Unsupervised Part of Speech Induction Using Paradigmatic Representations of Word Context

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We investigate paradigmatic representations of word context in the domain of unsupervised part of speech induction. Paradigmatic representations of word context are based on potential substitutes of a word in contrast to syntagmatic representations based on its neighbors. In preliminary experiments we find that clustering word contexts (without considering target words) gives limited results and conclude that it is important to consider the co-occurrence of a word and its context for part-of-speech induction. We model the joint probability of words and their contexts (as represented by potential substitutes) using the S-CODE framework (Maron, Lamar, and Bienenstock 2010). S-CODE maps target words, their potential substitutes and other features to high dimensional Euclidean vectors. These vectors aggregate into clusters that largely match the traditional part-of-speech boundaries and give state-of-the-art results in unsupervised part-of-speech induction, including 80% many-to-one accuracy on the Penn Treebank and statistically significant improvements over best published results on 17 out of 19 corpora in 15 languages.

1. Introduction

Grammar rules apply not to individual words (e.g. dog, eat) but to part-of-speech categories (e.g. noun, verb). Thus learning part-of-speech categories (also known as lexical or syntactic categories) is one of the fundamental problems in language acquisition.

Linguists identify part-of-speech categories based on semantic, syntactic, and morphological properties of words. There is also evidence that children use prosodic and phonological features to bootstrap part-of-speech category acquisition (Ambridge and Lieven 2011). However there is as yet no satisfactory computational model that match human performance. Thus identifying the best set of features and best learning algorithms for part-of-speech induction is still an open problem.

Relationships between linguistic units can be classified into two types: syntagmatic (concerning positioning), and paradigmatic (concerning substitution). Syntagmatic relations determine which units can combine to create larger groups and paradigmatic relations determine which units can be substituted for one another. Figure 1 illustrates the paradigmatic vs syntagmatic axes for words in a simple sentence and their possible substitutes.

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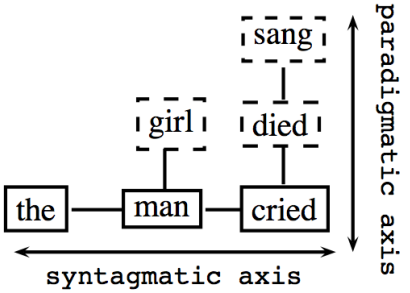

Figure 1: Syntagmatic vs. paradigmatic axes for words in a simple sentence (Chandler 2007).

Part-of-speech categories represent groups of words that can be substituted for one another without altering the grammaticality of a sentence. In this paper we explore models of part-of-speech induction based on potential substitutes of words. We build *substitute word distributions* for each position in the text which specify the probability of every vocabulary word in that position. Table 1 gives substitute distributions for an example sentence.

Table 1: The substitute word distributions (with probabilities in parentheses) for some of the positions in the example sentence *"Pierre Vinken, 61 years old, will join the board as a nonexecutive director Nov. 29."* based on an n-gram language model.

will:	*will* (0.9985), *would* (0.0007), *to* (0.0006), *also* (0.0001), . . .
join:	*join* (0.6528), *leave* (0.2140), *oversee* (0.0559), *head* (0.0262), *rejoin* (0.0074), . . .
the:	*its* (0.9011), *the* (0.0981), *a* (0.0006), . . .
board:	*board* (0.4288), *company* (0.2584), *firm* (0.2024), *bank* (0.0731), *strike* (0.0030), . . .

Note that the substitute word distribution for a position (e.g. the second position in Fig. 1) is a function of the context only (i.e. *"the ___ cried"*), and does not depend on the word that actually appears there (i.e. "man"). Thus substitute distributions represent *individual word contexts*, not word types. We refer to representations based on substitute distributions as *paradigmatic representations of word context*.

We expect words used in similar contexts (with similar substitute distributions) to share the same part-of-speech. Thus part-of-speech induction depends on which contexts are considered similar, and context similarity in turn is a function of the features used to represent word context. Paradigmatic representations, using features of the substitute distribution, uncover latent similarities between contexts that on the surface seem to have little in common. This makes paradigmatic representations more robust to data sparsity, compared to syntagmatic representations which use neighboring words as features. Our empirical results demonstrate that paradigmatic representations significantly outperform syntagmatic ones when compared using similar part-of-speech induction algorithms on identical datasets. Section 2 presents alternative representations of word context and discusses paradigmatic representations in more detail.

2. Representing Word Context

In this section we demonstrate the different contextual representations in the part-of-speech induction (aka. syntactic word categorization) literature and introduce the substitute words as

an alternative to the current context representations. In the rest of the paper the words in the vocabulary are referred as *types* and the instances of types are referred as *tokens*.

The contextual representations can be categorized into three groups based on the way they incorporate the local context information of the target type or token: (1) syntagmatic representation, (2) Hidden Markov Models (HMM) and (3) paradigmatic representation. These representations can be further subdivided into two subgroups based on whether they group the types or the tokens.

Syntagmatic Representation. The syntagmatic representation represents the context using the neighboring words, typically co-occurrences with a single word on the left or a single word on the right word called a "frame" (e.g., **the** *dog* **is***;* **the** *cat* **is**). (Schütze and Pedersen 1993; Redington, Crater, and Finch 1998; Mintz 2003; St Clair, Monaghan, and Christiansen 2010; Lamar et al. 2010; Maron, Lamar, and Bienenstock 2010). Turney and Pantel (2010) give a broad overview of syntagmatic approaches and their applications within the Vector Space Modeling framework. Depending on the way they incorporate co-occurences these models can group types or tokens.

Schütze (1993) represented the context of a word type by concatenating its left and right co-occurences vectors. These vectors were calculated for each type by using the left and the right neighbors of the type instances therefore they characterize the distribution of the left and right neighboring tokens of the type. One constraint of this representation is that it represents types rather than tokens thus it is not possible to group the instances of any type into the separate categories.

Mintz (2003) showed on a subset of child directed speech corpus (CHILDES) (MacWhinney 2000) that non-adjacent high frequent bigram frames are useful for the language learners on the syntactic categorization of the tokens. For example, the tokens that are observed at "_" in the frame "**the _ is**" are assigned to the same category. Using the top-45 frequent frames Mintz achieved an average of 98% unsupervised accuracy[1]. The main limitation of the top-45 frequent frames is that they could only analyze the 6% of the tokens on average due to the sparsity. Another drawback is that the tokens with only one common neighbors could not exchange information.

St Clair et al. (2010) extended the work of Mintz (2003) and introduced the flexible bigram frames which represent the context by using the left and the right bigrams separately. As a result tokens with a common left or right bigram can exchange information and might be grouped together. For instance, two tokens that are observed at "'_" in "**the _ is**" and "**a _ is**" can be categorized together due to the shared right bigram "**is**". Using a feed forward connectionist model they showed that the flexible frames are statistically better than the frequent frames in terms of the supervised accuracy[2]. They also showed that representing token contexts only with the left or the right bigram is statistically better than the frequent frames but worse than the flexible frames in terms of supervised accuracy. Both Mintz (2003) and St Clair (2010) did not report any results with contexts larger than bigram since as the context is enriched, the re-occurrence frequency of a frame becomes lower which causes the data sparsity (Manning and Schütze 1999).

