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MSc in Information Technology with Web Development

"A Prototype Multi Criteria Group Decision Support System Based On The Analytic Hierarchy Process"

by

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B.2.4.
A Prototype Multi Criteria Group Decision Support System based on the Analytic Hierarchy Process

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Decision support systems are lately becoming a more widespread set of tools for the decision analyst and decision maker and utilized in an increasingly large number of companies and organizations. Furthermore, today's evolution that comes from the extensive use of the internet makes even more attractive and efficient to deploy group decision aid techniques so as to:

- **Achieve a more democratic operational environment on the productive and social activities that take place in an organization.**
- **Optimise the overall results, taking into consideration as many possible views coming form all the different actors involved.**
- **Keep up to date to the recent needs of the knowledge age, where specialization of each of the participating members involved in the decision making process, play an active role.**

The project involved the design and development of a prototype multi criteria group decision support system based on the Analytical Hierarchical Process proposed by Thomas L. Saaty, to obtain individual priorities of preferences based on relative pair wise comparisons among the involved criteria and alternatives. Thereafter, these individual priorities are aggregated and using a Social Choice Function an overall rank is obtained for all individuals participating in the decision process.

This prototype system was developed on three tier architecture taking advantage of the capabilities of the web environment and serving the needs for the geographical distribution of the involved partners (decision makers) over particular decisions.

Finally, the use of this system is examined and illustrated through various case studies concerning business decision problems. Whereby, a critical evaluation of the perspective uses and further development of this system is presented.

Keywords: Decision Support Systems, Social Choice Theory, Group Decision Support Systems, Multi Criteria Analysis

B.2.5.
Web – based Group Decision Support Systems A pilot wGDSS application

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A Web – Based Group Decision Support System (Wgdss) designed properly, brings its users in position for effective Decision Making (DM) from geographically dispersed places using the Internet as communication channel. The effectiveness of such a system depends seriously on the quality of proper web applications which support group decision making and software agents which secure the visualization of the on-line decision making process (session).

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ABSTRACT

Group decision making under multiple criteria in a democratic society include various voting and counting methodologies. The non-ranked voting method is by far the most commonly used method in political elections today. Each voter has one vote and no more on all the candidates who offer themselves on the voters' choice. Ideally, the voting procedure should be kept reasonably simple and straightforward so as to cause no difficulty to the voters. On the other hand, the primary concern of the counting process is accuracy and effectiveness. What is needed is a method that allows voters to indicate not only their chosen candidate, but also their order of preference by which all the candidates would be placed. The preferential voting method, first introduced by Chevalier Jean-Charles de Borda in 1770 proposed to add the ranks of a given alternative(candidate) on each of the criteria. For a given criterion one point is assigned to an alternative ranked last, two points to an alternative ranked second and so on. The social choice or the aggregated preorder is obtained by summing all the points assigned for each alternative and by ranking the first alternative with the most points, second the alternative with the immediately lower number of points and so forth. In general, group decision is understood to be a reduction of individual preferences among a set of criteria to a single collective preference or group choice. The results are clearly dependent on the counting method used. In 1785 Marquis de Condorcet discovers the paradox of voting, the fact that social choice processes based on the principle of the majority rule can give rise to nontransitive (cyclical) ranking amongst candidates (alternatives). To solve for the Condorcet effect the Social Choice Theory studies the problem of the counting process classified by a Social Choice Function, where voting is a group decision making method in a democratic society, an expression of the will of the majority. The counting methods (Social Choice Function) used in this project include the Eigenvector Function, proposed by Thomas L. Saaty in 1970, to obtain individual priorities of preferences and Borda's Function to obtain the group choice (ranking). The conclusions drawn in this project gives rise to questions about the very idea of democracy and proposes a new perspective to the whole methodology of group decision with an innovative user friendly interface.

