Week 9 Problems

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Question:	1	2	3	4	Total
Points:	1	1	1	1	4
Score:					

1. (1 point) Suppose U is a neighborhood of a point x and that $U \subseteq V$. Show that V is a neighborhood of x.

   Solution: U is a neighborhood of x that means there exists $\varepsilon > 0$ such that $\aleph_\varepsilon(x) \subseteq U$. Now, $U \subseteq V$ means $\aleph_\varepsilon(x) \subseteq V$. Therefore, V is a neighborhood of x. ■

2. (1 point) (a) Suppose U and V are neighborhoods of a point x. Show that $U \cap V$ is a neighborhood of x.

   Solution: U and V are neighborhoods of x that means there exists $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $\aleph_{\varepsilon_1}(x) \subseteq U$ and $\aleph_{\varepsilon_2}(x) \subseteq V$. Now, $x \in U \cap V$ we can define a $\varepsilon' = \min\{\varepsilon_1, \varepsilon_2\}$. $\aleph_{\varepsilon'}(x) \subseteq U \cap V$ is an ε-neighborhood of x. Therefore, $U \cap V$ is a neighborhood of x. ■

   (b) Show that (a) remains true for a finite collection of neighborhoods.

   Solution: Let $U_1, U_2, \ldots, U_n$ be neighborhoods of x then $\forall i \in \{1, 2, \cdots, n\}, \exists \varepsilon_i > 0 \ni \aleph_{\varepsilon_i}(x) \subseteq U_i$. Now, $x \in \bigcap_{i=1}^{n} U_i$, we can define a $\varepsilon' = \min\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\}$. $\aleph_{\varepsilon'}(x) \subseteq \bigcap_{i=1}^{n} U_i$ is an ε-neighborhood of x. Therefore, $\bigcap_{i=1}^{n} U_i$ is a neighborhood of x. ■

   (c) Does (a) remains true for an infinite collection of neighborhoods?

   Solution:

3. (1 point) (a) If S and T are bounded sets, show that $S \cap T$ are bounded.

Solution:

(b) If S and T are as in (a), show that $\sup(S \cup T) = \max\{\sup S, \sup T\}$ (be sure to justify use of max).

Solution:

(c) Is it true that $\sup(S \cap T) = \min\{\sup S, \sup T\}$?

Solution:

(d) Give a condition under which the equality in (c) would be true.

Solution:

(e) Let $\{S_\alpha : \alpha \in \mathcal{A}\}$ be a collection of bounded sets (where $\mathcal{A}$ is finite). Show that $\bigcup_{\alpha \in \mathcal{A}} S_\alpha$ is bounded.

Solution:

(f) Let $\{S_\alpha : \alpha \in \mathcal{A}\}$ be a collection of bounded sets (where $\mathcal{A}$ is infinite). Is $\bigcup_{\alpha \in \mathcal{A}} S_\alpha$ necessarily bounded?

Solution:

4. (1 point) (a) Suppose x_k is a real number for $k = 1, 2, \cdots$, and there is a positive number ε so that $x_k > \varepsilon$ for each k. If B is any real number, show that there is a natural number n so that $x_1 + x_2 + \cdots + x_n > B$.

Solution:

(b) Show that this need not be the case if we assume only that $x_n > 0$ for all n.

Solution: