LDL-precoded FTN Signaling with Power Allocation in The Block Fading Channel

Y. Li, J. Wang, S. Xiao, G. Wu, and W. Tang

Presenter: Yuan Li

National Key Lab on Communications

University of Electronic Science and Technology of China

Outline

- Motivation
- System Model
- Maximum average information rate
- Outage capacity
- Simulation Results
- Summary

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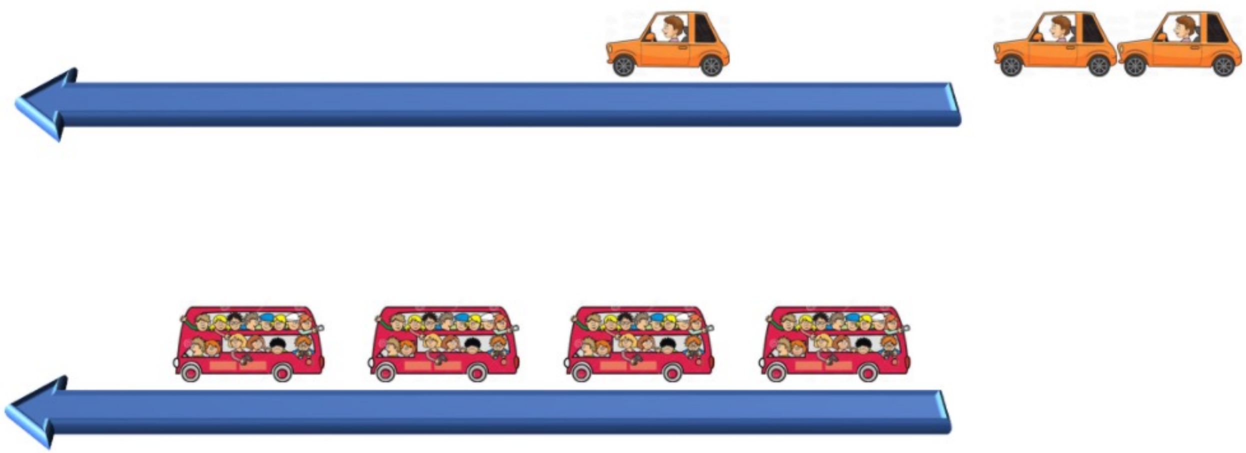
Background

Spectrum efficiency is a crucial quota in communication systems.

$$SE = \frac{R}{B}$$

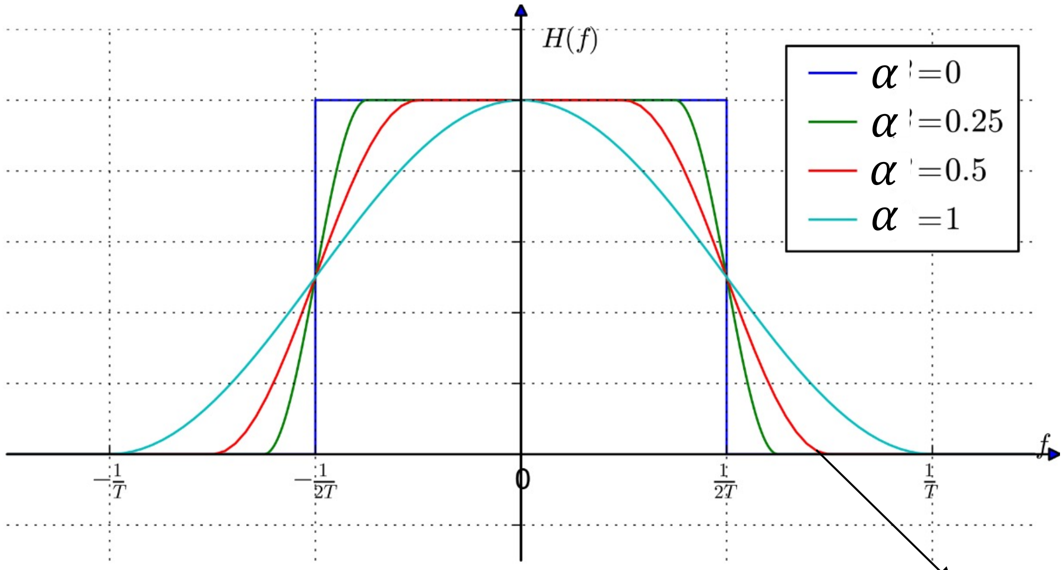
where R is the information rate in bps and B is the bandwidth in Hz.



