Speculations on Test-Time Scaling

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Cornell

Outline

Introduction

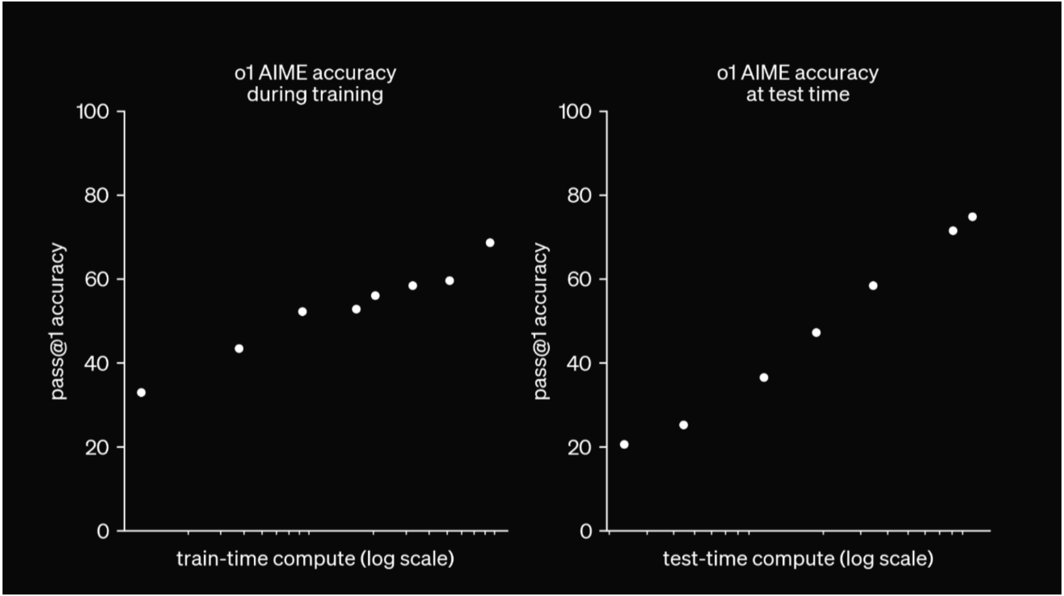
The Clues

Notation

The Suspects

No Verifier

Conclusions


o1 AIME accuracy
during training
100
80
60
40
20
0
pass@1 accuracy
train-time compute (log scale)
o1 AIME accuracy
at test time
100
80
60
40
20
0
pass@1 accuracy
test-time compute (log scale)

Context

- LLM (2018-2024) driven by training scaling

- Speculation: Benefit of static data running out

Implication

- Breakthrough in large-scale RL Training
-

What have we seen?

- Public demo model
- Strong result in constrained domains.

This Talk

- Survey of the public literature
- Synthesis of discussions with expert
- Gossip and hearsay

Thanks

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What we know

Our large-scale **reinforcement learning algorithm** teaches the model how to think productively using its **chain of thought** in a highly **data-efficient** training process.

What we know

- RL - Signal from verifiable problems
- CoT - “Thinking” occurs in token stream
- Data Efficient - Fixed set of good problems

From Gossip

- Single final model
- Not learned from expert examples
-

Chain of Thought

o1 learns to hone its chain of thought and refine the strategies it uses. It learns to recognize and **correct its mistakes**. It learns to **break down tricky steps** into simpler ones. It learns to try a **different approach** when the current one isn't working.

Review: Chain of Thought

-

Planning

Backtracking

Strategies

Summary

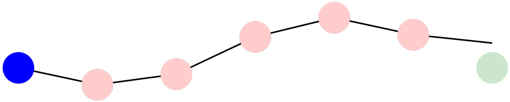
- Solves problems by very long CoT
- CoT includes “thinking” (search / planning)
- Core novelty: Inducing this behavior

Notation - Test-Time (No learning yet!)

