Exploiting the Essential Assumptions of Analogy-Based Effort Estimation

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Abstract—*Background*: There are too many design options for software effort estimators. How can we best explore them all? *Aim*: We seek aspects on general principles of effort estimation that can guide the design of effort estimators. *Method*: We identified the essential assumption of analogy-based effort estimation, i.e., the immediate neighbors of a project offer stable conclusions about that project. We test that assumption by generating a binary tree of clusters of effort data and comparing the variance of supertrees versus smaller subtrees. *Results*: For 10 data sets (from Coc81, Nasa93, Desharnais, Albrecht, ISBSG, and data from Turkish companies), we found: 1) The estimation variance of cluster subtrees is usually *larger* than that of cluster supertrees; 2) if analogy is restricted to the cluster trees with lower variance, then effort estimates have a significantly lower error (measured using MRE, AR, and Pred(25) with a Wilcoxon test, 95 percent confidence, compared to nearest neighbor methods that use neighborhoods of a fixed size). *Conclusion*: Estimation by analogy can be significantly improved by a dynamic selection of nearest neighbors, using only the project data from regions with small variance.

Index Terms—Software cost estimation, analogy, k-NN.

1 Introduction

Software effort estimates are often wrong by a factor of four [1] or even more [2]. As a result, the allocated funds may be inadequate to develop the required project. In the worst case, overrunning projects are canceled and the entire development effort is wasted. For example, NASA canceled its incomplete Check-out Launch Control System project after the initial $200 M estimate was exceeded by another $200 M [3].

It is clear that we need better ways to generate project effort estimates. However, it is not clear how to do that. For example, later in this paper, we document thousands of variations for analogy-based effort estimation (ABE). Effort estimation is an active area of research [4], [5], [6], [7] and more variations are constantly being developed. We expect many more variations of ABE and other effort estimation methods to appear in the very near future.

Recent publications propose data mining toolkits for automatically exploring this very large (and growing) space of options for generating effort estimates. For example, in 2006, Auer et al. [8] proposed an extensive search to learn the best weights to assign different project features. Also in that year, Menzies et al.'s [9] COSEEKMO tool explored thousands of combinations of discretizers, data preprocessors, feature subset selectors, and inductive learners. In 2007, Baker proposed an exhaustive search of all possible project features, learners, etc. He concluded that such an exhaustive search was impractical [10].

The premise of this paper is that we can do better than a COSEEKMO-style brute-force search through the space of all variants of effort estimators. Such studies are computationally intensive (the COSEEKMO experiments took two days to terminate). With the ready availability of cheap CPU farms and cloud computing, such CPU investigations are becoming more feasible. On the other hand, data sets containing historical examples of project effort are typically small.[1] In our view, it seems misdirected to spend days of CPU time just to analyze a few dozen examples. These CPU-intensive searches can generate gigabytes of data. Important general properties of the estimation process might be missed, buried in all that data. As shown below, if we exploit these aspects, we can significantly improve effort estimates.

This paper proposes an alternative to brute-force and heuristic search. According to our *easy path* principle for designing an effort predictor:

> *Find the situations that confuse estimation. Remove those situations.*

(Later in this paper, in Section 3.2, we will offer a precision definition of "confuse estimation." For now, we need only say that confused estimates are highly inaccurate.)

The easy path is not standard practice, i.e., it is a *heuristic*. Usually, prediction systems are matured by adding mechanisms to handle the harder cases (cases for whom estimation accuracy is lower). For example, the AdaBoost

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1. For example, the effort estimation data sets used in Mendes et al. [11], Auer et al. [8], Baker [10], this study, and Li et al. [12] have median size (13, 15, 31, 33, 52), respectively.

algorithm generates a list of learners, and each learner focuses on the examples that were poorly handled by the one before [13].

Focusing on just the easy cases (cases for whom estimation accuracy is higher) could be problematic. If we only explore the easy cases, we could perform badly on the hard test cases. On the other hand, if the easy path works, it finds shortcuts that simplifies future effort estimation work. Also, it avoids COSEEKMO's brute-force search since, according to this principle, we only explore the options that challenge the essential assumptions of the predictor.

The rest of this paper uses the easy path to build and evaluate an effort estimator called TEAK (short for "Test Essential Assumption Knowledge" and available at http://unbox.org/wisp/tags/teak/). In keeping with the easy path, we only explored design options that commented on TEAK's essential assumptions; specifically: 1) case subset selection and 2) how many training examples should be used for estimation.

TEAK's design applied the easy path in five steps:

1. Select a prediction system.
2. Identify the predictor's essential assumption(s).
3. Recognize when those assumption(s) are violated.
4. Remove those situations.
5. Execute the modified prediction system.

On evaluation, we found that for the data sets studied here, TEAK generated significantly *better* estimates than comparable methods.

More generally, the success of the easy path principle recommends it for future research. When designing a predictor, it is useful to *first* try optimizing for the situations where prediction is easy, *before* struggling with arcane and complex mechanisms to handle the harder situations. For example, in future work, we will apply steps 1, 2, 3, 4, 5 to other aspects of effort estimation like feature weighting and similarity measures.

The rest of this paper is structured as follows: After a review of the general field of effort estimation, we will focus on ABE. For ABE, we will work through the above five steps to design TEAK. TEAK's performance will then be compared against six other ABE systems. Our conclusion will be to recommend TEAK for effort estimation.

The paper uses the notation of Fig. 1.

Symbol	Explanation
ABE	Analogy Based Estimation.
ABE0	A baseline ABE method.
NNet	A neural net prediction system with one hidden layer.
LR	Linear regression.
GAC	Greedy Agglomerative Clustering.
TEAK	Test Essential Assumption Knowledge.
GAC1, GAC2	First and second GAC trees within TEAK.
x, y	Depending on the context, x and y can refer to two instances/projects in a dataset or alternatively to two vertices in a GAC tree.
x_i, y_i	i^{th} features of projects x and y respectively.
w_i	Feature weight for the difference of features x_i and y_i in Euclidean distance function.
L_x, L_y, L_z	Leaves of the sub-trees whose roots are x, y and z respectively.
k_x, k_y, k_z	The number of leaves in L_x, L_y, L_z respectively.
k-NN	k Nearest Neighbors.
k	An italic k alone refers to analogies, i.e. selected similar projects.
b_i,b_j, c_i, c_j	All these symbols are related to discretization of continuous columns. b_i and b_j refer to breakpoints i and j, which in return produce discrete bins that have counts of c_i and c_j instances within themselves.
Best(K)	A procedure that heuristically finds the best k value for a dataset.
σ_x^2, σ_{yz}^2	Assuming that x, y and z are vertices in a GAC tree and x is the parent of y and z, σ_x^2 refers to the variance of instances in x and σ_{yz}^2 refers to the weighted sum of the variances of y and z.
α, β, γ, R, $max(\sigma^2)$	These symbols are associated with different pruning policies. α, β and γ keep user-defined values to fine-tune pruning. R is a random variable that can have values from 0 to 1. $max(\sigma^2)$ refers to maximum variance of all sub-trees in a GAC tree.
T, N	T refers to a given dataset and N refers to a test set out of this dataset.
$predicted_i$	The effort of test instance $N_i \in N$ predicted by some induced prediction system.
$actual_i$	The actual effort seen in test instance $N_i in N$
AR	Absolute residual. $\lvert actual_i - predicted_i \rvert$
MRE	magnitude of relative error. $\frac{AR}{actual_i}$
PRED(X)	The percentage of estimates that are within X% of the actual value.
win_i, tie_i, $loss_i$	The total number of wins, ties and losses of a variant in comparison to other variants according to Wilcoxon signed rank test.