Just like using high-order modulations in communication systems!

Source: https://www.bluedanube.com/blog/2018/07/23/why-does-spectral-efficiency-matter

Background



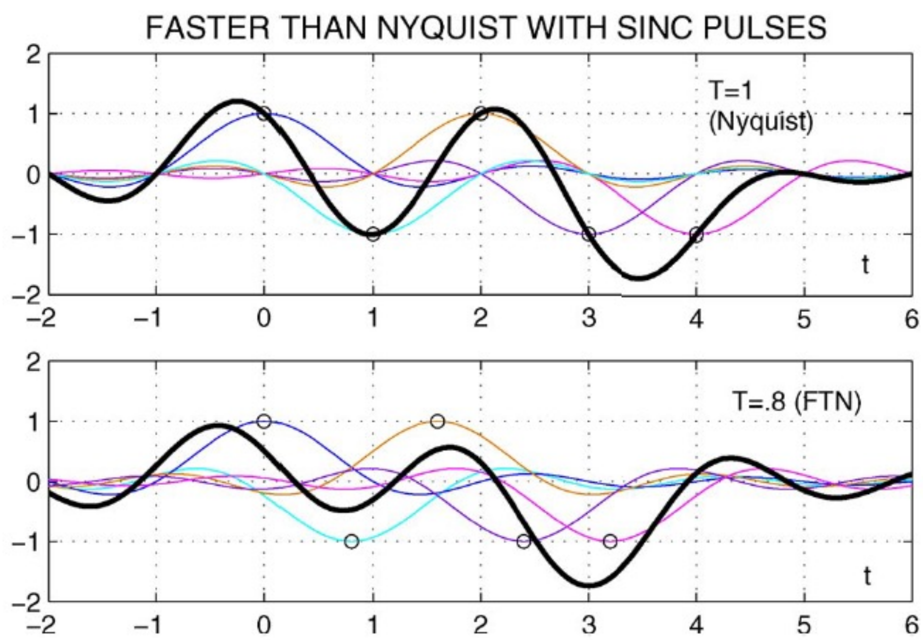
$$\frac{1+\alpha}{2T} = (1+\alpha)W$$

Source: https://en.wikipedia.org/wiki/Raised-cosine_filter

Background

- **What is faster-than-Nyquist (FTN) signaling?**

$$x(t) = \sum_n x_n \phi(t - n\tau T) \quad \tau \in (0,1)$$



Source: J. B. Anderson, F. Rusek and V. Owall, "Faster-than-Nyquist signaling," *Proc. IEEE*, vol.101, no. 8, pp. 1817-1830, Aug. 2013.

Background

- **Drawbacks of decoding algorithms for FTN signaling at receiver side**
- Complexity increases exponentially as the considered ISI taps
- Detection complexity may be excessive for low τ
- **Typical schemes for precoded FTN signaling**
- Singular-value decomposition (SVD) based method
- Square root decomposition based method
- Cholesky decomposition based method

Background

- **Complexity of typical schemes for precoded FTN signaling**

Scheme	Formulation	flops
SVD	$\mathbf{G} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^T$	$O(2N^3)$
Square-root decomposition	$\mathbf{G} = \mathbf{G}^{\frac{1}{2}}\mathbf{G}^{\frac{1}{2}}$	$O(N^3)$
Cholesky decomposition	$\mathbf{G} = \mathbf{L}\mathbf{L}^T$	$O(N^3/3)$

Wide application range, but high complexity

limited application range, but low complexity

✓ **LDL decomposition**

Background

- **LDL decomposition**

For a given positive definite matrix $\mathbf{G}$, it can be decomposed as $\mathbf{G} = \mathbf{LDL}^T$, where $\mathbf{L}$ is a lower triangular matrix and $\mathbf{D}$ is a diagonal matrix.

