Unsupervised Part of Speech Induction Using Paradigmatic Representations of Word Context

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We investigate paradigmatic representations of word context in the domain of unsupervised part of speech induction. Paradigmatic representations of word context are based on potential substitutes of a word in contrast to syntagmatic representations based on its neighbors. We model the joint probability of words and their contexts (as represented by potential substitutes) using the S-CODE framework. S-CODE maps target words, their potential substitutes and other features to high dimensional Euclidean vectors. These vectors aggregate into clusters that largely match the traditional part-of-speech boundaries and give state-of-the-art results in unsupervised part-of-speech induction, including 80% many-to-one accuracy on the Penn Treebank and statistically significant improvements over best published results on 17 out of 19 corpora in 15 languages.

1. Introduction

Grammar rules apply not to individual words (e.g. dog, eat) but to part-of-speech categories (e.g. noun, verb). Thus learning part-of-speech categories (also known as lexical or syntactic categories) is one of the fundamental problems in language acquisition.

Linguists identify part-of-speech categories based on semantic, syntactic, and morphological properties of words. There is also evidence that children use prosodic and phonological features to bootstrap part-of-speech category acquisition (Ambridge and Lieven 2011). However there is as yet no satisfactory computational model that match human performance. Thus identifying the best set of features and best learning algorithms for part-of-speech induction is still an open problem.

Relationships between linguistic units can be classified into two types: syntagmatic (concerning positioning), and paradigmatic (concerning substitution). Syntagmatic relations determine which units can combine to create larger groups and paradigmatic relations determine which units can be substituted for one another. Figure 1 illustrates the paradigmatic vs syntagmatic axes for words in a simple sentence and their possible substitutes.

Part-of-speech categories represent groups of words that can be substituted for one another without altering the grammaticality of a sentence. In this paper we explore models of part-of-speech induction based on potential substitutes of words. We build *substitute word distributions*

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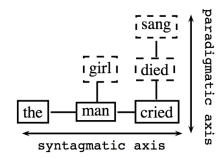


Figure 1: Syntagmatic vs. paradigmatic axes for words in a simple sentence (Chandler 2007).

for each position in the text which specify the probability of every vocabulary word in that position. Table 2 gives substitute distributions for an example sentence.

Table 1: The substitute word distributions (with probabilities in parentheses) for some of the positions in the example sentence "Pierre Vinken, 61 years old, will join the board as a nonexecutive director Nov. 29." based on an n-gram language model.

```
      will:
      will (0.9985), would (0.0007), to (0.0006), also (0.0001), ...

      join:
      join (0.6528), leave (0.2140), oversee (0.0559), head (0.0262), rejoin (0.0074), ...

      the:
      its (0.9011), the (0.0981), a (0.0006), ...

      board:
      board (0.4288), company (0.2584), firm (0.2024), bank (0.0731), strike (0.0030), ...
```

Note that the substitute word distribution for a position (e.g. the second position in Fig. 1) is a function of the context only (i.e. "the ___ cried"), and does not depend on the word that actually appears there (i.e. "man"). Thus substitute distributions represent *individual word contexts*, not word types. We refer to representations based on substitute distributions as *paradigmatic representations of word context*.

We expect words used in similar contexts (with similar substitute distributions) to share the same part-of-speech. Thus part-of-speech induction depends on which contexts are considered similar, and context similarity in turn is a function of the features used to represent word context. Paradigmatic representations, using features of the substitute distribution, uncover latent similarities between contexts that on the surface seem to have little in common. This makes paradigmatic representations more robust to data sparsity, compared to syntagmatic representations which use neighboring words as features. Our empirical results demonstrate that paradigmatic representations significantly outperform syntagmatic ones when compared using similar part-of-speech induction algorithms on identical datasets. Section 1.1 presents alternative representations of word context and discusses paradigmatic representations in more detail.

1.1 Representing Word Context

In this section we demonstrate the different contextual representations in the part-of-speech induction (aka. syntactic word categorization) literature and introduce the substitute words as an alternative to the current context representations. In the rest of the paper the words in the vocabulary are referred as *types* and the instances of types are referred as *tokens*.

The contextual representations can be categorized into three groups based on the way they incorporate the local context information of the target type or token: (1) syntagmatic

representation, (2) Hidden Markov Models (HMM) and (3) paradigmatic representation. These representations can be further subdivided into two subgroups based on whether they group the types or the tokens.

Syntagmatic Representation. In syntagmatic representation the context is defined with the neighboring words, typically co-occurrences with a single word on the left or a single word on the right word called a "frame" (e.g., **the** dog **is**; **the** cat **is**) (Schütze and Pedersen 1993; Redington, Crater, and Finch 1998; Mintz 2003; St Clair, Monaghan, and Christiansen 2010; Lamar et al. 2010; Maron, Lamar, and Bienenstock 2010). Turney and Pantel (2010) give a broad overview of syntagmatic approaches and their applications within the Vector Space Modeling framework. Depending on the way they incorporate co-occurences, these models can perform hard (type based) or soft (token based) clustering.

Schütze (1993) represented the context of a word type by concatenating its left and right co-occurrence vectors. These vectors were calculated for each type by using the left and the right neighbors of the type instances therefore they characterize the distribution of the left and right neighboring tokens of the type. One constraint of this representation is that it represents types rather than tokens thus it is not possible to group the instances of any type into the separate categories.

Mintz (2003) showed on a subset of child directed speech corpus (CHILDES) (MacWhinney 2000) that non-adjacent high frequent bigram frames are useful for the language learners on the syntactic categorization of the tokens. For example, the tokens that are observed at "_" in the frame "the _ is" are assigned to the same category. Using the top-45 frequent frames Mintz achieved an average of 98% unsupervised accuracy\(^1\). The main limitation of the top-45 frequent frames is that they could only analyze the 6% of the tokens on average due to the sparsity. Another drawback is that the tokens with only one common neighbors could not exchange information.

St Clair et al. (2010) extended the work of Mintz (2003) and introduced the flexible bigram frames which represent the context by using the left and the right bigrams separately. As a result tokens with a common left or right bigram can exchange information and might be grouped together. For instance, two tokens that are observed at "_" in "the _ is" and "a _ is" can be categorized together due to the shared right bigram "is". Using a feed forward connectionist model they showed that the flexible frames are statistically better than the frequent frames in terms of the supervised accuracy². They also showed that representing token contexts only with the left or the right bigram is statistically better than the frequent frames but worse than the flexible frames in terms of supervised accuracy. Both Mintz (2003) and St Clair (2010) did not report any results with contexts larger than bigram since as the context is enriched, the re-occurrence frequency of a frame becomes lower which causes the data sparsity (Manning and Schütze 1999).

HMM. Prototypical HMM uses a bigram structure where tokens are generated by latent categories and learns the latent category sequence that generates the given word sequence instead of clustering tokens directly (Brown et al. 1992; Blunsom and Cohn 2011; Goldwater and Griffiths 2007; Johnson 2007; Ganchev et al. 2010; Berg-Kirkpatrick and Klein 2010; Lee, Haghighi, and Barzilay 2010). The POS induction literature focused on the first and second order HMMs since the higher order HMMs have additional complicating factors³ and require more complex training

¹ Unsupervised accuracy was defined as the number of hits (when two intervening tokens that observed in the frame are from the same category) divided by number of false alarms (when two intervening tokens that observed in the frame are from different categories).

² In order to perform meaningful comparisons they used all of the frequent frames instead of the top-45 ones.

³ The number of parameters in a prototypical HMM quadratically increases as the HMM order increases.

procedures (Johnson 2007). Depending on the design and the training procedure HMM models can group types or tokens which are detailed in Section 5.

Paradigmatic Representation. In the paradigmatic representation context is defined as the distribution of the substitute words in that context. Schütze (1995) incorporates paradigmatic information by concatenating the left co-occurrence and the right co-occurrence vectors of the right and the left tokens, respectively and grouped the tokens that have similar vectors. The vectors from the neighbors include potential substitutes. Yatbaz et al. (2012) calculate the most likely substitute words of a word in a given context and clusters the types that have similar substitutes.

