Multi-Objective Optimization

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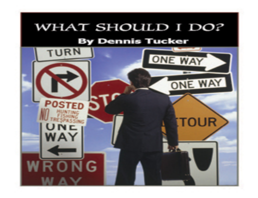
LECTURE 8

Outline

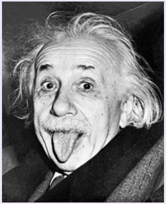
Recent Results and Open Problems



Introduction

After 32 years of existence, and with so much work done, EMO may seem intimidating to some people. If so many people have worked in this area for the last 20 years, what remains to be done?

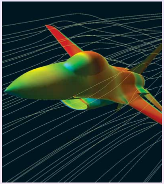
Recent Results and Open Problems



Luckily, there still are many opportunities to do research in this area, even within topics that seem to have been visited a lot in the past.

Imagination is more important than knowledge.
Albert Einstein

Recent Results and Open Problems



MOEAs for Expensive Objective Functions

Many real-world problems have objective functions which are very expensive to evaluate (e.g., in aeronautical engineering). Few MOEAs currently exist to deal with such problems.

MOEAs for Expensive Objective Functions

- Currently, there are three main lines of algorithmic design to deal with these problems:
  1. Use of “clever” approaches to save objective function evaluations. The most common sort of approach is the use of fitness approximation schemes.
  2. Use of surrogate methods, which adopt an approximation of the objective function which is cheap to evaluate and then adjust the error of the model by performing few evaluations of the real objective function(s).
  3. Use of parallel MOEAs. Although you can get very creative here, few people have actually proposed interesting approaches.

Fitness Approximation

The idea here is to estimate the quality of some of the individuals based on an approximation model of the fitness landscape. Thus, for the single-objective case, fitness approximation schemes estimate the fitness of an individual based on the previously observed objective function values of its neighboring individuals.

Fitness Approximation

The schemes can go from very simple ideas, such as **fitness inheritance** in which an offspring's fitness is estimated by using the weighted average fitness of its parents, to the use of neural networks or statistical models built from a few points that are adopted to predict the fitness of new individuals.

Taking this to the multiobjective case is not trivial, and is currently an interesting line of research.

Surrogate Methods

There is a lot of work done on surrogate methods in the engineering literature. Even for the multiobjective case, researchers have tested approaches such as radial basis functions, kriging, regression models, among others. Normally, kriging is considered to be the best choice, if we can afford its high computational cost.

The implementation of MOEAs that incorporate surrogate methods is not trivial!

ParEGO

It's probably the best known MOEA based on surrogate methods (at least for the EMO community). It requires only between 100 and 250 objective function evaluations to produce reasonably good approximations in problems of low dimensionality (up to 10 decision variables).

Parallel MOEAs

Although parallel EAs have been relatively popular in the literature, relatively few research exists on designing parallel MOEAs that really exploit parallel architectures. A number of interesting issues arise when designing parallel MOEAs. For example: should we use local external archives or only a global one? Is it possible to explore non-overlapping portions of the search space with each deme? What about GPU-based implementations (can we do this in an efficient and effective way)?

Parallel MOEAs

A few interesting parallel MOEAs currently exist:

- **The Predator-Prey Model**: It places solutions (preys) on the vertices of an undirected connected graph, defining a neighborhood. Such preys are caught by predators that perform random walks within a certain neighborhood. Predators are only interested in preys that are good in a particular objective. Thus, preys that are good in all the objectives are given a higher chance of survival and can produce more descendants. An interesting approach that shares similarities with the predator-prey model is the cellular multi-objective genetic algorithm proposed by Alba et al. (2003).

DRMOGA: The *Divided Range Multi-Objective Genetic Algorithm* (Hiroyasu, 2000) was one of the earliest attempts to divide the effort of the search done by a parallel MOEA in a clever way. Zhu (2002) and Deb (2003) proposed other approaches that also aimed to divide the search effort of a parallel MOEA.