Keywords: *Decision Support Systems, Social Choice Theory, Social Choice Functions, Group Decision Support Systems, Borda's Positional Method, Analytic Hierarchy Process, Hierarchon, Fundamental Scale, Eigenvalue Equation, Principal Maximum Eigenvalue, Right Eigenvector, Consistency Ratio, Rank Preservation.*

INTRODUCTION AND BACKGROUND

The problems of group decision making under multiple criteria are widely varied, yet share some common characteristics. Firstly, each problem is composed of multiple criteria/objectives/alternatives. Each decision maker produces their relevant criteria/objectives/alternatives, where they may share some, none or all of decision maker i's criteria. Secondly, multiple criteria usually conflict with one another. For example, in selecting a new car, the objective of increasing quality of service might also increase the cost of the car. Finally, there is always a group of persons who arrive at a decision by means of voting, refered to as the committee. The committee may use different techniques to arrive at a final decision, such as the social choice theory, which utilizes voting methods, the expert judgment/group participation analysis which discusses the advantages and disadvantages of a project or even the game theory approach where each decision maker has her/his own strategy. The project will be involved with the social choice theory which basically derives the social choice function (methodology for counting the votes). The voting process is carried out by all the members of the committee, the counting process on the other hand is carried out by a select group of the committee under expert direction, supervision and checking.

The simple majority decision rule using the non-ranked voting method for two candidates (alternatives) depends only upon individual preferences with respect to this pair of preferences. Therefore, the pattern of group choice may be obtained. Assume n individuals and two alternatives x and y. We symbolize the i_{th} individual prefers x to y by $x\ P_i\ y$, the i_{th} individual is indifferent to x and y by $x\ I_i\ y$ and the i_{th} individual sees that x is at least as good as y by $x\ R_i\ y$. Arrow's theorem on the method of majority decision, states : A group decision function is the method of simple majority decision in which x R y holds if and only if (iff) the number of individuals such that $x\ R_i\ y$ is at least as great as the number of individuals such that $y\ R_i\ x$. Hence,

$$x\ P\ y \qquad \#(i\!: x\ P_i\ y) > \#\{i\!: y\ P_i\ x), \qquad i \in n$$

$$x\ R\ y \qquad \#(i\!: x\ R_i\ y) \geq \#\{i\!: y\ R_i\ x), \qquad i \in n$$

$$x\ I\ y \qquad \#(i\!: x\ I_i\ y) = \#\{i\!: y\ I_i\ x), \qquad i \in n$$

defines the necessary conditions for simple majority decision. Such that, P, R and I represent binary relations of weak simple majority, strict simple majority and of tie under simple majority rule.

When the number of alternatives are increased then the Condorcet effect comes into action. Suppose an electoral body of 60 individuals voted for a teaching position from a field of three candidates (alternatives) a, b, and c in the following manner:

23 have given the order:	a P c P b
19 have given the order:	b P c P a
16 have given the order:	c P b P a
2 have given the order:	c P a P b

This is a clear example of the Condorcet effect. Indeed, the result depends on the method of counting used. Any of the three candidates could be elected: candidate a by the plurality method, candidate b by the second ballot of the majority and candidate c by the Condorcet principle. In another example the simple majorities could be intransitive where a beats b, b beats c and c beats a, which is the paradox of voting as the committee would have a circular preference amongst the alternatives and would not be able to arrive at a transitive ranking. In analyzing the voting, the wish of the majority can be determined by comparing the respective merits of the candidates two by two. In comparing candidate a with b, we have:

23 + 2 = 25 votes for a P b

19 + 16 = 35 votes for b P a

hence, the opinion of the majority is b P a.

Similarly, comparing a and c, we have:

23 votes for a P c

19 + 16 + 2 = 37 votes for c P a

hence, the majority is c P a.

Finally, comparing b to c, we have:

19 votes for b P c

23 + 16 + 2 = 41 votes for c P b

hence, the majority is c P b.

The result of the analysis shows that the will of the majority has three judgements:

c P b, b P a, c P a, that is the order c P b P a .