Fig. 1. The explanations of symbols that are used in our research are summarized here. Symbols that are related to each other are grouped together.

2 Background

2.1 Scope

This paper is *not* a detailed comparison of analogy-based estimation to other estimation methods (for such a large scale comparison of many different methods, the reader is referred to [9], [14]). While we compare our proposed new technique to a limited number of other estimation methods (specifically, neural networks and regression), that comparison is only to ensure that analogy-based estimation is not noticeably worse than other methods in widespread use.

The main point of this paper is as follows: Analogy-based effort estimation is a widely used and widely studied technique [8], [12], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25]. This paper reports a novel method to improve that technique. Specifically, when estimating via analogy, it is *best to first prune all subsets of the data with high variance*. We will argue that a new *variance heuristic* is a better way to select analogies:

- Without this heuristic, analogies are selected by their distance to the test instance.
- With this heuristic, a preprocessor prunes the space of possible analogies, removing the subsets of the data with high variance (in high variance subsets, training data offers highly variable conclusions).

This heuristic works, we believe, since if a test instance falls into such subsets, then (by definition) minor changes to the test will lead to large changes in the prediction (due to the variability in that region). We show below that, by removing those problematic subsets, effort estimation by analogy can be improved.

2.2 Analogous Approaches

While the variance heuristic is novel and unique in the effort estimation literature, analogous proposals can be found in the requirements engineering literature, dating back to the 1990s. In the seminal paper "To Be *and* Not To Be," Nuseibeh [26] discusses a spectrum of methods for handling inconsistent specifications. One method is *circumvent*, i.e., instead of expending effort resolving regions of contradiction, add "pollution markers" that screen the problematic regions away from the rest of the system. Note that our variance heuristic (that prunes the data subsets

with high variance) is something like a pollution marker since it guides the reasoning away from problematic training data.

Another use for such marks is to mark any segments that human agents need to explore. Turning back from requirements engineering (which was the focus of Nuseibeh's discussion) back to effort estimation (which is the focus of this paper), we could utilize pollution marks to highlight regions where more data collection might be beneficial. Note that this approach is not explored here since our premise is that we must make the best use possible of fixed data. However, this might be a promising area of future research.

2.3 Effort Estimation

Having set the context for this paper, we now turn to the details.

After Shepperd [6], we say that software project effort estimation usually uses one of three methods:

- human-centric techniques (a.k.a. expert judgment);
- model-based techniques including:
  - algorithmic/parametric models such COCOMO [1], [27];
  - induced prediction systems.

Human-centric techniques are the most widely used estimation method [28], but are problematic. If an estimate is disputed, it can be difficult to reconcile competing human intuitions, e.g., when one estimate is generated by a manager who is senior to the other estimator. Also, Jorgensen and Gruschke [29] report that humans are surprisingly poor at reflecting and improving on their expert judgments.

One alternative to expert judgment is a model-based estimate. Models are a reproducible methods for generating an estimate. This is needed for, e.g., US government software contracts that require a model-based estimate at each project milestone [9]. Such models are used to generate and audit an estimate, or to double check a human-centric estimate.

Model-based estimates can be generated using an algorithmic/parametric approach or via induced prediction systems. In the former, an expert proposes a general model, then domain data are used to tune that model to specific projects. For example, Boehm's 1981 COCOMO model [1] hypothesized that development effort was exponential on LOC and linear on 15 *effort multipliers* such as analyst capability, product complexity, etc. Boehm defined a *local calibration* procedure to tune the COCOMO model to local data.

Induced prediction systems are useful if the available local training data do not conform to the requirements of a predefined algorithmic/parametric model such as COCOMO. There are many induction methods, including linear regression, neural nets, and analogy, just to name a few [9], [30]. Analogy-based estimation is discussed in detail in the next section. In order to give the reader some context, we offer here some notes on none-analogy methods (estimation methods other than analogy-based estimation). Regression assumes that the data fit some function. The parameters of that function are then adjusted to minimize the difference between the values predicted by the model and the actual values in the training data. For example, in linear regression, the model is assumed to be of the form

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots,$$

where x_i are model inputs, y are the model outputs, and β_i are the coefficients adjusted by a linear regression induction system.

Neural nets are useful when the data distributions are not simple linear functions [31], [32], [33]. An input layer of project details is connected to zero or more "hidden" layers which, in turn, connect to an output node (the effort prediction). The connections are weighted directed edges. If the signal arriving to a node sums to greater than some threshold value, the node is said to "fire" and a weight is propagated across the network. Learning in a neural net compares the output value to the expected value, then applies some correction method to improve the edge weights (e.g., the "back propagation" algorithm first invented by Bryson and Ho in 1969 [34] and made popular by Rumelhart et al. in the 1980s [35]).

All induction systems require a bias in order to decide what details can be safely ignored. For example, linear regression assumes that the effort data fit a straight line. When data do not match the bias of the induction system, various patches have been proposed. Boehm [1, p. 526-529] and Kitchenham and Mendes [36] advocate taking the logarithms of exponential distributions before applying linear regression. Selecting the right patch is typically a manual process requiring an analyst experienced in effort estimation.

2.4 Analogy-Based Estimation

In ABE, effort estimates are generated for a *test* project by finding similar completed software projects (a.k.a. the *training projects*). Following Kadoda et al. [19], Mendes et al. [11], and Li et al. [12], we define a baseline ABE called ABE0, as follows:

ABE0 executes over a table of data where

- each row contains one project;
- columns contain *independent* variables (features) in the projects and *dependent* variables (features) that store, for example, effort and duration required to complete one project.

After processing the training projects, ABE0 inputs one test project then outputs an estimate for that project. To generate that estimate, a *scaling measure* is used to ensure all independent features have the same degree of influence on the distance measure between test and training projects. Also, a *feature weighting* scheme is applied to remove the influence of the less informative independent features. For example, in feature subset selection [37], some features are multiplied by zero to remove redundant or noisy features.

The similarity between the target project case and each case in the case-based repository is determined by a similarity measure. There are different methods of measuring similarity that have been proposed for different measurement contexts. A similarity measure is measuring the closeness or the distance between two data objects in an n-dimensional feature space; the result is usually

presented in a distance matrix (or similarity matrix) identifying the similarity among all cases in the data set. The euclidean distance metric is the most commonly used in ABE for its distance measures, and it is suitable for continuous values such as software size, effort, and duration of a project. It is based on the principle of the Pythagorean Theorem to derive a straight line distance between two points in n-dimensional space.

In general, the unweighted euclidean distance between two points $P = (p_1, p_2, \ldots, p_n)$ and $Q = (q_1, q_2, \ldots, q_n)$, and can be defined and calculated as

$$\begin{aligned}&\sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2 + \cdots + (p_n - q_n)^2} \\ &\quad = \sqrt{\sum_{i=1}^{n} (p_i - q_i)^2}.\end{aligned} \tag{1}$$

An alternative is to apply different weights to each individual project feature to reflect the its relative importance in the prediction system. The weighted euclidean distance can be calculated as

$$\begin{aligned}&\sqrt{w_1(p_1 - q_1)^2 + w_2(p_2 - q_2)^2 + \cdots + w_n(p_n - q_n)^2} \\ &\quad = \sqrt{\sum_{i=1}^{n} w_i(p_i - q_i)^2},\end{aligned} \tag{2}$$

where w_1 and w_n are the weights of the first and nth project features. Note that in the special case of $w_i = 1$ (i.e., equal weighting) the equations are identical.

The above euclidean distance functions are suitable for general problems, particularly when values are of continuous nature. There are other different distance metrics for noncontinuous variable; these include, but are not limited to, Jaccard distance for binary distance [38] and Gower distance, described by Gower and Legendre [39]. In this paper, we only consider the euclidean distance measure, which is most relevant to the context of software cost estimation.

Irrespective of the similarity measure used, the objective is to rank similar cases from the data set to the target case and utilize the known solution of the nearest k cases. The value of k in this case has been the subject of debate in the ABE research community [19], [17]. Shepperd and Schofield [17] suggested the ideal value for k is 3, that is, only three closest neighboring cases will be considered. These k cases will be adjusted or adapted to better fit the target problem by predefined rules, a human expert, or more commonly, using a simple mean or median of the selected k cases.

2.5 Alternatives to ABE0

Within the space of ABE methods, ABE0 is just one approach. Based on our reading of the literature, we see other variants that take different approaches to

- the selection of relevant features,
- the similarity function,
- the weighting method used in similarity function,
- the case subset selection method (a.k.a selected analogies or k value), and
- the adaption strategy (a.k.a solution function).

Not every paper explores every option. For example:

- In [19], the focus of Kadoda et al. is the impact of the selected number of analogies.
- In [12], Li et al. study the effects of relevant subset selection in training set (i.e., historical data) as well as feature weighting in the similarity function.
- Auer et al. propose an optimal weight finding mechanism by means of extensive search in [8].
- In [15], Walkerden and Jeffery investigate selected analogies and compare the performance of human experts to that of tools,
- Finally, in [11], Mendes et al. limit historical data to a single domain and compare different ABE configurations to non-ABE methods.

Generalizing from the above, the following notes try to map the space of options within current research [8], [11], [12], [15], [19]. Since researchers are developing new technologies for effort estimation all the time, such as AQUA [40], AQUA+ [20], and COSEEKMO [9], this map is incomplete. However, it does illustrate our general point that there are thousands of possible variants to ABE.

2.5.1 Three Case Subset Selectors

A *case subset selection* is sometimes applied to improve the set of training projects. These selection mechanisms are characterized by how many cases they remove:

- *Remove nothing:* Usually, effort estimators use all training projects [17]. ABE0 is using this variant.
- *Outlier* methods prune training projects with (say) suspiciously large values [24]. Typically, this removes a small percentage of the training data.
- *Prototype* methods find, or generate, a set of representative examples that replace the training cases. Typically, prototype generation removes most of the training data. For example, Chang's prototype generators [41] explored three data sets A, B, C of size 514, 150, 66 instances, respectively. He converted these into new data sets A', B', C' containing 34, 14, 6 prototypes, respectively. Note that the new data sets were very small, containing only 7, 9, 9 percent of the original data.

2.5.2 Eight Feature Weighting Methods

In other work, Li et al. [12] and Hall and Holmes [37] review eight different feature weighting schemes. Li et al. use a genetic algorithm to learn useful feature weights. Hall and Holmes review a variety of methods ranging from WRAPPER (a $O(2^F)$ search through all subsets of F features) to various filters methods (that run much faster than WRAPPER), including their preferred correlation-based method.

In our own work, we have developed yet another feature weighted scheme. The fundamental assumption underlying ABE0 is that projects that are similar with respect to project features will be also similar with respect to project effort. To formally evaluate this hypothesis, Keung et al. [24] developed a more comprehensive solution toward ABE0, called Analogy-X (a.k.a AX). For example, given two distance matrices constructed from the selected predictor variables and the response variable, we can correlate the two matrices and show their distance correlation function.

However, different ordering of the matrix elements may result different matrix correlations; AX applies Mantel's technique that randomly permutes the distance matrix elements 1,000 times to produce randomization statistic distribution. Based on the Mantel correlation, AX selects the project features that improves overall Mantel correlation and uses a set of procedures similar to that of stepwise regression to select the project features that are statistically relevant to the solution space, effectively removing the need for brute-force feature selection in the classical ABE0 proposed in [17]. More importantly, AX provides a statistical justification as to whether ABE should be used for the data set under investigation. Keung et al.'s [24] study also concludes that data set quality and variance within the data set are influential factors; removing data points with large variance will improve prediction performance.

2.5.3 Five Discretization Methods

Some feature weighting schemes require an initial *discretization* of continuous columns. Discretization divides a continuous range at break points $b_1, b_2, \ldots$, each containing a count $c_1, c_2, \ldots$ of numbers [42]. There are many discretization policies in the literature, including

- equal frequency, where $c_i = c_j$;
- equal width, where $b_{i+1} - b_i$ is a constant;
- entropy [43];
- PKID [44];
- do nothing at all.

2.5.4 Six Similarity Measures

Mendes et al. [11] discuss three similarity measures, including the weighted euclidean measure described above, an unweighted variant (where $w_i = 1$), and a "maximum distance" measure that focuses on the single feature that maximizes interproject distance. Frank et al. [45] offer a fourth similarity measure that uses a triangular distribution that sets to the weight to zero after the distance is more than "k" neighbors away from the test instance. A fifth and sixth similarity measure are the Minkowski distance measure used in [46] and the mean value of the ranking of each project feature used in [15].

2.5.5 Four Adaption Mechanisms

With regard to adaptation, the literature reports many approaches, including

- report the median effort value of the analogies;
- report the mean-dependent value;
- summarize the adaptations via a second learner, e.g., regression [10], model trees [9], [47] or neural network [48];
- report a weighted mean where the nearer analogies are weighted higher than those further away [11].

2.5.6 Six Ways to Select Analogies

Li et al. [12] comment that there is much discussion in the literature regarding the number of analogies to be used for estimation. Numerous methods are proposed, which we divide into *fixed* and *dynamic*.

