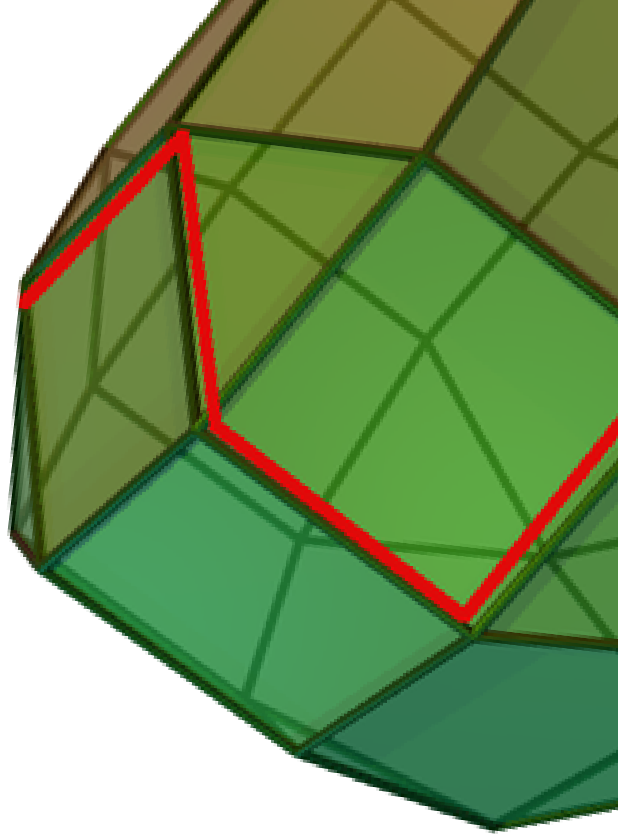
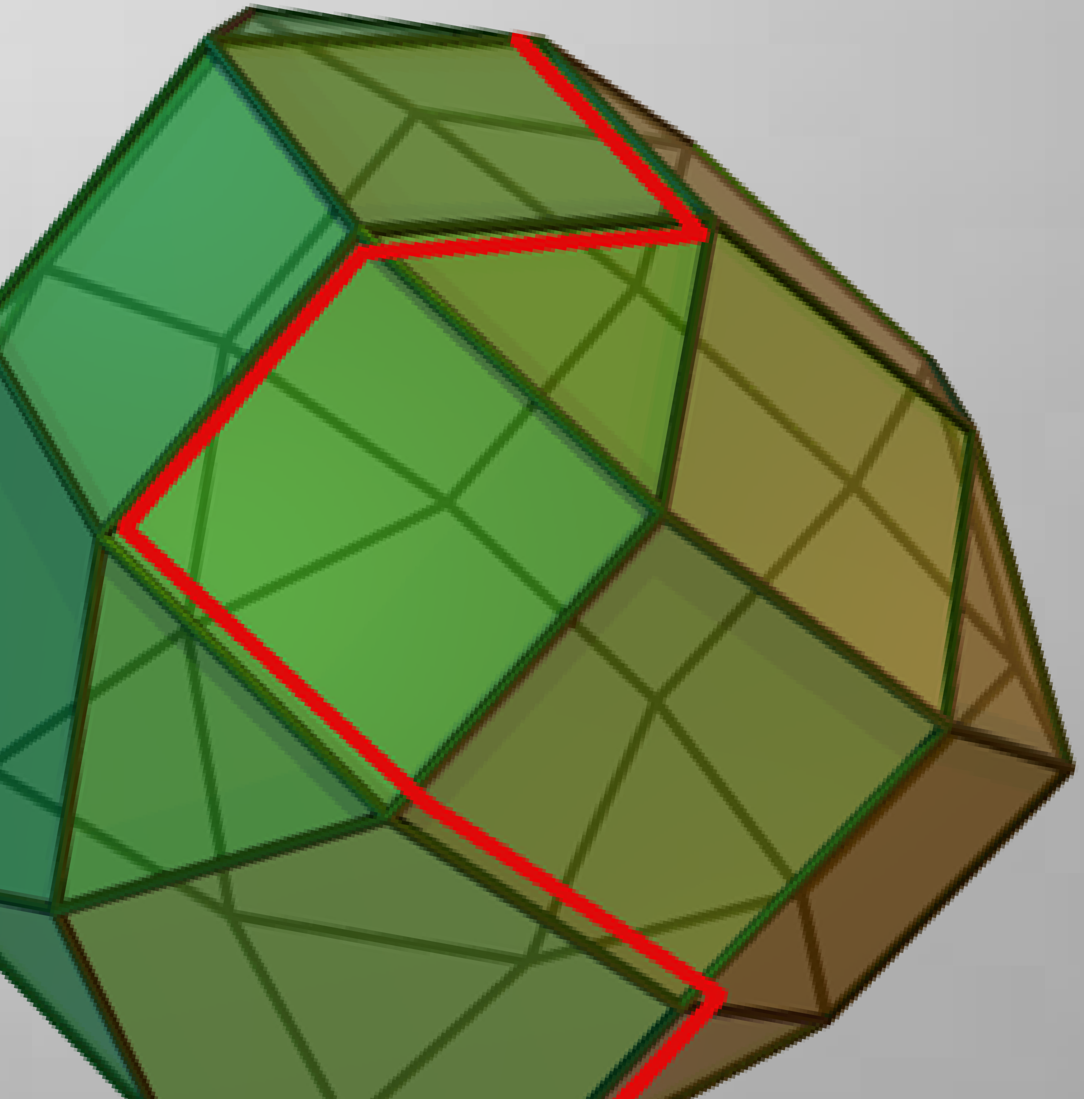


Построение глобальных траекторных гипотез

AI integration



Постановка задачи

	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	0	1	1	1	0	1	1	1
2		0	0	0	1	1	1	1	0	1	1
3			0	0	1	1	1	1	1	0	1
4				0	1	1	1	1	1	1	0
5					0	0	0	1	1	0	1
6						0	0	1	1	1	0
7							0	1	1	1	1
8								0	1	1	0
9									0	1	1
10										0	1
11											0

	TH1	TH2	TH3	TH4	TH5	TH6	TH7	TH8	TH9	TH10	TH11	W
GH1	1	0	0	0	1	0	0	0	1	0	1	3.73
GH4	0	1	0	0	1	0	0	1	0	0	0	3.69
GH5	0	1	0	0	1	0	0	0	0	0	1	3.69
GH6	0	1	0	0	0	1	0	1	0	1	0	3.63
GH9	0	0	1	0	1	0	0	1	1	0	0	3.61
GH10	0	0	1	0	1	0	0	0	1	0	1	3.61
GH2	1	0	0	0	0	1	0	0	1	1	0	3.52
GH3	1	0	0	0	0	0	1	0	1	1	1	3.45
GH7	0	1	0	0	0	0	1	1	0	1	0	3.41
GH8	0	1	0	0	0	0	1	0	0	1	1	3.41
GH11	0	0	1	0	0	1	0	1	1	0	0	3.40
GH14	0	0	0	1	1	0	0	1	1	0	0	3.33
GH15	0	0	0	1	0	1	0	1	1	1	0	3.27
GH12	0	0	1	0	0	0	1	1	1	0	0	3.18
GH13	0	0	1	0	0	0	1	0	1	0	1	3.18
GH16	0	0	0	1	0	0	1	1	1	1	0	3.05

