UNIVERSITY of HOUSTON
CULLEN COLLEGE of ENGINEERING

ANTS LAB

Towards Better Generalization of Adaptive Gradient Methods

Yingxue Zhou, Belhal Karimi, Jinxing Yu, Zhiqiang Xu and Ping Li

Baidu Research

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First-order gradient optimizers for deep learning

- SGD (Robbins & Monro, 1951)
  - + Momentum (Qian, 1999)
  - + Nesterov (Nesterov, 1983)
- AdaGrad (Duchi et al., 2011)
- RMSprop (Tieleman & Hinton, 2012)
- Adam (Kingma & Lei Ba, 2015)

Name	Update Rule
SGD	$\Delta\theta_t = -\alpha g_t$
Momentum	$m_t = \gamma m_{t-1} + (1-\gamma) g_t,$ $\Delta\theta_t = -\alpha\, m_t$
Adagrad	$G_t = G_{t-1} + g_t^2,$ $\Delta\theta_t = -\alpha\, g_t G_t^{-1/2}$
RMSprop	$v_t = \beta_2 v_{t-1} + (1-\beta_2) g_t^2,$ $\Delta\theta_t = -\alpha\, g_t v_t^{-1/2}$
Adam	$m_t = \beta_1 m_{t-1} + (1-\beta_1) g_t,$ $v_t = \beta_2 v_{t-1} + (1-\beta_2) g_t^2,$ $\hat{m}_t = m_t/(1-\beta_1^t),$ $\hat{v}_t = v_t/(1-\beta_2^t),$ $\Delta\theta_t = -\alpha\, \hat{m}_t \hat{v}_t^{-1/2}$

	Acceler (e.g. SGD, Nester	t Methods n, RMSProp)
Fast convergence		
Good generalization		
Stability for complex settings such as GAN	✗	✓

Stochastic non-convex optimization

★ Minimize the *population loss* $f(\mathbf{w})$ given n i.i.d. samples $\mathbf{z}_1, \ldots, \mathbf{z}_n$ from unknown distribution $\mathcal{P}$:

$$\min_{\mathbf{w} \in \mathcal{W}} f(\mathbf{w}) \triangleq \mathbb{E}_{\mathbf{z} \sim \mathcal{P}}[\ell(\mathbf{w}, \mathbf{z})]$$

- $\ell : \mathcal{W} \times \mathcal{Z} \mapsto \mathbb{R}$: non-convex loss function
- $\mathbf{z} \in \mathcal{Z}$: data point following unknown distribution $\mathcal{P}$