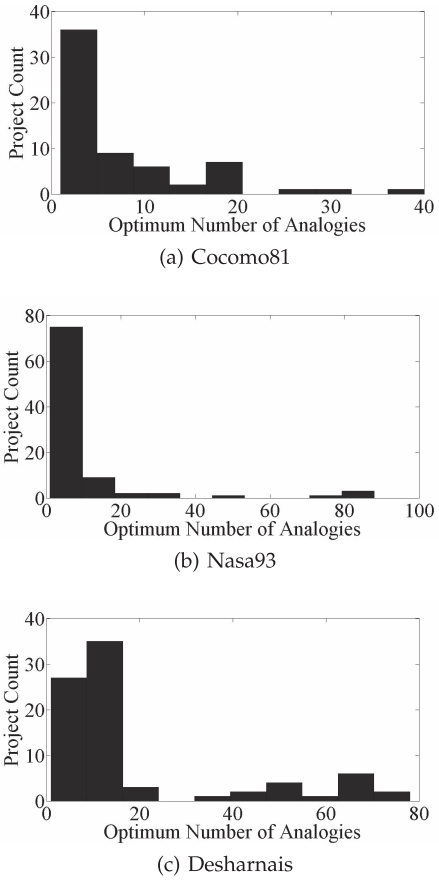

Fig. 2. Distribution of k after removing each project instance, then applying Best(K) on the remaining data. The y-axis counts the number of times a particular k value was found by Best(K).

Fixed methods use the same number of analogies for all items in the test set. For example, Li et al. [12] report that a standard fixed method is to always use $1 \leq k \leq 5$ nearest projects:

- $k = 1$ is used by Lipowezky [49] and Walkerden and Jeffery [15];
- $k = 2$ is used by Kirsopp and Shepperd [50];
- $k = 1, 2, 3$ is used by Mendes el al. [11].

Dynamic methods adjust the number of analogies, according to the task at hand. For example, following advice from Martin Shepperd,[2] Baker [10] tuned k to a particular training set using the following "Best(K)" procedure:

1. Select $N \subseteq T$ training projects at random.
2. For each $k \in 1..T - N$, compute estimates for $n \in N$.
3. Find the k value with least error in step 2.
4. When estimating, use the k-nearest neighbors, where k is set by step 3.

As shown in Fig. 2, Best(K) recommends k values that are very different from those seen in the standard fixed methods. These results come from three commonly used data sets (Desharnais, NASA93, and the original COCOMO data set from [1]: For notes on these data sets, see the document available at http://bit.ly/feimyA).

While ABE systems differ on many aspects, they all use analogy selection. Fig. 2 results suggest that there may be

2. Personal communication.

something suboptimal about standard, widely used, fixed selection methods. Hence, the rest of this paper takes a closer look at this aspect of ABE.

3 Designing TEAK

The above sample of the literature describes

$$3 \times 8 \times 5 \times 6 \times 4 \times 6 > 17{,}000$$

ways to implement similarity, adaptation, weighting, etc. Some of these ways can be ruled out, straight away. For example, at $k = 1$, all the adaptation mechanisms return the same result. Also, not all the feature weighting techniques require discretization, decreasing the space of options by a factor of five. However, even after discarding some combinations, there are still thousands of possibilities to explore. How might we explore all these variations?

The rest of this paper applies the easy path to design and evaluate an ABE system called TEAK. TEAK is an ABE0, with the variations described below.

3.1 Select a Prediction System

First, we *select a prediction system*. We use ABE since:

- It is widely studied [8], [12], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25].
- It works even if the domain data are sparse [51].
- Unlike other predictors, it makes no assumptions about data distributions or an underlying model.
- When the local data do not support standard algorithmic/parametric models like COCOMO, ABE can still be applied.

The easy path limits the space of design options to just *those that directly address the essential assumptions of the predictor*. As shown below, for ABE this directs us to issues of case subset selection and the number of analogies used for estimation.

3.2 Identify Essential Assumption(s)

The second step is to *identify the essential assumptions of that prediction system*. Although it is usually unstated, the basic hypothesis underlying the use of analogy-based estimation is that projects that are similar with respect to project and product factors will be similar with respect to project effort [25]. On the other hand, projects from a high variance region are likely to have very different project effort values and can decrease the accuracy in estimation, which we quote as to *"confuse estimation"* in our research. In other words:

Assumption One: Locality implies homogeneity (for $k > 1$).

This assumption holds for project training data with the following property:

- The k-nearest training projects with effort values $E_1, E_2, .., E_k$ have a mean value $\mu = (\sum_i^k E_i)/k$ and a variance $\sigma^2 = (\sum_i^k (E_i - \mu)^2)/(k-1)$.
- By *Assumption One*, decreasing k also decreases σ^2.
- If all estimates always have the same distance to the mean μ, then *Assumption One* always fails since in $(\sum_i^k (E_i - \mu)^2)/(k-1)$, if the numerator is constant, then decreasing k will always *increase* the variance.

The core of ABE is the premise that in the neighborhood of training instances, the reductions seen in $(\sum_i^k (E_i - \mu)^2)$ dominate over the increases due to $1/(k-1)$. As we shall see, this sometimes holds (and sometimes it does not). For example, let us assume that a node x has a left child y and a right child z. Let us further assume that each child has two leaves/instances that contain the effort values of $leaves(y) \in \{1{,}253,\ 1{,}440\}$ staff hours and $leaves(z) \in \{1{,}562,\ 5{,}727\}$ staff hours, which means that x contains effort values of $\{1{,}253,\ 1{,}440,\ 1{,}562,\ 5{,}727\}$. With these values, the variance (σ^2) of each node would be as follows: $\sigma_x^2 = 4.6523e6$, $\sigma_y^2 = 1.7485e4$, and $\sigma_z^2 = 8.6736e6$. In that scenario, going from parent node x to child nodes will create two cases:

1. Going to y: Reduction seen in $(\sum_i^k (E_i - \mu)^2)$ dominates over the increase due to $1/(k-1)$, i.e., the variance decreases when the number of instances decreases.
2. Going to z: Reduction seen in $(\sum_i^k (E_i - \mu)^2)$ cannot dominate over the increase due to $1/(k-1)$, i.e., the variance increases when the number of instances decreases.

As can be seen in this example, it is not necessarily true that moving to a smaller set of neighbors decreases variance. As shown below, it can improve prediction accuracy if ABE takes this matter into account.

3.3 Identify Assumption Violation

The third step is to *recognize situations that violate the essential assumption*. Implementing this step requires some way to compare the variance of larger k estimates to smaller k estimates. One way to achieve this is to use some clustering method that generates a tree of clusters, where each subtree contains training data that are closer together than the supertree.

There are many algorithms for generating trees of clusters. The basic method, called greedy agglomerative clustering (GAC), is used in various fields (data mining [52], databases [53], bioinformatics [54]). GAC executes bottom up by grouping together at a higher level $(i+1)$ the closest pairs found at level i. The algorithm terminates when some level i is found with only one node. GAC is "greedy" in that it does not pause to consider optimal pairings for vertices with very similar distances, i.e., it never backtracks looking for (say) better pairings at level i to reduce the distance between nodes at level $i+1$.

The result of GAC is a tree like Fig. 3. Note that, in this tree, the original training data are found at the leaves of the tree. All other nodes are nodes artificially generated by GAC to represent the median of pairs of the leaves, the median of the medians, and so on (recursively).