Our paradigmatic representation is related to the second order co-occurrences used in (Schütze 1995). Our method improves on his foundation from three aspects: (1) it can cluster both the types and tokens (2) it uses a 4-gram language model rather than bigram statistics, (3) it includes the whole 78,498 word vocabulary rather than the most frequent 250 words. More importantly, rather than simply concatenating the vectors that represent the target word with vectors that represent the context we use a co-occurrence modeling algorithm.

Similarly, Schütze and Pedersen (1993) define the words that frequently co-occur together as the *syntagmatic associates* and words that have similar left and right neighbors as the *paradigmatic parallels*. We find that representing the paradigmatic axis more directly using substitute vectors rather than frequent neighbors improves part of speech induction.

Sahlgren (2006) gives a detailed analysis of paradigmatic and syntagmatic relations in the context of word-space models used to represent the word meanings. Sahlgren's paradigmatic model represents word types using co-occurrence counts of their frequent neighbors, in contrast to his syntagmatic model that represents word types using counts of contexts (documents, sentences) they occur in. Our substitute vectors do not represent word types at all, but *contexts of word tokens* using probabilities of likely substitutes. Sahlgren finds that in word-spaces built by frequent neighbor vectors, more nearest neighbors share the same part of speech compared to word-spaces built by context vectors.

The two examples below illustrate the advantages of paradigmatic representation in uncovering similarities where no overt similarity that can be captured by a syntagmatic representation exists. The word "director" from the first sentence and the word "chief" from the second one have no common neighbors in their 4-gram neighborhood. The paradigmatic representation captures the similarity of these words by suggesting the same top substitutes for both (the numbers in parentheses give substitute probabilities):

```
(1) "Pierre Vinken, 61 years old, will join the board as a nonexecutive director Nov. 29." director: chairman (.8242), director (.0127), directors (.0127)...
(2) "... Joseph Corr was succeeded by Frank Lorenzo, chief of parent Texas Air."
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chief: chairman (.09945), president (.0031), directors (.0012) . . .

The high probability substitutes reflect both semantic and syntactic properties of the context. Top substitutes for "director" and "chief" are not only nouns, but specifically nouns compatible with the semantic context. Top substitutes for the word "the" in the first example consist of words that can act as determiners: its (.9011), the (.0981), a (.0006),

2. Algorithm

In this section we describe the components of our algorithm and the flow of data between these components. The algorithm predicts the syntactic category of a word in a given context based on its random substitutes sampled from a statistical language model. First, we construct a pairwise co-occurrence representation of words and their substitutes. Next, we map each word and each substitute in the co-occurrence data to real vectors (embeddings) on an n-dimensional sphere using the S-CODE algorithm (Maron, Lamar, and Bienenstock 2010). S-CODE places the vectors for words that frequently co-occur with the same substitutes (and substitutes that co-occur with the same words) close to each other on the sphere. We then apply k-means clustering to group word and substitute embeddings and we induce word categories from the resulting groups. In Section 2.1 we detail the representation of words and their substitutes as co-occurrence data, in Section 2.2 we describe the embedding algorithm, and finally in Section 2.3 we describe the different ways in which words and substitutes can be clustered to give us categorizations of word types or tokens.

2.1 Context Representation

We represent the context of a word with random substitutes that are likely to occupy the same position as the word. We sample random substitutes (with replacement) from a substitute word distribution for the context calculated based on an n-gram language model. The sample space of the substitute word distribution is the vocabulary of the language model. In effect, we are using substitute word distributions and the sampled random substitutes as *contextual features* that represent properties of a word's position. Table 2 shows the substitute word distributions for some positions in an example sentence.

Table 2: The substitute word distributions (with probabilities in parentheses) for some of the positions in the example sentence "Pierre Vinken, 61 years old, will join the board as a nonexecutive director Nov. 29." based on a 4-gram language model.

```
        will:
        will (0.9985), would (0.0007), to (0.0006), also (0.0001), ...

        join:
        join (0.6528), leave (0.2140), oversee (0.0559), head (0.0262), rejoin (0.0074), ...

        the:
        its (0.9011), the (0.0981), a (0.0006), ...