MRMOGA: This approach uses an island model in which migration is unidirectional, and is done in such a way that the resolution gets finer as we move to a new deme (Jaimes & Coello, 2005).

MOEAs for Expensive Objective Functions

There are, of course, other possible ideas to deal with expensive functions:

- In my research group, we have developed MOEAs based on differential evolution and particle swarm optimization that can solve the ZDT test problems and several of the DTLZ test problems, with less than 4,000 objective function evaluations without using surrogates. The main idea of these approaches is to increase a lot the selection pressure at the expense of sacrificing diversity. Then, using a handful of points located on the Pareto front (or very close to it), we can rebuild the rest of the front by using powerful local search engines (e.g., rough sets).

MOEAs for Expensive Objective Functions

Other ideas:

- Hybridization of MOEAs with gradient-based methods. There are a lot of interesting issues here, but if the hybridization is cleverly done, very efficient algorithms can be designed (e.g., Hernandez-Diaz et al., 2008, Lara et al., 2010).
- We could also hybridize MOEAs with direct search methods (e.g., Nelder and Mead) so that we don't have to estimate the derivatives (see for example, Zapotecas & Coello, 2008).

Self-Adaptation and Online Adaptation

Whatever happened with self-adaptation (and online adaptation) and the dream of creating a parameterless multi-objective evolutionary algorithm? After a few isolated efforts, such as the mutation operator based on Kohonen's self-organizing maps (Büche, 2002), the adaptive parameters of IMOEA (Tan et al., 2001) and the parameterless microGA^2 (Toscano & Coello, 2003), not much work has been done in this regard.

Self-Adaptation and Online Adaptation

The main problem for designing self-adaptive (or online adaptive) MOEAs lies on the difficulty to define convergence criteria, as well as the population behavior that can be used to guide the search in a proper manner. Although performance measures and Pareto optimality can be used for these tasks, it is evident that the parameters setting of a MOEA is much more difficult than that of a single-objective evolutionary algorithm.

Stopping Criteria

Another missing mechanism to get to a parameterless MOEA is a well-established stopping criterion. There have been some attempts in this direction:

- The rank-histograms from Kumar & Rockett (1997,2002).
- The micro-GA^2 (Toscano & Coello, 2003) stops when the solutions generated cannot be improved after a certain number of iterations.

Stopping Criteria

- Martí et al. (2007) proposed a mechanism that gathers information about the solutions obtained so far. This information is accumulated and updated using a discrete Kalman filter. This accumulated information is used to decide when to stop a MOEA.
- Trautmann et al. (2008,2009) proposed a convergence criterion based on statistical testing. There is also an interesting survey on this topic that was presented at EMO'2011 (Wagner et al., 2011).

Self-Adaptation and Online Adaptation

The other problem with self-adaptation is its extra computational cost (i.e., the additional objective function evaluations that it requires), which may be unaffordable in certain applications.

Self-Adaptation and Online Adaptation

Another important issue is the definition of the parameters that are worth self-adapting (e.g., mutation and crossover rates, population size, and even the encoding to be used).

Many-Objectivity

MOEAs that adopt a selection mechanism based on Pareto optimality do not scale properly: the number of nondominated solutions grows exponentially with the number of objectives (Farina, 2004). This makes the selection mechanism completely useless, since all the nondominated solutions are considered equally good!

Recent Results and Open Problems

It has been empirically shown that a random search is more effective than the NSGA-II when dealing with more than 10 objectives (Mostaghim & Schmeck, 2008).

There is also another interesting problem related to scalability: as we increase the number of objectives, the number of solutions required to sample the Pareto front, also grows exponentially.

Many-Objective Problems

In the current literature we can identify two approaches commonly adopted to cope with many-objective problems:

- Adopt or propose a preference relation that induces a finer grain order on the solutions than that induced by the Pareto dominance relation (Di Pierro, 2007; Farina, 2002; Sülflow, 2007; Sato, 2007).
- To reduce the number of objectives of the problem during the search process (Brockhoff, 2006), or *a posteriori*, during the decision making process (Deb, 2006; Brockhoff, 2007; Lopez Jaimes, 2008).