A GAC tree can be viewed as a tree of clusters where each node at height i is the centroid of the subclusters at height $i-1$. Given T initial instances, GAC builds a trees of maximum height $log_2(T)$. Since the number of vertices is halved at each next level, building a GAC tree requires the following number of distance calculations:

$$\left(\sum_i^{log_2(T)} \left(\frac{T}{2^{i-1}} \right)^2 \right) = \frac{4}{3}(T^2 - 1),$$

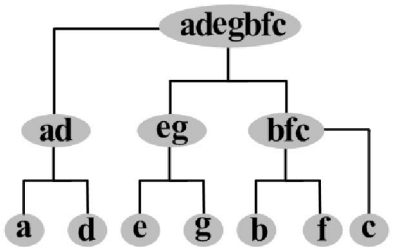

Fig. 3. A sample GAC tree built from seven instances: a, b, c, d, e, f, and g. In each level, the closest two nodes are coupled up to produce the nodes in one higher level. However, odd numbered instances result in unbalanced trees. In this figure, when all leaf nodes are paired up (pairing of letters here is random and just for illustration), c remains alone. The lone remaining node at any level will find the closest node to itself in one higher level and will merge to that node. For example, c merges to the node containing b and f.

This $O(T^2)$ computation is deprecated for large T. However, for this study, GAC construction takes less than a second.[3] Our runtimes were fast because effort estimation data sets are usually very small: Fig. 4 shows all our training projects contain less than 100 examples.

There are many other, faster, algorithms for generating trees of clusters including bisecting k-means [55] which, at each level, calls k-means[4] with $k = 2$ several times to split one cluster into two. The division leading to the clusters with the best intracluster similarities is then stored and used in subsequent splits. Other approaches, like MESO [56], recursively divide the data into smaller and smaller spheres containing *close* instances. MESO uses an incremental method to learn and update what *close* means for a particular data set.

We use GAC (with the distance measure of 1) rather than other methods like bisecting k-means or MESO for two reasons. First, these other methods use various heuristics to improve their runtimes. Our data sets are so small that such heuristic methods are not necessary. Second, our results (described below) with GAC are so promising that we are not motivated to experiment beyond GAC.

Using a GAC tree, finding the k-nearest neighbors in project data can be implemented using the following TRAVERSE procedure:

1. Place the test project at the root of the GAC tree.
2. Move the test project to the nearest child (where "nearest" is defined by (1)).
3. Go to step 2.

Clearly, a $k = 1$ nearest neighbor estimate comes from TRAVERSE-ing to a leaf, then reporting the effort of that leaf. More generally, a $k = N$ nearest neighbor estimate comes from TRAVERSE-ing to a subtree with N leaves, then reporting the median efforts of those leaves.

TRAVERSE can test *Assumption One*. Let some current vertex x have children y and z. We say that

- the subtrees starting at x, y, z have leaves L_x, L_y, L_z (and $L_x = L_y \cup L_z$).
- The number of subtree leaves is $k_x = k_y + k_z$.
- The variance of the leaves' efforts are $\sigma_x^2, \sigma_y^2, \sigma_z^2$.
- After C4.5 [57], we say the variance of the trees below x (denoted σ_{yz}^2) is the weighted sum

$$\sigma_{yz}^2 = \frac{k_y}{k_x}\sigma_y^2 + \frac{k_z}{k_x}\sigma_z^2.$$

Parent trees have the nodes of their children (plus one). If we TRAVERSE from a parent x to a child, then the subtree size k decreases. That is, TRAVERSEing moves into progressively smaller subtrees.

Assumption One holds if, when TRAVERSEing from all vertices x with children y and z, the subtree variance *decreases*. That is

$$\forall x \in T : \sigma_x^2 > \sigma_{yz}^2. \tag{3}$$

Note one special case of the above: In the case of $k = 1$, variance is zero since, by definition, all members of a sample of size one are the same. Hence, under that scenario TEAK would mostly return leaf nodes, which would lead to an erroneous execution. This scenario was realized before the implementation phase and it was addressed with a control mechanism. The control mechanism lets the test instance go down the GAC tree at most until one level higher than the leaves. Therefore, the variance value that plays a decisive role on the movement of test instances always comes from a population of at least two instances.

3.4 Remove Violations

The fourth step in TEAK's design is to *remove the situations that violate the essential assumption*. We instrumented TRAVERSE to report examples where (3) was violated, i.e., where it recursed into subtrees with a larger variance than the parent tree. We found that this usually occurs if a supertree contains mostly similar effort values, but one subtree has a minority of outliers. For example:

- Suppose some vertex x has children y, z.
- Let each child start subtrees whose leaves contain the effort values $leaves(y) \in \{1{,}253,\ 1{,}440\}$ staff hours and $leaves(z) \in \{1{,}562,\ 5{,}727\}$ staff hours.

In this example:

- The leaves of the parent tree x have similar effort values: 1,253 and 1,562 and 1,440 staff hours.
- But the leaves of the subtree z have outlier values, i.e., 5,727.
- TRAVERSEing from the supertree x to the subtree z increases the variance by two orders of magnitude.

A *subtree pruning policy* is used to prune subtrees with a variance that violates the essential assumption. We experimented with various policies that removed subtrees if they had

1. more than α times the parent variance;
2. more than $\beta * max(\sigma^2)$;
3. more than $R^\gamma * max(\sigma^2)$, where R is a random number $0 \leq R \leq 1$.

3. Using Matlab on a standard Intel x86 dual core notebook running LINUX with 4 GB of ram.

4. The k-means clustering algorithm selects centroids at random, labels each instance by its nearest centroid, then updates the centroid position to the central position of all instances with the same label. The algorithm repeats till the centroid position stabilizes.

Dataset	Features	$T = \|Projects\|$	Content	Historical Effort Data					
				Units	Min	Median	Mean	Max	Skewness
Cocomo81	17	63	NASA projects	months	6	98	683	11400	4.4
Cocomo81e	17	28	Cocomo81 embedded projects	months	9	354	1153	11400	3.4
Cocomo81o	17	24	Cocomo81 organic projects	months	6	46	60	240	1.7
Nasa93	17	93	NASA projects	months	8	252	624	8211	4.2
Nasa93c2	17	37	Nasa93 projects from center 2	months	8	82	223	1350	2.4
Nasa93c5	17	40	Nasa93 projects from center 5	months	72	571	1011	8211	3.4
Desharnais	12	81	Canadian software projects	hours	546	3647	5046	23940	2.0
SDR	22	24	Turkish software projects	months	2	12	32	342	3.9
Albrecht	7	24	Projects from IBM	months	1	12	22	105	2.2
ISBSG-Banking	14	29	Banking projects of ISBSG	hours	662	2355	5357	36046	2.6
		Total: 448							

Fig. 4. The 448 projects used in this study come from 10 data sets. Indentation in column one denotes a data set that is a subset of another data set. For notes on this data, see the document available at http://bit.ly/feimyA.

In order to avoid overfitting, our pruning policy experiments were restricted to one data set (Boehm's COCOMO embedded projects [1]) then applied, without modification, to the others. The randomized policy (#3) produced lowest errors, with smallest variance. The success of this randomized policy suggests two properties of effort estimation training data:

- The boundary of "good" training projects is not precise. Hence, it is useful to sometimes permit random selection of projects either side of the boundary.
- The policy tuning experiments recommended $\gamma = 9$. This selects for subtrees with less than 10 percent of the maximum variance.[5] This, in turn, suggests that the above example is typical of effort estimates, i.e., subtree outliers are usually a few large effort values.