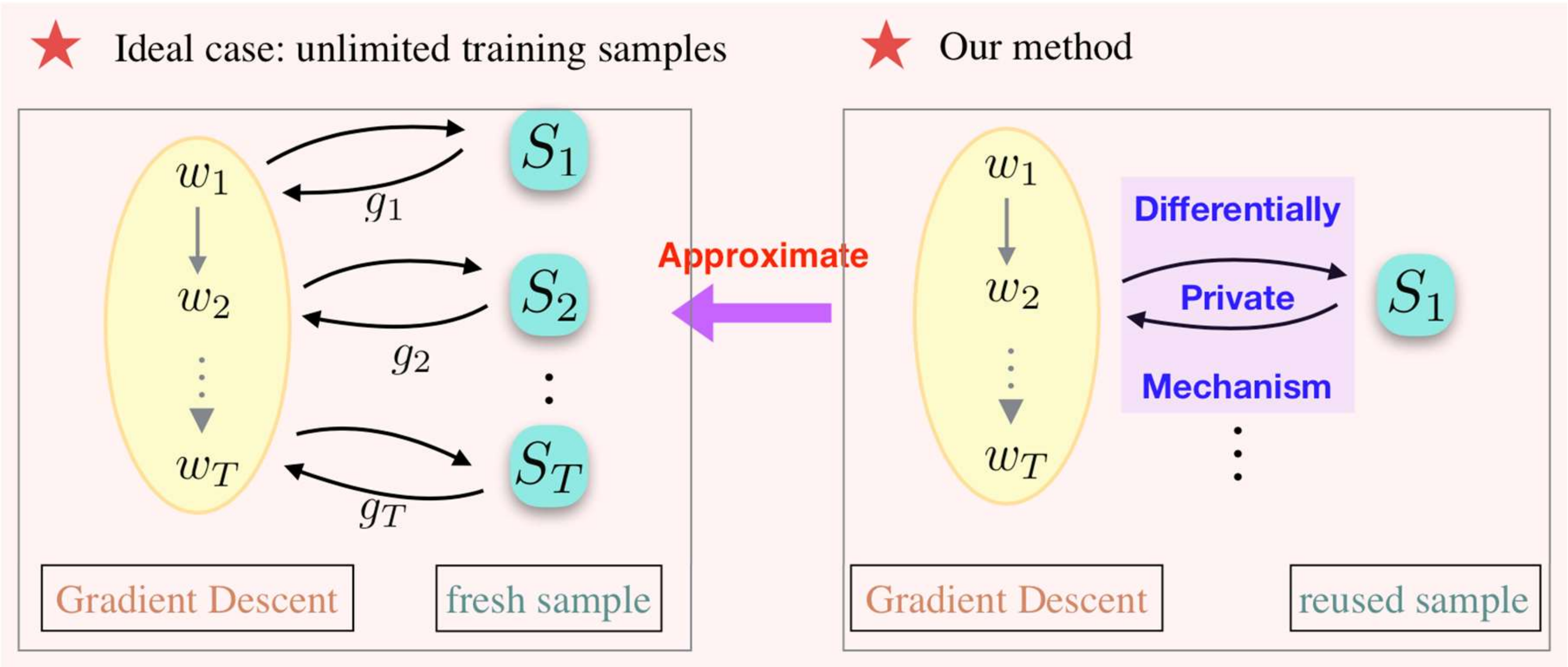
★ Minimize the empirical risk (ERM):

$$\min_{\mathbf{w} \in \mathcal{W}} \hat{f}(\mathbf{w}) \triangleq \frac{1}{n} \sum_{j=1}^{n} \ell\left(\mathbf{w}, \mathbf{z}_j\right)$$

★ Adaptive Gradient Methods: AdaGrad, RMSprop, Adam, AdaBound, etc

- Optimization bounds for the training objective, e.g., norm of the *empirical gradient*.
- Generalization bound, e.g., norm of the *population gradient*.

