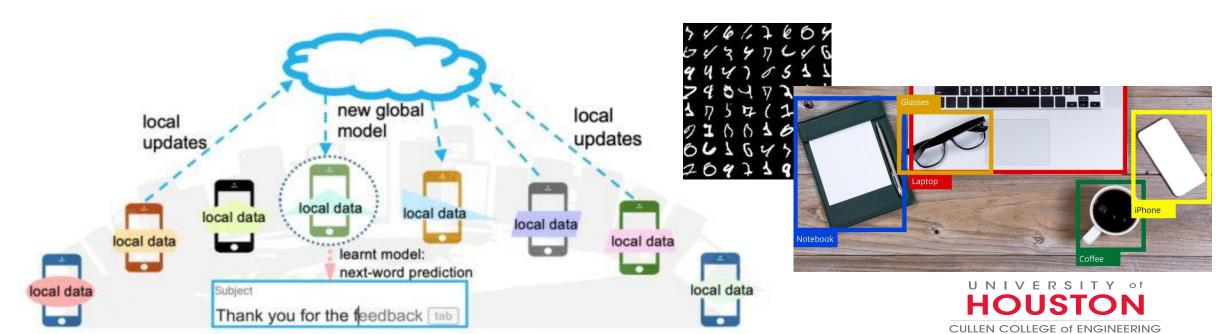
- Introduction
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### Federated Learning:

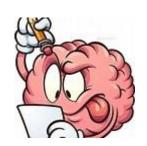
- > Server sends the current global model to users
- Each user update the model with its local data, and send it to server
- > Server average (FedAvg) the deltas and updates global model



## Learning-based user identification

- > Examples: face, voiceprint, fingerprint, etc.
- ➤ Goal: learn discriminative features
- > Challenge: large dataset and privacy concerns

Multiple users?



**Distributed data?** 

Sensitive data?

**Use federated learning!!!** 



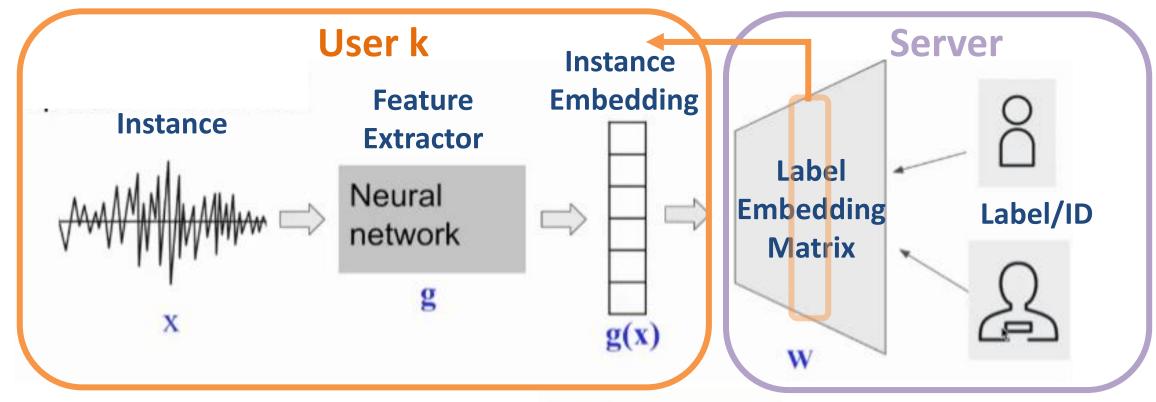




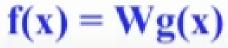




### Federated learning for user identification











# Score or logit: f(x) = Wg(x)

- > One wants large scores for positive instance and label pairs
- > One want small scores for negative instance and label pairs

$$\ell_{\text{cl}}(f(\boldsymbol{x}), y) = \underbrace{\alpha \cdot \left(\boldsymbol{d}(g_{\boldsymbol{\theta}}(\boldsymbol{x}), \boldsymbol{w}_{y})\right)^{2}}_{\ell_{\text{cl}}^{\text{pos}}(f(\boldsymbol{x}), y)} + \underbrace{\beta \cdot \sum_{c \neq y} \left(\max\left\{0, \nu - \boldsymbol{d}(g_{\boldsymbol{\theta}}(\boldsymbol{x}), \boldsymbol{w}_{c})\right\}\right)^{2}}_{\ell_{\text{cl}}^{\text{neg}}(f(\boldsymbol{x}), y)}$$







If only trained on positive loss...

$$g(x) = w_1 = ... = w_K$$
 for any x.

$$\ell_{\text{cl}}(f(\boldsymbol{x}), y) = \underbrace{\alpha \cdot \left(\boldsymbol{d}(g_{\boldsymbol{\theta}}(\boldsymbol{x}), \boldsymbol{w}_{y})\right)^{2}}_{\ell_{\text{cl}}^{\text{pos}}(f(\boldsymbol{x}), y)} + \underbrace{\beta \cdot \sum_{c \neq y} \left(\max\left\{0, \nu - \boldsymbol{d}(g_{\boldsymbol{\theta}}(\boldsymbol{x}), \boldsymbol{w}_{c})\right\}\right)^{2}}_{\ell_{\text{cl}}^{\text{neg}}(f(\boldsymbol{x}), y)}$$