Alternative Preference Relations

- **Average Ranking Method**: This method computes for each solution a different rank considering each objective independently. The final rank of a solution is obtained by summing all theirs ranks on each objective (Bentley, 1997).
- *k*-**Optimality**: It's a relaxed form of Pareto dominance that takes into account the number of improved objectives between two solutions (Farina & Amato, 2004).

Alternative Preference Relations

- **Preference Order Ranking**: A point $\mathbf{x}^*$ is efficient in order k if $f(\mathbf{x}^*)$ is not dominated by any point for any of the k-element subsets of the objectives. The condition of efficiency of order can be used to help reduce the number of points in a set by retaining only those that are regarded as "best compromises" (di Pierro, 2006).

Alternative Preference Relations

- **Control of the dominance area**: Sato et al. (2007) proposed a method to control the dominance area of solutions. This method can control the degree of expansion or contraction of the dominance area by adopting a user-defined parameter.

Alternative Preference Relations

- **Favour Relation**: It consists of a new relation called *favour*. This technique requires no user interaction and can handle infeasible solutions. x is *favoured* to y ($x <_f y$) iff i components of x are better than the corresponding components of y and only j components of y are better than the corresponding components of x.

Dimensionality Reduction

The main idea is to identify the redundant objectives (or redundant to some degree) in order to discard them. A redundat objective is one that can be removed without changing the dominance relation.

Deb and Saxena (2005) proposed a method for reducing the number of objectives based on principal component analysis. The main assumption is that if two objectives are negatively correlated (taking the generated Pareto front as the data set), then these objectives are in conflict with each other.

Dimensionality Reduction

- Brockhoff and Zitzler (2006) defined two kinds of objective reduction problems and two corresponding algorithms to solve them. Here, the conflict is defined using the change in the dominance relation induced by the set of objectives over a solution set in the objective space. That is, if the dominance relation among the vectors does not change when an objective is discarded, then that objective is not in conflict with the other objectives and therefore is considered as redundant.

Other approaches are, of course possible:

- Use of linear or nonlinear aggregating functions.
- Use of alternative schemes (e.g., adopting machine learning techniques, such as in the *multiobjective neural estimation of distribution algorithm* (MONEDA) (Martí et al., 2008)).
- Use of alternative archiving techniques (e.g., the Two-Archive MOEA, which uses one archive for convergence and another for diversity (Praditwong & Yao, 2006)).

What about scalability in decision variable space?

There is almost no work on this topic, which also deserves attention. Zhang & Lim (2007) performed a small study with a single problem that was scaled up to 100 decision variables. Durillo et al. (2008) did a more thorough study, adopting from 8 to 2048 decision variables. Perhaps the most remarkable finding was that PAES was the most salient technique from the several compared (NSGA-II, SPEA2, MOCell, OMOPSO and PESA-II). OMOPSO did very well up to 256 decision variables and ranked second between 512 and 1024 decision variables.

The Challenges of this Century

After 32 years of activity, from which about 20 have been quite intense, we can say that this area has lost its innocence. Today, it's not enough to design a slight variation of an existing algorithm to get it published in a specialized journal. We can say that the age of the straightforward problems is over. We now experience the growing pains, since we are in transition towards an age in which we start seeing a lot of work "by analogy" and few novel ideas.

The Challenges of this Century

However, several problems remain that are worth studying, although the solution of most of them may consume a considerable amount of time. This has made the discipline less friendly than before and may scare away those who got here out of curiosity. Let's see if these problems (some of which are briefly described next) are studied in sufficient depth in the years to come.

The Challenges of this Century

- **Sources of Difficulty**: What makes a problem difficult for a MOEA? Clearly, this is a fundamental question for this discipline, but today we have little information in this regard. We know, for example, that a disconnection in decision variable space causes more trouble than a disconnection in objective function space.