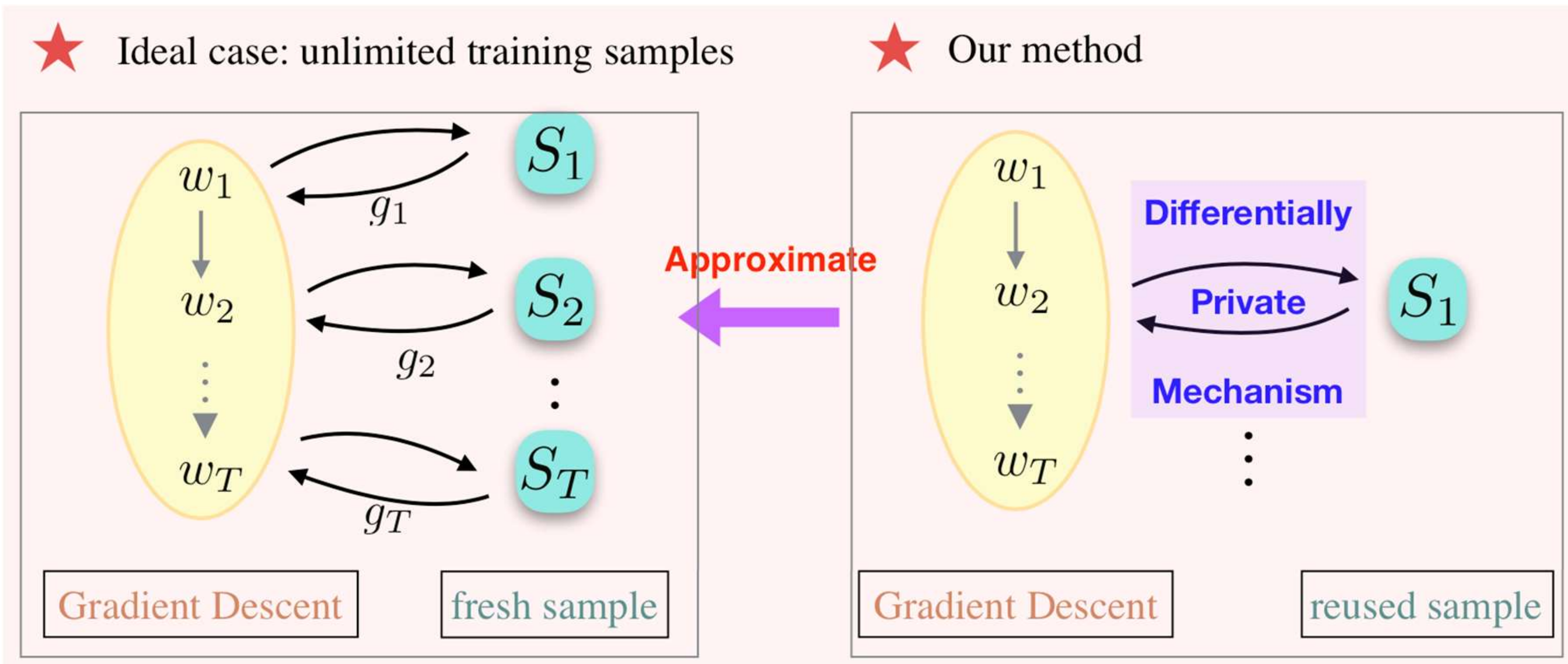
Main Idea



Ideal case: we have access to fresh samples in each iteration

- Sample gradients stay close to the population gradient across all iterations
- Leading to high probability bounds on the population stationary point