In theory, stochastic methods like policy #3 introduce a degree of instability in the performance of the induction system. In practice, this is not an issue with TEAK. In the evaluation section, described below, we repeat our analysis of TEAK 20 times using various performance measures and experimental rigs. When we compare the results of those repeated trials against just running TEAK once, we can see no major performance differences.

3.5 Execute the Modified System

The final step in the design of TEAK is to build a new prediction system. TEAK executes as follows:

- Apply GAC to the training projects to build a tree called GAC1.
- Prune GAC1 using the subtree pruning policy described above. The remaining leaves are the *prototypes* to be used in effort estimation.
- Apply GAC to the prototypes to build a second tree called GAC2.
- Place the test project at the root of GAC2. Compute an estimate from the median value of the GAC2 projects found by TRAVERSE2. TRAVERSE2 is a variant of TRAVERSE that ensures the essential assumption is never violated. It stops recursing into GAC2 subtrees when (3) is violated.

4 Comparisons

Recall the preexperimental concern expressed above: Taking the easy path might ignore important design issues, to the detriment of the predictions. To address that concern, this section compares TEAK to a range of other ABE0 variants as well the other induced prediction systems described in Section 2, i.e., neural nets and linear regression (we selected this particular range of algorithms at the suggestion of reviewers of this paper).

As discussed above in Section 2.1, a detailed comparison of analogy-based estimation to other estimation methods is not the fundamental aim of this research. To observe the results of such a detailed comparison, we found [9] and [14] particularly helpful. In this research, we compare TEAK to other estimation methods (specifically, neural networks and regression), merely to ensure that analogy-based estimation is not significantly *worse* than other methods in widespread use.

In the following comparisons, TEAK will be assessed using

- two different experimental rigs (leave-one-out and n-way cross validation: see Section 4.1);
- three different performance measures (absolute residual (AR), MRE, PRED(25), see Section 4.2);

4.1 Randomized Trials

Recall that TEAK uses a randomized method for subtree pruning. Any evaluation of such a randomized method must be repeated multiple times. Appealing to the central limit theorem, we used 20 repeats.

- Twenty times, for each data set, we randomized the order of the rows in that data set.
- Next, we conducted *both* a Leave-One-Out study and a three-way cross-validation study.

In Leave-One-Out, given T projects, then $\forall t \in T$ use t as the test project and the remaining $T - 1$ projects for training. In three-way cross validation, the data set of T projects is divided into three bins; bin_i is used for testing while the remaining $T - bin_i$ projects are used for testing.

Since some of our data sets are very small (e.g., the 24 instances of Cocomo81o), we used a three-way cross validation (and not the 10-way used by, say, Quinlan [57]). Some thought was given to using three-way for small data sets and 10-way for larger data sets. However, this would introduce a complication into the analysis that is neither recommended by the literature nor handled by any statistical technique that we are aware of.

We use both Leave-one-out and N-way cross validation since the effort estimation literature is ambiguous on which is most appropriate. In the Kitchenham et al. survey [7], all

5. The mean of $rand()^9 \approx 0.1$.

the projects reviewed in their Table 3 used N-way cross validation. However, other prominent studies prefer leave-one-out [22].

For these studies, we used all independent features when computing similarities. We applied 20 randomized trials using the 448 projects from 10 data sets of Fig. 4 (for notes on this data, see the document available at http://bit.ly/feimyA). In all, these randomized trials generated a total of 8,990 training/test set pairs, where $20 * 448 = 8{,}960$ of the pairs come from Leave-One-Out and another $10 * 3 = 30$ pairs come from $10 * 3$-way.

4.2 Details

For each of these 8,990 pairs of training/test, estimates were generated by TEAK, neural networks, regression, and six other ABE0 variants:

- Five variants returned the median effort seen in the kth nearest neighbors for $k \in \{1, 2, 4, 8, 16\}$.
- The other variant returned the median effort seen in k neighbors found using Baker's Best(K) procedure. From Section 2.5, recall that Best(K) adjusts k to each data set by reflecting over all the training projects.
- For notes on regression and NNet please see Section 2.3.

Since this is paired data (same train and test data passed through multiple treatments), we applied a Wilcoxon signed rank test (95 percent confidence) to rank the resulting estimates. Ranked[6] statistical tests like the Wilcoxon are useful if it is not clear that the underlying distributions are Gaussian [58] because ranked statistics mitigate the problem of effort estimation results that sometimes contain a small number of very large errors [59].

We collected information on three performance metrics: AR, MRE, PRED(25). The magnitude of the absolute residual is computed from the difference between predicted and actual:

$$AR = |actual_i - predicted_i|. \tag{4}$$

We prefer AR to other performance measures since we share Shepperd's concern [60] that anything other than the simplest evaluation statistic can introduce analysis issues. Nevertheless, other measures are more common in the effort estimation literature (e.g., see [7, Table 3]) such as MRE and PRED(25). MRE is the magnitude of the relative error. It is calculated by expressing AR as a ratio of the actual prediction:

$$MRE = \frac{|actual_i - predicted_i|}{actual_i}. \tag{5}$$

AR and MRE are calculated for every item in a test set. PRED(X), on the other hand, is a summary statistic that reports behavior over an entire test suite. PRED(X) reports the average percentage of the N estimates in the test set that were within X percent of the actual values: For example, PRED(30) = 50 percent means that half the estimates are within 30 percent of the actual value. Chulani and Boehm assesses their models using PRED(30) [61]. We use the stricter criteria of PRED(25) since that is more common in the literature, e.g., [17], [33], [62].

6. In a ranking analysis, the raw results (10.2, 21.3, 22.1, 24, 25, 30, 100) are replaced with their ranks in the sort order, i.e., (1, 2, 3, 4, 5, 6, 7).

```
win_i = 0, tie_i = 0, loss_i = 0
win_j = 0, tie_j = 0, loss_j = 0
if WILCOXON(P_i, P_j) says they are the same then
   tie_i = tie_i + 1;
   tie_j = tie_j + 1;
else
   if median(P_i) < median(P_j) then
      win_i = win_i + 1
      loss_j = loss_j + 1
   else
      win_j = win_j + 1
      loss_i = loss_i + 1
   end if
end if
```

Fig. 5. Pseudocode for win-tie-loss calculation between variants i and j with performance measures P_i and P_j. Note here that only for Pred(25) is the comparison based on actual values ($Pred(25)_i$, $Pred(25)_j$) rather than median values ($\text{median}(P_i)$, $\text{median}(P_j)$).

In order to summarize the results of the Wilcoxon comparisons of the MRE, AR, PRED(25) measures, we use the following win-tie-loss procedure. For each iteration of the randomized trials, each data set generated $20 * 7 = 140$ (MRE, PRED(25), AR) distributions for each induced prediction system (neural nets, regression, the ABE0 variants). To calculate the win-tie-loss values, we first checked if two distributions i, j are statistically different according to the Wilcoxon test. If not, then we incremented tie_i and tie_j. On the other hand, if they turned out to be different, we updated win_i, win_j and $loss_i, loss_j$ after a numerical comparison of their median values. The pseudocode for win-tie-loss calculation is given in Fig. 5.