HMM. Prototypical HMM uses a bigram structure where tokens are generated by latent categories and learns the latent category sequence that generates the given word sequence instead of clustering token directly (Brown et al. 1992; Blunsom and Cohn 2011; Goldwater and Griffiths

1 Unsupervised accuracy was defined as the number of hits (when two intervening tokens observed in the frame from the same category) divided by number of false alarms (when two intervening tokens observed in the frame from different categories).

2 The number of analyzed tokens was same in their experiments since they used all the frequent frames instead of the top-45 ones.

2007; Johnson 2007; Ganchev et al. 2010; Berg-Kirkpatrick and Klein 2010; Lee, Haghighi, and Barzilay 2010). As a result The POS induction literature focused on the first and second order HMMs since the higher order HMMs have additional complicating factors[3] and require more complex training procedures (Johnson 2007). Depending on the design and the training procedure HMM models can be trained to cluster types or tokens which are detailed in Section **??**.

Paradigmatic Representation. In the paradigmatic representation context is defined as the distribution of the substitute words in that context. Schütze (1995) incorporated paradigmatic information by concatenating the left co-occurrence and the right co-occurrence vectors of the right and the left tokens, respectively and grouped the tokens that have similar vectors. Yatbaz et al. (2012) calculated the most likely substitute words of a word in a given context and grouped the types that have similar substitutes.

Our paradigmatic representation is also related to the second order co-occurrences used in (Schütze 1995). Schütze concatenates the left and right context vectors for the target word type with the left context vector of the right neighbor and the right context vector of the left neighbor. The vectors from the neighbors include potential substitutes. Our method improves on his foundation from three aspects: (1) it can cluster both the types and tokens (2) it uses a 4-gram language model rather than bigram statistics, (3) it includes the whole 78,498 word vocabulary rather than the most frequent 250 words. More importantly, rather than simply concatenating the vectors that represent the target word with vectors that represent the context we use a co-occurrence modeling algorithm.

Similarly, Schütze and Pedersen (1993) define the words that frequently co-occur together as the *syntagmatic associates* and words that have similar left and right neighbors as the *paradigmatic parallels*. Turney and Pantel (2010) give a broad overview of syntagmatic approaches and their applications within the Vector Space Modeling framework. We find that representing the paradigmatic axis more directly using substitute vectors rather than frequent neighbors improves part of speech induction.

Sahlgren (2006) gives a detailed analysis of paradigmatic and syntagmatic relations in the context of word-space models used to represent the word meanings. Sahlgren's paradigmatic model represents word types using co-occurrence counts of their frequent neighbors, in contrast to his syntagmatic model that represents word types using counts of contexts (documents, sentences) they occur in. Our substitute vectors do not represent word types at all, but *contexts of word tokens* using probabilities of likely substitutes. Sahlgren finds that in word-spaces built by frequent neighbor vectors, more nearest neighbors share the same part of speech compared to word-spaces built by context vectors.

The two examples below illustrate the advantage of paradigmatic representations in uncovering similarities where no overt similarity that can be captured by a syntagmatic representation exists. The word "board" from the first sentence and the word "council" from the second sentence have no common neighbors except the determiner "the". The paradigmatic representation captures the similarity of these words by suggesting the same top substitutes for both (the numbers in parentheses give substitute probabilities):

(1) *"Pierre Vinken, 61 years old, will join the* **board** *as a nonexecutive director Nov. 29."*
board: board (.4288), company (.2584), firm (.2024), bank (.0731), . . .

3 The number of parameters in a prototypical HMM quadratically increases as the HMM order increases.

(2) *". . . and hold only 25 % of the seats on the* **council** *."*
council: board (.6591), company (.0795), firm (.0542), bank (.0154), . . .

The high probability substitutes reflect both semantic and syntactic properties of the context. Top substitutes for "board" and "council" are not only nouns, but specifically nouns compatible with the semantic context. Top substitutes for the word "the" in the first example consist of words that can act as determiners: its (.9011), the (.0981), a (.0006), . . .

3. Abstract Algorithm

In this section we describe the components of our algorithm and their relationship with each other. The algorithm predicts the syntactic category of a word in a given context based on its random substitutes. In other words first we construct the co-occurrence representation of words and their substitutes with the help of a language model and then map each value in the co-occurrence data to a corresponding embedding on a n-dimensional sphere using the S-CODE algorithm. Finally, we apply k-means clustering to categorize the word embeddings by which we induced the word categories. In the next subsection we detail the representation of word contexts as co-occurrence data, in Subsection 3.2 we explain the embedding calculation and finally in Subsection 3.3 we describe the different ways of embedding clustering.

3.1 Context Representation

Word contexts are represented by random substitutes that are sampled from the corresponding substitute word distributions. Random substitutes are sampled with replacement from the substitute distributions that are calculated based on an n-gram language model. The sample space of the substitute word distributions is the vocabulary of the language model.[4] It is possible (and beneficial, see Section 4) to sample more than one substitutes and generate more pairs from the same substitute distribution as in Table 2. The calculation of substitute distributions and random substitute sampling are detailed in Appendix A. To capture the relation between each word and its context we construct a co-occurrence representation by pairing the words with their random substitutes. Table 2 shows random substitutes of each word and their co-occurrence representation on an example sentence. A target word might appear as a word or a random substitute therefore to clarify this ambiguity we concatenate "W" and "C" to words and contexts (i.e., random substitutes), respectively, in the co-occurrence data.

The next section explains the S-CODE algorithm which takes the co-occurrence data as its input and calculates the embeddings of the words and their substitutes on an n-dimensional sphere. In the rest of the paper we use the term "substitutes" and "random substitutes" interchangeably.

3.2 Co-occurence Embedding

The S-CODE algorithm maps each word and substitute value in the co-occurrence data to an embedding on an n-dimensional sphere as detailed in Appendix B. The basic idea of the mapping is that words and substitutes that are frequently observed as pairs in the co-occurrence data will have close embeddings while unobserved pairs will have embeddings that are apart from each other.

4 Sampled substitutes might include the unknown word tag "_unk_" since it is in the language model vocabulary. For example substitutes of proper nouns usually include "_unk_" as a substitute.

Table 2: The left table shows three possible substitutes, seperated with "/", sampled for each position in the example sentence *"Pierre Vinken, 61 years old, will join the board as a nonexecutive director Nov. 29 ."* based on a 4-gram language model. The right table represents the input sentence as a co-occurrences of words and their substitutes. Thus words on the left column presents the target word while words on the right column represents the context of the correponding target word.

Word	**Random Substitutes**
Pierre	*Mr. / Pierre / Mr.*
Vinken	*_unk_ / Beregovoy / Cardin*
,	*, / , / ,*
61	*48 / 52 / 41*
years	*years / years / years*
old	*old / old / old*
,	*, / , / ,*
will	*will / will / will*
join	*head / join / leave*
the	*its / its / the*
board	*board / company / firm*
as	*as / as / as*
a	*a / a / a*
nonexecutive	*nonexecutive / non-executive / nonexecutive*
director	*chairman / chairman / director*
Nov.	*April / May / of*
29	*16 / 29 / 9*
.	*. / . / .*

Word	**Context**
W:Pierre	*C:Mr.*
W:Pierre	*C:Pierre*
W:Pierre	*C:Mr.*
W:Vinken	*C:unk*
W:Vinken	*C:Beregovoy*
W:Vinken	*C:Cardin*
...	
W:join	*C:head*
W:join	*C:join*
W:join	*C:leave*
W:the	*C:its*
W:the	*C:its*
W:the	*C:the*
...	
W:director	*C:chairman*
W:director	*C:chairman*
W:director	*C:director*
...	

The co-occurrence data in Figure 2 consists of pairs such as (*W:director*, *C:chairman*) and (*W:chief*, *C:chairman*) therefore S-CODE forces the embeddings of *W:director* and *W:chief* to be close to the embedding of *C:chairman*. Similar to the former case the embeddings of *W:Pierre* and *W:Frank* will be close to the embedding *C:Mr.* becouse of the frequently observed pairs. As a results the final embeddings of *W:director* and *W:chief* will be close to each other due to the common substitute *C:chairman* and will be apart from *W:Pierre* and *W:Frank* due to the lack of common substitute as shown on Figure 2 (similarly the embeddings of *W:Pierre* and *W:Frank* will be close to each other due to *C:Mr.*).

S-CODE constructs embeddings on n-dimensional sphere for each unique value of words and substitutes. Thus each pair in the co-occurrence data can be represented in three different ways by using the output of S-CODE: (1) word embedding ($\mathbf{W}$) which represents the word type information, (2) substitute embedding ($\mathbf{C}$) which represents the context information, and (3) word and substitute embeddings concatenated ($\mathbf{W} \oplus \mathbf{C}$). In the next section we apply k-means clustering to these three representation and analyze the characteristic of final clusters.

3.3 Embedding Clustering

Each target word in the original input sentence is represented with word–substitute pairs and each unique word and substitute value are represented with embeddings on an n dimensional sphere. Therefore clustering the embeddings means clustering the target words in the original input. We run instance weighted k-means algorithm to cluster the final embeddings constructed

Word	Context
...	...
W:director	C:chairman
W:chief	C:chairman
...	...
W:Pierre	C:Mr.
W:Frank	C:Mr.
...	...

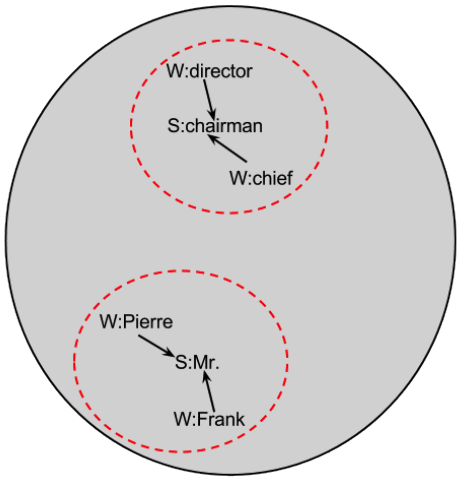

Figure 2: The figure on the right is the final embeddings of the input co-occurrence data given on the left table after S-CODE converges. Dashed circles represent the possible groupings of the embeddings on the sphere.

by S-CODE. The target words can be represented in three ways as described in the previous section. Clustering each representation with k-means will end up with a clusters with different characteristics.

*Word embeddings (***W***).* All of the target word instances are represented with the same embedding. For instance, although we sample three substitutes per target word in Table 2, each unique target word has only one embedding in **W**. Thus clustering target words based on this representation employ the one-tag-per-word assumption from the beginning. The one-tag-per-word assumption is suitable for the part of speech induction problem given that 93.69% of the word occurrences in the human labeled PTB data are tagged with their most frequent part of speech (Toutanova et al. 2003). However the clustering performance on the ambiguous words (words that have more than one tag) degrades due to the one-tag-per-word assumption. For example the word "thought" has two possible ... . [!! ask volkan for this case]

*Context embeddings (***C***).* Each target word instance is represented with its substitutes embeddings instead of its word embedding therefore clustering them does not employ or force the one-tag-per-word assumption. For example, the word "Pierre" in Table 2 will be represented with the embeddings of *S:Mr.*, *S:Pierre* and *S:Mr.* however another occurrence of the word "Pierre" in a different context might be represented with different substitutes. It is possible to represent context with several substitutes, in such a case k-means might group substitute embeddings into different clusters which leads to an ambiguity on the final cluster of the target word. To solve this issue the target word is assigned to the cluster in which the majority of its substitute embeddings are present[5].

5 Ties are broken randomly.

Concatenation word and context embeddings ($\mathbf{W} \oplus \mathbf{C}$*).* This representation concatenates the word and its substitute embeddings such that each target word is represented with r vectors where r is the number of random substitutes per target word. Therefore it explicitly forces the one-tag-per-word assumption while handling the ambiguity. For instance, the target word "Pierre" in Table 2 will be represented with three vectors that will be the concatenation of the embedding of *W:Pierre* with the embeddings of *C:Mr.*, *C:Pierre* and *C:Mr.*, seperately. Similar to the former case we run k-means clustering on these concatenated vectors and use majority voting to assign the target word cluster.

The first representation applies the one-tag-per-word assumption from the beginning and clusters word types instead of tokens. On the other hand the second one relaxes the one-tag-per-word assumption and clusters word tokens. The final one also clusters tokens however it represents each token with the concatenation of the word and context embeddings therefore it forces the one-tag-per-word assumption while handling the ambiguity. Section 4 compares the performance of these three representations on the word category induction problem.

4. Experiments

Section 4.1 details the test corpus to be induced and experiment parameters used in the rest of this work.

Section 4.3 and Section 4.4.2 aim to compare our paradigmatic representation of the word context to previously described syntagmatic representations of the word context for word types and word tokens, respectively. Section 4.3 also replicates the bigram based S-CODE results from (Maron, Lamar, and Bienenstock 2010).

Section 4.5 explores morphological and orthographic features as additional sources of information for POS induction of word types and its results improve the state-of-the-art in the field of POS induction.

4.1 Experimental Settings

To make a meaningful comparison to the previous works the Wall Street Journal Section of the Penn Treebank (PTB) (Marcus et al. 1999) was used as the test corpus (1,173,766 tokens, 49,206 types) to be induced. The treebank uses 45 part-of-speech tags which is the set we used as the gold standard for comparison in our experiments.

To compute substitutes in a given context we trained a language model using approximately 126 million tokens of Wall Street Journal data (1987-1994) extracted from CSR-III Text (Graff, Rosenfeld, and Paul 1995) (excluding sections of the PTB). We used SRILM (Stolcke 2002) to build a 4-gram language model with Kneser-Ney discounting. Words that were observed less than 20 times in the language model training data were replaced by UNK tags, which gave us a vocabulary size of 78,498. The perplexity of the 4-gram language model on the test corpus is 96. For computational efficiency only the top 100 substitutes and their unnormalized probabilities were computed for each positions in the PTB using the FASTSUBS algorithm (Yuret 2012)[6]. The probability vectors for each position were normalized to add up to 1.0 giving us the final substitute distributions used in the rest of this study.

The experiments were run using the following default settings (unless otherwise stated): (i) each word was kept with its original capitalization, (ii) the learning rate parameters were set to $\varphi_0 = 50$, $\eta_0 = 0.2$ for faster convergence in log likelihood, (iii) the number of S-CODE iterations were set to 50 million, (iv) the S-CODE dimensions and Z were set to 25 and 0.166, respectively,

6 The substitutes with unnormalized log probabilities can be downloaded from `http://goo.gl/jzKH0`.

Table 3: Summary of results in terms of the MTO and VM scores. Standard errors are given in parentheses when available. Starred entries have been reported in the review paper (Christodoulopoulos, Goldwater, and Steedman 2010). Distributional models use only the identity of the target word and its context. The models on the right incorporate orthographic and morphological features.