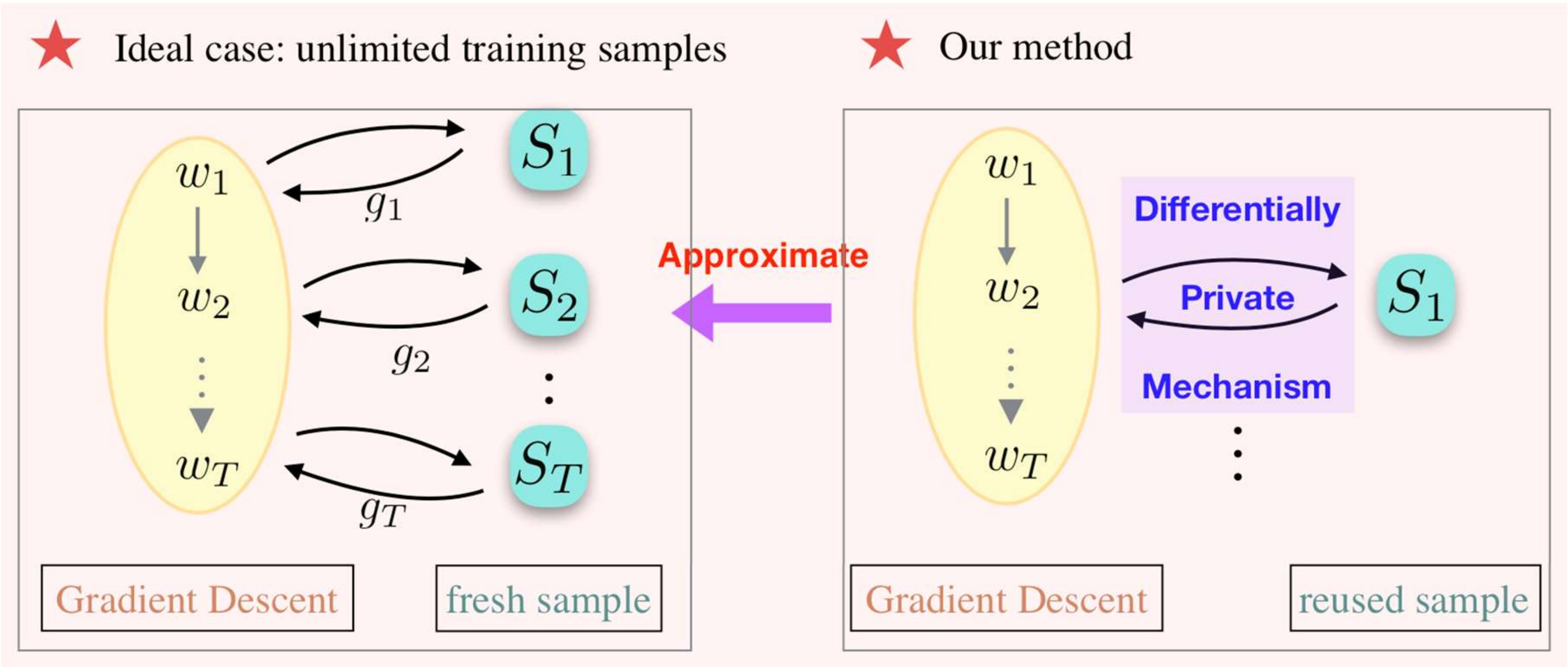
Main Idea



Our method: Stable Adaptive Gradient Descent Algorithm (SAGD)

- Training set S_t maintains the statistical nature of fresh data
- StGD is running multiple passes over the training data, but not doing ERM.

Main Idea



SAGD guarantees:

- ☑ Sample gradients concentrate to population gradients across all iterations
- ☑ Norm of population gradient converges with high probability
- ☑ An upper bound on the number of iterations

Differential Privacy

Main Point: There is no much difference between the output of the algorithm over two datasets that differ in one data element

Definition:

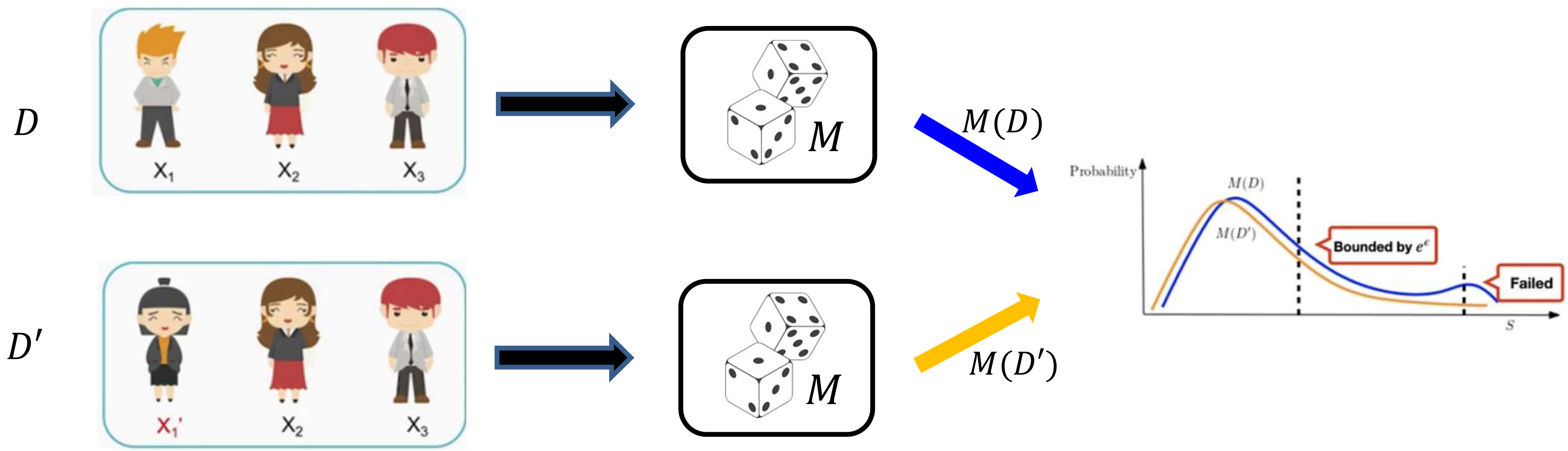
A randomized algorithm M is called (ϵ, δ)-differentially private if for all neighboring datasets D, D' and every outcome $S \subseteq Range(M)$

$$\Pr[\mathcal{M}(D) \in S] \leq e^{\epsilon} \Pr[\mathcal{M}(D') \in S] + \delta.$$

Privacy budget

Probability of failure

Differential Privacy



Basic properties of DP

- For any (ϵ, δ)-DP algorithm $M(\cdot)$, $B(M(\cdot))$ is still (ϵ, δ)-DP for any post-processing $B(\cdot)$