Positive Loss = 0
Negative Loss max
Not work



Minimize positive loss while keeping label embeddings spread-out





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# Algorithm

## Federated Averaging with Spreadout (FedAwS):

- > The trained label embeddings should be geometric saperated.
- > Add regulation term to spread them out by a margin v
- ➤ User sends the updated feature extractor and own label embedding to server. Server averages the feature extractor and compute the regulation.

$$\operatorname{reg}_{\operatorname{sp}}(W) = \sum_{c \in [C]} \sum_{c' \neq c} \left( \max \left\{ 0, \nu - d(\boldsymbol{w}_c, \boldsymbol{w}_{c'}) \right\} \right)^2.$$





# Algorithm

### FedAwS: two challenges

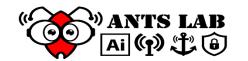
- > hyperparameter v is hard to determine
- > the number of user (C) is huge => expensive computation

Solution: stochastic "hard" negative mining

Choose Top-k closest label embeddings in one subset Choose v to be Top-(k+1) closest distance

$$\operatorname{reg}_{\operatorname{sp}}(W) = \sum_{c \in [C]} \sum_{c' \neq c} \left( \max \left\{ 0, \nu - \boldsymbol{d}(\boldsymbol{w}_c, \boldsymbol{w}_{c'}) \right\} \right)^2.$$

$$\operatorname{reg}_{\operatorname{sp}}^{\operatorname{top}}(W) = \sum_{c \in \mathcal{C}^t} \sum_{\substack{y \in \mathcal{C}', \\ y \neq c}} -\boldsymbol{d}^2(\boldsymbol{w}_c, \boldsymbol{w}_y) \cdot [\![y \in \mathcal{N}_k(c)]\!],$$



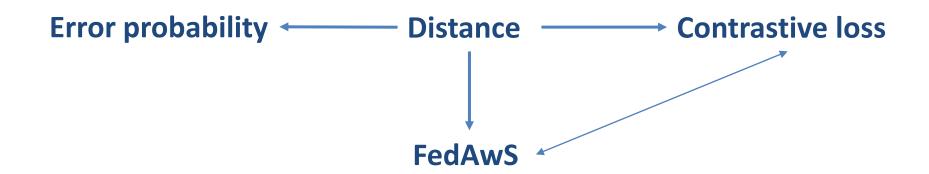


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- I. Reason why a simple spreadout will work
- II. Similar in shape, the cosine contrastive loss
- III. Relate FedAwS with cosine contrastive loss







### I. Reason why a simple spreadout will work

**Proposition 1.** Let the minimum distance between the class embeddings be  $\rho := \inf_{i \neq j} \boldsymbol{d}(\boldsymbol{w}_i, \boldsymbol{w}_j)$ , and the expected distance between the embeddings of an instance  $\boldsymbol{x}$  and its true class y be  $\epsilon = \mathbb{E}_{(\boldsymbol{x},y) \sim \mathrm{P_{XY}}} \boldsymbol{d}(g_{\boldsymbol{\theta}}(\boldsymbol{x}), \boldsymbol{w}_y)$ . Then, the probability of misclassification satisfies

$$P(\exists z \neq y \text{ s.t. } d(g_{\theta}(x), w_y) \geq d(g_{\theta}(x), w_z)) \leq 2\epsilon/\rho.$$

Proof:

$$\begin{split} \mathrm{P} \big( \exists z \neq y \text{ s.t. } \boldsymbol{d}(g_{\boldsymbol{\theta}}(\boldsymbol{x}), \boldsymbol{w}_y) \geq \boldsymbol{d}(g_{\boldsymbol{\theta}}(\boldsymbol{x}), \boldsymbol{w}_z) \big) \\ \leq \mathrm{P} \big( \boldsymbol{d}(g_{\boldsymbol{\theta}}(\boldsymbol{x}), \boldsymbol{w}_y) \geq \frac{\rho}{2} \big) \\ \text{Markov} \\ \text{Inequality} \quad \leq \frac{2 \mathbb{E}_{(\boldsymbol{x}, y) \sim \mathrm{P_{XY}}} \boldsymbol{d}(g_{\boldsymbol{\theta}}(\boldsymbol{x}), \boldsymbol{w}_y)}{\rho} = \frac{2\epsilon}{\rho}. \end{split}$$

The error probability is bounded by the intra-class distance divided by the minimum inter-class distance!