In contrast, if $\mathbf{G}$ is indefinite, $\mathbf{MGM}^T = \mathbf{LBL}^T$ holds, where $\mathbf{M}$ is the permutation matrix and $\mathbf{B}$ is the block diagonal matrix with $1{\times}1$ and $2{\times}2$ blocks on its diagonal.

For the sake of unity, we use $\mathbf{G} = \mathbf{LDL}^T$ to represent these two forms.

R. Horn and C. Johnson, Matrix Analysis, Cambridge Univ. Press, 1985.
G. Golub, and C. Loan, Matrix Computations, 3rd ed. Baltimore: Johns Hopkins Univ. Press, 1996.

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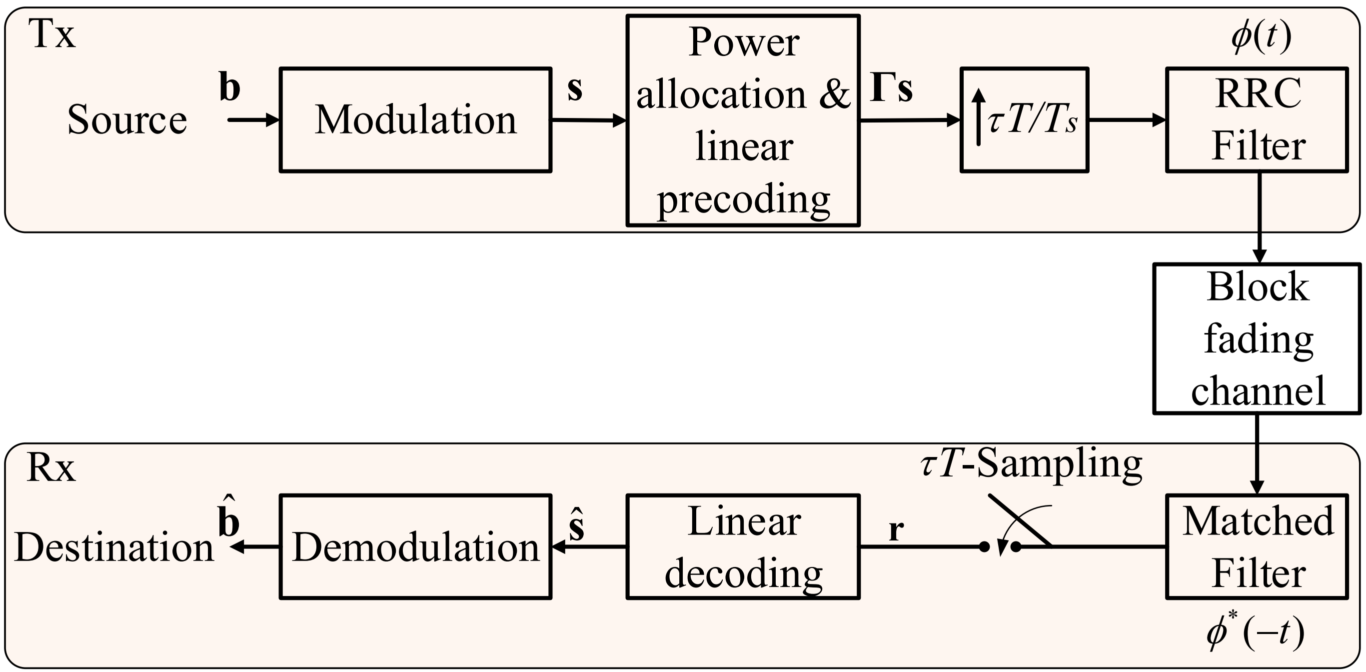
System Model

- ✓ An LDL-precoded FTN communication system with power allocation
- ✓ Block fading channel



System Model

- **Power allocation and linear precoding**

$$\boldsymbol{\Gamma} = (\mathbf{L}^T)^{-1}\mathbf{P},$$

where $\mathbf{L}$ is obtained by $\mathbf{G} = \mathbf{LDL}^{\boldsymbol{T}}$, $\mathbf{P} = diag\{\sqrt{p_0}, \cdots, \sqrt{p_{N-1}}\}$.