- For any query $q(\cdot)$, adding Gaussian noise $M(\cdot) = q(\cdot) + \mathcal{N}(0,\sigma^2)$ is (ϵ, δ)-DP when $\sigma \propto \frac{\Delta\sqrt{\log 1/\delta}}{\epsilon}$, where $\Delta = \max_{d(D,D')=1} \|q(D) - q(D')\|_2$ is the ℓ_2-norm sensitivity of $q(\cdot)$

- For any query $q(\cdot)$, adding Laplacian noise $M(\cdot) = q(\cdot) + Lap(\sigma)$ is ϵ-DP when $\sigma \propto \frac{\Delta}{\epsilon}$,where $\Delta = \max_{d(D,D')=1} \|q(D) - q(D')\|_1$ is the ℓ_1-norm sensitivity

SAGD with Laplace mechanism

★ SAGD with Laplace mechanism

Algorithm 1 SAGD with DGP-LAP

1: **Input**: Dataset S, certain loss $\ell(\cdot)$, initial point $\mathbf{w}_0$ and noise level σ.
2: Set noise level σ, iteration number T, and stepsize η_t.
3: **for** $t = 0, ..., T-1$ **do**
4: DPG-LAP: Compute full batch gradient on S:
$$\hat{\mathbf{g}}_t = \frac{1}{n}\sum_{j=1}^{n} \nabla \ell(\mathbf{w}_t, z_j).$$
5: Set $\tilde{\mathbf{g}}_t = \hat{\mathbf{g}}_t + \mathbf{b}_t$, where $\mathbf{b}_t^i$ is drawn i.i.d from $\mathrm{Lap}(\sigma)$ for all $i \in [d]$.
6: $\mathbf{m}_t = \tilde{\mathbf{g}}_t$ and $\mathbf{v}_t = (1-\beta_2)\sum_{i=1}^{t} \beta_2^{t-i} \tilde{\mathbf{g}}_i^2$.
7: $\mathbf{w}_{t+1} = \mathbf{w}_t - \eta_t \mathbf{m}_t / (\sqrt{\mathbf{v}_t} + \nu)$.
8: **end for**

- SAGD with DPG-LAP (Alg. 1) is $\left(\frac{\sqrt{T\ln(1/\delta)}G_1}{n\sigma}, \delta\right)$ -differentially private.

SAGD with Laplace mechanism

Lemma 1. *Let $\mathcal{A}$ be an (ϵ, δ)-differentially private gradient descent algorithm with access to training set S of size n. Let $\mathbf{w}_t = \mathcal{A}(S)$ be the parameter generated at iteration $t \in [T]$ and $\hat{\mathbf{g}}_t$ the empirical gradient on S. For any $\sigma > 0$, $\beta > 0$, if the privacy cost of $\mathcal{A}$ satisfies $\epsilon \leq \sigma/13$, $\delta \leq \sigma\beta/(26\ln(26/\sigma))$, and sample size $n \geq 2\ln(8/\delta)/\epsilon^2$, we then have*

$$\mathbb{P}\left\{|\hat{\mathbf{g}}_t^i - \mathbf{g}_t^i| \geq G\sigma\right\} \leq \beta\,, \quad \forall i \in [d] \text{ and } \forall t \in [T]\,.$$

- If the privacy cost ϵ is bounded by the estimation error, the differential privacy mechanism enables the reused training sample set to maintain statistical guarantees as if they were fresh samples

- SAGD with DPG-LAP (Alg. 1) is $\left(\frac{\sqrt{T\ln(1/\delta)}G_1}{n\sigma}, \delta\right)$ -differentially private.
- Upper bound on T: $\sqrt{T\ln(1/\delta)}G_1/(n\sigma) \leq \sigma/13$

SAGD with Laplace mechanism

★ High-probability bound: noisy gradient approximates population gradient.

$$\mathbb{P}\left\{\|\tilde{\mathrm{g}}_t - \mathrm{g}_t\| \geq \sqrt{d}\sigma(G+\mu)\right\} \leq d\beta + d\exp(-\mu), \ \forall t \in [T], \ \beta > 0 \ \text{ and } \mu > 0.$$

★ Non-asymptotic convergence rate (population gradient):

$$\min_{1\leq t\leq T} \|\nabla f(\mathbf{w}_t)\|^2 \leq \mathcal{O}\left(\frac{d\rho_{n,d}^2}{n^{2/3}}\right) \qquad \rho_{n,d} \triangleq \mathcal{O}(\ln n + \ln d)$$

with probability at least $1 - \mathcal{O}\left(1/\left(\rho_{n,d} n\right)\right)$.