- x - the problem specification
- $z \in \mathcal{S}^T$ - the chain of thought (CoT)
- $y \in \mathcal{Y}$ - the final answer
- $p(y|x) = \mathbb{E}_{z \sim p(z|x)} p(y|x, z)$ - model

Warm-up: Ancestral Sampling

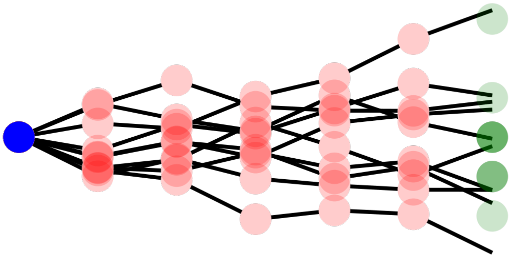
- $\tilde{z} \sim p(z|x)$
- $\tilde{y} \sim p(y|x, z = \tilde{z})$



$|\tilde{z}|$ is the amount of test-time compute

Warm-up: Monte-Carlo (Self-Consistency)

- $\tilde{z} \sim p(z|x)$
- $\tilde{y} \sim p(y|x, \tilde{z})$



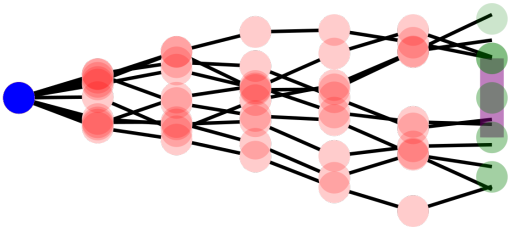
Pick majority choice $\tilde{y}^i$

Notation - Verifier

- $\text{Ver} : \mathcal{Y} \to \{0, 1\}$, tells us if an answer is correct or not
- Examples: Regular expression for math, unit test for code.

Warm up: Rejection Sampling / Best-of-N

- $\tilde{z} \sim p(z|x)$
- $\tilde{y} \sim p(y|x, \tilde{z})$



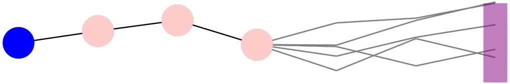
We only keep the correct subset of $\tilde{y}$, $\{\tilde{y} : Ver(\tilde{y})\}$

Variants:

Warm up: Monte-Carlo Roll-Outs

- Given $x, z_{1:t}$ (a partial chain of thought) define expected reward as

$$\mathbb{E}_{y \sim p(y|z,x), z_{t:T} \sim p(z|z_{1:t},x)}[Ver(y)]$$



- Roll-outs apply MC to this expectation.

Goal: Learning

- $\max_\theta \sum \log p(y|x;\theta)$

- Intractable expectation over latent CoT

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The Suspects

- Guess + Check
- Guided Search
- AlphaZero
- Learn to Search
- Wildcard

Informal: Guess + Check

- Sample N CoTs
- Check if successful
- Train on good ones

Formalization: Rejection Sampling EM

$$\max_\theta \sum \log p(y|x;\theta) = \sum \log E_z p(y,z|x)$$

- E-Step: Sample $\mathcal{Z} = \{\tilde{z}_i\}_{i=1}^N$ from the posterior with Rejection Sampling

$$\tilde{z} \sim p(z|\mathsf{Ver}(y) = 1, x)$$

- M-Step: Fit $\theta' \leftarrow \arg\max_\theta \sum_{\tilde{z}\in\mathcal{Z}} \log p(\tilde{z}|x;\theta)$

Terminology

[Zelikman et al., 2022, Gulcehre et al., 2023, Singh et al., 2023]

- STaR
- ReST
- ReST-EM
- Filtered Rejection Sampling
- Best-of-N Training

Batched

- Batched -> Compute trajectories first, then train with behavioral cloning

- Online -> Use policy gradient-like steps to update after each example

Empirical Results

[Singh et al., 2023]