        board:
        board (0.4288), company (0.2584), firm (0.2024), bank (0.0731), strike (0.0030), ...
```

To capture the relation between each word and its context we construct a co-occurrence representation by pairing the words with randomly sampled substitutes. Table 3 shows random substitutes of each word and their pairwise co-occurrence representation input to S-CODE for an example sentence. It is possible (and beneficial) to sample more than one substitute and generate multiple pairs for the same word-context pair as seen in Table 3. A target word might appear both as a word and a random substitute therefore to clarify this ambiguity we prepend "W:" and "S:" to words and substitutes, respectively, in the co-occurrence data. The calculation of substitute distributions and random substitute sampling are detailed in Appendix A.

The next section describes the S-CODE algorithm which takes the pairwise co-occurrence data as its input and calculates the embeddings of the words and their substitutes on an n-dimensional sphere.

2.2 Co-occurrence Embedding

The S-CODE algorithm maps each unique word and substitute in the co-occurrence data to a real vector (embedding) on an *n*-dimensional sphere as detailed in Appendix B. The basic idea of the mapping is that words and substitutes that are frequently observed as pairs in the co-occurrence data will have close embeddings while pairs not observed together will have embeddings that are far apart from each other.

Table 3: The table on the left shows three possible substitutes sampled with replacement for each position in an example sentence based on a 4-gram language model. The table on the right is the pairwise co-occurrence data fed to the S-CODE algorithm derived from these samples. The prefixes "W:" and "S:" are used to distinguish target words and substitutes. Sampled substitutes might include the unknown word tag "<unk>" representing words outside the fixed size vocabulary of the language model.

Word	Random Substitutes
Pierre	Mr. / Pierre / John
Vinken	<unk> Beregovoy Cardin</unk>
,	, / , / ,
61	48 / 52 / 41
years	years / years / years
old	old / old / old
,	, / , / ,
will	will / will / will
join	head / join / leave
the	its / its / the
board	board / company / firm
as	as / as / as
a	a / a / a
nonexecutive	nonexecutive non-executive nonexecutive
director	chairman / chairman / director
Nov.	April May of
29	16 / 29 / 9
	././.

Word	Substitute
W:Pierre	S:Mr.
W:Pierre	S:Pierre
W:Pierre	S:John
W:Vinken	<i>S:</i> < <i>unk</i> >
W:Vinken	S:Beregovoy
W:Vinken	S:Cardin
W:join	S:head
W:join	S:join
W:join	S:leave
W:the	S:its
W:the	S:its
W:the	S:the
W:director	S:chairman
W:director	S:chairman
W:director	S:director

Word	Substitute
W:director W:chief	S:chairman
W:Pierre W:Frank	S:John S:John

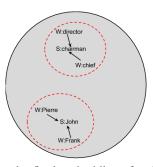


Figure 2: The figure on the right represents the final embeddings for the words and substitutes from the co-occurrence data on the left after S-CODE converges. Dashed circles represent the possible groupings of the embeddings on the sphere.

The co-occurrence data in Figure 2 consists of pairs such as (W:director, S:chairman) and (W:chief, S:chairman) therefore S-CODE forces the embeddings of W:director and W:chief to be close to the embedding of S:chairman. Similarly the embeddings of W:Pierre and W:Frank will be close to the embedding of S:John because they are frequently co-occurring pairs. As a result the final embeddings of W:director and W:chief will be close to each other due to the common substitute S:chairman and will be apart from W:Pierre and W:Frank due to the lack of common

substitutes (similarly the embeddings of *W:Pierre* and *W:Frank* will be close to each other due to *S:John*).

The coordinates of the embeddings for each unique word and substitute constitute the input to the clustering stage as described in the next subsection.

2.3 Clustering

At this stage, each unique word in the text and each unique substitute sampled to represent their contexts is mapped to a real vector embedding on an n dimensional sphere⁴. We apply the instance weighted k-means clustering algorithm to three different representations derived from these embeddings, each with its own advantages and disadvantages:

Clustering word embeddings (\mathbf{W}). In the first setting, we ignore substitute embeddings and apply clustering only to target word embeddings. Each target word *type* has a single embedding, and gets assigned to a single cluster. Thus clustering target words based on this representation can only assign a single tag to each word type and cannot represent multiple parts of speech for ambiguous words (words that have more than one tag).

For example, the word *offer* is tagged as NN(399), VB(105) and VBP(34) in its 538 PTB occurrences⁵. When all instances of *offer* are assigned to the same cluster, the MTO upper bound of *offer* will be 399/538=.74 using the most frequent tag NN.

Clustering word embeddings was previously explored in (Yatbaz, Sert, and Yuret 2012), which achieved the best results to date (80% MTO) for English. Sections 3.2 and 3.5 summarize these experiments for completeness. The surprising success of the one-tag-per-word assumption in English part of speech induction is partly due to the fact that 93.69% of the word occurrences in the human labeled PTB data are tagged with their most frequent part of speech (Toutanova et al. 2003). However the clustering performance on ambiguous words is bounded and the utility of a part of speech induction model which cannot handle ambiguity is questionable.

Clustering substitute embeddings (S). In a second set of experiments, we ignore target word embeddings and apply clustering only to substitute embeddings, associating each substitute with a unique cluster. We then categorize target word *tokens* based on what cluster the majority of their substitutes belong. It is important to note that in this setting we are ignoring the identity and features of the target words and in effect clustering word contexts (substitutes are determined by the context and are conditionally independent of the target word).

For example, the target word *W:board* in Table 3 will be represented with the embeddings of *S:board*, *S:company* and *S:firm* while another occurrence of the word *W:board* in a different context might be represented with embeddings of different substitutes such as *S:embark* or *S:enter*. Each occurrence of the target word "board" is assigned to the cluster in which the majority of its substitute embeddings are present⁶. This approach generally results in higher accuracy for highly ambiguous words like *offer* where clustering substitute embeddings achieves .82 MTO compared to the one-tag-per-word upper bound of .74. Section 3.3 presents the results of experiments clustering substitute embeddings.

Unfortunately such highly ambiguous words do not constitute a significant portion of the corpus and the overall accuracy suffers (.64 compared to .80 MTO for word clustering). We also observed similar results in our preliminary experiments without S-CODE trying to cluster

⁴ In fact many words that appear in the text also appear as substitutes and thus get two embeddings.

⁵ NN, VB and VBP are three part-of-speech tags from the Penn Treebank corpus and they correspond to singular noun, verb in base form and non-3rd person singular verb in present tense, respectively.

⁶ Ties are broken randomly.

contexts directly (using the Kullback-Leibler divergence between their substitute distributions). Many common words that occur in similar contexts and that have similar substitutes, e.g. *his* and *the*, belong to different parts of speech. In addition, words that are generally not substitutable like "do" and "put" are placed in the same category by the PTB. This suggests that the identity and features of the target word are indispensable and that a purely substitutability based linguistic definition is insufficient for inducing parts of speech as tagged in the PTB.

Clustering the concatenation of word and substitute embeddings ($\mathbf{W} \oplus \mathbf{S}$). In a third set of experiments we concatenate the n-dimensional embedding vector for each sampled substitute with the n-dimensional embedding vector of its target word and apply clustering to the resulting 2n-dimensional vectors. We then categorize target word tokens based on what cluster the majority of their substitute-concatenated vectors belong. For instance, the target word "Pierre" in Table 3 will be represented with three vectors that will be the concatenation of the embedding of W:Pierre with the embeddings of S:Mr, S:Pierre and S:John, separately. Models that are based on this representation do not employ the one-tag-per-word assumption, they can cluster word tokens, and they can represent ambiguity. Clusters that are constructed according to this representation tend to assign fewer categories to each word type than substitute clustering due to the concatenation of \mathbf{W} . The advantage in highly ambiguous words is still retained (e.g. the MTO for offer is .89) and the overall result is significantly improved (.70 MTO) compared to substitute clustering. Section 3.4 presents our experiments clustering concatenated word-substitute vectors.

Summary. The first setting applies the one-tag-per-word assumption from the beginning and clusters word types instead of tokens. The second setting clusters word contexts (as represented by substitutes) and is able to categorize individual word tokens. However it ignores the identity of the target word. The third setting also clusters word tokens but represents each token with a set of concatenated word and substitute embeddings. Representing the identity of the target word without enforcing the one-tag-per-word assumption improves the results on highly ambiguous words. Section 3 compares the performance of these three settings on the part of speech induction problem, as well as experiments with different features and languages.

3. Experiments

In this section we present our experiments categorizing word types and word tokens using the three different settings that were described in Section 2.3 to determine their performance on the unsupervised part of speech induction problem. We extend the best performing model with orthographic and morphological features to improve its performance further. Finally, we apply our best performing model to languages other than English to test its cross-language applicability.

Section 3.1 details the test corpus and the experimental parameters used in the English experiments. Section 3.2 presents experiments that cluster word *types* based on target word embeddings \mathbf{W} . Section 3.3 presents experiments that cluster word *tokens* based on substitute embeddings \mathbf{S} . Section 3.4 presents experiments that also cluster word tokens based on a concatenation of target word and substitute embeddings $\mathbf{W} \oplus \mathbf{S}$. Section 3.5 explores morphological and orthographic features as additional sources of information for POS induction of word types. Section 3.6 compares our paradigmatic representation of word context to previously described syntagmatic representations of word context for POS induction in word types to validate our hypothesis that the improvement we get is due to our original representation of word context. Finally, Section 3.7 extends the language and corpus coverage by applying the best performing model to 19 corpora in 15 languages.

Our word type clustering results, first presented in (Yatbaz, Sert, and Yuret 2012), are still state of the art for English POS induction. However in this study we show that the performance

on ambiguous words can be further improved by clustering word tokens (Sections 3.3 and 3.4), we improve the model that incorporates morphological and orthographic features (Section 3.5 and Appendix C), and we demonstrate the applicability of our methods to other languages (Section 3.7).

3.1 Experimental Settings

To make a direct comparison with previously published results, the Wall Street Journal Section of the Penn Treebank (PTB) (Marcus et al. 1999) was used as the test corpus (1,173,766 tokens, 49,206 types) for English experiments. The treebank uses 45 part-of-speech tags which is the set we used as the gold standard for comparison in our experiments.

To compute substitutes in a given context we trained a language model using approximately 126 million tokens of Wall Street Journal data (1987-1994) extracted from CSR-III Text (Graff, Rosenfeld, and Paul 1995) (excluding sections of the PTB). We used SRILM (Stolcke 2002) to build a 4-gram language model with Kneser-Ney discounting. Words that were observed less than 20 times in the language model training data were replaced by <unk> tags, which gave us a vocabulary size of 78,498. The perplexity of the 4-gram language model on the test corpus is 96. For computational efficiency only the top 100 substitutes and their unnormalized probabilities were computed for each position in the PTB using the FASTSUBS algorithm (Yuret 2012)⁷. The probability vectors for each position were normalized to add up to 1.0 giving us the final substitute distributions used in the following experiments.

The experiments were run using the following default settings (unless otherwise stated): (i) each word was kept with its original capitalization, (ii) the learning rate parameters for S-CODE (see Appendix B) were set to $\varphi_0 = 50$, $\eta_0 = 0.2$, (iii) the number of S-CODE iterations were set to 50 million, (iv) the S-CODE dimensions and \tilde{Z} were set to 25 and 0.166, respectively, (v) a modified k-means algorithm with smart initialization was used (Arthur and Vassilvitskii 2007), and (vi) the number of k-means restarts were set to 128 to improve clustering and reduce variance.

Each experiment was repeated 10 times with different random seeds and the results are reported with standard errors in parentheses or error bars in graphs. Table 4 summarizes all the results reported in this section and the ones we cite from the literature.

3.2 Clustering Word Embeddings (W)

In order to categorize word types, we cluster the embeddings (i.e. 25-dimensional vectors) ϕ_w assigned to each word type w by S-CODE. S-CODE uses stochastic gradient ascent (see Appendix B) to find the ϕ_w , ψ_c embeddings for words and substitutes on a 25-dimensional unit sphere. The algorithm cycles through the word-substitute pair data for approximately 50 million updates. The resulting ϕ_w vectors are clustered using an instance weighted k-means algorithm (each ϕ_w vector is weighted in proportion to the number of times w appears in the data). The resulting cluster-id for each ϕ_w becomes the predicted category for word type w. Using the default settings and sampling 64 substitutes for each token the many-to-one accuracy is .7667 (.0056) and the V-measure is .6819 (.0029).

Sensitivity Analysis. To analyze the sensitivity of this result to our specific parameter settings we ran a number of experiments where each parameter was varied over a range of values.

⁷ The substitutes with unnormalized log probabilities can be downloaded from http://goo.gl/jzKHO.

Table 4: Summary of results in terms of the MTO and VM scores. Standard errors are given in parentheses when available. Starred entries have been reported in the review paper (Christodoulopoulos, Goldwater, and Steedman 2010). Distributional models use only the identity of the target word and its context. The models on the right incorporate orthographic and morphological features.

Distributional Models	MTO	VM	Models with Additional Features	MTO	VM
Lamar et al. (2010)	.708	-	Clark (2003)*	.712	.655
Brown et al. (1992)*	.678	.630	Christodoulopoulos et al. (2011)	.728	.661
Goldwater et al. (2007)*	.632	.562	Berg-Kirkpatrick et al. (2010)	.755	-
Ganchev et al. (2010)*	.625	.548	Christodoulopoulos et al. (2010)	.761	.688
Maron et al. (2010)	.688 (.0016)	-	Blunsom and Cohn (2011)	.775	.697
Substitutes(Sparse-tokens) (Sec. 3.4)	.7030 (.0070)	.6006 (.0071)	Substitutes and Features (Sec. 3.5)	.8002 (.0070)	.7163 (.0040)
Bigrams (Sec. 3.6)	.7319 (.0088)	.6554 (.0039)			
Substitutes(Types) (Sec. 3.2)	.7667 (.0056)	.6819 (.0029)			

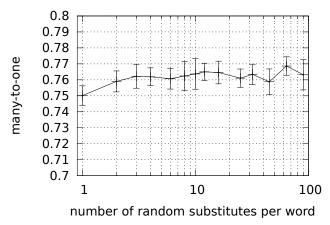


Figure 3: MTO is not sensitive to the number of random substitutes sampled per word token.

Figure 3 illustrates that the result is fairly robust with respect to the number of random substitutes sampled for each target word token, as long as the training algorithm can observe more than a few random substitutes per word.

Figure 4 shows that at least 10 embedding dimensions are necessary to get within 1% of the best result, but there is no significant gain from using more than 25 dimensions.

Figure 5 shows that the constant \tilde{Z} approximation can be varied within two orders of magnitude without a significant performance drop in the many-to-one score. For uniformly distributed points on a 25 dimensional sphere, the expected $Z\approx 0.146$. In the experiments where we tested we found the real Z always to be in the 0.140-0.170 range. When the constant \tilde{Z} estimate is too small the attraction in Eq. B.3 dominates the repulsion in Eq. B.4 and all points tend to converge to the same location. When \tilde{Z} is too high, it prevents meaningful clusters from coalescing.

3.3 Clustering Substitute Embeddings (S)

In the previous section we group word types rather than word tokens by clustering the word embeddings. In this section we remove this one-tag-per-word restriction and group word tokens

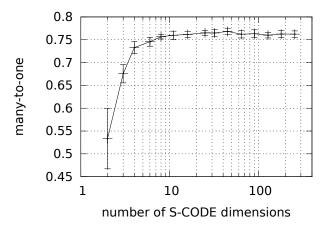


Figure 4: MTO falls sharply for less than 10 S-CODE dimensions, but more than 25 do not help.

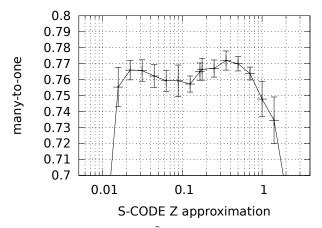


Figure 5: MTO is fairly stable as long as the \tilde{Z} constant is within an order of magnitude of the real Z value.

according to the embeddings of their substitutes. We sample 64 random substitutes for each word token and input them to S-CODE as word (\mathbf{W}) – substitute (\mathbf{S}) pairs. The resulting embeddings of the substitutes are clustered using the instance weighted k-means algorithm. The process yields 64 cluster-ids for each target word token's context. The predicted category for the target word token is chosen to be the majority cluster-id among these 64 cluster-ids. Ties for the majority are broken randomly. In effect, we are using random substitutes as features of the context, and we are clustering contexts of individual word tokens. The many-to-one accuracy is .6366 (.0023) and the V-measure is .4865 (.0051) which is lower than word type clustering. However, we show that the performance on highly ambiguous words improve significantly.

In order to explain the merit of the token based POS induction, we first define the gold-tag perplexity for word types as follows:

$$GP(w) = 2^{H(p_w)} = 2^{-\sum_t p_w(t)log_2 p_w(t)}$$
(1)