- **Received signal**

$$r(t) = h\sum_n x_n g(t - n\tau T) + \eta(t)$$

$$\mathbf{r} = h\mathbf{Gx} + \boldsymbol{\eta} = h\mathbf{G\Gamma s} + \boldsymbol{\eta} \Longrightarrow \text{colored noise, } E[\boldsymbol{\eta}\boldsymbol{\eta}^H] = N_0\mathbf{G}$$

- **Linear decoding**

$$\boldsymbol{\Phi} = \boldsymbol{\Gamma}^T$$

- **Estimate of symbol vector**

$$\hat{\mathbf{s}} = \boldsymbol{\Phi}\mathbf{r}$$

$$= h\mathbf{PDPs} + \boldsymbol{\eta}_u \Longrightarrow E[\boldsymbol{\eta}_u\boldsymbol{\eta}_u^H] = N_0\mathbf{PDP}$$

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System Model

- **The positive definiteness of G**

As the block length $N \to \infty$, $\mathbf{G}$ is positive definite for $\tau \in \left[\frac{1}{1+\alpha}, 1\right)$, $\mathbf{G}$ is indefinite for $\tau \in (0, \frac{1}{1+\alpha})$.

As mentioned before, $\mathbf{G} = \mathbf{LDL}^{\boldsymbol{T}}$, where $\mathbf{L}$ is a (permutated) lower triangular matrix and $\mathbf{D}$ is the block diagonal matrix with $1\times1$and 2×2 blocks on its diagonal. The $1\times1$and 2×2 diagonal blocks can be represented as d_k and $\begin{bmatrix} d_l & d_{l,l+1} \\ d_{l+1,l} & d_{l+1} \end{bmatrix}$, respectively. $k \in A, l \in B, and\ |A| + 2|B| = N$

Y. J. Daniel Kim, ``Properties of faster-than-Nyquist channel matrices and folded-spectrum, and their applications," *2016 IEEE Wireless Commun. and Netw. Conf.*, Doha, 2016, pp. 1-7.

Lagrange Method

- **Problem formulation**
  - Objective: maximize the average information rate
  - Variables: allocated power for each symbol
  - Constraint: block energy N
- **Instantaneous mutual information**

$$I_{ins}(\mathbf{s};\hat{\mathbf{s}}) = h(\hat{\mathbf{s}}) - h(\boldsymbol{\eta}_u)$$

$$\leq \frac{1}{2}\log_2\left((2\pi e)^N \det\left(\mathrm{Cov}(\hat{\mathbf{s}})\right)\right) - \frac{1}{2}\log_2\left((2\pi e)^N \det(N_0\mathbf{PDP})\right)$$

$$= \log_2(\det(\frac{||h||_2^2\mathbf{PD}^T\mathbf{P}}{N_0} + \mathbf{I}_N))$$

$$= \sum_{k=0}^{|A|-1}\log_2\left(1 + \frac{||h||_2^2 d_k p_k}{N_0}\right) + \sum_{k=0}^{|B|-1}\log_2(\rho_1 - \rho_2),$$

where $\rho_1 = (1 + \frac{||h||_2^2 d_l p_l}{N_0})(1 + \frac{||h||_2^2 d_{l+1} p_{l+1}}{N_0})$ and $\rho_2 = \frac{||h||_2^4 d_{l+1,l} d_{l,l+1} p_l p_{l+1}}{N_0^2}$

Lagrange Method

- **Transmit block energy**

$$E_{blk} = E[(\mathbf{\Gamma s})^H \mathbf{G\Gamma s}] = \mathrm{E}[\mathbf{s}^H \mathbf{PDPs}] = \sum_{i=0}^{N-1} d_i p_i$$

- **Lagrange Function**

$$\sum_{k=0}^{|A|-1} \log_2\left(1 + \frac{||h||_2^2 d_k p_k}{N_0}\right) + \sum_{k=0}^{|B|-1} \log_2(\rho_1 - \rho_2) + \beta(\sum_{i=0}^{N-1} d_i p_i - N)$$