Формализация

$$w \in \mathbb{R}^n \quad w_i \geq 0 \;\; \forall\, i \in 1, \ldots, n \quad x \in \mathbf{B}^n \quad \mathbf{B} = \{0, 1\}$$

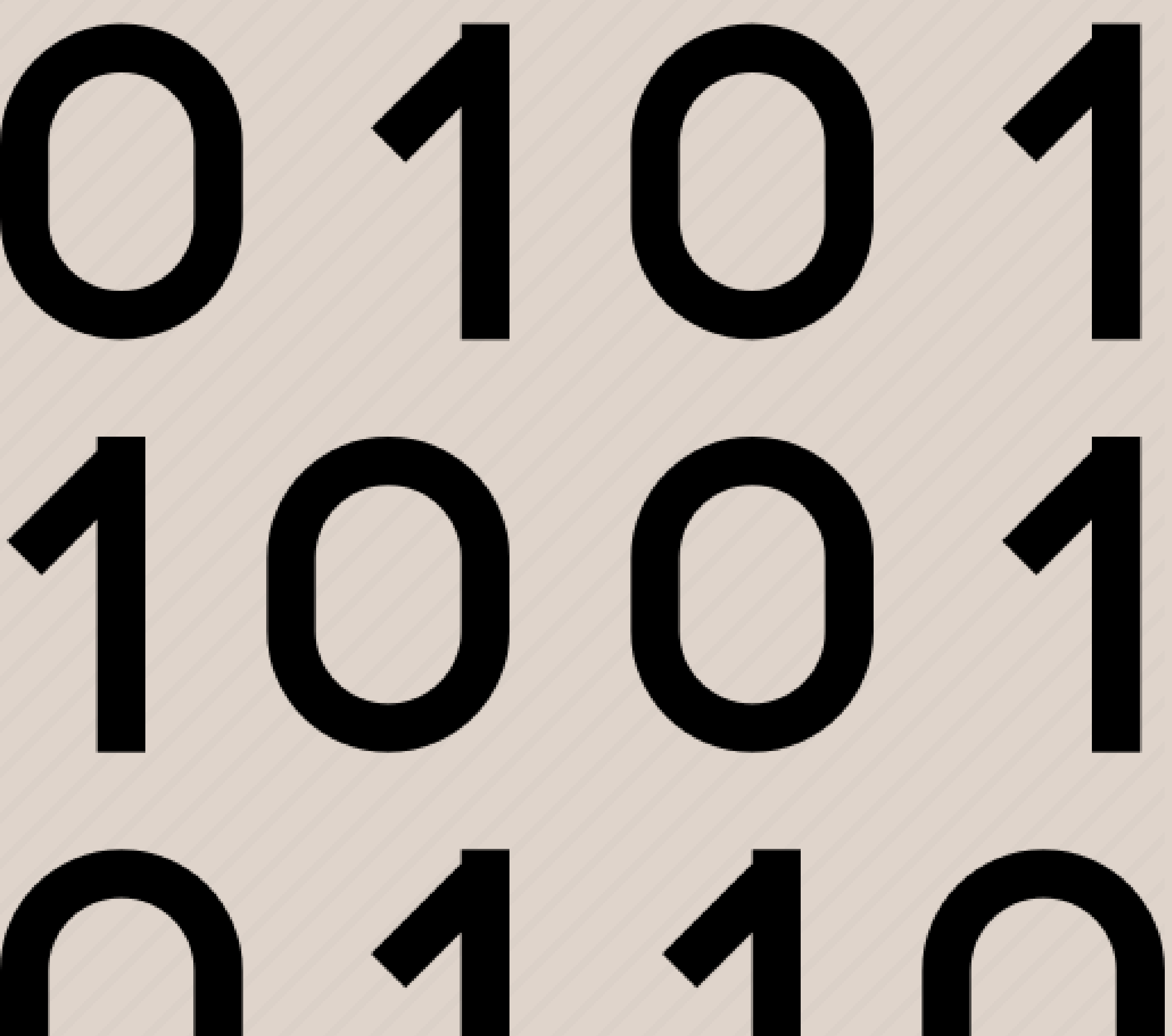
$$\max \quad w \cdot x$$

$$x_i + x_j \leq 1 \quad \forall\, i, j : \; a_{ij} = 0 \;\; i < j \;\; (x_i \wedge x_j = 0)$$