- Given n samples, previous approaches can achieve $\mathcal{O}(1/\sqrt{n})$
- This paper can achieve $\mathcal{O}(1/n^{2/3})$

SAGD with Laplace mechanism

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- SAGD with DPG-LAP (Alg. 1) is $\left(\frac{\sqrt{T\ln(1/\delta)}G_1}{n\sigma}, \delta\right)$ -differentially private.
- Upper bound on T: $\sqrt{T\ln(1/\delta)}G_1/(n\sigma) \leq \sigma/13$

SAGD with Sparse vector technique

★ SAGD with Sparse vector technique

Algorithm 2 SAGD with DPG-SPARSE

Input: Dataset S, certain loss $\ell(\cdot)$, initial point $\mathbf{w}_0$.
Set noise level σ, iteration number T, and stepsize η_t.
Split S randomly into S_1 and S_2.
for $t = 0, ..., T-1$ **do**
    DPG-SPARSE: Compute full batch gradient on S_1 and S_2:
$$\hat{\mathbf{g}}_{S_1,t} = \frac{1}{|S_1|}\sum_{\mathbf{z}_j \in S_1} \nabla \ell(\mathbf{w}_t, \mathbf{z}_j), \qquad \hat{\mathbf{g}}_{S_2,t} = \frac{1}{|S_2|}\sum_{\mathbf{z}_j \in S_2} \nabla \ell(\mathbf{w}_t, \mathbf{z}_j).$$
    Sample $\gamma \sim \text{Lap}(2\sigma)$, $\tau \sim \text{Lap}(4\sigma)$.
    if $\|\hat{\mathbf{g}}_{S_1,t} - \hat{\mathbf{g}}_{S_2,t}\| + \gamma > \tau$ **then**
        $\tilde{\mathbf{g}}_t = \hat{\mathbf{g}}_{S_1,t} + \mathbf{b}_t$, where $\mathbf{b}_t^i$ is drawn i.i.d from $\text{Lap}(\sigma)$, for all $i \in [d]$.
    else
        $\tilde{\mathbf{g}}_t = \hat{\mathbf{g}}_{S_2,t}$
    end if
    $\mathbf{m}_t = \tilde{\mathbf{g}}_t$ and $\mathbf{v}_t = (1-\beta_2)\sum_{i=1}^{t} \beta_2^{t-i} \tilde{\mathbf{g}}_i^2$.
    $\mathbf{w}_{t+1} = \mathbf{w}_t - \eta_t \mathbf{m}_t / (\sqrt{\mathbf{v}_t} + \nu)$.
end for
Return: $\tilde{\mathbf{g}}_t$.

SAGD with Sparse vector technique

- SAGD with DPG-SPARSE is $\left(\frac{\sqrt{C_s \ln(2/\delta)}2G_1}{n\sigma}, \delta\right)$ -differentially private.
- C_s - the number of times the validation fails, i.e., $\|\hat{\mathrm{g}}_{S_1,t} - \hat{\mathrm{g}}_{S_2,t}\| + \gamma > \tau$ is true, over T iterations in SAGD.
- Imply an improved upper bound on T: $\sqrt{C_s \ln(1/\delta)}G_1/(n\sigma) \leq \sigma/13$

if $C_s = \mathcal{O}(\sqrt{T})$, the upper bound of T can be improved from $T \leq \mathcal{O}(n^2)$ to $T \leq \mathcal{O}(n^4)$