Distributional Models	MTO	VM	Models with Additional Features	MTO	VM
Lamar et al. (2010)	.708	-	Clark (2003)*	.712	.655
Brown et al. (1992)*	.678	.630	Christodoulopoulos et al. (2011)	.728	.661
Goldwater et al. (2007)*	.632	.562	Berg-Kirkpatrick et al. (2010)	.755	-
Ganchev et al. (2010)*	.625	.548	Christodoulopoulos et al. (2010)	.761	.688
Maron et al. (2010)	.688 (.0016)	-	Blunsom and Cohn (2011)	.775	.697
Substitutes(Tokens) (Sec. **??**)	.7030 (.0070)	.6006 (.0071)	Substitutes and Features (Sec. 4.5)	.8002 (.0070)	.7163 (.0040)
Bigrams (Sec. **??**)	.7319 (.0088)	.6554 (.0039)			
Substitutes(Types) (Sec. **??**)	.7667 (.0056)	.6819 (.0029)			

(v) a modified k-means algorithm with smart initialization was used (Arthur and Vassilvitskii 2007), and (vi) the number of k-means restarts were set to 128 to improve clustering and reduce variance.

Each experiment was repeated 10 times with different random seeds and the results are reported with standard errors in parentheses or error bars in graphs. Table 3 summarizes all the results reported in this section and the ones we cite from the literature.

4.2 Clustering Word Embeddings (W)

The S-CODE uses stochastic gradient ascent (see Appendix **??**) to find the ϕ_w, ψ_s embeddings for word and random-substitute in these pairs on a single 25-dimensional sphere. The algorithm cycles through the data until we get approximately 50 million updates. The resulting ϕ_w vectors are clustered using an instance weighted k-means algorithm. Cluster-id for each ϕ_w is assigned as the predicted cluster-id for word type W. Using the default settings and sampling 64 substitutes for each token the many-to-one accuracy is .7667 (.0056)and the V-measure is .6819 (.0029).

To analyze the sensitivity of this result to our specific parameter settings we ran a number of experiments where each parameter was varied over a range of values.

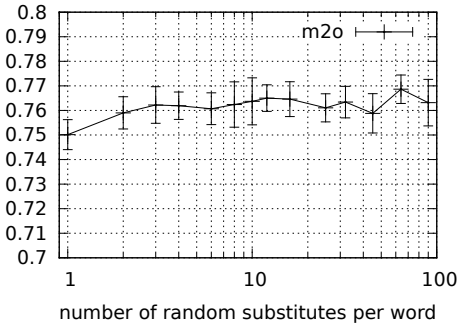

Figure 3: MTO is not sensitive to the number of random substitutes sampled per word token.

Figure 3 illustrates that the random-substitute result is fairly robust as long as the training algorithm can observe more than a few random substitutes per word.

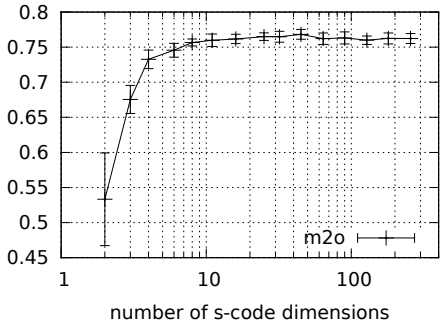

Figure 4: MTO falls sharply for less than 10 S-CODE dimensions, but more than 25 do not help.

Figure 4 shows that at least 10 embedding dimensions are necessary to get within 1% of the best result, but there is no significant gain from using more than 25 dimensions.

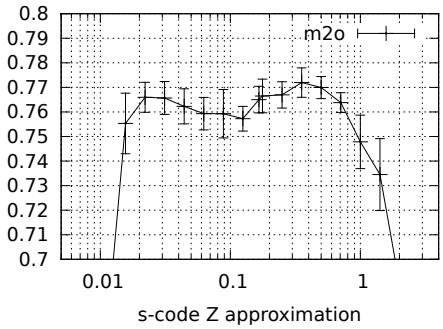

Figure 5: MTO is fairly stable as long as the $\tilde{Z}$ constant is within an order of magnitude of the real Z value.

Figure 5 shows that the constant $\tilde{Z}$ approximation can be varied within two orders of magnitude without a significant performance drop in the many-to-one score. For uniformly distributed points on a 25 dimensional sphere, the expected $Z \approx 0.146$. In the experiments where we tested we found the real Z always to be in the 0.140-0.170 range. When the constant $\tilde{Z}$ estimate is too small the attraction in Eq. B.3 dominates the repulsion in Eq. B.4 and all points tend to converge to the same location. When $\tilde{Z}$ is too high, it prevents meaningful clusters from coalescing.

We find our model with random substitute to be fairly robust to different parameter settings and the resulting many-to-one score significantly better than the state-of-the-art distributional models.

4.3 Paradigmatic vs Syntagmatic Representations of Word Context

To get a direct comparison of the paradigmatic and syntagmatic context representations we feed 4 different co-occurrences defined in Section 2 into the S-CODE algorithm. The first model accepts word (X) - right bigram (Y) pairs as the input, the second model accepts word (X) - left bigram(Y) pairs as the input, the third model accepts word (X) - concatenation of the left

and right bigrams (Y) pairs (Mintz 2003) as the input and the final model accepts words (X) - left bigram(Y_1) and right bigram (Y_2) tuples (St Clair, Monaghan, and Christiansen 2010) as the input to the S-CODE. At the end we cluster the word types (X) with k-means algorithm and report the results on Table 4.

Table 4: Summary of results in terms of the MTO and VM scores of the S-CODE algorithm when the paradigmatic or syntagmatic representations are feed as an input. Standard errors are given in parentheses when available. Results of the statistically best performing system are written in bold. We do not report the original results of Maron et al. (2010) since our replication achieves higher accuracies.

Input	MTO	VM
X (word) - Y (right bigram)	.6625 (.0115)	.5809 (.0066)
X (word) - Y (left bigram)	.6604 (.0054)	.5983 (.0028)
X (word) - Y (left and right bigram concatenation)	.7268 (.0091)	.6416 (.0052)
X (word) - Y_1, Y_2 (left and right bigrams)	.7173 (.0061)	.6381 (.0032)
Maron et al. (2010)(replication)	.7319 (.0088)	.6554 (.0039)
X (word) - Y (random substitutes)	**.7667 (.0056)**	**.6819 (.0029)**

To replicate the work of Maron et al. (2010) we feed word (X) - right bigram (Y) pairs as the input. At the end each word w in the vocabulary ends up with two points on the sphere, a ϕ_w point representing the behavior of w as the left word of a bigram and a ψ_w point representing it as the right word. The two vectors for w are concatenated to create a 50-dimensional representation at the end. These 50-dimensional vectors are clustered using the k-means algorithm. Maron et al. (2010) report many-to-one scores of .6880 (.0016) for 45 clusters and .7150 (.0060) for 50 clusters (on the PTB). Using our default settings the bigram model achieves .7319 (.0088) MTO and .6554 (.0039) VM accuracies. Table 4 summarizes all the results and shows that the paradigmatic representation accuracies are significantly higher than the syntagmatic representation MTO and VM accuracies.