$$x_i \geq 0 \quad \forall\, i \in 1, \ldots, n$$

$$x_i \in \{0, 1\} \quad \forall\, i \in 1, \ldots, n$$

Zero-one linear programming



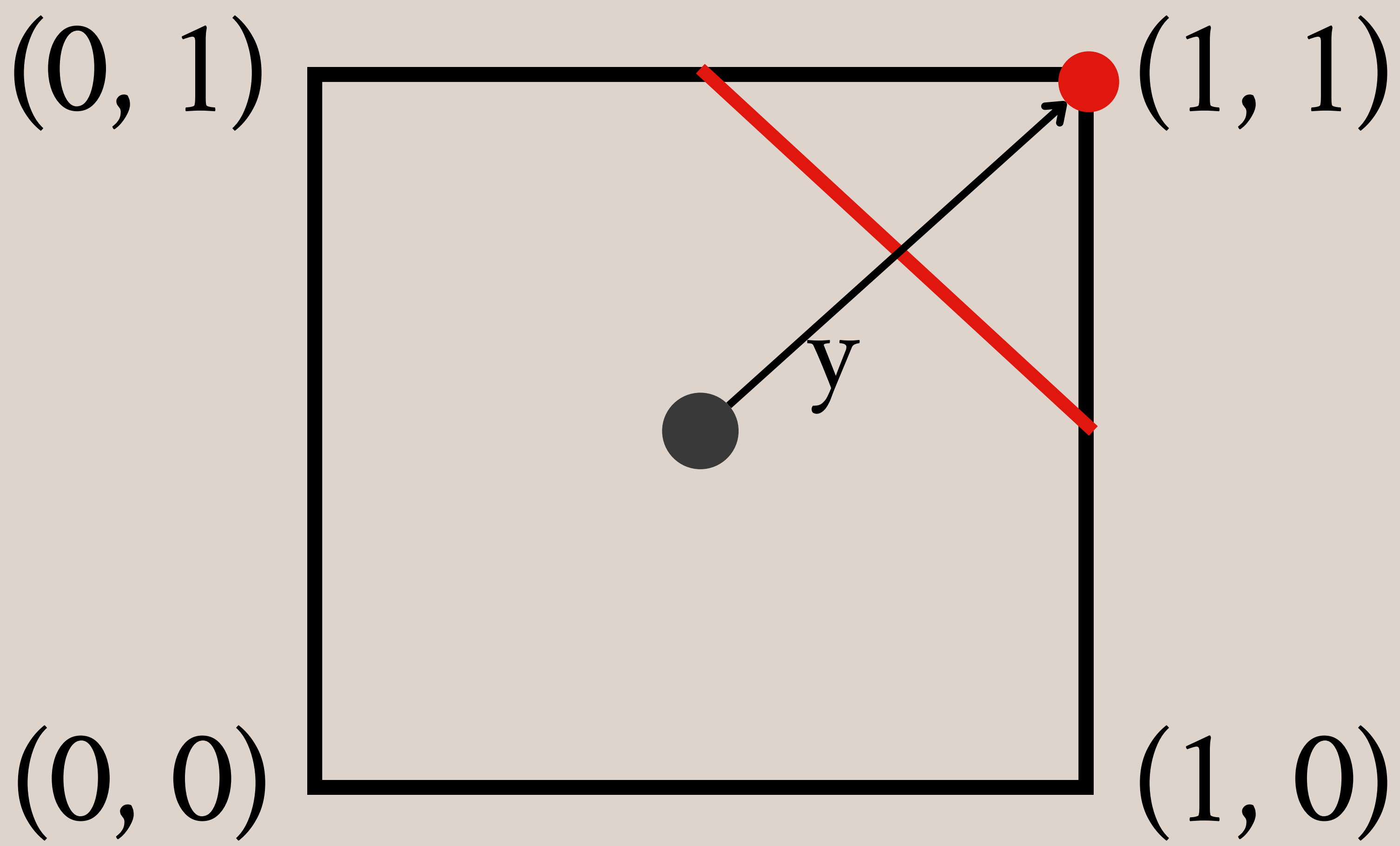
1. Branch and bounds
2. Cutting plane methods
3. Parallel shifts of the objective function hyper-plane
4. Boolen algebra methods
5. Combinatorial methods
6. Lagrangian method

Лемма. Пусть $x^* \in \mathbf{B}^n$. Тогда $\exists m \in \mathbb{R},\ y \in \mathbb{R}^n$:

$\forall x \in \mathbb{B}^n,\ x \neq x^*\ \ x \cdot y \leq m, \quad x^* \cdot y > m.$

Доказательство: $y = x^* - 0.5(1, \ldots, 1),\ m = y \cdot \tilde{x}^*$

$\tilde{x}^* = (0.5, x_2^*, \ldots, x_n^*)$ (далее см. приложение).



K-BLP:

$$k,\ n,\ m,\ w,\ A_1,\ b_1$$

$$for\ i \in 1, \ldots, k:$$

$$x_i^* = BLP(w, A_i, b_i)$$

$$\tilde{x}_i^*[2, \ldots, n] = x_i^*$$

$$\tilde{x}_i^*[1] = 0.5$$

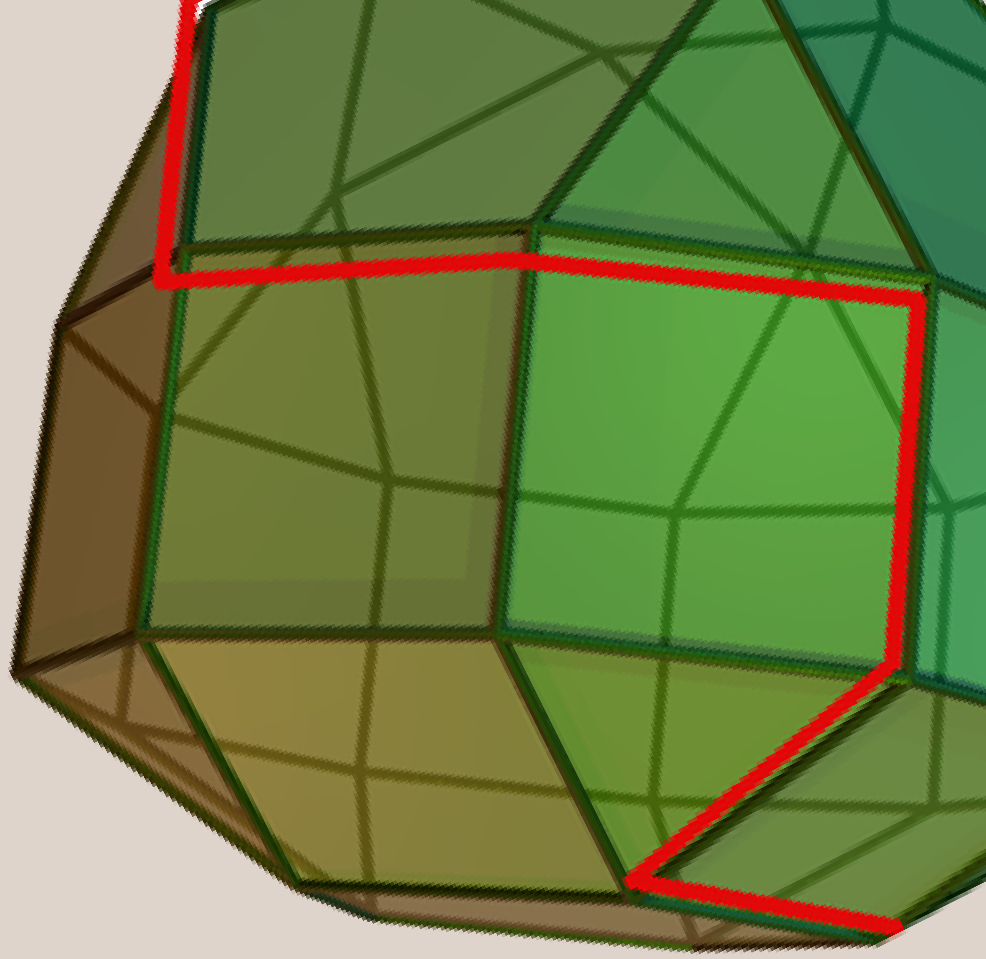
$$y_i = x^* - 0.5(1, \ldots, 1)$$

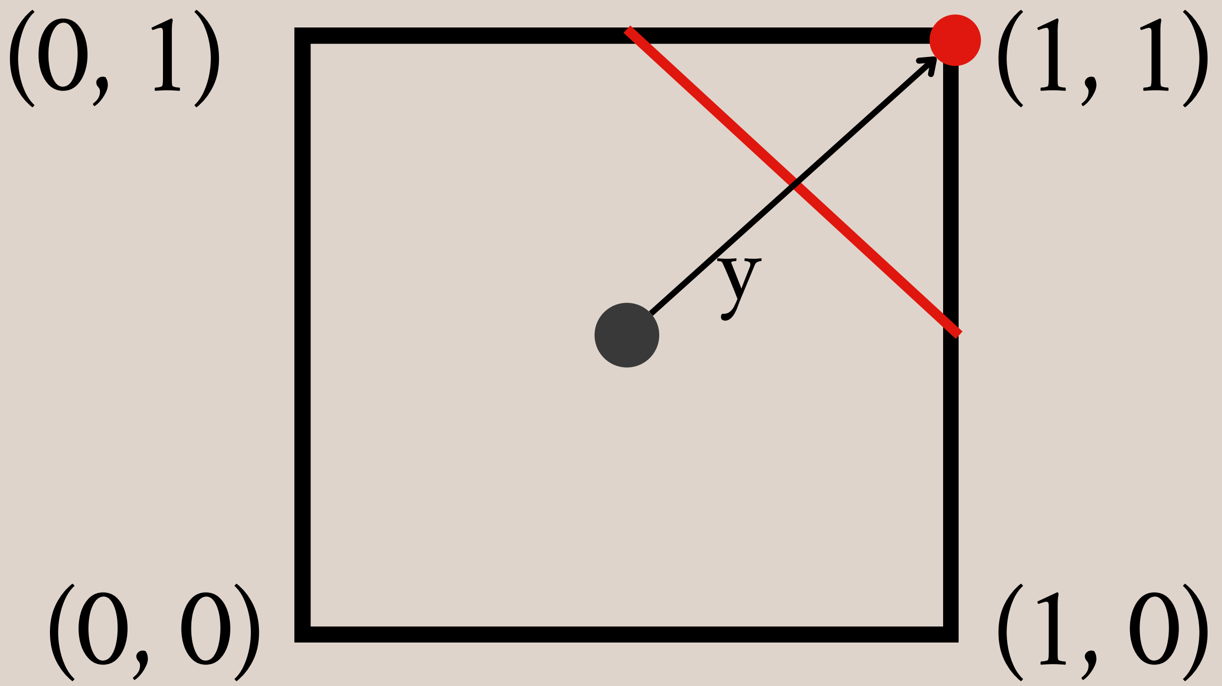
$$A_{i+1}[m+i-1, n] = A_i$$

$$A_{i+1}[m+i] = y_i$$