where w is a word, t is a tag, p_w is the gold POS tag distribution of the word type w and $H(p_w)$ is the entropy of the p_w distribution. The gold-tag perplexity (GP) is used to determine the POS ambiguity of a word type, relating how often a word type is associated with different POS tags in the test corpus. A GP of 1 for a word type w indicates w is associated with same POS tag throughout the test corpus, meaning the word type w's POS is unambiguous. A word with N equally probable tags would have a RP of RP increases the ambiguity of a word type increases and this poses a handicap for induction models that limits tag variety for the word types. To display the limitations, we split the test corpus into two subsets: word types with RP less than 1.75 and word types with RP equal to or greater than 1.75. We performed MTO evaluation on our induction output and obtained the induced-tag – gold-tag mappings. Using the mappings obtained over the test corpus, we evaluated the accuracy in the two subsets.

Table 5: The MTO accuracy of W, S and $W \oplus S$ based models on two subsets that consist of words with GP smaller and larger than 1.75, respectively. The percentage of each subset in the test data is reported in the title bar. The average GP of each clustering over the whole corpus is reported in the last column. Each score is an average of 10 random starts of our algorithm and the standard error of each one is reported in parenthesis while statistically the best MTO score of each column is reported in bold.

Model	GP < 1.75 89%	$GP \ge 1.75$ 11%	$GP \ge 1$ 100%	Average GP
Clustering W embeddings (Type based)	.8054 (.0065)	.4383 (.0104)	.7667 (.0056)	1.0 (.0)
Clustering W ⊕ S embeddings (Sparse-token based)	.7322 (.0079)	.4671 (.0174)	.7030 (.0070)	1.3406 (.0057)
Clustering S embeddings (Token based)	.6620 (.0051)	.4309 (.0093)	.6366 (.0023)	1.5318 (.0076)

The performance of our algorithm clustering the S embeddings is summarized in Table 5. Due to the one-tag-per-word nature of POS induction, the type based model outperforms the token based one on the unambiguous words. The token based model achieves statistically comparable results with the type based model on the ambiguous words. Type based model can not handle words with ambiguity while the token based model can. In order to take advantage of both models we apply our algorithm on concatenation of W and S embeddings in the next section.

3.4 Clustering Concatenation of Word and Context Embeddings (W \oplus S)

Two models presented in earlier sections perform POS induction either by assuming (Section 3.2) or discarding (Section ??) the one-tag-per-word assumption. In this section we define a sparse-token based model which clusters the concatenation of **W** and **S** embeddings. This model not only tends to put instances of a word type into the same cluster but also performs token based clustering by incorporating the word type and context information together.

Similar to the previous models, we generate W-S pairs as the input to S-CODE. For each observed W-S pair in the S-CODE input, corresponding 25-dimensional ϕ_w and ψ_c embeddings are concatenated to create a 50-dimensional representation. We used the same experimental setting of the previous section and predict the token clusters according to the majority cluster-id of the corresponding pairs. The many-to-one accuracy of this model is .7030 (.0070) and the V-measure is .6006 (.0071).

Table 5 presents the performance of the $W \oplus S$ based model over the subsets and it achieves statistically better MTO than both of the W and S based models on ambiguous words. Due to the

bias towards to the sparse clustering, sparse-token based model statistically improves the MTO accuracy on unambiguous words compared to the S based model but it still can not achieve the performance of the W based model. The $W \oplus S$ based model constructs token based clusters that tend to assign instances of a word type into the same cluster which leads to a smaller average GP than the S based model as shown in Table 5.

3.5 Morphological and Orthographic Features

Clark (2003) demonstrates that using morphological and orthographic features significantly improves part of speech induction with an HMM based model. Section 5 describes a number of other approaches that show similar improvements. We integrate additional features together with substitutes by using the model described in Appendix C.

The orthographic features we used are similar to the ones in (Berg-Kirkpatrick et al. 2010) with small modifications:

- Initial-Capital: this feature is generated for capitalized words with the exception of sentence initial words.
- Number: this feature is generated when the token starts with a digit.
- Contains-Hyphen: this feature is generated for lowercase words with an internal hyphen.
- Initial-Apostrophe: this feature is generated for tokens that start with an apostrophe.

We generated morphological features using the unsupervised algorithm Morfessor (Creutz and Lagus 2005). Morfessor was trained on the WSJ section of the Penn Treebank using default settings, and a perplexity threshold of 1. In our model, a word type consists of two parts: a stem and a suffix part. The suffix part is used as the morphological feature thus each word type has only one morphological feature⁸. The program induced 5575 suffix types that are present in a total of 19223 word types. Table 6 presents the co-occurrence tuples of the example sentence after incorporating the orthographic and morphological features.

Using the training settings of the previous section, the addition of morphological and orthographic features increased the many-to-one score of the random-substitute model to .8002 (.0070) and V-measure to .7163 (.0040). Both these results improve the state-of-the-art in part of speech induction significantly as seen in Table 4.

3.6 Paradigmatic vs Syntagmatic Representations of Word Context

To get a direct comparison of the paradigmatic and syntagmatic context representations we feed 4 different co-occurrences defined in Section 1.1 into the S-CODE algorithm. The first model accepts word (W) - right bigram (C) pairs as the input, the second model accepts word (W) - left bigram (C) pairs as the input, the third model accepts word (W) - concatenation of the left and right bigrams (C) pairs (Mintz 2003) as the input and the final model accepts words (W) - left bigram (C_1) and right bigram (C_2) tuples (St Clair, Monaghan, and Christiansen 2010) as the input to the S-CODE. Finally, we cluster the word type embeddings (W) with k-means algorithm and report the results on Table 7.

⁸ We extracted the stem part by concatenating the splits until including the first "STM" labeled split and the suffix part by concatenating rest of the splits.

Table 6: The words of input sentence "Pierre Vinken, 61 years old, will join the board as a nonexecutive director Nov. 29." is represented with their substitutes and features. Words on the left column presents the target word, words on the second column represents the context and tokens on the rest of the columns are the features of the corresponding target word. Features without values are unobserved therefore set to null.

Word	Word Context		Initial Capital	Number	Contains Hypen	Initial Apostrophe
W:Pierre	C:Mr.		F:IC			
W:Vinken	C: <unk></unk>		F:IC			
W:,	C:,					
W:61	C:48			F:N		
W:years	C:years	F: s				
W:old	C:old					
W:join	C:head					
W:the	C:its					
W:board	C:company					
W:as	C:as					
W:a	C:a					
W:nonexecutive	C:non-executive					
W:director	C:chairman	F:or				
W:Nov.	C:May		F:IC			
W:29	C:9			F:N		
W.	<i>C</i> :.					

Table 7: Summary of results in terms of the MTO and VM scores of the S-CODE algorithm when the paradigmatic or syntagmatic representations are feed as an input. Standard errors are given in parentheses when available. Results of the statistically best performing system are written in bold. We do not report the original results of Maron et al. (2010) since our replication achieves higher accuracies.

Input	MTO	VM
W (word) - C (right bigram)	.6625 (.0115)	.5809 (.0066)
W (word) - C (left bigram)	.6604 (.0054)	.5983 (.0028)
W (word) - C (left and right bigram concatenation)	.7268 (.0091)	.6416 (.0052)
W (word) - C_1 , C_2 (left and right bigrams)	.7173 (.0061)	.6381 (.0032)
Maron et al. (2010)(replication)	.7319 (.0088)	.6554 (.0039)
W (word) - C (random substitutes)	.7667 (.0056)	.6819 (.0029)

To replicate the work of Maron et al. (2010) we feed word (W) - right bigram (C) pairs as the input. At the end each word w in the vocabulary ends up with two points on the sphere, a ϕ_w point representing the behavior of w as the left word of a bigram and a ψ_w point representing it as the right word. The two vectors for w are concatenated to create a 50-dimensional representation at the end. These 50-dimensional vectors are clustered using the k-means algorithm. Maron et al. (2010) report many-to-one scores of .6880 (.0016) for 45 clusters and .7150 (.0060) for 50 clusters (on the PTB). Using our default settings the bigram model achieves .7319 (.0088) MTO and .6554 (.0039) VM accuracies. Table 7 summarizes all the results and shows that the paradigmatic representation accuracies are significantly higher than the syntagmatic representation MTO and VM accuracies.

3.7 Multilingual Experiments

We performed experiments with a range of languages and three different feature configurations to establish both the robustness of our model across languages and to observe the effects of different features. Following Christodoulopoulos et al. (2011), in addition to the PTB we extend our experiments to 8 languages from MULTEXT-East (Bulgarian, Czech, English, Estonian, Hungarian, Romanian, Slovene and Serbian) (Erjavec 2004) and 10 languages from the CoNLL-X shared task (Bulgarian, Czech, Danish, Dutch, German, Portuguese, Slovene, Spanish, Swedish and Turkish) (Buchholz and Marsi 2006). For all experiments, we use the best performing model of Section 3.2 (i.e. clustering the word embeddings) with default settings. To perform meaningful comparisons with the previous work we train and evaluate our models on the training section of MULTEXT-East⁹ and CONLL-X languages (Lee, Haghighi, and Barzilay 2010).