In median and mean performance measures used in other studies (see [7, Table 3]), the entire distribution is summarized by its central tendency (measured in terms of median or mean), then two methods are compared solely in terms of those two central points. In the approach of Fig. 5, on the other hand, with the use of Wilcoxon the variance around centrality is also considered, which is not the case with single-point assessment methods. Such single point assessments are also deprecated in the literature, e.g., see Foss et al.'s scathing critique of mean MRE [63]. Therefore, we adopt the approach of Fig. 5 over single-point assessments.

4.3 Results

Initially, our intention was to report results using all the data sets of Fig. 4. However, we found that the Albrecht data set was producing a very large number of ties (over 98 percent). On closer inspection, we found that in our rig, Albrecht was a data set in which all our treatments generated very similar results (the plots of the MREs generated by our eight methods was indistinguishable). Since Albrecht was mostly unable to distinguish between the different treatments, we excluded it from the rest of our analysis.

The resulting Win/Loss/Ties values from a Leave-One-Out study that measured MRE are shown in Fig. 6. When ranked in terms of $win - loss$, in $\frac{6}{9}$ data sets, TEAK is the *top ranked* method, i.e., it always ranked first on that performance score. The next best method was linear regression, which is found in the top rank in only $\frac{3}{9}$ data sets.

Data set	Variant	Win	Tie	Loss	Win - Loss
Cocomo81	TEAK	87	73	0	87
	Best(K)	49	110	1	48
	k=16	42	107	11	31
	k=8	41	100	19	22
	k=4	28	96	36	-8
	NNet	37	76	47	-10
	k=1	28	88	44	-16
	k=2	26	82	52	-26
	LR	7	18	135	-128
Cocomo81e	TEAK	55	105	0	55
	NNet	43	117	0	43
	k=8	32	126	2	30
	k=16	32	126	2	30
	Best(K)	32	126	2	30
	k=4	18	113	29	-11
	k=1	8	97	55	-47
	k=2	4	101	55	-51
	LR	11	59	90	-79
Cocomo81o	TEAK		136	0	24
	k=16	9	151	0	9
	k=8	8	152	0	8
	Best(K)	8	152	0	8
	NNet	9	150	1	8
	k=4	7	151	2	5
	LR	7	145	8	-1
	k=2	2	128	30	-28
	k=1	1	125	34	-33
Nasa93c5	TEAK	40	120	0	40
	LR	25	135	0	25
	k=16	17	141	2	15
	Best(K)	17	139	4	13
	k=8	16	134	10	6
	NNet	10	144	6	4
	k=4	10	127	23	-13
	k=2	7	110	43	-36
	k=1	3	100	57	-54
SDR	TEAK	67	93	0	67
	k=1	43	97	20	23
	NNet	25	123	12	13
	k=4	26	118	16	10
	k=8	18	132	10	8
	k=2	20	126	14	6
	Best(K)	16	126	18	-2
	k=16	13	120	27	-14
	LR	0	49	111	-111
ISBSG-Banking	TEAK	30	130	0	30
	NNet	24	136	0	24
	LR	23	137	0	23
	k=16	22	138	0	22
	k=8	19	141	0	19
	Best(K)	21	137	2	19
	k=4	14	112	34	-20
	k=1	8	106	46	-38
	k=2	4	73	83	-79
Nasa93	LR	72	88	0	72
	TEAK	26	134	0	26
	NNet	16	143	1	15
	k=16	13	133	14	-1
	k=8	15	128	17	-2
	Best(K)	14	128	18	-4
	k=4	6	122	32	-26
	k=2	4	113	43	-39
	k=1	6	107	47	-41
Nasa93c2	LR	158	2	0	158
	TEAK	36	106	18	18
	k=16	25	115	20	5
	NNet	17	123	20	-3
	k=8	15	116	29	-14
	Best(K)	15	116	29	-14
	k=4	11	101	48	-37
	k=2	5	95	60	-55
	k=1	6	90	64	-58
Desharnais	LR	63	97	0	63
	NNet	51	109	0	51
	TEAK	37	121	2	35
	k=16	25	129	6	19
	k=8	22	124	14	8
	Best(K)	16	120	24	-8
	k=4	14	116	30	-16
	k=2	6	80	74	-68
	k=1	1	74	85	-84

Fig. 6. MRE-based win-loss-tie results from the 20*Leave-One-Out experiments. For each data set, results are sorted by win minus loss values. Gray cells indicate variants with zero losses. The performance of the various induced prediction systems is summarized in top-left corner of Fig. 7.

These scores are summarized at top left of Fig. 7 (see the tables for "MRE"). There is not enough space in this paper to repeat Fig. 4 for every combination of (Leave-One-Out, Cross-Val)*(MRE, AR, PRED(25)), i.e., six times in all. Hence, we present a summary of those results in Fig. 7. In all cases:

1. Best(K) and $K \in \{1, 2, 4, 8, 16\}$ rarely appeared in the top-ranked methods. That is, standard analogy selection mechanism performed comparatively worse than applying TEAK's variance heuristic.
2. While nonanalogy methods sometimes did better on certain data sets, overall TEAK's extension to analogy-based reasoning was competitive with non-analogy methods.

From result 1, we recommend variance pruning for analogy estimation since, unequivocally, of all the analogy variants studied here TEAK is the superior system.

As to result 2, we hesitate to conclude, just from this sample, that TEAK is *always* the best effort estimation method. However, its results are encouraging and should motivate continued research into analogy-based methods. In our review of effort estimation [9], we commented that best practices include generating estimates from multiple sources. Certainly, these results offer no reason to *exclude* analogy as one of those sources.

4.4 But Why Does It Work?

In discussions about TEAK, we are sometimes asked if it is wise to use variance to assess the suitability of neighborhood for providing donor cases. The argument goes as follows: While a high variance for a given neighborhood of k suggests that this is a *bad* neighborhood, a low variance does not necessarily imply that the neighborhood is *good*.

In reply, we note that TEAK does not *only* use variance to select the donor cases. TRAVERSE2 pushes *away* from regions with high variance while pushing *toward* regions with similar features to the test instance. TRAVERSE2 pushes away from high variance regions since

- it executes over a space of training data which high variance regions pruned away;
- its recursive descent terminates if it enters a region of increasing variance.

At the same time, TRAVERSE2 pushes toward regions with similar features as follows:

- It descends a binary tree of clusters.
- At each step, the test instance is moved toward the subtree whose euclidean distance is closest to the test instance.

The above experiments show, we argue, that this policy does better than just pushing toward regions with higher similarity. That is, *augmenting* nearest neighbor algorithms with variance avoidance does better than just applying nearest neighbor.

4.5 Threats to Validity

Internal validity questions to what extent the cause-effect relationship between dependent and independent variables hold [64].