If it was necessary to choose only one candidate, candidate c is chosen. Condorcet noticed that when the non-ranked voting method was used, it would have given 23 votes for candidate a, 19 votes for candidate b and 18 votes for candidate c, where candidate a wins; giving a wrong idea of the collective preference. In the non-ranked voting method we do not give the respective merits of the other candidates, therefore, the judgment is incomplete.

Using the method of the second ballot, in the majority representation system, there must be a run off election between a and b by the second ballot. The preference orders remain unchanged and 23 +2 =25 express a P b and 19 + 16 = 35 voters express b P a. In this case b is the winner.

Furthermore, suppose the 60 voters vote slightly differently as shown:

23 votes:	a P b P c
17 votes:	b P c P a
2 votes:	b P a P c
10 votes:	c P a P b
8 votes:	c P b P a

The analysis based on the respective merits of all candidates comparing two by two gives:

b P c:	23 + 17 + 2 = 42;	c P b:	10 + 8 = 18
c P a:	17 + 10 + 8 = 35;	a P c:	23 + 2 = 25
a P b:	23 + 10 = 33;	b P a:	17 + 2 + 8 = 27

By the law of majority:

b P c, c P a and a P b

They are distinct, yet none of the voters belong to all these majorities.

The Condorcet Effect

Considering a committee which has a consistent set of preferences for three alternatives. Six possible opinion categories are produced:

a P b P c

a P c P b

c P a P b

c P b P a

b P c P a

b P a P c

Let n be the total number of the committee members and n_i the number of the committee member in each of the six categories, where i = 1,2,3,…,6. The collective preference is obtained using the Condorcet principle; comparing a with b. The six categories are regrouped into two classes:

$$\#(i\text{: a } P_i \text{ b}) = \#(1) \text{ and } \#(2) \text{ and } \#(3) = n_1 + n_2 + n_3$$

$$\#(i\text{: b } P_i \text{ a}) = \#(4) \text{ and } \#(5) \text{ and } \#(6) = n_4 + n_5 + n_6$$

and similarly for the other alternatives:

$$\#(i\text{: a } P_i \text{ c}) = \#(1) \text{ and } \#(2) \text{ and } \#(6) = n_1 + n_2 + n_6$$

$$\#(i\text{: c } P_i \text{ a}) = \#(3) \text{ and } \#(4) \text{ and } \#(5) = n_3 + n_4 + n_5$$

$$\#(i\text{: b } P_i \text{ c}) = \#(1) \text{ and } \#(5) \text{ and } \#(6) = n_1 + n_5 + n_6$$

$$\#(i\text{: c } P_i \text{ b}) = \#(2) \text{ and } \#(3) \text{ and } \#(4) = n_2 + n_3 + n_4$$

Inconsistency will occur if we have a P b, b P c and c P a or b P a, c P b and a P c. The first case occurs when the three inequalities happen simultaneously:

$$n_1 + n_2 + n_3 > n_4 + n_5 + n_6$$

$$n_1 + n_5 + n_6 > n_2 + n_3 + n_4$$

$$n_3 + n_4 + n_5 > n_1 + n_2 + n_6$$

Conditions of the second case can be written by reversing the inequalities above. If we choose:

$$n1 = n3 = n5 = 1$$

$$n2 = n4 = n6 = 0$$

We satisfy the three inequality conditions. This is a collective preference of three persons who hold opinions (1), (3) and (5). The first of the three voters can adopt any of the six opinions and likewise for the second and third, so the result is 6 x 6 x 6 = 216 possibilities. Among these 216 possibilities there are 12 (twelve) which give rise to the Condorcet effect, a little less than 6 %. These twelve possibilities are:

(1) (3) (5)

(1) (5) (3)

(3) (1) (5)

(3) (5) (1)

(5) (1) (3)

(5) (3) (1)

and six analogous arrangements of (2), (4) and (6).

More precisely, computational analysis show that the Condorcet effect represent only a small fraction of the possibilities:

3 voters: 5.6%

5 voters: 7.0%

9 voters: 7.8%

The probability of a no majority winner is a function of the number of voters and alternatives. In the table below the probability of a no majority winner is varied accordingly to the number of alternatives, when the number of voters is very large.

Alternatives	Probability
1	0,0000
2	0,0000
3	0,0877
4	0,1755
5	0,2513
6	0,3152
7	0,3692

For example if the number of alternatives are 3 the probability is 8.77%, when it is increased to 4 then the probability is 17.55%. In the table below the probability of a no majority winner is presented for different combinations of number of alternatives and individuals.

	Individuals											
Alternatives	3	5	7	9	11	13	15	21	25	29	59	∞
3	0,0556	0,0694	0,0750	0,7800	0,7980	0,8110	0,8200	0,8360	0,0843	0,0848	0,0863	0,0877
4	0,1111	0,1400	0,1500									0,1755
5	0,1600	0,2000	0,2200									0,2513
6	0,2000	0,2500	0,2700									0,3152

We observe that as the number of alternatives are increased, the probabilities of cyclical majorities increase toward 1, with little sensitivity to the number of voters for a given number of alternatives. The paradox of voting has be formalized and studied by Arrow K. J. and procedures to handle this paradox has been one of the more popular preoccupations of social science theory in the past century. Social Choice Theory defines such functions to solve for the Condorcet effect.

Social Choice Function

A social choice function can be considered as an aggregation procedure based on preferential voting system. It is a mapping which assigns a non empty subset of the potential feasible subset to each ordered pair consisting of a potential feasible subset of alternatives and a profile of the committee preferences. Here we may define the Condorcet Principle as follows:

For any nonempty finite set of A alternatives, n-voters give each one's preference order of the alternatives, so a profile on A is any n-tuple of linear orders on A. For any situation and alternatives x, y ϵ A, we let $\#(i:\ x\ P_i\ y)$ be the number of voters that have x preferred to y. Hence, $\#(i: x\ P_i\ y) + \#(i: y\ P_i\ x) = n$ when $x \neq y$. Then the simple majority win relation on A is defined by:

$$x\ P\ y \text{ iff } \#(i: x\ P_i\ y) > \#(i: y\ P_i\ x) \qquad \text{(iff – if and only if)}$$

alternatives x and y are tied under simple majority if

$$\#(i: x\ P_i\ y) = \#(i: y\ P_i\ x)$$

For all situations candidate (alternative) x will be the winner whenever x ϵ A and x P y for all y ϵ A\{x}.

(A\{x} implies candidate x is exluded from A).

The Condorcet principle is based on an argument involving probabilities of correct judgments and embodies the democratic rule of the will of the majority. Also, the majority candidate (alternative) constitutes a stable equilibrium in that it cannot be beaten by a challenger in a direct majority vote between the two.

A binary relation R on a set A is a subset of A x A, defined as the set of all ordered pairs (x, y) such that x and y belong to A. Let x R y represent a binary relation between x and y, that is, x is at least as good as y, or x is greater than y, that is, ~ (x R y) if x is as not as good as y or x is not greater than y. For all x and y, x R y if and only if the number of individuals for whom x R_i y is at least as large as the number for whom y R_i x where subscript i is index of individuals. This is the relation of weak preference of individual i as R_i. The properties of binary relation R over a set a is defined as:

1. R is **reflexive** over A iff for all x belonging to A, x R x, that is, x R x, $\forall x \in A$
2. R is **irreflexive** over A iff for all x belonging to A not x R x, that is, ~(x R x), $\forall x \in A$
3. R is **connected** (complete) over A iff for all x and y ($x \neq y$) belonging to A, x R y or y R x, that is, [x R y U y R x], $\forall x, y \in A$
4. R is **symmetric** over A for all x and y belonging to A, x R y implies y R x, that is, [x R y $\rightarrow$ y R x], $\forall x, y \in A$
5. R is **asymmetrical** over A iff for all x and y belonging to A, x R y implies not y R x, that is, [x R y $\rightarrow$~ y R x], $\forall x, y \in A$
6. R is **antisymmetric** over A iff for all x and y belonging to A x R y and y R x implies x is the same as y, that is, [x R y and y R x] $\rightarrow$ (x = y), $\forall x, y \in A$
7. R is **transitive** over A iff for all x and y and z belonging to A, x R y and y R x implies x R z, that is, [x R y and y R z] $\rightarrow$ x R z, $\forall x, y, z \in A$
8. R is **negatively** transitive over A iff for all x, y and z belonging to A not x R y and not y R x and not y R z implies not x R z, that is, [~x R y and ~y R z] $\rightarrow$ ~x R z, $\forall x, y, z \in A$

Reflexivity means that the alternative is as good as itself, the relation is connected if whenever alternative x is not the winner, then alternative y will be the winner. It is transitive, if alternative x is at least as good as alternative y, which is itself as least as good as alternative z, and must imply that alternative x is as least as good as alternative z. Symmetry means that every alternative has an equal chance to be selected.

To define the properties of group decision rule, we assume that there are n individuals in the group, for all x and y with each individual we associate a variable D_i that takes the values -1, 0, 1 according whether the individual i prefers y to x , y P_i x, is indifferent to x and y , x I_i y, or prefers x to y, x P_i y respectively. For the group we write F(D) = {-1, 0, 1}. For the set of all individual preference profile, we write, $\mathcal{D} = \{-1, 0, 1\}^n$.

In other words, an element D ∈ $\mathcal{D}$ where D = (D_1, D_2, D_3,…, D_n). Then a social choice function is defined to be a function F(D) = f(D_1, D_2, D_3,…, D_n) for all D ∈ $\mathcal{D}$, that is F: $\{-1, 0, 1\}^n \rightarrow \{-1, 0, 1\}$. These properties are:

1. **Decisiveness**: F: $\mathcal{D} \rightarrow \{-1, 0, 1\}$ is decisive iff $D \neq \underline{0} \rightarrow F(D) \neq 0$. It is weakly decisive if $\{D: F(D) = 0\} = \{\underline{0}\}$, and strongly decisive if {D: F(D) = 0} = 0.
2. **Neutrality:** (duality): $f(-D_1, -D_2, -D_3,\ldots, -D_n) = -f(D_1, D_2, D_3,\ldots, D_n)$
3. **Anonymity:** (equality): $f(D_1, D_2, D_3,\ldots, D_n) = f(D_{\sigma(1)}, D_{\sigma(2)}, D_{\sigma(3)},\ldots, D_{\sigma(n)})$, if σ is a permutation on (1, 2, 3,…, n)
4. **Monotonicity:** (positive responsiveness): If D ≥ D' then F(D) ≥ F(D')
5. **Unnanimity** (weak Pareto criterion): f(1,…,1) = 1, or f(-1,…,-1) = -1
6. **Homogeneity**: F(mD) = F(D) for all positive integers of m
7. **Weak Pareto optimality:** If Di = 1 for all i, then F(D) = 1, or if Di = -1 for all i, then F(D) = -1.
8. **Strong Pareto Optimality:** If Di = {1,0} for all I and D, k = 1 for some k, then F(D) = 1, and if Di = 0 for all I, then F(D) = 0

Considering a preferential voting system, where each voter expresses her/his preference by ordering the candidates, then decisiveness means that a social choice function is such that each of the voters' preferences leads to a defined and unique solution. Neutrality prevents a build in favoritism for either candidate, since the social choice function will be reversed if the voters reverse their votes, that is, each candidate is treated equally. Anonymity means that the system gives equal rights to each voter. Monotonicity assures that if a voter moves x upwards in her/his ranking leaving the relative standing of the others unchanged, then candidate x will stand at least as well relative to each other candidate as before. Unanimity prescribes that x wins when everyone prefers x to y and that y wins when everyone prefers y to x. Homogeneity, prescribes that if a voter is indifferent between x and y, she/he is replaced by two voters each with the same preference as the original except that one prefers x to y and the other prefers y to x. The Pareto optimality means that if every voter thinks that x is better y (or at least as good as) then so does society.