4.4 POS Induction of Word Tokens

4.4.1 Random Substitutes. We extend the random-substitutes algorithm presented earlier to perform POS induction for the word tokens rather than the word types. We generate word (X) – random-substitute (Y) pairs as the input to the S-CODE. For each observed $X - Y$ pair in the S-CODE input, corresponding 25-dimensional ϕ_x and ψ_y embedding vectors are concatenated to create a 50-dimensional representation. The resulting 50-dimensional vectors are clustered using the instance weighted k-means algorithm with 128 restarts. The process yields 64 cluster-ids (for every pair generated from word token's context) for each word token's context. The cluster-ids tokens are predicted by the majority cluster-id of the corresponding pairs. Ties for the majority are broken randomly. The many-to-one accuracy is .7030 (.0070) and the V-measure is .6006 (.0071) .

In order to demonstrate the merit in the token based POS induction, we first define the gold-tag perplexity for the word types as following:

$$GP(w) = 2^{H(p_w)} = 2^{-\sum_x p_w(x) log_2 p_w(x)} \quad (1)$$

Where p_w is the gold POS tag distribution of the word type w and $H(p_w)$ is the entropy of the p_w distribution.

Gold-tag perplexity (GP) is used to determine the POS ambiguity of the word types, relating how often a word type is associated with different POS tags in the test corpus. A GP of 1 for a

Table 5: Accuracy in the gold-tag perplexity separated subsets.

Model	$GP < 1.75$ 89%	$GP \geq 1.75$ 11%
Substitutes(Type)	.8054 (.0065)	.4383 (.0104)
Substitutes(Tokens)	.7322 (.0079)	.4671 (.0174)

Table 6: Accuracies of the token based S-CODE models on the gold-tag perplexity separated subsets.

Model	$GP < 1.75$ 89%	$GP \geq 1.75$ 11%	$GP \geq 1.0$ 100%
X (word) - Y (left bigram)	.5950 (.0051)	.4783 (.0005)	.5821 (.0041)
X (word) - Y (right bigram)	.6239 (.0049)	.3075 (.0153)	.5891 (.0046)
X (word) - Y (left and right bigram concatenation)	.7523 (.0065)	.4492 (.0240)	.7190 (.0049)
X (word) - Y_1, Y_2 (left and right bigrams)	.6697 (.0065)	.4579 (.0052)	.6464 (.0051)
X (word) - Y (random substitutes)	.7322 (.0079)	.4671 (.0174)	.7030 (.0073)

word type w indicates w is associated with same POS tag throughout the test corpus, meaning the word type w's POS is unambiguous. As the GP increases ambiguity for the word types increases and poses a handicap for induction models that limits tag variety for the word types. To display the limitations, we split the test corpus in two subsets: word types with GP less than 1.75 and word types with GP equal or greater than 1.75. We performed MTO evaluation on the the whole test corpus induction output and obtained the induced-tag – gold-tag mappings. Using the mappings obtained over the test corpus, we evaluated the accuracy in the subsets. Table 5 presents the evaluation over the subsets.

4.4.2 Paradigmatic vs Syntagmatic Representations of Word Context. In order to compare the token clustering performance of the paradigmatic and the syntagmatic context representations we use the same 4 models defined in Section 4.3. Following the previous section we concatenate the 25-dimensional ϕ_x and ψ_y (ψ_{y_1} and ψ_{y_2} in the fourth model) embeddings of the corresponding observed pairs (tuples in the fourth model) and represent the first three models outputs with a 50-dimensional vectors (75-dimensional vectors in the fourth model). The resulting vectors are clustered using k-means algorithm with 128 restarts.

4.5 Morphological and Orthographic Features

Clark (2003) demonstrates that using morphological and orthographic features significantly improves part of speech induction with an HMM based model. Section **??** describes a number of other approaches that show similar improvements. We integrate additional features together with substitutes by using the model decsribed in Appendix C.

The orthographic features we used are similar to the ones in (Berg-Kirkpatrick et al. 2010) with small modifications:

- Initial-Capital: this feature is generated for capitalized words with the exception of sentence initial words.
- Number: this feature is generated when the token starts with a digit.

- Contains-Hyphen: this feature is generated for lowercase words with an internal hyphen.
- Initial-Apostrophe: this feature is generated for tokens that start with an apostrophe.

We generated morphological features using the unsupervised algorithm Morfessor (Creutz and Lagus 2005). Morfessor was trained on the WSJ section of the Penn Treebank using default settings, and a perplexity threshold of 1. In our model, a word type consists of two parts: a stem and a suffix part. The suffix part is used as the morphological feature thus each word type has only one morphological feature[7]. The program induced 5575 suffix types that are present in a total of 19223 word types.

Using the training settings of the previous section, the addition of morphological and orthographic features increased the many-to-one score of the random-substitute model to .8002 (.0070) and V-measure to .7163 (.0040). Both these results improve the state-of-the-art in part of speech induction significantly as seen in Table 3.

5. Multilingual Experiments

We performed experiments with a range of languages and three different feature configurations to establish both the robustness of our model across languages and to observe the effects of different features. Following Christodoulopoulos et al. (2011), in addition to the PTB we extend our experiments to 8 languages from MULTEXT-East (Bulgarian, Czech, English, Estonian, Hungarian, Romanian, Slovene and Serbian) (Erjavec 2004) and 10 languages from the CoNLL-X shared task (Bulgarian, Czech, Danish, Dutch, German, Portuguese, Slovene, Spanish, Swedish and Turkish) (Buchholz and Marsi 2006). For all experiments, we use the best performing model of Section **??** (i.e. the random substitute model) with default settings. To perform meaningful comparisons with the previous work we train and evaluate our models on the training section of MULTEXT-East[8] and CONLL-X languages (Lee, Haghighi, and Barzilay 2010).

Section 5.1 details the language model and feature statistics of each language. Section 5.2 summarizes the results of our models for all of the languages in our corpora. In the rest of this section we refer to the MULTEXT-East and CONLL-X corpora as the testing corpora and the language model training corpora as the training corpora.

5.1 Random Substitute and Features

To sample substitutes we calculate the probabilities of top 100 substitutes of each position, we train a 4-gram language model with the corresponding training corpora of each language as described in Section 4.1. Table 7 presents statistics related to the language model training and testing corpora. For all languages except Serbian, English and Turkish, we train the language models by using the corresponding Wikipedia dump files[9].

Serbian shares a common basis with Croatian and Bosnian therefore we trained 3 different language models using Wikipedia dump files of Serbian together with these two languages and measured the perplexities on the MULTEXT-East Serbian corpus. We chose the Croatian

7 We extracted the stem part by concatenating the splits until including the first "STM" labeled split and the suffix part by concatenating rest of the splits.

8 Languages of MULTEXT-East corpora do not tag the punctuation marks, thus we add an extra tag for punctuation to the tag-set of these languages.

9 Latest Wikipedia dump files are freely available at `http://dumps.wikimedia.org/` and the text in the dump files can be extracted using WP2TXT (`http://wp2txt.rubyforge.org/`)

Table 7: Summary of language model training and test corpora statistics for each language in the test set. Last two columns present the number of induced suffix parts and word types with these suffix parts after the morphological feature extraction.