The Future

The Challenges of This Century

- However, we do not know what features are desirable in a MOEA so that it can efficiently solve real-world problems (or at least a particular subclass). Current test problems are much more challenging than those existing 10 years ago, but that provides no indication regarding the suitability of today's MOEAs for solving real-world problems.

The Challenges of This Century

- **Efficiency**: What is the efficiency limit that we can reach with a MOEA? Today, there are MOEAs that require less than 5,000 objective function evaluations to generate reasonably good approximations of the Pareto front of problems with 10 or more decision variables. Can we reduce this figure to 1,000? Can we design such a MOEA with a “robust” behavior? What is robustness in a multi-objective context?

The Challenges of This Century

- **Constraint-Handling**: In its origins, evolutionary multi-objective optimization did not pay much attention to constraints. Over the years, simple constraint-handling methods based on straightforward modifications to the Pareto dominance rules were adopted. As of today, very few approaches exist to explicitly handle constraints in multi-objective optimization problems.

The Challenges of This Century

- Ironically, multi-objective optimization concepts have inspired constraint-handling techniques used for single-objective optimization (see for example [Coello, 2000; Yen, 2003]). However, the desdain for constrained problems is reflected even in the current benchmarks adopted for testing new MOEAs.

The Challenges of This Century

- **Back to our Roots**: I'm firmly convinced that we have considerably underestimated the potential of hybridizing evolutionary algorithms with mathematical programming techniques. There used to be algorithms based on game theory, min-max optimality, and the ε-constraint method, but many researchers considered them "politically incorrect". In recent years, however, several researchers have emphasized the advantages of hybridizing MOEAs with mathematical programming techniques (see for example (Shukla, 2007; Lara et al., 2010; Zapotecas & Coello, 2010; Bosman, 2012)).

The Challenges of This Century

- **Uncertainty and Dynamism**: Topics such as *stochastic* dominance or stochastic multi-objective combinatorial problems, have been scarcely dealt with in the specialized literature and can give rise to new MOEAs. The same can be said about dynamic problems, although there is more work done in this topic (see for example (Farina & Deb, 2004)).

The Challenges of This Century

- **Visualization**: Another interesting topic is the visualization of a Pareto front in a high-dimensional space. There are some recent proposals in this regard (e.g., Pryke et al., 2006; Obayashi & Sasaki, 2003). There is also some interesting work done in the MCDM community in this regard (Lotov, 2004).

The Challenges of This Century

- **Test Problems**: The ZDT and the DTLZ test problems have been widely used, although there are several other benchmarks which have been less popular. We have, for example, Okabe's problems (Okabe, 2004) and the WFG test problems from Huband et al. (2005 and 2006), which are among the hardest proposed so far. However, these test problems, regardless of how difficult they are, are loosely related to the actual difficulties that real-world problems have.

The Challenges of This Century

- **The Third Generation**: One day, somebody will have an idea sufficiently powerful for others to follow him/her. This will probably be an algorithm, although it could be simply an operator or a specific mechanism. I wonder how big will this community be by then.

Evolutionary multi-objective optimization still has a lot to offer regarding research work. However, the new age that we are currently living in this discipline, requires more commitment and the generation of more profound ideas. Probably some simple problems still remain, but we have to look for them more carefully. Clearly, the design of algorithms tailored for a particular problem (of a high degree of difficulty) will remain as an active research area during a long time.

To know more about evolutionary multi-objective optimization

Please visit our EMOO repository located at:

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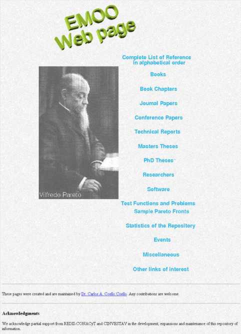

To know more about evolutionary multi-objective optimization

The EMOO repository currently contains:

- Over 10560 bibliographic references including 293 PhD theses, 45 Masters theses, over 4890 journal papers and over 3920 conference papers.
- Contact information of 79 EMOO researchers.
- Public domain implementations of SPEA, NSGA, NSGA-II, the microGA, MOGA, ϵ-MOEA, MOPSO and PAES, among others.