Maximum average information rate

✓ $\tau \in \left[\frac{1}{1+\alpha}, 1\right)$ **LDL-PA-FTN**

$|B| = 0$. Solving the Lagrange function, we have

$$p_n = \frac{1}{d_n}$$

Let $\gamma = \frac{||h||_2^2 P\tau T}{N_0}$ be the instantaneous signal to noise ratio (SNR) and substitute $p_n = \frac{1}{d_n}$ into $I_{ins}(\mathbf{s}; \hat{\mathbf{s}})$, we have

$$\begin{aligned}\bar{R} &= \int_0^\infty \lim_{N\to\infty} \frac{\max\{I_{ins}(\mathbf{s}; \hat{\mathbf{s}})\}}{N\tau T} \Pr(\gamma) \mathrm{d}\gamma \\ &= \int_0^\infty \frac{1}{\tau T} \log_2(1+\gamma) \Pr(\gamma) \mathrm{d}\gamma \\ &\leq \frac{2W}{\tau} \log_2(1+\bar{\gamma}) \text{ bits/s}\end{aligned}$$

where $W = \frac{1}{2T}$ and $\bar{\gamma} = E[\gamma]$.

T. Ishihara and S. Sugiura, "SVD-precoded faster-than-Nyquist signaling with optimal and truncated power allocation," *IEEE Trans. Wireless Commun.*, vol. 18, no. 12, pp. 5909-5923, Dec. 2019.

Maximum average information rate

$$\sum_{k=0}^{|A|-1} \log_2\left(1+\frac{||h||_2^2 d_k p_k}{N_0}\right)+\sum_{k=0}^{|B|-1} \log_2(\rho_1-\rho_2)+\beta(\sum_{i=0}^{N-1} d_i p_i - N)$$

- ✓ $\tau \in (0, \frac{1}{1+\alpha})$ **LDL-TPA-FTN**

B is not empty. Ignore B and set a threshold in A !

$$\sum_{k=0}^{|A|-1} \log_2\left(1+\frac{||h||_2^2 d_k p_k}{N_0}\right)+\beta(\sum_{k=0}^{|A|-1} d_k p_k - N)$$

In the same manner, we have

$$p_k = \begin{cases} \frac{N}{N_a d_k}, d_k \geq \theta \\ 0, \quad d_k < \theta \end{cases}$$

$$\bar{R}_1 \leq \frac{2W N_a}{\tau N} \log_2(1+\bar{\gamma}) \text{ bits/s}$$

Number of active symbols

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Outage capacity

$$\gamma = \frac{\left\| h \right\|_2^2 P\tau T}{N_0} \qquad p(\gamma) = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}$$

- **LDL-PA-FTN**

Given required minimum information rate R_{min}, we have

$$P_o(R_{min}) = \Pr\left[\frac{2\mathrm{W}}{\tau}\log_2(1+\gamma) \le R_{min}\right]$$

$$= \int_0^{2^{\frac{R_{min}\tau}{2W}}-1} p(\gamma)d\gamma$$

$$= 1 - \exp\left(-\frac{2^{\frac{R_{min}\tau}{2W}}-1}{\bar{\gamma}}\right)$$

Given the outage probability ε, we have

$$C_\varepsilon = \frac{2W}{\tau}\log_2\left(1 + \frac{F^{-1}(1-\varepsilon)P\tau T}{N_0}\right) \text{ bits/s}$$

where $F(v) = \Pr[\left\| h \right\|_2^2 > v]$.