		Language Model		Test set				
	Language	Source	Word Count	Word Count	Perplexity (ppl)	Unknown Word	Suffix Parts	Word Types with Suffix parts
WSJ	English	News	126,170,376	1,173,766	79.926	0.012	5575	19223
MULTEXT-East	Bulgarian	Wikipedia	32,511,616	101,173	655.202	.0565	609	4209
	Czech	Wikipedia	59,698,049	100,368	1,069.67	.0299	2787	12848
	English	News	126,170,376	118,424	265.246	.0288	1251	4783
	Estonian	Wikipedia	14,513,571	94,898	871.765	.0654	4448	13638
	Hungarian	Wikipedia	66,069,788	98,426	742.676	.0449	5423	15995
	Romanian	Wikipedia	35680870	118,328	666.855	.1074	2064	9445
	Slovene	Wikipedia	18,969,846	112,278	658.711	.0389	2093	11834
	Serbian	Wikipedia	17,129,679	108,809	804.962	.0580	2722	12476
CoNLL-X Shared Task	Bulgarian	Wikipedia	32,511,616	190,217	538.972	.0430	926	8225
	Czech	Wikipedia	59,698,049	1,249,408	1,233.95	.0250	12443	85673
	Danish	Wikipedia	35,863,945	94,386	351.24	.0393	3708	10897
	Dutch	Wikipedia	159,978,524	195,069	390.818	.0476	5250	13407
	German	Wikipedia	437,777,863	699,610	680.036	.0487	15219	45414
	Portuguese	Wikipedia	150,099,154	206,678	378.656	.0861	5033	15721
	Slovene	Wikipedia	18,969,846	28,750	663.053	.0414	1257	4781
	Spanish	Wikipedia	332,311,650	89,334	274.418	.0424	2648	9316
	Swedish	Wikipedia	32,004,538	191,467	1,233.95	.0250	2897	12725
	Turkish	Web	491,195,991	47,605	868.829	.0508	5651	14227

language model since it achieved the lowest perplexity score and unknown word ratio on the MULTEXT-East Serbian corpus.

To train the statistical language model of English, we use Wall Street Journal data (1987-1994) extracted from CSR-III Text (Graff, Rosenfeld, and Paul 1995) (excluding sections of the PTB) and for the Turkish language modeling we use the web corpus collected from Turkish news and blog sites (Sak, Güngör, and Saraçlar 2008).

Language model training files vary across the languages in terms of quality and quantity. In order to reduce the unknown word ratio of resource poor languages and to standardize the process we set the vocabulary threshold to 2 for all languages except English. English has relatively low unknown word ratio therefore we set the threshold to 20 instead of 2.

We use the same set of orthographic features described in Section 4.5 except we add an "Only-Punctuation" feature to the languages of MULTEXT-East corpora. The "Only-Punctuation" feature is generated when a token only consists of punctuation characters.

Morphological features are extracted by the method described in Section 4.5 using the training sections of each language in MULTEXT-East and CoNLL-X corpora[10]. Language specific morphological feature statistics are summarized in Table 7.

10 We don't use the language model corpora to extract morphological features.

Table 8: The MTO and VM scores on 19 corpora in 15 languages together with the number of types and tags in gold–set which equals to number of induced clusters in all languages. Best published results are from ‡(Blunsom and Cohn 2011), ⋆(Christodoulopoulos, Goldwater, and Steedman 2011) and †(Clark 2003). Bold results represent the best MTO and VM accuracies of the corresponding language with at least 90% confidence level. MULTEXT-East corpora do not tag the punctuation marks, thus we add an extra tag for punctuation and represent it with "+1".

	Language	Types	Tags	Best Published	Syntagmatic Bigram	DIS	DIS+O	DIS+O+M
WSJ	English	49,190	45	.775 / .697‡	.7314 / .6558	.7667 / .6819	.7820 / .7020	**.8002 / .7163**
MULTEXT-East	Bulgarian	16,352	12+1	.665 / .556⋆	.6732 / .4119	.6927 / .5341	.6964 / .5469	**.7027** / .5513
	Czech	19,115	12+1	.642 / **.539**⋆	.6269 / .4586	.7025 / .5020	.7022 / .5047	**.7045** / .5096
	English	9,773	12+1	.733 / .633⋆	.7690 /. 6131	.8239 / .6631	.8246 / .6696	**.8329 / .6769**
	Estonian	17,845	11+1	.644 / **.533**⋆	.6089 / .4119	.6612 / .4469	**.6704** / .4658	.6445 / .4452
	Hungarian	20,321	12+1	.682 / **.548**⋆	.6181 / .4514	.6900 / .4972	.6963 / .5173	**.7254** / .5402
	Romanian	15,189	14+1	.611 / .523⋆	.6565 / .5202	.6412 / .5004	**.6607** / .5262	.6432 / .5127
	Slovene	17,871	12+1	.679 / **.567**⋆	.6772 / .5044	.6914 / .4951	**.6966*** / .4998	.6823 / .4938
	Serbian	18,095	12+1	.641 / **.510**†	.6267 / .4510	.6311 / .4536	.6317 / .4557	.6370 / .4648
CoNLL-X Shared Task	Bulgarian	32,439	54	.704 / **.596**†	.6972 / .5532	.7328 / .5781	**.7348** / .5844	.7321 / .5835
	Czech	130,208	12	.701‡ / .484⋆	.6944 / .5036	.6739 / .4838	**.7176* / .5336**	.7039 / .5118
	Danish	18,356	25	.761‡ / .591⋆	.6757 / .5290	.7236 / .5583	.7538 / .5962	.7417 / .5919
	Dutch	28,393	13	.711‡ / .547⋆	.6703 / .5205	.6957 / .5331	**.7401 / .5986**	.7210 / .5919
	German	72,326	54	.744⋆ / .630†	.7525 / .6285	.7669 / .6306	**.7799 / .6575**	.7557 / .6395
	Portuguese	28,931	22	.785‡ / .639⋆	.7031 / .5617	.7439 / .5798	.7901 / .6316	.7861 / .6353
	Slovene	7,128	29	.642⋆ / **.539**†	.6384 / .4976	.6513 / .4957	**.6545** / .5093	.6543 / .5031
	Spanish	16,458	47	**.788**‡ / .632⋆	.7086 / .5844	.7479 / .6086	.7712 / .6346	.7588 / .6287
	Swedish	20,057	41	.682 / **.589**†	.6721 / .5558	.6962 / .5674	**.6962** / .5721	.6675 / .5628
	Turkish	17,563	30	.628 / .408⋆	.6069 / .3551	.6239 / .3823	.6372 / .4098	**.6487 / .4206**

5.2 Results

For each language we report results of three models: (1) distributional (DIS), (2) distributional with orthographic features (DIS+O) and (3) distributional with both orthographic and morphological features (DIS+O+M). Similar to the settings used in Section **??**, we use the 25 dimensional sphere with 64 substitutes for all languages. For each language the number of induced clusters is set to the number of tags in the gold-set as presented in Table 9.

As a baseline model we chose the syntagmatic bigram version of S-CODE described in Section **??** which is a very strong baseline compared to the ones used in (Christodoulopoulos, Goldwater, and Steedman 2011). Table 9 summarizes the MTO and VM scores of our models together with the syntagmatic bigram baseline and the best published accuracies on each language corpus.

DIS significantly outperforms the syntagmatic bigram baseline in both MTO and VM scores on 14 languages. DIS+O+M has the state-of-the-art MTO and VM accuracy on the PTB. DIS+O and DIS+O+M achieve the highest MTO scores on all languages of MULTEXT-East corpora while scoring the highest VM accuracies on English and Romanian. On the CoNLL-X languages our models perform better than the best published MTO or VM accuracies on 10 languages.