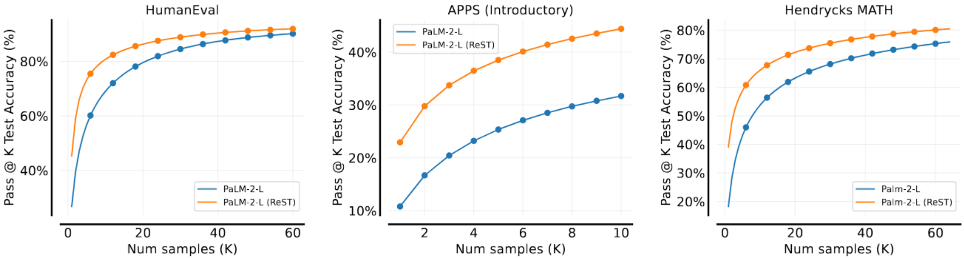


Figure 5 | **Pass@K results** for PaLM-2-L pretrained model as well as model fine-tuned with ReST^{EM}. For a fixed number of samples K, fine-tuning with ReST^{EM} substantially improves Pass@K performance. We set temperature to 1.0 and use nucleus sampling with $p = 0.95$.

Why might this be right?

- Extremely simple and scalable

- Good baseline in past work

- No evidence this learns to correct, plan

- Well-explored in literature with marginal gains

Deeper

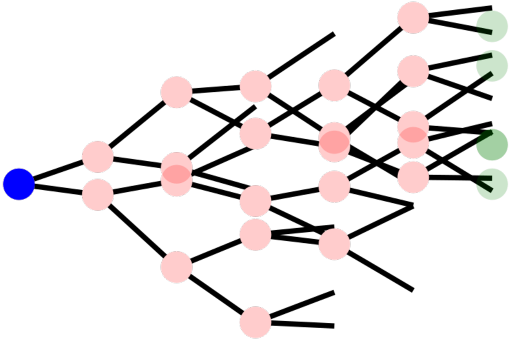
- Rejection sampling may be really inefficient.
- Particularly on hard problems, may get no signal

Informal: Guided Search

- During CoT sampling, use a “guide” to correct trajectories
- Check if final versions are successful
- Train on good ones

Beam Search with Guide

- $r : S^T \to \mathbb{R}$ is the guide/reward function
- $\mathcal{Z}_{1:t-1} = \{z^i_{1:t-1}\}_{i=1}^m$, current partial CoT candidates
- Sample k potential $\tilde{z}_t$ from $p(z_t | x, z^i_{1:t-1})$ for each $z^i_{1:t-1}$
- Keep the top m samples, ordered by $r(\tilde{z}_t)$, repeat



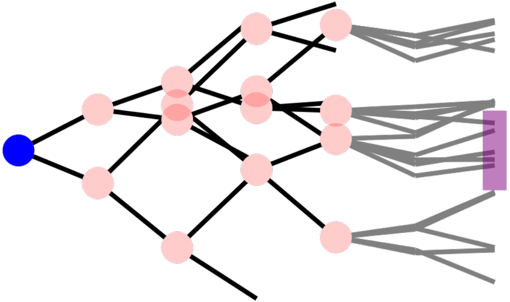
What to use as Guide?

- Monte Carlo Roll-outs
- Learned Reward Model

Beam Search with Roll-Outs

- For a potential $\tilde{z}_t$, sample n solutions $\tilde{y}^{i,j}$ from $p(y, z_{t+1:T} | x, z_{1:t-1}, \tilde{z}_t)$

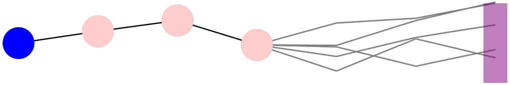
- $r_{MC}(\tilde{z}_t) = \frac{1}{n} \sum_{j=1}^{n} Ver(\tilde{y}^{i,j})$



[Wang et al., 2023]

Amortized Roll-Outs

- Rollouts are costly, so instead learn a model $r_\psi(z_t)$ to approximate rollouts
- Use r_{MC} to determine labels to train r_ψ



What about test time?