Outage capacity

- **LDL-TPA-FTN**

Given required minimum information rate R_{min}, we have

$$Pt_o(R_{min}) = 1 - \exp\left(-\frac{2^{\frac{R_{min}\tau N}{2WN_a}-1}}{\bar{\gamma}}\right)$$

Given the outage probability ε, we have

$$Ct_\varepsilon = \frac{2WN_a}{\tau N}\log_2\left(1 + \frac{F^{-1}(1-\varepsilon)P\tau T}{N_0}\right) \text{ bits/s}$$

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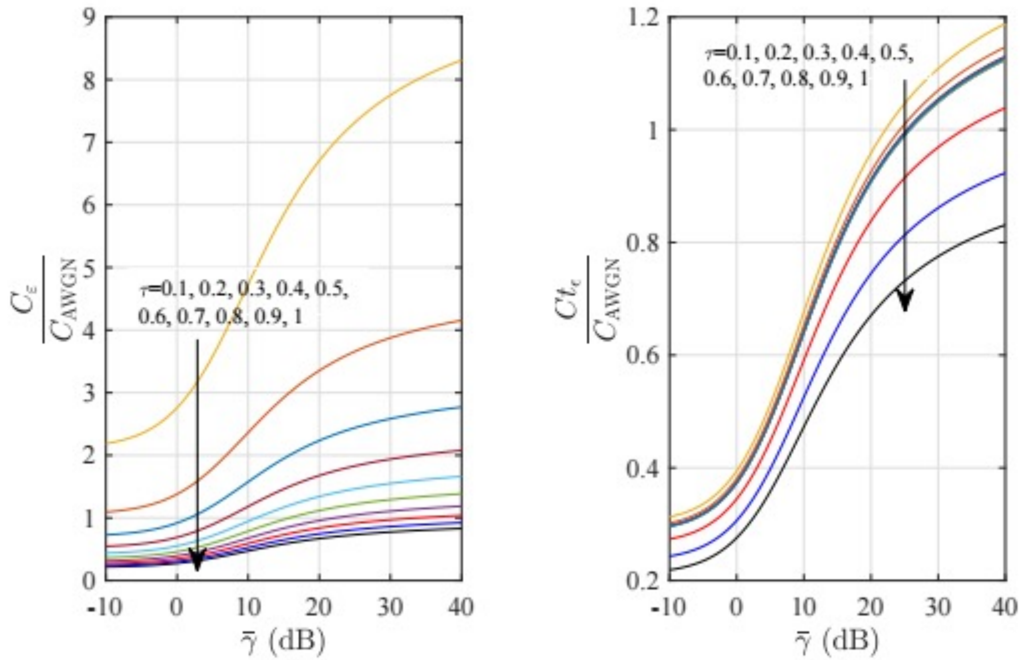
Simulation: Parameters

Name	Value
Orthogonal symbol duration (T)	1s
Sample interval (Ts)	0.01s
Modulation type	BPSK
Acceleration factor (τ)	{0.1, 0.2, … , 1}
Shape filter type	Normal RRC filter/ Blackman windowed RRC filter
Shape filter order	1000/10000
Roll-off factor (α)	0.3
Channel type	Rayleigh block fading channel
Block length (N)	1000
Outage probability (ε)	0.1