Table 9: The MTO and VM scores of X+Y token clustering S-CODE on 19 corpora in 15 languages together with the number of types and tags in gold–set which equals to number of induced clusters in all languages. MULTEXT-East corpora do not tag the punctuation marks, thus we add an extra tag for punctuation and represent it with "+1".

	Language	Types	Tags	S-CODE X	S-CODE X+Y
WSJ	English	49,190	45	.7667 / .6819	.7030 / .6006
MULTEXT-East	Bulgarian	16,352	12+1	.6927 / .5341	.6551 / .4644
	Czech	19,115	12+1	.7025 / .5020	.6145 / .4031
	English	9,773	12+1	.8239 / .6631	.7697 / .5977
	Estonian	17,845	11+1	.6612 / .4469	.5934 / .3509
	Hungarian	20,321	12+1	.6900 / .4972	.6235 / .4152
	Romanian	15,189	14+1	.6412 / .5004	.5931 / .4185
	Slovene	17,871	12+1	.6914 / .4951	.6466 / .4119
	Serbian	18,095	12+1	.6244 / .4473	.5622 / .3466
CoNLL-X Shared Task	Bulgarian	32,439	54	.7328 / .5781	.6615 / .4821
	Czech	130,208	12	.6739 / .4838	.6082 / .3566
	Danish	18,356	25	.7236 / .5583	.6210 / .4351
	Dutch	28,393	13	.6957 / .5331	.6168 / .4000
	German	72,326	54	.7669 / .6308	.6484 / .5083
	Portuguese	28,931	22	.7479 / .5798	.6734 / .4859
	Slovene	7,128	29	.6513 / .4957	.5918 / .4164
	Spanish	16,458	47	.7479 / .6086	.6739 / .5142
	Swedish	20,057	41	.6962 / .5674	.5779 / .4385
	Turkish	17,563	30	.6239 / .3823	.5910 / .3309

6. Discussion

In this section we perform further analysis on the clustering output of our best model and indicate the possible reasons of comparably low VM scores. To illustrate how words are distributed in the induced clusters, we compare the output of our model with gold-tags of the PTB. We also discuss the effect of coarse gold-tag sets on our model performance.

Figure 6 is the Hinton diagram of the PTB showing the relationship between the most frequent tags and clusters from the experiment in Section 4.5. In general the errors seem to be the lack of completeness (multiple large entries in a row), rather than lack of homogeneity (multiple large entries in a column). The algorithm tends to split large word classes into several clusters. Some examples are:

- Titles like Mr., Mrs., and Dr. are split from the rest of the proper nouns in cluster (39).
- Auxiliary verbs (10) and the verb "say" (22) have been split from the general verb clusters (12) and (7).
- Determiners "the" (40), "a" (15), and capitalized "The", "A" (6) have been split into their own clusters.
- Prepositions "of" (19), and "by", "at" (17) have been split from the general preposition cluster (8).

Nevertheless there are some homogeneity errors as well:

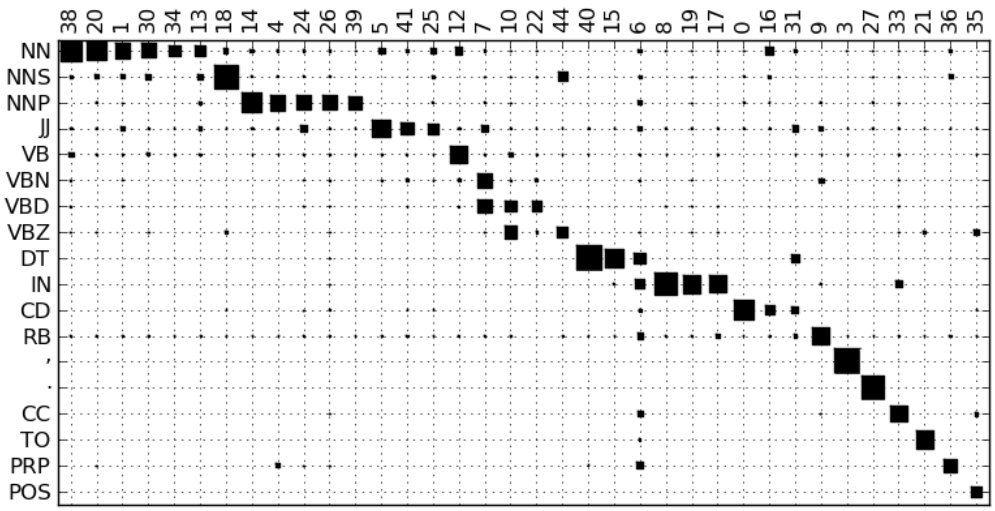

Figure 6: Hinton diagram comparing most frequent tags and clusters. Area of each square is proportional to the joint probability of the given tag and cluster.

- The adjective cluster (5) also has some noun members probably due to the difficulty of separating noun-noun compounds from adjective modification.
- Cluster (6) contains capitalized words that span a number of categories.

Most closed-class items are cleanly separated into their own clusters as seen in the lower right hand corner of the diagram.

The completeness errors become more noticeable on languages with coarse tag-sets thus our models perform worse than the best published models on 6 of MULTEXT-East languages in terms of VM scores while achieving the state-of-the-art MTO scores on same languages as shown on Table 9. On CONLL-X languages the effect of completeness errors is less noticeable since all languages except Czech and Dutch have fine grained tag-sets.

The completeness errors are not surprising given that the words that have been split are not generally substitutable with the other members of their gold-tag set category. Thus it can be argued that metrics that emphasize homogeneity such as MTO are more appropriate in this context than metrics that average homogeneity and completeness such as VM as long as the number of clusters is controlled.

There are two concerns inherent in all distributional methods: (i) words that are generally substitutable like “the” and “its” are placed in separate categories (DT and PRP$) by the gold standard, (ii) words that are generally not substitutable like “do” and “put” are placed in the same category (VB). Freudenthal et al. (2005) point out that categories with unsubstitutable words fail the standard linguistic definition of a syntactic category and children do not seem to make errors of substituting such words in utterances (e.g. *“What do you want?”* vs. **“What put you want?”*). Whether gold standard part of speech tags or distributional categories are better suited to applications like parsing or machine translation can be best decided using extrinsic evaluation. In this study we evaluate our results by comparing them to gold standard part of speech tags and leave the extrinsic evaluation of the induced tags for future work.

7. Contributions

Our main contributions can be summarized as follows:

- We introduced substitute vectors as paradigmatic representations of word context and demonstrated their use in unsupervised part of speech induction on 19 corpora in 15 languages.
- We demonstrated that using paradigmatic representations of word context and modeling co-occurrences of word and context types with the S-CODE learning framework give superior results when compared to a syntagmatic bigram model.
- We extended the S-CODE framework to incorporate morphological and orthographic features and improved the state-of-the-art many-to-one accuracy in unsupervised part of speech induction on 17 out of 19 corpora.
- All our code and data, including the substitute vectors for the PTB, MULTEXT-East and CoNLL-X shared task corpora are available at the authors' website at `xxx.xxx.xxx`.