$$b_{i+1}[1, \ldots, m+i-1] = b_i$$

$$b_{i+1}[m+i] = y_i \cdot \tilde{x}_i^*$$

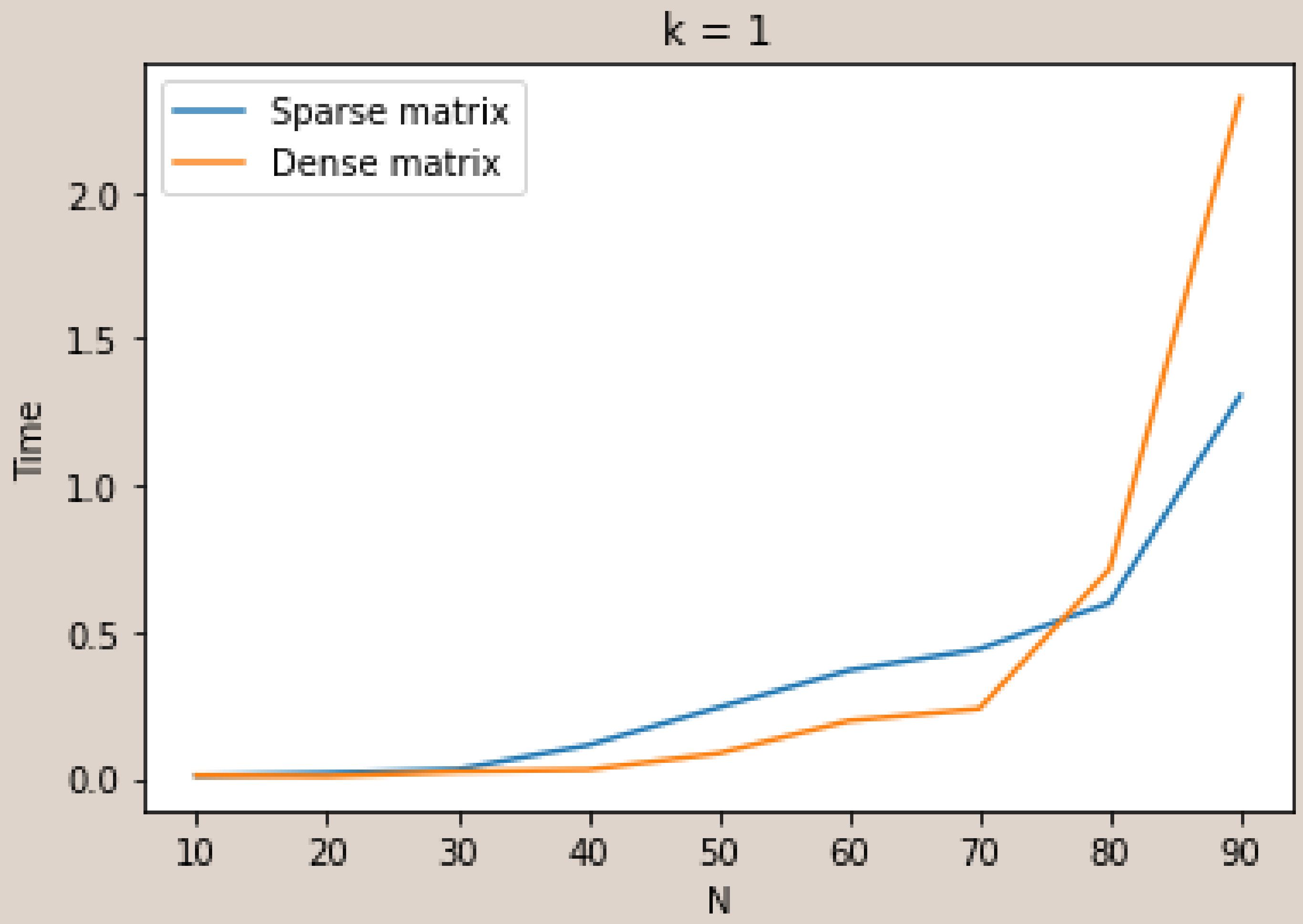


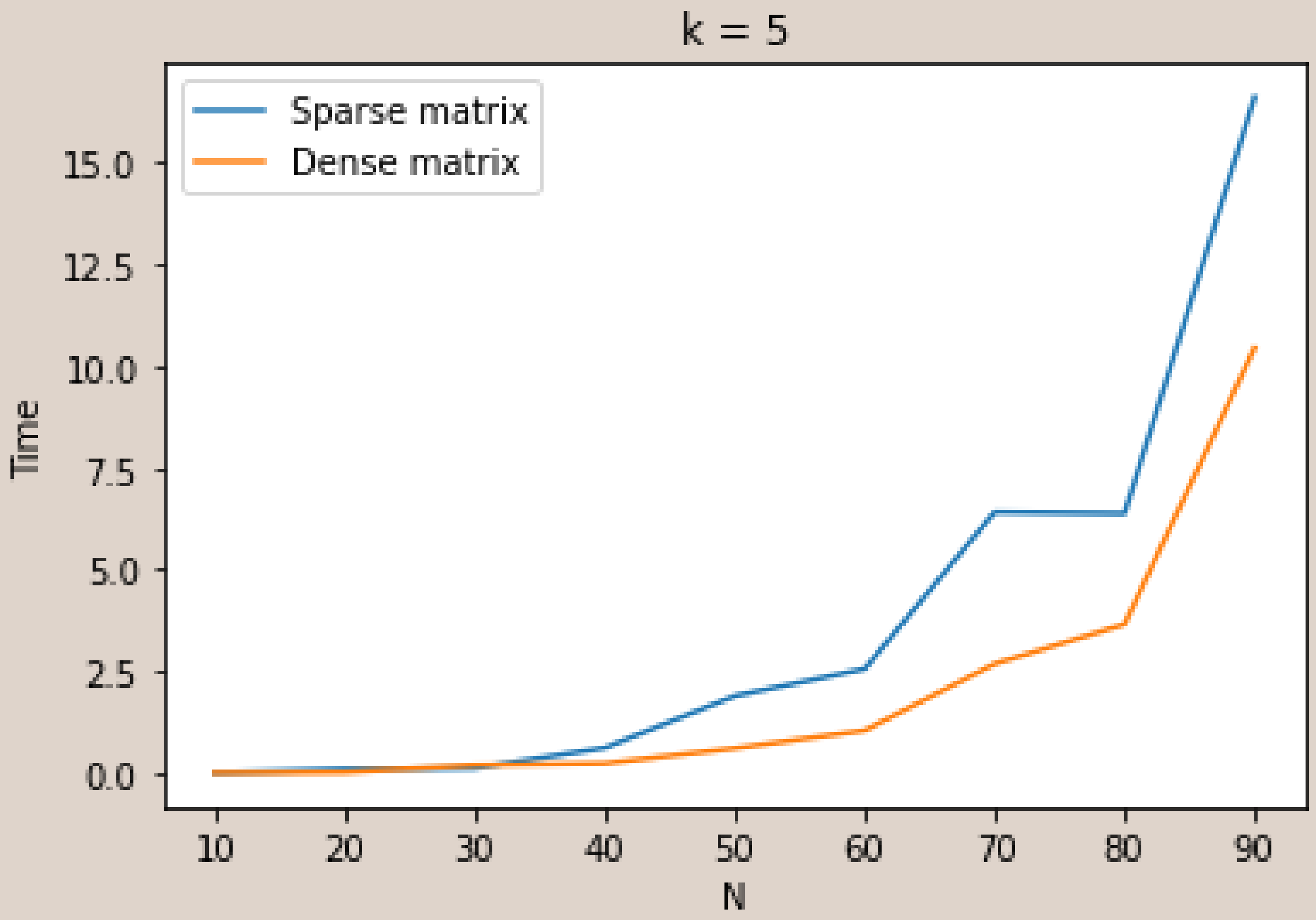


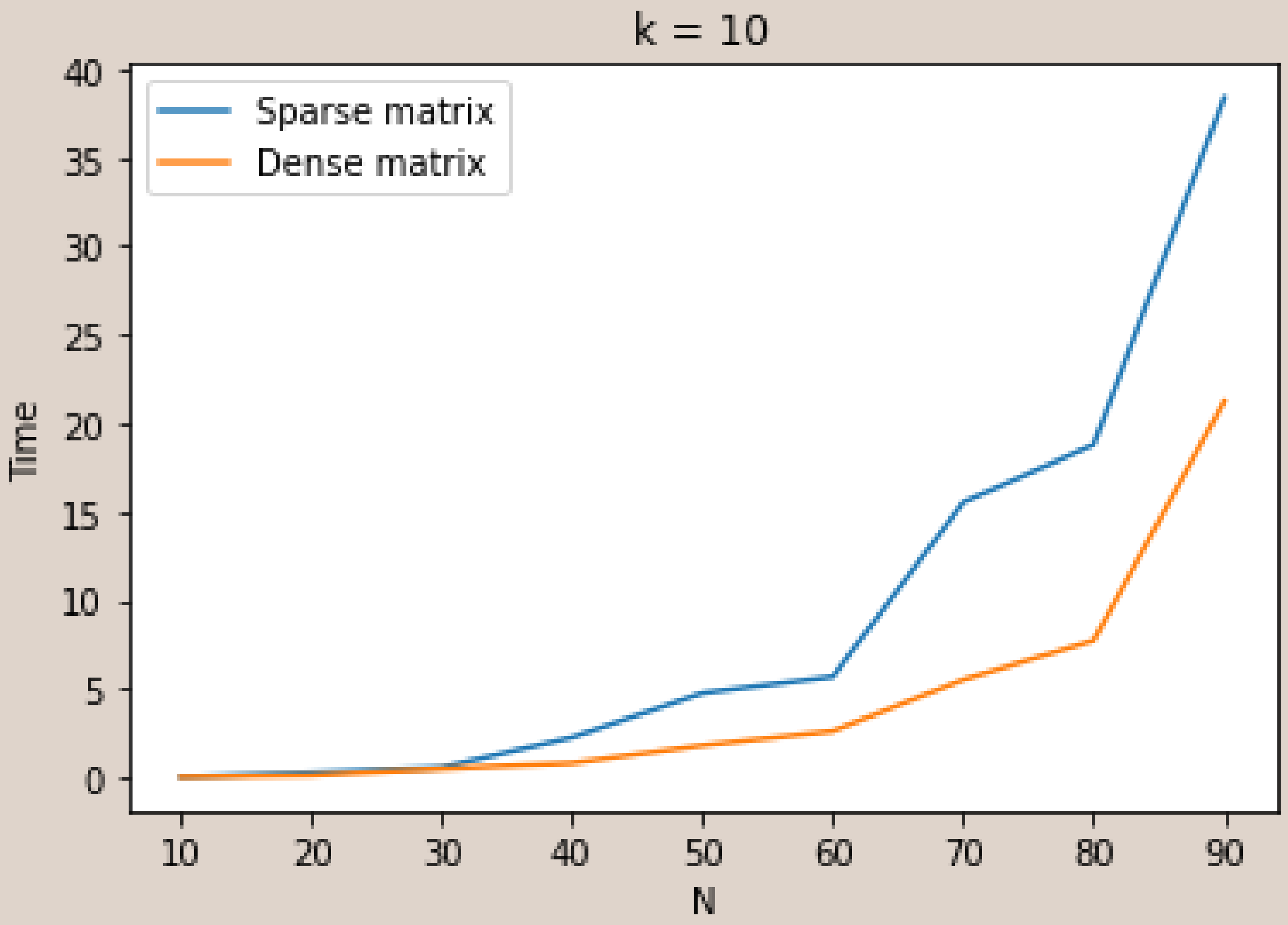
Теорема. Результатом алгоритма K-BLP является набор k наилучших решений оптимизационной задачи $x_1^*, \ldots, x_k^*$.