The Condorcet function is given by:

$$f_c(x) = \min_{y \in A \setminus \{x\}} \#(i : xP_i y)$$

(A\{x} implies candidate x is exluded from A).

where, the alternatives are ranked in the order of the values of f_c.

Considering the previous example of the 60 voters who vote differently so as to produce cyclical majorities, the analysis of the voting results by pairwise comparison of all candidates gives:

#(i: a P_i b) = 33;	#(i: b P_i a) = 27
#(i: a P_i c) = 25;	#(i: c P_i a) = 35
#(i: b P_i c) = 42;	#(i: c P_i b) = 18

The values of the above pairwise comparison can be represented by the following matrix:

	a	b	C	f_c
a	--	33	25	25
b	27	--	42	27
c	35	18	--	18

As f_c measures the worst against any other candidate, Condorcet's function is a maximum function as it chooses those candidates whose worst showing against the others is as good as possible. Therefore, according to Condorcet's Social (counting) function, the social preference ordering of the candidates is b P a P c, since:

$$f_c(b) > f_c(a) > f_c(c)$$

Other Social Choice Functions include:

1. Borda's Function
2. Copeland's Function
3. Nanson's Function
4. Dogson's Function
5. Kemeny's Function
6. Cook and Seiford's Function
7. Fishburn's Function
8. Eigenvector Function
9. Bernardo's Assignment Function
10. Cook and Seiford's Ordinal Intersection Method

This project uses the Eigenvector function, proposed by Thomas L. Saaty in 1970, to obtain the individual priorities of preferences and Borda's Function to obtain the group ranking from the collective preferences of the committee. Kemeny's and, Cook and Seidel's Function is also discussed so as to propose improvements over Borda's Function.

Eigenvector Function

Let x_i (i = 1, 2, 3,…, m) be a nonempty finite set A of alternatives and n be the number of individuals which give their preference order for the alternatives, so that a profile on A is any n – tuple of linear orders on A. A situation is any ordered pair of a set of alternatives and a profile on that set. For such situations and alternatives, x_i , x_j $\in$ A, we let n_{ij} be the number of voters that have x_i preferred to x_j. The analysis of the voting results by pairwise comparisons of all alternatives gives the following matrix:

$$D = \begin{array}{c|cccc} & x_1 & x_2 & \cdots & x_m \\ x_1 & 1 & \frac{n_{12}}{n_{21}} & \cdots & \frac{n_{1m}}{n_{m1}} \\ x_2 & \frac{n_{21}}{n_{12}} & 1 & \cdots & \\ . & & & & \\ . & & & & \\ . & & & & \\ x_m & \frac{n_{m1}}{n_{1m}} & & \cdots & 1 \end{array}$$

This is a reciprocal matrix which has all positive elements and obeys the reciprocal rule:

$d_{ij} = 1 / d_{ji}$

but $d_{ij} = n_{ik} / n_{jk}$, $j \neq k$

An element of the matrix $d_{ij} = (n_{ij} / n_{ji})$ gives the relative strength of priorities xi versus alternative xj, which provides a scaling formulation of the principal maximum eigenvalue problem giving a single overall priority for all alternatives. The associated unique eigenvector is the vector of priorities (weights) normalized where the entries sum to unity. If we regard the value of the pairwise comparison as the relative priorities (weights), then the above matrix becomes:

$$\begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_m} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_m} \\ . & & & \\ . & & & \\ . & & & \\ \frac{w_m}{w_1} & \frac{w_m}{w_2} & \cdots & \frac{w_m}{w_m} \end{bmatrix}$$