Simulation: ε −outage capacity to AWGN capacity ratio

$\varepsilon = 0.1$

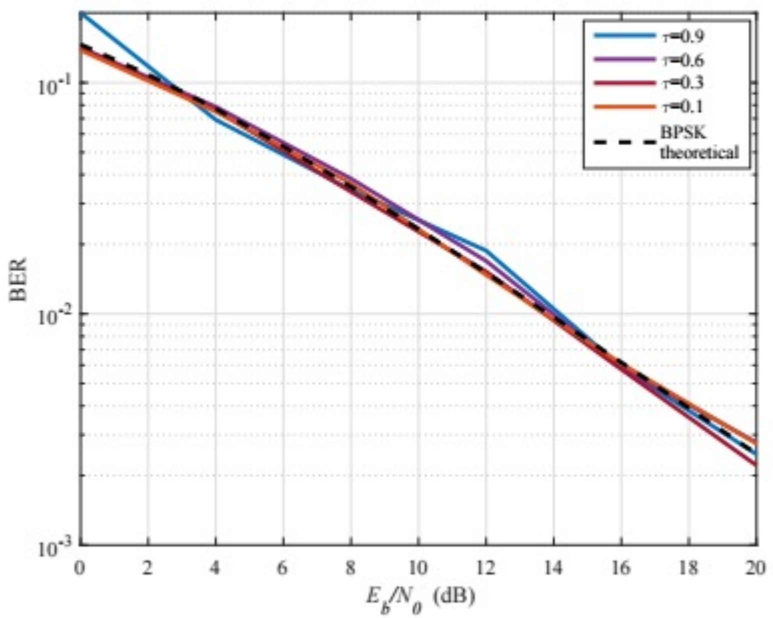


(a) LDL-PA-FTN (b) LDL-TPA-FTN

$\theta = 10^{-8}$

- The gain in (a) is not able to obtain.
- Ct_ε/C_{AWGN} increases as τ decreases, but its improvement is small for $\tau \leq 0.7$

Simulation: BER of LDL-PA/TPA-FTN in the block fading channel



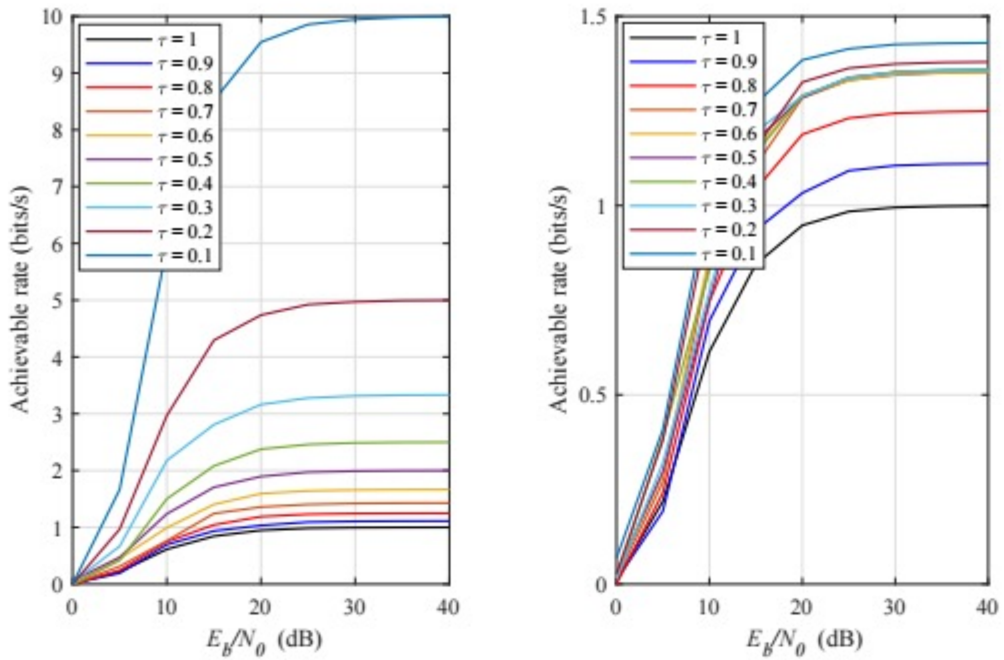
- The incurred ISI by FTN is well handled.