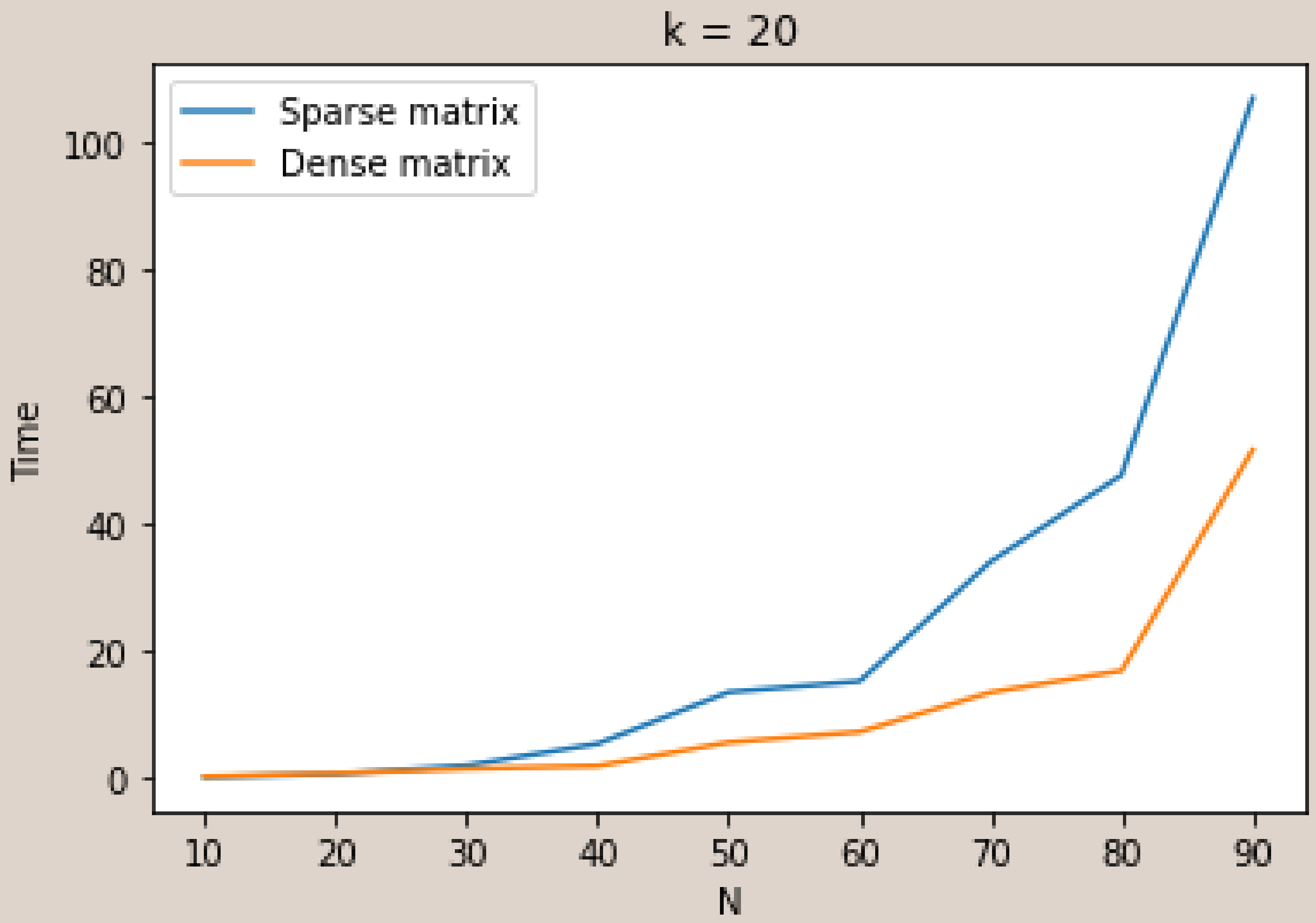
Замечание. Если не будет вырождения оптимизационной задачи.