If we multiply this matrix by the transpose of the vector $\underline{w}^T = (w_1, w_2, w_3,\ldots, w_m)$, we obtain the vector m $\underline{w}$ or :

$$D\,\underline{w} = m\,\underline{w}$$

where, $\underline{w}$ is given. Suppose we only had D and we wanted to find $\underline{w}$, then we solve for:

$$(D - m\,I)\,\underline{w} = 0$$

where I is an identity matrix. The above equation has a non zero solution iff m is an eigenvalue of D, that is, it is a root of the characteristic equation of D, det(D – λI) = 0. Now, as D has unit rank since every row is a constant multiple of the first row, thus all eigenvalues λ_i , i = 1, 2, 3,…, m (sum of diagonal elements) of D are zero except for one. Also,

$$\sum_{i=1}^{m} \lambda_i = m$$

Therefore, only one element, λ_{max} equals m, that is, $\lambda_i = 0$, $\lambda_i = \lambda_{max}$. The result is a unique solution and we have recovered the scale from the matrix of ratios.

The eigenvector social choice function, f_E , satisfies the property of neutrality as it treats all alternatives equally as it is based on pairwise comparisons of individuals (voters) between a pair of alternatives. As the matrix gives equal rights to the individuals (voters), then the property of anonymity is also satisfied. Also, since each voter expresses her/his preference by ordering, each of the voters' preferences leads to a defined and unique solution, it satisfies the property of monotonicity. The Homogeneity property is also satisfied as the associated eigenvector is split into m voters, each has the same preference as the original. Finally, if every voter thinks alternative i is better than k (or at least as good as) then so does society, satisfying the Pareto optimality.

Borda Function

The method of Borda proposes the rank order method, with alternatives (candidates) in A assigned marks of m -1, m -2,…, 1 for each individual (voter). The Borda score for each alternative is the sum of the voters mark for that alternative. The alternative with the highest Borda score is considered the winner. The Borda score of an alternative x is equivalent to the sum of the number of voters that have x preferred to y for all $y \in A \setminus \{x\}$, that is,

$$f_B(x) = \sum_{y \in A} \#(i : xP_i y)$$

where the alternatives are ranked in the order of f_B.

Using Borda's method to the second example of Condorcet, we obtain a matrix for the values of pairwise comparison and Borda's counting function:

	a	b	c	f_B
a	--	33	25	58
b	27	--	42	69
c	35	18	--	53

or assigning 2, 1 and 0 marks to the first ranked, second ranked and third ranked alternative, we obtain:

a: 2 x 23 + 1 x (2 + 10) + 0 x (17 + 8) = 58

b: 2 x (17 +2) + 1 x (23 + 8) + 0 x 10 = 69

c: 2 x (10 + 8) + 1 x 17 + 0 x (23 + 2) = 53

The social preference ordering of the alternatives is b P a P c, since,

$$f_B(b) > f_B(a) > f_B(c)$$

The Borda function is also homogeneous, monotonic, Pareto optimal , monotonic and neutral.

Multiple Criteria

The sequential procedures in decision making under multiple criteria include the following phases:

1. Preparatory – advertising specifically the objective and the criteria of the decision process.
2. Screening – methods of elimimating alternatives that bear no relation to the objective.
3. Evaluating – reviewing related alternatives and weighting to obtain priorities.
4. Decision – produce rankings of priorities

The evaluating and decision phases of group decision making are represented mathematically and systematically under multiple criteria, which can be classified either as quantitative e.g. job experience which is measurable or qualitative e.g. dependability, which is judgmental and difficult to measure. The most commonly used evaluation techniques are:

1. Ranking
2. Rating
3. Scoring
4. Utility (Value) Function

The above techniques indicate preference with regard to a group of alternatives under consideration. The ordinal approach involves the ranking of alternatives. The cardinal approach involves the ranking and scoring of alternatives, that is, the position and the value. If an alternative is distinct from the rest then the above approaches need not be applied. However, when there are several alternatives whose overall characteristics are similar, then either of the approaches may be applied.