We train a 4-gram language model with the corresponding training corpora of each language as described in Section 3.1. To sample substitutes we calculate the probabilities of the top 100 substitutes for each position by using the corresponding language model. Morphological features of each language are extracted by the method described in Section 3.5. The details of the language model training and feature extraction are detailed in Appendix D. For each language we

Table 8: The MTO and VM scores on 19 corpora in 15 languages together with the number of types and tags in the gold–set which equals to number of induced clusters in all languages. Best published results are from [‡](Blunsom and Cohn 2011), *(Christodoulopoulos, Goldwater, and Steedman 2011) and [†](Clark 2003). Bold results represent the best MTO and VM accuracies of the corresponding language with at least 90% confidence level. MULTEXT-East corpora do not tag the punctuation marks, thus we add an extra tag for punctuation and represent it with "+1".

	Language	Types	Tags	Best Published	Syntagmatic Bigram	CLU-W	CLU-W+O	CLU-W+O+M
WSJ	English	49,190	45	.775 / .697 [‡]	.7314 / .6558	.7667 / .6819	.7820 / .7020	.8002 / .7163
	Bulgarian	16,352	12+1	.665 / .556*	.6732 / .4119	.6927 / .5341	.6964 / .5469	.7027 / .5513
asl	Czech	19,115	12+1	.642 / .539 *	.6269 / .4586	.7025 / .5020	.7022 / .5047	.7045 / .5096
먈	English	9,773	12+1	.733 / .633*	.7690 /. 6131	.8239 / .6631	.8246 / .6696	.8329 / .6769
FEXT-East	Estonian	17,845	11+1	.644 / .533 *	.6089 / .4119	.6612 / .4469	.6704 / .4658	.6445 / .4452
E	Hungarian	20,321	12+1	.682 / .548 *	.6181 / .4514	.6900 / .4972	.6963 / .5173	.7254 / .5402
	Romanian	15,189	14+1	.611 / .523*	.6565 / .5202	.6412 / .5004	.6607 / .5262	.6432 / .5127
MUL	Slovene	17,871	12+1	.679 / .567 *	.6772 / .5044	.6914 / .4951	.6966 / .4998	.6823 / .4938
	Serbian	18,095	12+1	.641 / .510 †	.6267 / .4510	.6311 / .4536	.6317 / .4557	.6370 / .4648
¥	Bulgarian	32,439	54	.704 / .596 †	.6972 / .5532	.7328 / .5781	.7348 / .5844	.7321 / .5835
Task	Czech	130,208	12	.701 [‡] / .484*	.6944 / .5036	.6739 / .4838	.7176 / .5336	.7039 / .5118
	Danish	18,356	25	.761 [‡] / .591*	.6757 / .5290	.7236 / .5583	.7538 / .5962	.7417 / .5919
Shared	Dutch	28,393	13	.711 [‡] / .547*	.6703 / .5205	.6957 / .5331	.7401 / .5986	.7210 / .5919
Sh	German	72,326	54	.744* / .630†	.7525 / .6285	.7669 / .6306	.7799 / .6575	.7557 / .6395
×	Portuguese	28,931	22	.785‡ / .639*	.7031 / .5617	.7439 / .5798	.7901 / .6316	.7861 / .6353
Ţ	Slovene	7,128	29	.642* / .539 †	.6384 / .4976	.6513 / .4957	.6545 / .5093	.6543 / .5031
CoNL	Spanish	16,458	47	.788 [‡] / .632*	.7086 / .5844	.7479 / .6086	.7712 / .6346	.7588 / .6287
υ Ο	Swedish	20,057	41	.682 / .589 †	.6721 / .5558	.6962 / .5674	.6962 / .5721	.6675 / .5628
	Turkish	17,563	30	.628 / .408*	.6069 / .3551	.6239 / .3823	.6372 / .4098	.6487 / .4206

report results of three models that cluster: (1) word embeddings (CLU-W), (2) word embeddings

⁹ Languages of MULTEXT-East corpora do not tag the punctuation marks, thus we add an extra tag for punctuation to the tag-set of these languages.

with orthographic features (CLU-W+O) and (3) word embeddings with both orthographic and morphological features (CLU-W+O+M). Similar to the settings used in Section 3.2, we use the 25 dimensional sphere with 64 substitutes for all languages. For each language the number of induced clusters is set to the number of tags in the gold-set as presented in Table 8.

As a baseline model we chose the syntagmatic bigram version of S-CODE described in Section 3.6 which is a very strong baseline compared to the ones used in (Christodoulopoulos, Goldwater, and Steedman 2011). Table 8 summarizes the MTO and VM scores of our models together with the syntagmatic bigram baseline and the best published accuracies on each language corpus.

CLU-W significantly outperforms the syntagmatic bigram baseline in both MTO and VM scores on 14 languages. CLU-W+O+M has the state-of-the-art MTO and VM accuracy on the PTB. CLU-W+O and CLU-W+O+M achieve the highest MTO scores on all languages of MULTEXT-East corpora while scoring the highest VM accuracies on English and Romanian. On the CoNLL-X languages our models perform better than the best published MTO or VM accuracies on 10 languages.

4. Discussion

In this section we perform further analysis on the clustering output of our best model and indicate the possible reasons of comparably low VM scores. To illustrate how words are distributed in the induced clusters, we compare the output of our model with gold-tags of the PTB. We also discuss the effect of coarse gold-tag sets on our model performance.

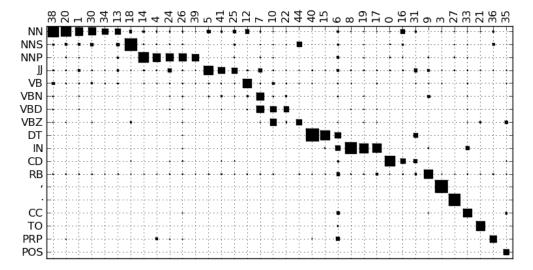


Figure 6: Hinton diagram comparing most frequent tags and clusters. Area of each square is proportional to the joint probability of the given tag and cluster.

Figure 6 is the Hinton diagram of the PTB showing the relationship between the most frequent tags and clusters from the experiment in Section 3.5. In general the errors seem to be the lack of completeness (multiple large entries in a row), rather than lack of homogeneity (multiple large entries in a column). The algorithm tends to split large word classes into several clusters. Some examples are:

- Titles like Mr., Mrs., and Dr. are split from the rest of the proper nouns in cluster (39).
- Auxiliary verbs (10) and the verb "say" (22) have been split from the general verb clusters (12) and (7).
- Determiners "the" (40), "a" (15), and capitalized "The", "A" (6) have been split into their own clusters.
- Prepositions "of" (19), and "by", "at" (17) have been split from the general preposition cluster (8).

Nevertheless there are some homogeneity errors as well:

- The adjective cluster (5) also has some noun members probably due to the difficulty of separating noun-noun compounds from adjective modification.
- Cluster (6) contains capitalized words that span a number of categories.

Most closed-class items are cleanly separated into their own clusters as seen in the lower right hand corner of the diagram.

The completeness errors become more noticeable on languages with coarse tag-sets thus our models perform worse than the best published models on 6 of MULTEXT-East languages in terms of VM scores while achieving the state-of-the-art MTO scores on the same languages as shown on Table 8. On CONLL-X languages the effect of completeness errors is less noticeable since all languages except Czech and Dutch have fine grained tag-sets.

The completeness errors are not surprising given that the words that have been split are not generally substitutable with the other members of their gold-tag set category. Thus it can be argued that metrics that emphasize homogeneity such as MTO are more appropriate in this context than metrics that average homogeneity and completeness such as VM as long as the number of clusters is controlled.

There are two concerns inherent in all distributional methods: (i) words that are generally substitutable like "the" and "its" are placed in separate categories (DT and PRP\$) by the gold standard, (ii) words that are generally not substitutable like "do" and "put" are placed in the same category (VB). Freudenthal et al. (2005) point out that categories with unsubstitutable words fail the standard linguistic definition of a syntactic category and children do not seem to make errors of substituting such words in utterances (e.g. "What do you want?" vs. *"What put you want?"). Whether gold standard part of speech tags or distributional categories are better suited to applications like parsing or machine translation can be best decided using extrinsic evaluation. In this study we evaluate our results by comparing them to gold standard part of speech tags and leave the extrinsic evaluation of the induced tags for future work.

5. Related Work

There are several good reviews of algorithms for unsupervised part of speech induction (Christodoulopoulos, Goldwater, and Steedman 2010; Gao and Johnson 2008) and models of syntactic category acquisition (Ambridge and Lieven 2011).

This work is to be distinguished from supervised part of speech disambiguation systems, which use labeled training data (Toutanova et al. 2003), unsupervised disambiguation systems, which use a dictionary of possible tags for each word (Yatbaz and Yuret 2010), or prototype driven systems which use a small set of prototypes for each class (Haghighi and Klein 2006). The problem of induction is important for studying under-resourced languages that lack labeled

corpora and high quality dictionaries. It is also essential in modeling child language acquisition because every child manages to induce syntactic categories without access to labeled sentences, labeled prototypes, or dictionary constraints.

Models of unsupervised part of speech induction fall into two broad groups based on the information they utilize. Distributional models only use word types and their context statistics. Word-feature models incorporate additional morphological and orthographic features.

5.1 Distributional models