Simulation: Achievable rate

Packet error rate

$$\frac{N_a(1-PER)}{\tau NT}$$

$\theta = 10^{-8}$

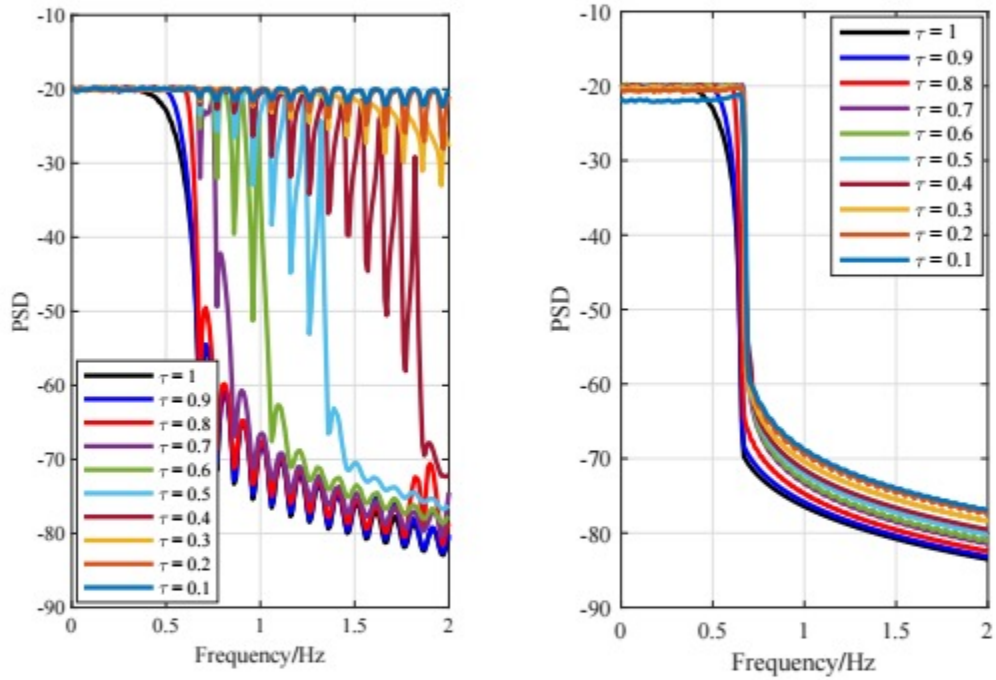


(a) Normal RRC filter with the order of 1000.

(b) Blackman windowed RRC filter with the order of 10000.

Simulation: PSD

$\theta = 10^{-8}$



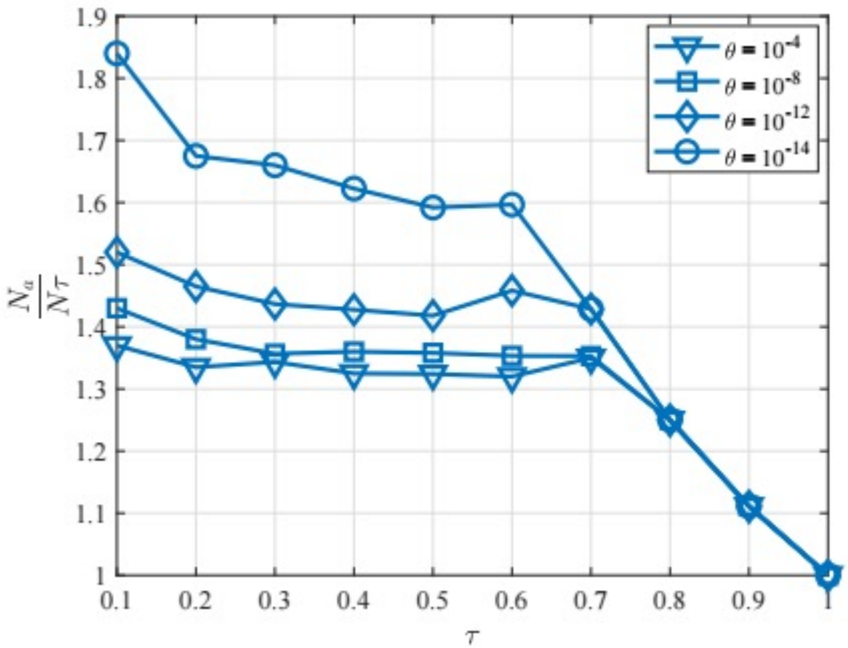
(a) Normal RRC filter with the order of 1000.

(b) Blackman windowed RRC filter with the order of 10000.

- A trade-off should be made between the side-lobe suppression of the PSD and achievable rate.

Simulation: Achievable rate

$$\frac{N_a}{\tau N}$$



- The performance gain of LDL-TPA-FTN depends on how small θ is.
- The precision of the decomposition of G should be considered carefully for LDL-TPA-FTN for small τ.

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Summary

- ✓ A low complexity precoded FTN scheme with power allocation
- ✓ The threshold selection in LDL-TPA-FTN may be troublesome in practice.
- ✓ The proposed scheme needs further evaluation, e.g., PAPR, IBI, high-order modulation.

Thank you!