- Learned rewards can improve test-time without verifier.
-

Terminology

[Uesato et al., 2022, Setlur et al., 2024, Wang et al., 2023, Lightman et al., 2023, Snell et al., 2024]

- Value
- PRM
- PAV
- Math Shepard.
- snell.

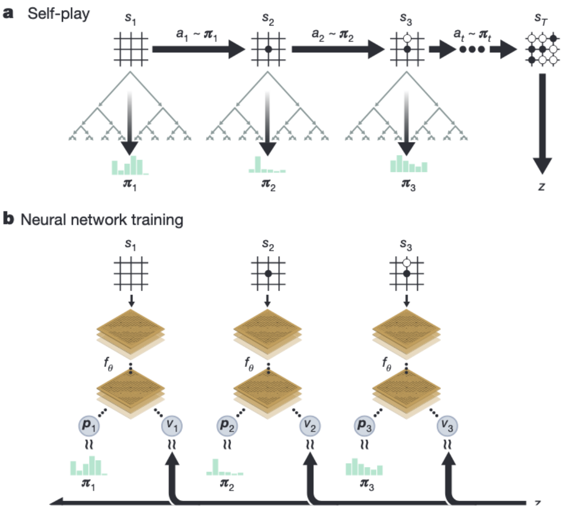
Why might this be right?

- OpenAI is exploring
- Makes RS more efficient.
- Learned rewards are effective
- Assumption: o1 is a single test-time model (although could train or distill-in)
- Not clear if it learns planning.

Deeper

- Improving search seems critical.

Reminder: AlphaZero



Informal: AlphaZero

- Self-play using guided-search with exploration
- Label final outcomes of self-play games
- Train both a reward model and policy

[Anthony et al., 2017]

Formalized: Expert Iteration

- Iterative algorithm combining learned model + expert search with a verifier.

- Generate samples using $p(y, z|x)$, reward model $r(z_t)$, and search algorithm (e.g. beam search)

- Label samples using $Ver(y)$

- Train $p(y, z|x)$, $r(z_t)$ on the labeled samples, and repeat

UCB for exploration

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Empirical Results

[Hosseini et al., 2024]

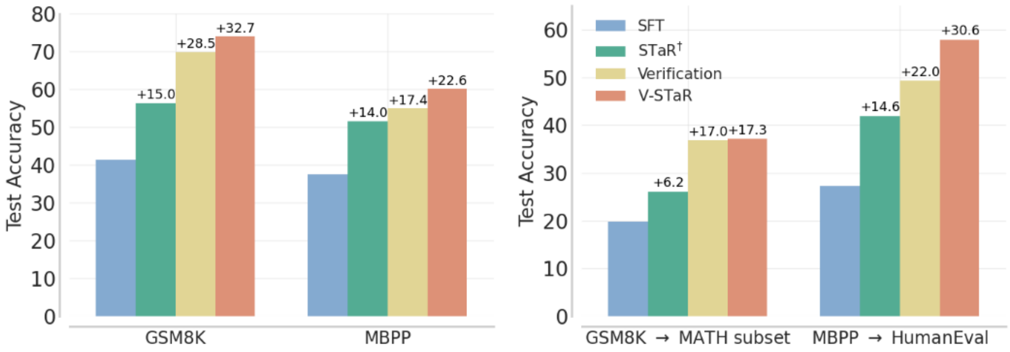

Figure 8: Test accuracy of 13B V-STaR compared to baselines. We report Best-of-64 for verification-based methods and Pass@1 for others. **(Left)** Test accuracy for training tasks. **(Right)** Transfer evaluation of GSM8K and MBPP trained models on MATH subset and HumanEval respectively.

Why might this be right?

- Major demonstrated RL result
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Deeper

- Can we force the model to search?

Informal: Learning to Correct

- Sample N Successful CoTs
- Edit $z \to z'$ to inject incorrect expansions before correct ones.
- Train on z' trajectories

Formalized: Stream of Search

[Gandhi et al., 2024]

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Empirical Results

Score results

Why might this be right?

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-

Why might this be wrong?

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-

[Brown et al., 2024]

No verifier

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