Distributional models can be further categorized into three subgroups based on the learning algorithm. The first subgroup represents each word type/token with its context vector and clusters these vectors accordingly (Schütze 1995). Work in modeling child syntactic category acquisition has generally followed this clustering approach (Redington, Crater, and Finch 1998; Mintz 2003). The second subgroup consists of probabilistic models based on the Hidden Markov Model (HMM) framework (Brown et al. 1992). A third group of algorithms constructs a low dimensional representation of the data that represents the empirical co-occurrence statistics of word types (Globerson et al. 2007), which is covered in more detail in Section 6.

Clustering. Clustering based methods represent the context using the neighboring words, typically a single word on the left and a single word on the right called a "frame" (e.g., the dog is; the cat is). They cluster word types rather than word tokens based on the frames they occupy thus employing one-tag-per-word assumption from the beginning (with the exception of (Mintz 2003; St Clair, Monaghan, and Christiansen 2010) and some methods in (Schütze 1995)). They may suffer from the data sparsity caused by the infrequent words and the infrequent contexts. The solutions suggested either restrict the set of words and set of contexts to be clustered to the most frequently observed, or use dimensionality reduction. Redington et al. (1998) define context similarity based on the number of common frames bypassing the data sparsity problem but achieve lower scores than the best performing systems. Mintz (2003) only uses the most frequent 45 frames to cluster tokens and achieves 98% unsupervised accuracy on the tokens observed in the most frequent 45 frames. Similar to Mintz's work, St Clair et al. (2010) show that systems that model the left and right frames of tokens seperately perform better than the frequent frames both interms of token clustering accuracy and the token coverage. Biemann (2006) contructs a graph based view of the most frequent 10,000 words using contexts formed from the most frequent 150-200 words and clusters the tokens. Schütze (1995) and Lamar et al. (2010) employ SVD to enhance similarity between less frequently observed word types and contexts. Lamar et al. (2010) represent each context by the currently assigned left and right tag (which eliminates data sparsity) and cluster word types using a soft k-means style iterative algorithm. They report the best clustering result to date of .708 many-to-one accuracy on the PTB.

HMMs. The prototypical bitag HMM model maximizes the likelihood of the corpus $w_1 \dots w_n$ expressed as $P(w_1|c_1)\prod_{i=2}^n P(w_i|c_i)P(c_i|c_{i-1})$ where w_i are the word tokens and c_i are their (hidden) tags. One problem with such a model is its tendency to distribute probabilities equally and the resulting inability to model highly skewed word-tag distributions observed in hand-labeled data (Johnson 2007). To favor sparse word-tag distributions one can enforce a strict one-tag-per-word solution (type clustering) (Brown et al. 1992; Clark 2003), use sparse priors in a Bayesian setting (Goldwater and Griffiths 2007; Johnson 2007), or use posterior regularization (Ganchev et al. 2010). Each of these techniques provide significant improvements over the standard HMM model: for example Gao and Johnson (2008) show that sparse priors can gain from 4% (.62 to .66 on the PTB) in cross-validated many-to-one accuracy. However

Christodoulopoulos et al. (2010) show that the older one-tag-per-word models such as (Brown et al. 1992) outperform the more sophisticated sparse prior and posterior regularization methods both in speed and accuracy (the Brown model gets .68 many-to-one accuracy on the PTB). Given that 93.69% of the word occurrences in human labeled data are tagged with their most frequent part of speech (Toutanova et al. 2003), this is probably not surprising; one-tag-per-word is a fairly good first approximation for induction.

5.2 Word-feature models

One problem with the algorithms in the previous section is the poverty of their input features. Of the syntactic, semantic, and morphological information linguists claim underlie syntactic categories, context vectors or bitag HMMs only represent limited syntactic information in their input. Experiments incorporating morphological and orthographic features into HMM based models demonstrate significant improvements. (Clark 2003; Berg-Kirkpatrick and Klein 2010; Blunsom and Cohn 2011) incorporate similar orthographic features and report improvements of 3, 7, and 10% respectively over the baseline Brown model.

(Clark 2003; Blunsom and Cohn 2011) cluster types by incorporating similar orthographic features and report improvements of 3 and 10% respectively over the baseline Brown model. Berg-Kirkpatrick et al. incorporate orthographic features into EM algorithm where they replace the multinomial components with miniature logistic regressions and cluster tokens while improving the Brown model by 7%.

Christodoulopoulos et al. (2010) use prototype based features as described in (Haghighi and Klein 2006) with automatically induced prototypes and report an 8% improvement over the baseline Brown model by clustering tokens. Abend et al. (2010) train a prototype-driven model with morphological features by first clustering the high frequent types as the landmarks and then assigning the remaining types to these landmark clusters. Christodoulopoulos et al. (2011) define a type-based Bayesian multinomial mixture model in which each word instance is generated from the corresponding word type mixture component and word contexts are represented as features. They achieve a .728 MTO score by extending their model with additional morphological and alignment features gathered from parallel corpora. To our knowledge, nobody has yet tried to incorporate phonological or prosodic features in a computational model for syntactic category acquisition.

5.3 Paradigmatic representations

Yatbaz et al. (2012) explore the paradigmatic representation of the word contexts by modeling the co-occurrence of words and their substitutes within the CODE framework. Their experiments on the PTB types shows that paradigmatic representation improves the state-of-the-art MTO and V-measure (VM) accuracies of both distributional models and models with additional word features. This paper builds on that preliminary work by (1) exploring induction of part-of-speechs at token level (in addition to type level), (2) improving the model for using additional features, and (3) experimenting with additional languages.

5.4 Evaluation

We report many-to-one and V-measure scores for our experiments as suggested in (Christodoulopoulos, Goldwater, and Steedman 2010). The many-to-one (MTO) evaluation maps each cluster to its most frequent gold tag and reports the percentage of correctly tagged instances. The MTO score naturally gets higher with increasing number of clusters but it is an intuitive metric when comparing results with the same number of clusters. The V-measure (VM) (Rosen-

berg and Hirschberg 2007) is an information theoretic metric that reports the harmonic mean of homogeneity (each cluster should contain only instances of a single class) and completeness (all instances of a class should be members of the same cluster). In Section 4 we argue that homogeneity is perhaps more important in part of speech induction and suggest MTO with a fixed number of clusters as a more intuitive metric.

6. Contributions

Our main contributions can be summarized as follows:

- We introduced substitute vectors as paradigmatic representations of word context and demonstrated their use in unsupervised part of speech induction on 19 corpora in 15 languages.
- We demonstrated that using paradigmatic representations of word context and modeling co-occurrences of word and context types with the S-CODE learning framework give superior results when compared to a syntagmatic bigram model.
- We extended the S-CODE framework to incorporate morphological and orthographic features and improved the state-of-the-art many-to-one accuracy in unsupervised part of speech induction on 17 out of 19 corpora.
- All our code and data, including the substitute vectors for the PTB, MULTEXT-East and CoNLL-X shared task corpora are available at the authors' website at xxx.xxx.xxx.

Appendix A: Computation of Substitute Distributions

In this study, we predict the syntactic category of a word in a given context based on its substitute distribution. The sample space of the substitute distribution is the vocabulary of the language model including the unknown word tag <unk>. Note that the substitute distribution is a function of the context only and is indifferent to the target word.

It is best to use both the left and the right context when estimating the probabilities for potential lexical substitutes. For example, in "He lived in San Francisco suburbs.", the token San would be difficult to guess from the left context but it is almost certain looking at the right context. We define c_w as the 2n-1 word window centered around the target word position: $w_{-n+1} \dots w_0 \dots w_{n-1}$ (n=4 is the n-gram order we have used). The probability of a substitute word w in a given context c_w can be estimated as:

$$P(w_{0} = w | c_{w}) \propto P(w_{-n+1} \dots w_{0} \dots w_{n-1})$$

$$= P(w_{-n+1})P(w_{-n+2} | w_{-n+1})$$

$$\dots P(w_{n-1} | w_{-n+1}^{n-2})$$

$$\approx P(w_{0} | w_{-n+1}^{-1})P(w_{1} | w_{-n+2}^{0})$$

$$\dots P(w_{n-1} | w_{0}^{n-2})$$

$$(A.1)$$

$$(A.2)$$

$$(A.2)$$