Mini-batch SAGD algorithm

Algorithm 3 Mini-Batch SAGD

1: **Input**: Dataset S, certain loss $\ell(\cdot)$, initial point $\mathbf{w}_0$.
2: Set noise level σ, epoch number T, batch size m, and stepsize η_t.
3: Split S into $B = n/m$ batches: $\{s_1, ..., s_B\}$.
4: **for** $epoch = 1, ..., T$ **do**
5: **for** $k = 1, ..., B$ **do**
6: Call DPG-LAP or DPG-SPARSE to compute $\tilde{\mathbf{g}}_t$.
7: $\mathbf{m}_t = \tilde{\mathbf{g}}_t$ and $\mathbf{v}_t = (1-\beta_2)\sum_{i=1}^{t} \beta_2^{t-i} \tilde{\mathbf{g}}_i^2$.
8: $\mathbf{w}_{t+1} = \mathbf{w}_t - \eta_t \mathbf{m}_t / (\sqrt{\mathbf{v}_t} + \nu)$.
9: **end for**
10: **end for**

★Non-asymptotic convergence rate (population gradient):

$$\min_{t=1,\ldots,T} \|\nabla f(\mathbf{w}_t)\|^2 \leq \mathcal{O}\left(\frac{\rho_{n,d}\left(f(\mathbf{w}_1) - f^\star\right)}{(mn)^{1/3}}\right) + \mathcal{O}\left(\frac{d\rho_{n,d}^2}{(mn)^{1/3}}\right)$$

- When $m = \sqrt{n}$, mini-batch SAGD achieves $\mathcal{O}(1/\sqrt{n})$ convergence rate
- When $m = n$, recover the full batch convergence rate

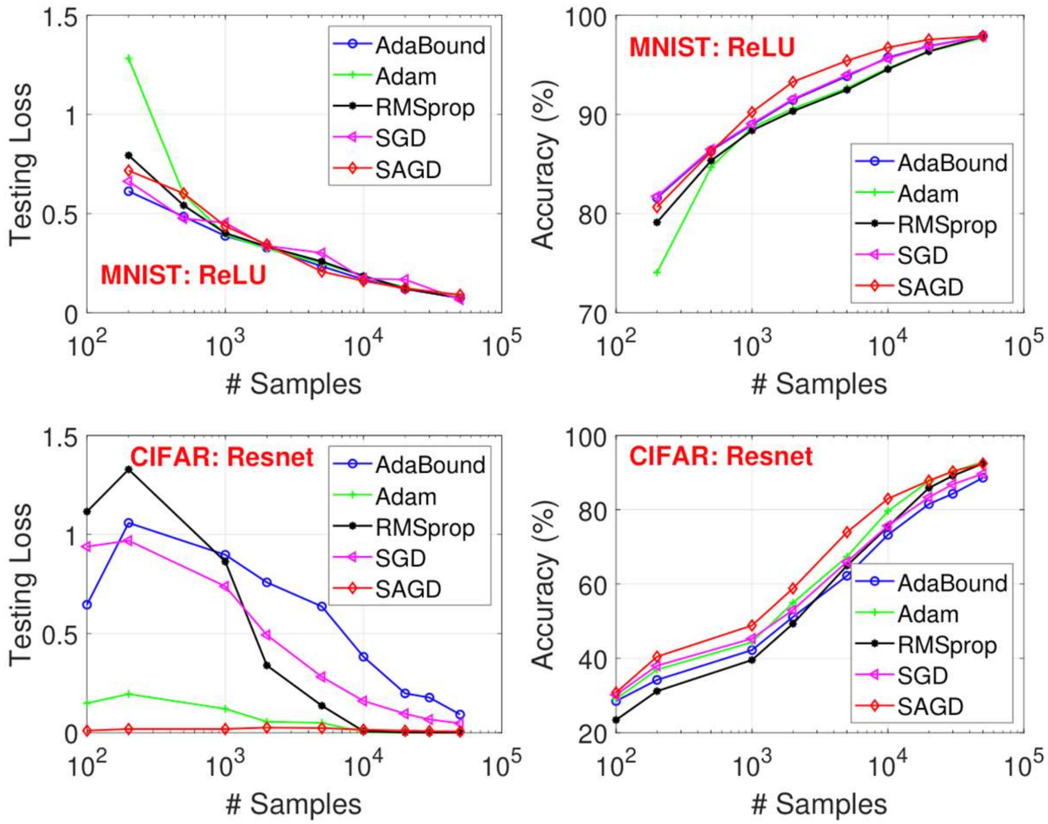
Experiments Settings

Dataset	Network Type	Architectures
MNIST	Feedforward	2-Layer with ReLU and 2-Layer with Sigmoid
CIFAR-10	Deep Convolutional	VGG-19 and ResNet-18
Penn Treebank	Recurrent	2-Layer LSTM and 3-Layer LSTM

For each task, construct multiple training sets of different size by sampling from the original training set.