The general internal validity issue is that data mining experiments (like those discussed above) do not collect new data, but only generate theories from historical data. Ideally, we should take a learned theory and apply it to

20 * LEAVE-ONE-OUT

	TEAK	LR	NNet	Best(K)	k=1	k=16	k=2	k=4	k=8
MRE									
Cocomo81	▲								
Cocomo81e	▲								
Cocomo81o	▲								
Nasa93		▲							
Nasa93c2		▲							
Nasa93c5	▲								
Desharnais		▲							
Sdr	▲								
ISBSG-Banking	▲								
Count	6	3	0	0	0	0	0	0	0
Pred(25)									
Cocomo81	▲								
Cocomo81e			▲						
Cocomo81o	▲								
Nasa93		▲							
Nasa93c2		▲							
Nasa93c5	▲								
Desharnais		▲							
Sdr	▲								
ISBSG-Banking	▲								
Count	5	3	1	0	0	0	0	0	0
AR									
Cocomo81	▲								
Cocomo81e	▲								
Cocomo81o	▲								
Nasa93		▲							
Nasa93c2		▲							
Nasa93c5	▲								
Desharnais		▲							
Sdr	▲								
ISBSG-Banking	▲								
Count	6	3	0	0	0	0	0	0	0

20 * 3-WAY CROSS-VALIDATION

	TEAK	LR	NNet	Best(K)	k=1	k=16	k=2	k=4	k=8
MRE									
Cocomo81	▲								
Cocomo81e	▲								
Cocomo81o	▲								
Nasa93		▲							
Nasa93c2		▲							
Nasa93c5	▲								
Desharnais		▲							
Sdr	▲								
ISBSG-Banking	▲								
Count	6	3	0	0	0	0	0	0	0
Pred(25)									
Cocomo81	▲								
Cocomo81e			▲						
Cocomo81o	▲								
Nasa93		▲							
Nasa93c2		▲							
Nasa93c5	▲								
Desharnais		▲							
Sdr	▲								
ISBSG-Banking	▲								
Count	5	3	1	0	0	0	0	0	0
AR									
Cocomo81				▲					
Cocomo81e	▲								
Cocomo81o	▲								
Nasa93		▲							
Nasa93c2		▲							
Nasa93c5	▲								
Desharnais		▲							
Sdr	▲								
ISBSG-Banking	▲								
Count	6	3	0	1	0	0	0	0	0

Fig. 7. Summary of the random trials, e.g., Fig. 6, is summarized top-left in "MRE." This figure displays the top performing inducted predictive system, measured via $(win - loss)$. This is repeated for all the performance measures (MRE, PRED(25), AR) and both experimental rigs (leave-one-out on the left and three-way cross validation on the right). The last row of each table shows the sum of times a method appeared as the top performing variant. In the majority of cases, TEAK appears as the top-ranked predictive system.

some new situation, then observe if the predicted effect occurs in practice. Note that if no explicit theory is generated, then it cannot be applied outside of the learning system. That is, all ABE systems suffer from issues of internal validity since they do not generate an explicit theory. However, it is possible to mitigate this problem by simulating how an ABE system might be applied to a new situation. Note that the Leave-One-Out approach used in this paper generates estimates using test data that is not used in training.

Construct validity (i.e., face validity) assures that we are measuring what we actually intended to measure [65]. In our research, we are using a variety of performance measures (AR, MRE, PRED(25)) and a pair of evaluation experiments (leave-one-out and cross val). This was done to increase the construct validity of this study. MRE is widely used for assessing the performance of competing software effort estimation models [63], [66], [67]. Foss et al. [63] have provided an extensive discussion demonstrating that using only MRE itself may be leading to incorrect evaluation. Hence, we take care to apply multiple performance measures and a pair of evaluation experiments.

External validity is the ability to generalize results outside the specifications of that study [68]. To ensure the generalizability of our results, we studied a large number of projects. Our data sets contain a wide diversity of projects in terms of their sources, their domains, and the time period they were developed in. For example, we used data sets composed of software development projects from different organizations around the world to generalize our results [69]. Our reading of the literature is that this study uses more project data, from more sources, than numerous other papers. All the papers we have read, as well as Table 4 of [7], list the total number of projects in all data sets used by other studies. The median value of that sample is 186, which is less than half the 448 projects used in our study.

4.6 Future Work

In this paper, we have applied the easy path principle to design a new method for case and analogy selection. In future work, we will apply the easy path to similarity measures, feature weighting, and adaption. For example:

- After grouping together rows with similar estimates, we might weight features by their variance within each group (and higher variance means lower weight).
- Alternatively, Lipowezky [49] observes that feature and case selection are similar tasks (both remove cells in the hypercube of all cases times all columns). Under this view, it should be possible to convert our case selector to a feature selector.

Our investigations in this area are very preliminary and, at this time, we have no conclusive results to report.

Another promising avenue to explore is variations on the GAC clustering. Since our results have so far been quite promising, we have not explored alternatives to GAC. For example, we form links between quite distinct clusters merely because they have the minimum average-linkage-clustering (mean value of the distance between instances in each cluster). Perhaps another, more sophisticated, clustering algorithm would be a better way to group data.

5 Conclusion

In response to the growing number of options for designing software effort estimators, various researchers (e.g., [9], [10], [12]) have proposed elaborate and CPU-intensive search tools for selecting the best set of design options for some local data. While useful, these tools offer no insight into the effort estimation task: They report *what* the design is in simplifying future effort estimation tasks, but not *why* they were useful. Such insights are useful for reducing the complexity of future effort estimations.

In order to avoid the computation cost of these tools and to find the insights that simplify effort estimation, we design TEAK using an *easy path* principle. The easy path has five steps:

1. *Select a prediction system:* Analogy-based effort estimation is a widely studied method that works on sparse data sets. Hence, we selected ABE as our prediction system.
2. *Identify the predictor's essential assumption(s):* The essential assumption of ABE is that *locality implies homogeneity* (for $k > 1$), i.e., the closer the test project approaches the training projects, the smaller the variance in that neighborhood.
3. *Recognize when those assumption(s) are violated:* Mathematically, this can be tested by recursively clustering project data into a tree whose leaves contain historical effort data and whose internal nodes are medians of pairs of child nodes. When descending this tree, the essential ABE assumption is violated when subtrees have a larger variance than the parents.
4. *Remove those situations:* These assumptions can be removed by pruning subtrees with the larger variances.
5. *Execute the modified prediction system:* TEAK builds a second tree of clusters using just the projects not found in high variance subtrees. Estimates are generated from this second tree by a recursive descent algorithm that stops before the subtree variance is higher than the supertree variance. The leaves of terminating subtree are then accessed and the estimate is calculated from the median of the effort values in those leaves.

A pre-experimental concern with the easy path was that, in ignoring the hard training cases, we would miss important aspects of the data. Our experiments do not support that concern. TEAK never lost against other ABE methods and always won the most. Also, TEAK performed at least as well (if not better) than certain other nonanalogy-based estimation methods.

Our conclusions are twofold:

- For those using analogy-based estimation, we strongly recommend pruning instances from regions of high variance prior to generating estimates.
- For those designing new data algorithms, we conclude that it may be detrimental to obsess on the hard cases. Rather, it may be better to enhance what a predictor does best. For example, in the case of ABE, case selection via variance significantly improved the estimates.

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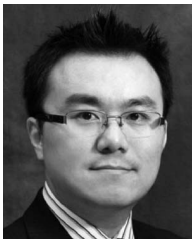

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