### II. Cosine contrastive loss

**Definition 1** (Cosine contrastive loss). Given an instance and label pair  $(\mathbf{x}, y)$  and the scorer  $f(\mathbf{x})$  in (1), the cosine contrastive loss takes the following form.

$$\ell_{\text{ccl}}(f(\boldsymbol{x}), y) = (\boldsymbol{d}_{\text{cos}}(g_{\boldsymbol{\theta}}(\boldsymbol{x}), \boldsymbol{w}_y))^2 + \sum_{c \neq y} (\max\{0, \nu - \boldsymbol{d}_{\text{cos}}(g_{\boldsymbol{\theta}}(\boldsymbol{x}), \boldsymbol{w}_c)\})^2. \quad (11)$$

$$d_{\cos}(\boldsymbol{u}, \boldsymbol{u}') = 1 - \boldsymbol{u}^{\top} \boldsymbol{u}' \quad \forall \, \boldsymbol{u}, \, \boldsymbol{u}' \in \mathbb{R}^d.$$

$$\ell_{\text{ccl}}(f(\boldsymbol{x}), y) = (1 - s_y)^2 + \sum_{c \neq y} (\max\{0, \nu - 1 + s_c\})^2$$





### III. Relate FedAwS with cosine contrastive loss

FedAwS objective:

$$\ell_{\rm sp}(f(\boldsymbol{x}), y) = (1 - s_y)^2 + \sum_{c \neq y} \left( \max \left\{ 0, \nu - 1 + \boldsymbol{w}_y^{\mathsf{T}} \boldsymbol{w}_c \right\} \right)^2,$$

Cosine contrastive loss:

$$\ell_{\text{ccl}}(f(\boldsymbol{x}), y) = (1 - s_y)^2 + \sum_{c \neq y} (\max\{0, \nu - 1 + s_c\})^2$$

$$|\Delta_c| \leq 2(1+2\nu) \cdot \left| \boldsymbol{w}_c^{\top} \boldsymbol{r}_{\boldsymbol{x},y} \right|.$$
 where  $\boldsymbol{r}_{\boldsymbol{x},y} = \boldsymbol{w}_y - g_{\boldsymbol{\theta}}(\boldsymbol{x})$  approach 0 during local training





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# **Evaluation**

**Evaluation Method:** classification with one class settings

Dataset: [CIFAR-10, CIFAR-100], [AmazonCat, WIKIAHRC, Amazon670K]

#### **Baseline:**

- 1. Training with only positive loss
- 2. Training with positive loss wit fixed label embeddings (avoid collapsing)
- 3. Softmax (oracle)

Model architecture: resnet, embedding dimension [64/512]

Training setup: 4K labels and users are selected in one round





# **Evaluation**

#### **Result: small dataset**

Dataset	Model	Baseline-1	Baseline-2	FedAwS	Softmax (Oracle)
CIFAR-10	RESNET-8	10.7	83.3	86.3	88.4
CIFAR-10	RESNET-32	9.8	92.1	92.4	92.4
CIFAR-100	RESNET-32	1.0	65.1	67.9	68.0
CIFAR-100	RESNET-56	1.1	67.5	69.6	70.0

### **Observation:**

- Training on positive labels gives very poor result due to collapse
- But once label embeddings are fixed, even randomly chosen, results are surprisingly good
- The proposed FedAwS outperforms two baselines and approaches softmax





# **Evaluation**

### Result: multi-lable dataset

K-10 \(\frac{1}{2} - 10\)

Dataset	#Features	#Labels	#TrainPoints	#TestPoints	Avg. #I/L	Avg. #L/I
AMAZONCAT	203,882	13,330	1,186,239	306,782	448.57	5.04
WIKILSHTC	1,617,899	325,056	1,778,351	587,084	17.46	3.19
AMAZON670K	135,909	670,091	490,449	153,025	3.99	5.45

Baseline 2 fails because the class# is too big to be separated

$K=10, \lambda=10$		rederated Learning with Only Positives			Oracie	
		Baseline-1	Baseline-2	FedAwS	Softmax	SLEEC
AMAZONCAT	P@1	3.4	64.1	92.1	92.1	90.5
	P@3	3.2	46.8	70.8	77.9	76.3
	P@5	3.1	32.6	58.7	62.3	61.5
Amazon670K	P@1	0.0	4.3	33.1	35.2	35.1
	P@3	0.0	2.8	29.6	31.6	31.3
	P@5	0.0	2.2	27.4	29.5	28.6
WIKILSHTC	P@1	7.6	7.9	37.2	54.1	54.8
	P@3	4.5	3.4	22.6	38.8	33.4
	P@5	2.8	2.6	16.2	29.9	23.9

Federated Learning with Only Positives



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# Conclusion

**Centralized** 

Share instance and label embedding

Learn from positive and negative labels

**FedAwS** 

Share label emb to server

Learn from positive, and spreadout labels

**FedUV** 

No emb shared to other users or server

Fit instance emb to ECC code (ensured to separate)





### Conclusion

- This work studied a novel learning setting, federated learning with only positive labels, and propose FedAwS that learn without negative instance and label pairs.
- It proves that strong geometric regulation can replace the negative sampling.
- The method achieves near oracle performance.