- For MNIST, training sets of size n $\in$ {50, 100, 200, 500, …} are constructed.
- For CIFAR10, training sets of size n $\in$ {200, 500, 1000, …} are constructed.

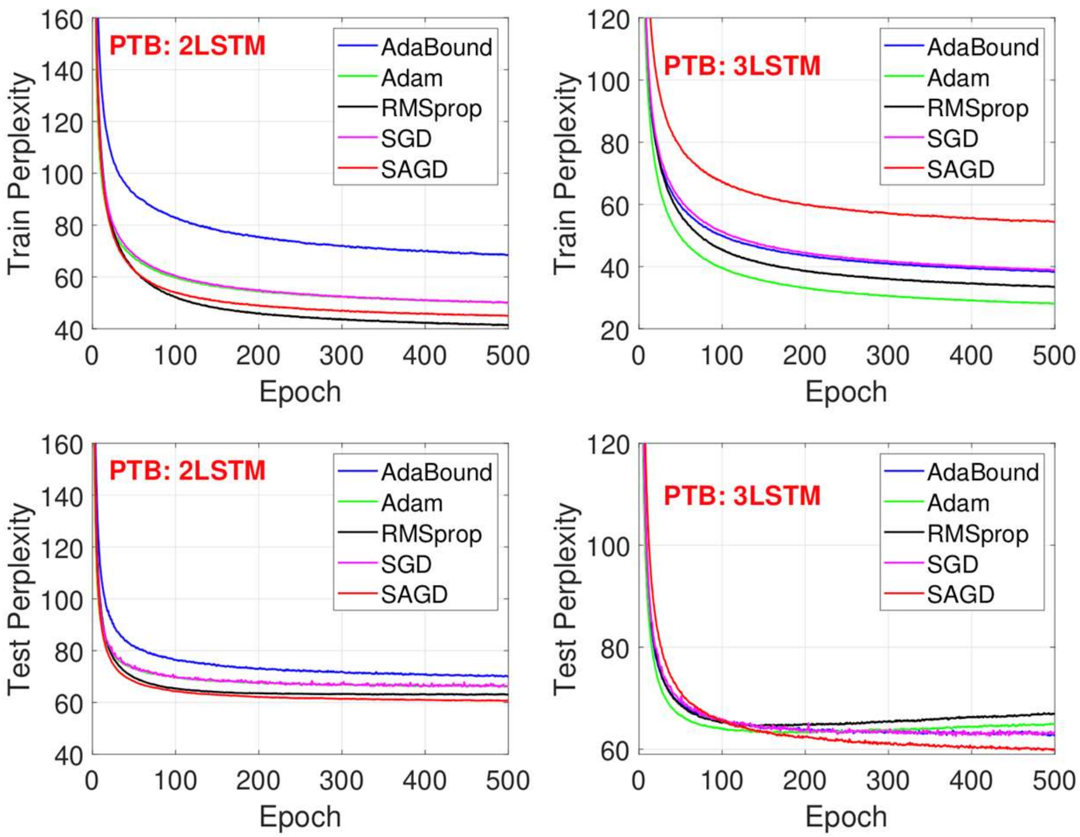
Experiments Results



- Test loss and test accuracy of ReLU neural network on MNIST
- Test loss and accuracy of ResNet-18 on CIFAR10

- SAGD obtains the best test accuracy among all the methods

Experiments Results



- Train and test perplexity of 2-layer LSTM (2LSTM), 3- layer LSTM (3LSTM)
- Even though some baseline optimizers achieve better training performance than SAGD, the latter performs the best in terms of test perplexity among all methods.

Conclusions

- Focus on the generalization ability of adaptive gradient methods
- Propose SAGD algorithms, which boost the generalization performance in both theory and practice through a novel use of differential privacy
- Experimental studies highlight that the proposed algorithms are competitive and often better than baseline algorithms for training deep neural networks