where w_i^j represents the sequence of words $w_i w_{i+1} \dots w_j$. In Equation A.1, $P(w|c_w)$ is proportional to $P(w_{-n+1} \dots w_0 \dots w_{n-1})$ because the words of the context are fixed. Terms without w_0 are identical for each substitute in Equation A.2 therefore they have been dropped in Equation

A.3. Finally, because of the Markov property of n-gram language model, only the closest n-1 words are used in the experiments.

Near the sentence boundaries the appropriate terms were truncated in Equation A.3. Specifically, at the beginning of the sentence shorter n-gram contexts were used and at the end of the sentence terms beyond the end-of-sentence token were dropped.

To obtain a discrete representation of the context, the random-substitutes algorithm pairs each word token with a substitute sampled from the pre-computed substitute distribution generated from the word token's context and then word (W) – random-substitute (S) pairs are fed to the S-CODE algorithm as input.

Appendix B: The CODE and S-CODE Models

In this section we review the unsupervised method that we use to model co-occurrence statistics: the Co-occurrence Data Embedding (CODE) (Globerson et al. 2007) method and its spherical extension (S-CODE) introduced by (Maron, Lamar, and Bienenstock 2010).

Let W and C be two categorical variables with finite cardinalities |W| and |C|. We observe a set of pairs $\{w_i, c_i\}_{i=1}^n$ drawn IID from the joint distribution of W and C. The basic idea behind CODE and related methods is to represent (embed) each value of W and each value of C as points in a common Euclidean space \mathbf{R}^d such that values that frequently co-occur lie close to each other. There are several ways to formalize the relationship between the distances and co-occurrence statistics, in this paper we use the following:

$$p(w,c) = \frac{1}{Z}\bar{p}(w)\bar{p}(c)e^{-d_{w,c}^2} \tag{B.1}$$

where $d_{w,c}^2$ is the squared distance between the embeddings of w and c, $\bar{p}(w)$ and $\bar{p}(c)$ are empirical probabilities, and $Z = \sum_{w,c} \bar{p}(w)\bar{p}(c)e^{-d_{w,c}^2}$ is a normalization term. If we use the notation ϕ_w for the point corresponding to w and ψ_c for the point corresponding to c then $d_{w,c}^2 = \|\phi_w - \psi_c\|^2$. The log-likelihood of a given embedding $\ell(\phi,\psi)$ can be expressed as:

$$\begin{split} \ell(\phi, \psi) &= \sum_{w,c} \bar{p}(w,c) \log p(w,c) \\ &= \sum_{w,c} \bar{p}(w,c) (-\log Z + \log \bar{p}(w) \bar{p}(c) - d_{w,c}^2) \\ &= -\log Z + const - \sum_{w,c} \bar{p}(w,c) d_{w,c}^2 \end{split}$$
 (B.2)

The likelihood is not convex in ϕ and ψ . We use gradient ascent to find an approximate solution for a set of ϕ_w , ψ_c that maximize the likelihood. The gradient of the $d_{w,c}^2$ term pulls neighbors closer in proportion to the empirical joint probability:

$$\frac{\partial}{\partial \phi_w} \sum_{w,c} -\bar{p}(w,c) d_{w,c}^2 = \sum_y 2\bar{p}(w,c) (\psi_c - \phi_w)$$
 (B.3)

The gradient of the Z term pushes neighbors apart in proportion to the estimated joint probability:

$$\frac{\partial}{\partial \phi_x} (-\log Z) = \sum_{u} 2p(w, c)(\phi_w - \psi_c)$$
(B.4)

Thus the net effect is to pull pairs together if their estimated probability is less than the empirical probability and to push them apart otherwise. The gradients with respect to ψ_c are similar. S-CODE (Maron, Lamar, and Bienenstock 2010) additionally restricts all ϕ_w and ψ_c to lie on the unit sphere. With this restriction, Z stays around a fixed value during gradient ascent. This allows S-CODE to substitute an approximate constant \tilde{Z} in gradient calculations for the real Z for computational efficiency. In our experiments, we used S-CODE with its sampling based stochastic gradient ascent algorithm and smoothly decreasing learning rate.

Appendix C: S-CODE with More than Two Variables

In order to accommodate multiple feature types the S-CODE model in the previous section needs to be extended to handle more than two variables. Globerson et. al (2007) suggest the following likelihood function:

$$\ell(\phi, \psi^{(1)}, \dots, \psi^{(K)}) = \bar{p}(w, c) \log p(w, c) + \sum_{i}^{K} \sum_{w, f^{(i)}} \bar{p}(w, f^{(i)}) \log p(w, f^{(i)}) \quad (C.1)$$

where $\bar{p}(w,c)$ is the empirical joint distribution of context C with $W, F^{(1)}, \ldots, F^{(K)}$ are extra K different variables whose empirical joint distributions with $W, \bar{p}(w,f^{(1)})\ldots\bar{p}(w,f^{(K)})$, are known. Eq. C.1 then represents a set of CODE models $p(w,f^{(k)})$ where each $F^{(k)}$ has an embedding $\psi_f^{(k)}$ but all models share the same ϕ_w embedding.

We adopt this likelihood function, let W represent a word, C represent a context (i.e., random substitute), and $F^{(1)},\ldots,F^{(K)}$ stand for morphological and orthographic features of the word thus each co-occurrence is a (K+1)-tuple, $(W,C,F^{(1)},\ldots F^{(K)})$. With this setup, the training procedure needs to change little: instead of sampling a word (w) – context (c), the word (w) – context (c) – features (f_1,\ldots,f_K) tuple is sampled and input to the gradient ascent algorithm. The gradient search algorithm updates the embeddings according to p(w,c) and $p(w,f^{(i)})$ where $i=1\ldots k$ and no updates are performed between c and $f^{(i)}$ s since they do not have any co-occurrence statistics and w is the only shared variable.

Tuples might have null values due to unobserved features. For example in the case of POS induction, the word "car" has no morphological or orthographic features therefore all the elements of the tuple have null value except the word type (w) and the context (c). We do not perform any pull or push updates on embeddings during the gradient search if the corresponding $f^{(k)}$ is null¹⁰.

Appendix D: Language Statistics

This section explains the language model training and feature extraction of each language that we apply our model in Section 3.7.

Statictical Language Modeling. For all languages except Serbian, English and Turkish, we train the language models by using the corresponding Wikipedia dump files¹¹. Serbian shares a common basis with Croatian and Bosnian therefore we trained 3 different language models using

 $^{10\,}$ In the POS induction problem w and c represents the word type and context therefore they are always observed.

¹¹ Latest Wikipedia dump files are freely available at http://dumps.wikimedia.org/ and the text in the dump files can be extracted using WP2TXT (http://wp2txt.rubyforge.org/)

Wikipedia dump files of Serbian together with these two languages and measured the perplexities on the MULTEXT-East Serbian corpus. We chose the Croatian language model since it achieved the lowest perplexity score and unknown word ratio on the MULTEXT-East Serbian corpus. To train the statistical language model of English, we use Wall Street Journal data (1987-1994) extracted from CSR-III Text (Graff, Rosenfeld, and Paul 1995) (excluding sections of the PTB) and for the Turkish language modeling we use the web corpus collected from Turkish news and blog sites (Sak, Güngör, and Saraçlar 2008).

In order to reduce the unknown word ratio of resource poor languages and to standardize the process we set the vocabulary threshold to 2 for all languages except English. English has a relatively low unknown word ratio therefore we set the threshold to 20 instead of 2. Table 1 summarizes the language model related statistics and scores that vary across the languages in terms of quality and quantity.

Feature extraction. Morphological features of each language are extracted using the training sections of the corresponding MULTEXT-East and CoNLL-X corpora. We don't use the language model corpora to extract morphological features. Number of morphological feature of each language is presented in Table 1. We use the same set of orthographic features described in Section 3.5 except we add an "Only-Punctuation" feature to the languages of MULTEXT-East corpora. The "Only-Punctuation" feature is generated when a token only consists of punctuation characters.

Table 1: Last two columns present the number of induced suffix parts and word types with these suffix parts after the morphological feature extraction.

		Langua	ige Model	Test set				
	Language	Source	Word Count	Word Count	Perplexity (ppl)	Unknown Word	Suffix Parts	Word Types with Suffix parts
WSJ	English	News	126,170,376	1,173,766	79.926	0.012	5575	19223
	Bulgarian	Wikipedia	32,511,616	101,173	655.202	.0565	609	4209
ast	Czech	Wikipedia	59,698,049	100,368	1,069.67	.0299	2787	12848
Ę	English	News	126,170,376	118,424	265.246	.0288	1251	4783
X	Estonian	Wikipedia	14,513,571	94,898	871.765	.0654	4448	13638
E	Hungarian	Wikipedia	66,069,788	98,426	742.676	.0449	5423	15995
MULTEXT-East	Romanian	Wikipedia	35680870	118,328	666.855	.1074	2064	9445
Ξ	Slovene	Wikipedia	18,969,846	112,278	658.711	.0389	2093	11834
	Serbian	Wikipedia	17,129,679	108,809	804.962	.0580	2722	12476
	Bulgarian	Wikipedia	32,511,616	190,217	538.972	.0430	926	8225
Task	Czech	Wikipedia	59,698,049	1,249,408	1,233.95	.0250	12443	85673
ΤĘ	Danish	Wikipedia	35,863,945	94,386	351.24	.0393	3708	10897
Shared	Dutch	Wikipedia	159,978,524	195,069	390.818	.0476	5250	13407
Sha	German	Wikipedia	437,777,863	699,610	680.036	.0487	15219	45414
×	Portuguese	Wikipedia	150,099,154	206,678	378.656	.0861	5033	15721
	Slovene	Wikipedia	18,969,846	28,750	663.053	.0414	1257	4781
CONLL	Spanish	Wikipedia	332,311,650	89,334	274.418	.0424	2648	9316
ပ္ပ	Swedish	Wikipedia	32,004,538	191,467	1,233.95	.0250	2897	12725
	Turkish	Web	491,195,991	47,605	868.829	.0508	5651	14227

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