K-BLP
ortools



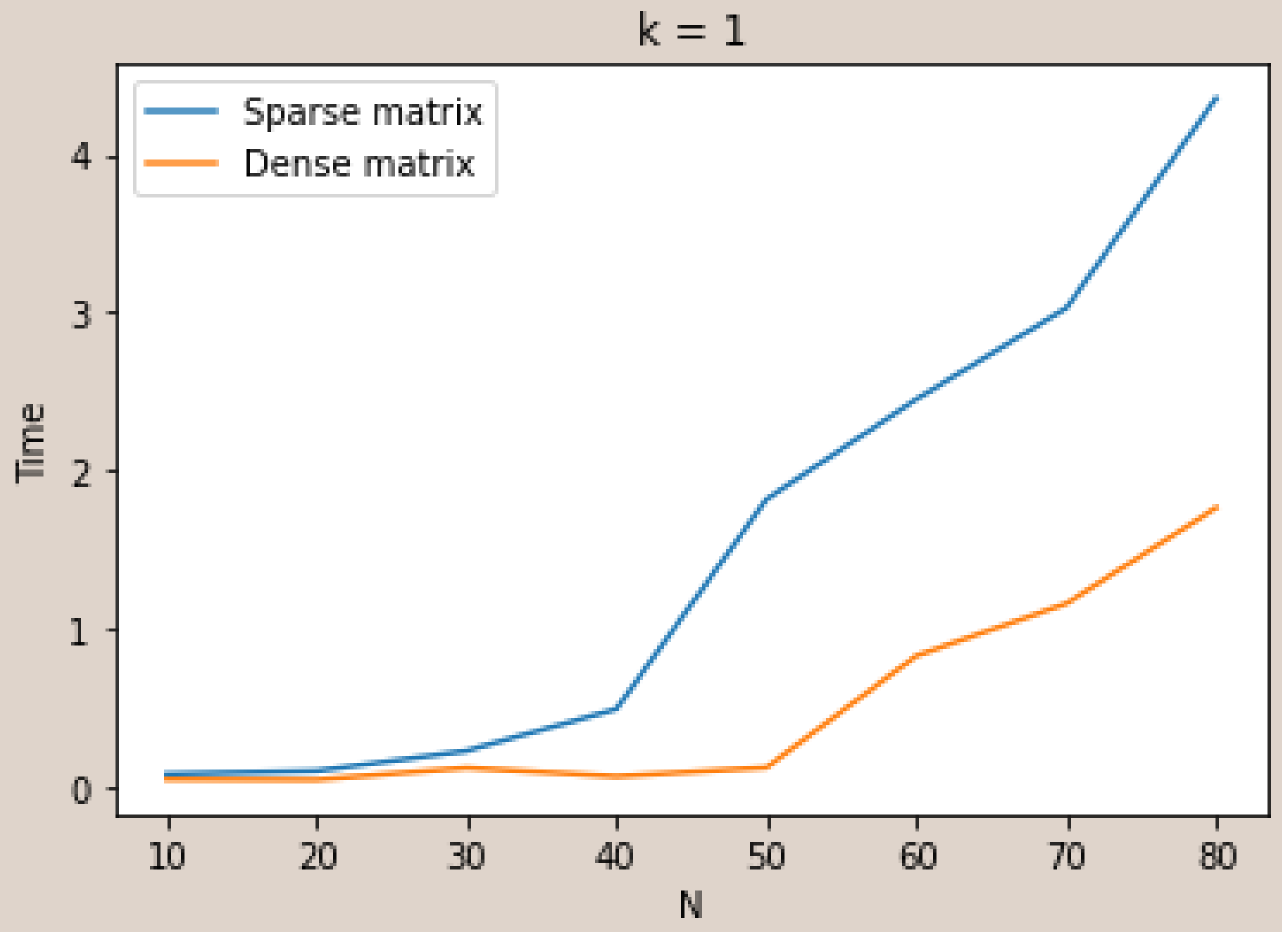


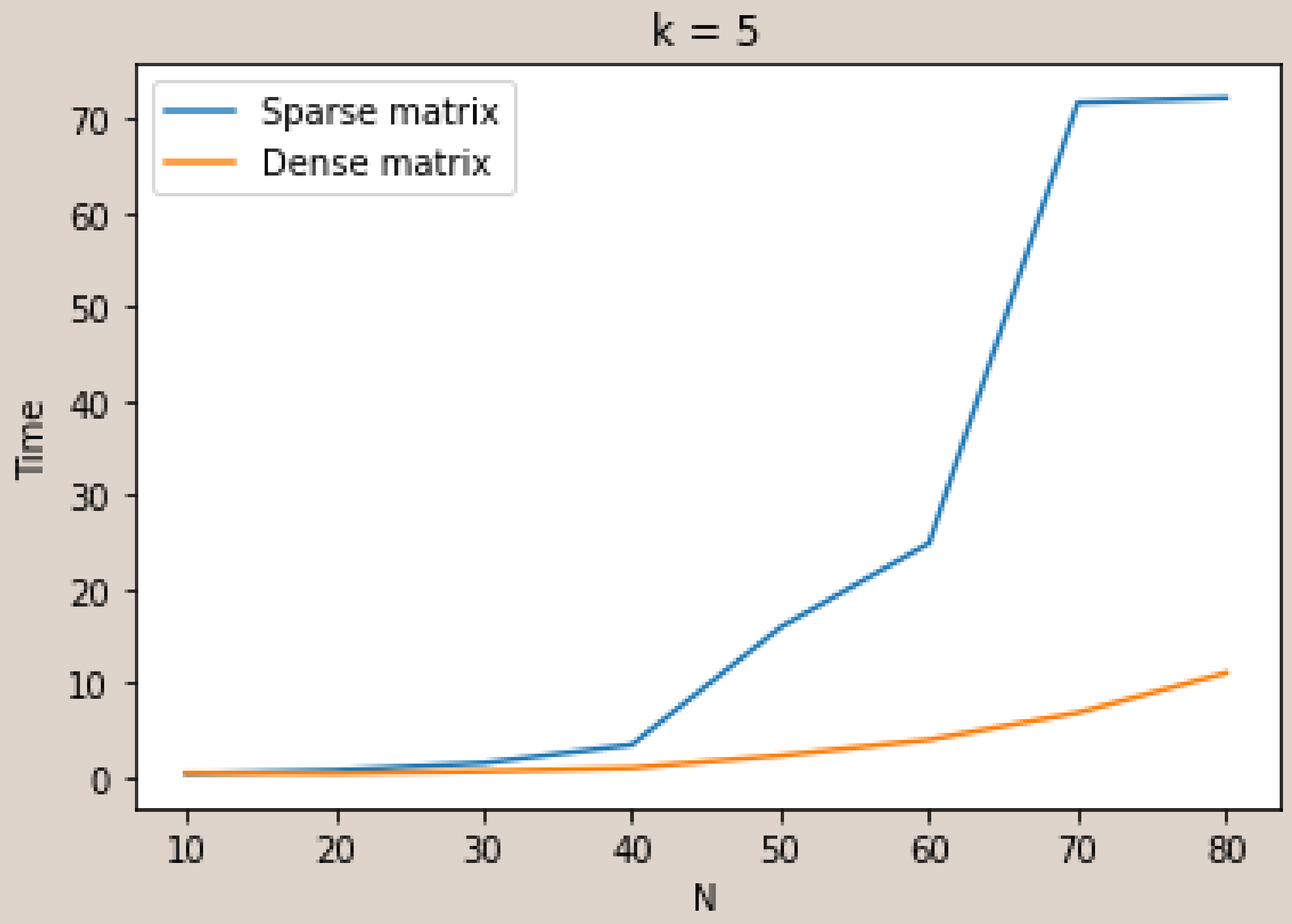


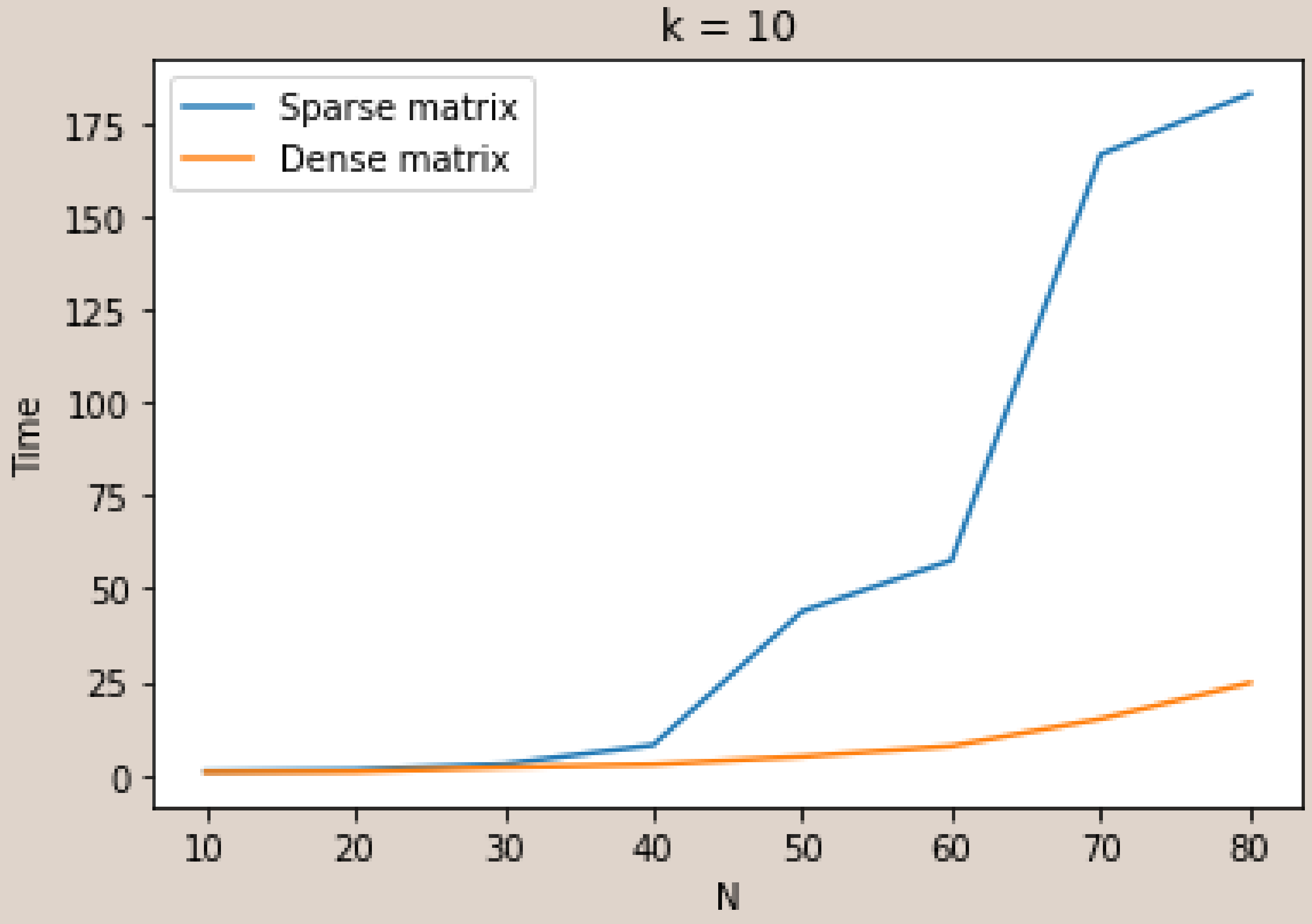


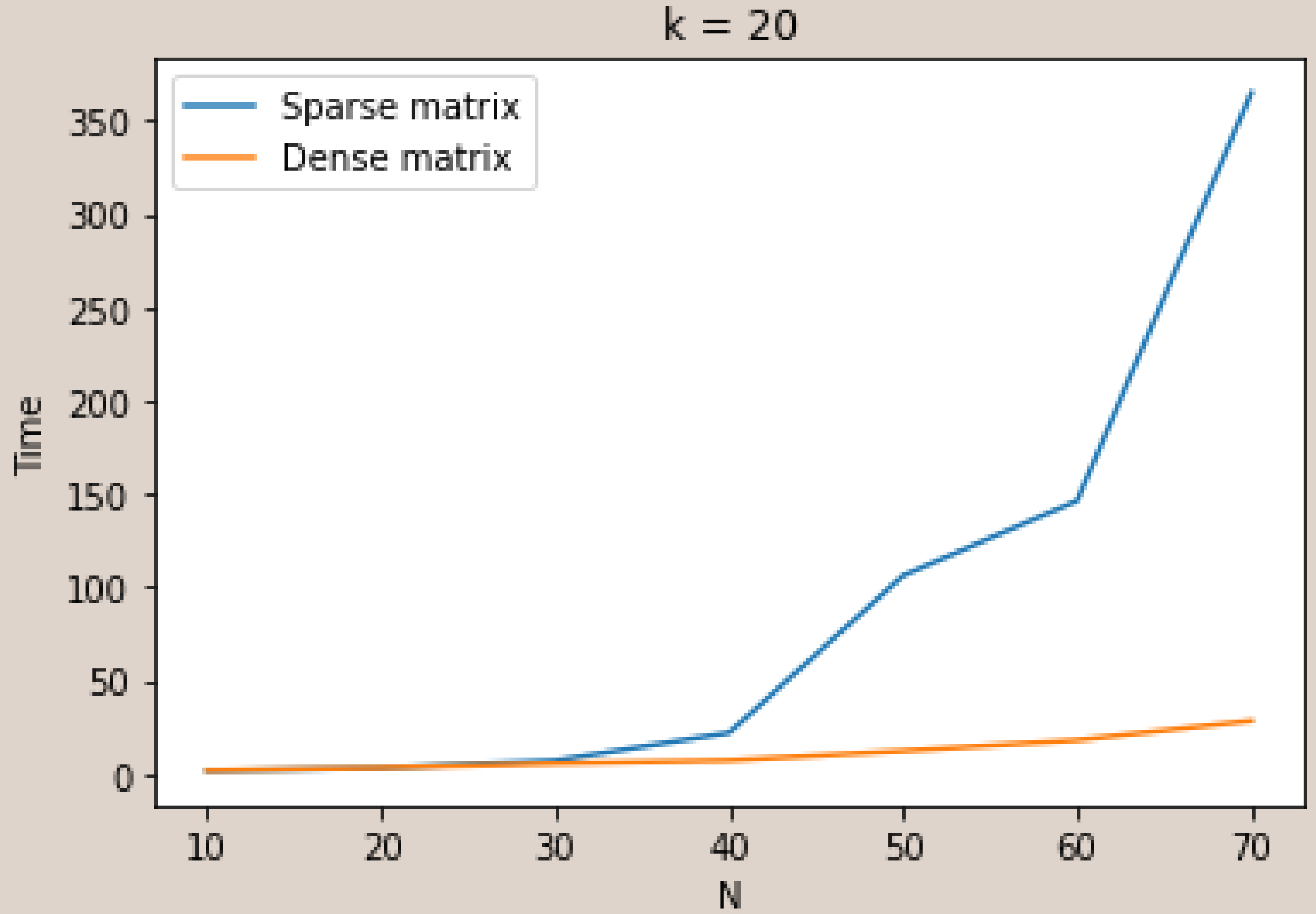
K-BLP

pulp


k = 1
Sparse matrix
Dense matrix
Time
N
0
1
2
3
4
10
20
30
40
50
60
70
80


k = 5
Sparse matrix
Dense matrix
Time
N
0
10
20
30
40
50
60
70
80


k = 10
Sparse matrix
Dense matrix
Time
N
0
25
50
75
100
125
150
175
10
20
30
40
50
60
70
80


k = 20
Sparse matrix
Dense matrix
Time
N
0
50
100
150
200
250
300
350
10
20
30
40
50
60
70

K-BLP	VS	gurobipy
• Гибкость		• Ограничения лицензии
• Blazingly fast		• Fast

Литература

- M. L. Balinski, (1965) Integer Programming: Methods, Uses, Computations. Management Science 12(3):253-313.
- Chitturi, R. V. (1961). A Master's Report. B.S. (Mech. Engg), Andhra University, Waltair, India.
- Chang, Y.-J., & Wah, B.W. (1995). Lagrangian techniques for solving a class of zero-one integer linear programs. In Proceedings Nineteenth Annual International Computer Software and Applications Conference (COMPSAC'95).