Appendix A: Computation of Substitute Distributions

In this study, we predict the syntactic category of a word in a given context based on its substitute distribution. The sample space of the substitute distribution is the vocabulary of the language model including the unknow word tag "_unk_". Note that the substitute distribution is a function of the context only and is indifferent to the target word.

It is best to use both the left and the right context when estimating the probabilities for potential lexical substitutes. For example, in *"He lived in San Francisco suburbs."*, the token *San* would be difficult to guess from the left context but it is almost certain looking at the right context. We define c_w as the $2n-1$ word window centered around the target word position: $w_{-n+1} \dots w_0 \dots w_{n-1}$ ($n = 4$ is the n-gram order). The probability of a substitute word w in a given context c_w can be estimated as:

$$P(w_0 = w|c_w) \propto P(w_{-n+1} \dots w_0 \dots w_{n-1}) \tag{A.1}$$

$$= P(w_{-n+1})P(w_{-n+2}|w_{-n+1}) \dots P(w_{n-1}|w_{-n+1}^{n-2}) \tag{A.2}$$

$$\approx P(w_0|w_{-n+1}^{-1})P(w_1|w_{-n+2}^{0}) \dots P(w_{n-1}|w_0^{n-2}) \tag{A.3}$$

where w_i^j represents the sequence of words $w_i w_{i+1} \dots w_j$. In Equation A.1, $P(w|c_w)$ is proportional to $P(w_{-n+1} \dots w_0 \dots w_{n-1})$ because the words of the context are fixed. Terms without w_0 are identical for each substitute in Equation A.2 therefore they have been dropped in Equation A.3. Finally, because of the Markov property of n-gram language model, only the closest $n-1$ words are used in the experiments.

Near the sentence boundaries the appropriate terms were truncated in Equation A.3. Specifically, at the beginning of the sentence shorter n-gram contexts were used and at the end of the sentence terms beyond the end-of-sentence token were dropped.

To obtain a discrete representation of the context, the random-substitutes algorithm pairs each word token with a substitute sampled from the pre-computed substitute distribution gener-

ated from the word token's context and then word (W) – random-substitute (S) pairs are fed to the S-CODE as input.

Appendix B: The CODE Model

In this section we review the unsupervised method that we use to model co-occurrence statistics: the Co-occurrence Data Embedding (CODE) (Globerson et al. 2007) method and its spherical extension (S-CODE) introduced by (Maron, Lamar, and Bienenstock 2010).

Let W and C be two categorical variables with finite cardinalities $|W|$ and $|C|$. We observe a set of pairs $\{w_i, c_i\}_{i=1}^n$ drawn IID from the joint distribution of W and C. The basic idea behind CODE and related methods is to represent (embed) each value of W and each value of C as points in a common Euclidean space $\mathbf{R}^d$ such that values that frequently co-occur lie close to each other. There are several ways to formalize the relationship between the distances and co-occurrence statistics, in this paper we use the following:

$$p(w,c) = \frac{1}{Z}\bar{p}(w)\bar{p}(c)e^{-d^2_{w,c}} \tag{B.1}$$

where $d^2_{w,c}$ is the squared distance between the embeddings of w and c, $\bar{p}(w)$ and $\bar{p}(c)$ are empirical probabilities, and $Z = \sum_{w,c}\bar{p}(w)\bar{p}(c)e^{-d^2_{w,c}}$ is a normalization term. If we use the notation ϕ_w for the point corresponding to w and ψ_c for the point corresponding to c then $d^2_{w,c} = \|\phi_w - \psi_c\|^2$. The log-likelihood of a given embedding $\ell(\phi, \psi)$ can be expressed as:

$$\begin{aligned}\ell(\phi,\psi) &= \sum_{w,c}\bar{p}(w,c)\log p(w,c) \qquad (B.2)\\ &= \sum_{w,c}\bar{p}(w,c)(-\log Z + \log\bar{p}(w)\bar{p}(c) - d^2_{w,c})\\ &= -\log Z + const - \sum_{w,c}\bar{p}(w,c)d^2_{w,c}\end{aligned}$$

The likelihood is not convex in ϕ and ψ. We use gradient ascent to find an approximate solution for a set of ϕ_w, ψ_c that maximize the likelihood. The gradient of the $d^2_{w,c}$ term pulls neighbors closer in proportion to the empirical joint probability:

$$\frac{\partial}{\partial\phi_w}\sum_{w,c} -\bar{p}(w,c)d^2_{w,c} = \sum_y 2\bar{p}(w,c)(\psi_c - \phi_w) \tag{B.3}$$

The gradient of the Z term pushes neighbors apart in proportion to the estimated joint probability:

$$\frac{\partial}{\partial\phi_x}(-\log Z) = \sum_y 2p(w,c)(\phi_w - \psi_c) \tag{B.4}$$

Thus the net effect is to pull pairs together if their estimated probability is less than the empirical probability and to push them apart otherwise. The gradients with respect to ψ_c are similar. S-CODE (Maron, Lamar, and Bienenstock 2010) additionally restricts all ϕ_w and ψ_c to lie on the unit sphere. With this restriction, Z stays around a fixed value during gradient ascent. This allows S-CODE to substitute an approximate constant $\tilde{Z}$ in gradient calculations for the real

Z for computational efficiency. In our experiments, we used S-CODE with its sampling based stochastic gradient ascent algorithm and smoothly decreasing learning rate.

Appendix C: S-Code with More than Two Variables

In order to accommodate multiple feature types the CODE model in the previous section needs to be extended to handle more than two variables. Globerson et. al(2007) suggest the following likelihood function:

$$\ell(\phi, \psi^{(1)}, \ldots, \psi^{(K)}) = \bar{p}(w, c) \log p(w, c) + \sum_{i}^{K} \sum_{w, f^{(i)}} \bar{p}(w, f^{(i)}) \log p(w, f^{(i)}) \quad \text{(C.1)}$$

where $\bar{p}(w, c)$ is the emprical joint distribution of context C with W, $F^{(1)}, \ldots, F^{(K)}$ are extra K different variables whose empirical joint distributions with W, $\bar{p}(w, f^{(1)}) \ldots \bar{p}(w, f^{(K)})$, are known. Eq. C.1 then represents a set of CODE models $p(w, f^{(k)})$ where each $F^{(k)}$ has an embedding $\psi_f^{(k)}$ but all models share the same ϕ_w embedding.

We adopt this likelihood function, let W represents a word, C represents a context (i.e., random substitute), and $F^{(1)}, \ldots, F^{(K)}$ stand for morphological and orthographic features of the word thus each word is a (K+1)-tuple, $(W, C, F^{(1)}, \ldots F^{(K)})$. With this setup, the training procedure needs to change little: instead of sampling a word (w) – context (c), the word (w) – context (c) – features ($f_1, \ldots, f_K$) tuple is sampled and input to the gradient ascent algorithm. The gradient search algorithm updates the embeddings according to $p(w, c)$ and $p(w, f^{(i)})$ where $i = 1 \ldots k$ and no updates are performed between c and $f^{(i)}$s since they do not have any co-occurrence statistics and w is the only shared variable.

Tuples might have null values due to the unobserved features. For example in the case of POS induction, the word "**car**" has no morphological or orthographic features therefore all the elements of the tuple have null value except the word type (w) and the context (c). We do not perform any pull or push updates on embeddings during the gradient search if the corresponding $f^{(k)}$ is null[11].

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11 In the POS induction problem w and c represents the word type and context therefore they are always observed.

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