The process of evaluating alternatives require certain criteria. Assume that a committee has to evaluate m candidates (alternatives) who are using p criteria (we assume the committee are using the same criteria), would yield the following matrix:

$$A^k = \left[a_{ij}\right]^k = \begin{bmatrix} a^1{}_{11} & a^2{}_{1j} & \dots & a^n{}_{1p} \\ a^1{}_{21} & a^2{}_{2j} & \dots & a^n{}_{2p} \\ . & & & \\ . & & & \\ . & & & \\ a^1{}_{m1} \quad \dots & a^2{}_{mj} & \dots & a^n{}_{mp} \end{bmatrix} \qquad (k = 1,\dots,n)$$

The symbol $A^k_i = [a_{i\,1},\dots,a_{i\,p}]^k$ means the alternative i is being evaluated by criteria from 1 to p by committee member k. The symbol $A^k_j = [a_{1\,j},\dots,a_{m\,j}]^k$ means that criterion j is being used by committee member k to evaluate all candidates from 1 to m. The solution to the problem is to have each candidate be evaluated by n number of committee members, using various p criteria, described by a mapping function:

$$\Psi = \{A^\kappa \mid \kappa = 1, \dots, n\} \rightarrow \{G\}$$

and can be obtained by the techniques discussed previously. This mapping function represents the various criteria the committee used in evaluating (judging) the alternatives (candidates). We may use the ordinal function for ranking and the cardinal function for rating or scoring. In either case we notice two approaches, the agreed criteria approach, where the committee have the same criteria, or the individual approach, where each member of the committee has their own set of criteria. This project is concerned with the agreed criteria approach.

In the ordinal case of the agreed criteria approach, the above matrix is used to obtain the Borda score for each candidate by each committee member. The first place candidate would receive a score of m -1, the second candidate wouls receive a score of m – 2, and so on. The candidate (alternative) with the highest Borda score, that is, the sum of all committee members' Borda scores will receive the first place, the second and so on. Thus, we obtain a collective ordered matrix that maps into {G}, that is,

$$G = \left[a'_{ij}\right] = \begin{bmatrix} a'^{1}_{11} & a'^{2}_{1j} & \ldots & a'^{k}_{1p} \\ a'^{1}_{21} & a'^{2}_{2j} & \ldots & a'^{k}_{2p} \\ . & & & \\ . & & & \\ . & & & \\ a'^{1}_{m1} & a'^{2}_{mj} & \ldots & a'^{k}_{mp} \end{bmatrix}$$

where $[a'^{k}_{ij}]$ is the ordering of candidate i under criterion j for committee member k..

As some criterions may be more important than others, the committee may wish to place more weight on that criterion, so we need a vector of weights, $\underline{W} = \{w_1, \ldots, w_p\}$, where w_i is the weight assigned to i^{th} criterion and

$$\sum_{i=1}^{p} w_i = 1.$$

the system of values (weights) can be obtained by the eigenvector function by which the committee members compare all criteria on a one to one basis. The agreement matrix Π, represents the number of orderings where the i^{th} candidate (alternative) is placed in the j^{th} position for a given criterion p, with entry elements Π_{ijp}. Thus, the collective agreement matrix:

$$G = [g_{ij} = \sum_{p=1}^{p} \Pi_{ijp} w_p]$$

where $\Pi_{ijp} = 1$, if the i^{th} candidate is placed in the j^{th} position, otherwise it is zero. If we want to match candidate i with rank number j so that the sum of the corresponding assigned weight value is as largest as possible, then we solve the assignment problem of linear programming:

$$Max \sum_{i=1}^{m} \sum_{j=1}^{m} g_{ij} x_{ij}$$

subject to:

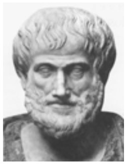
$$\sum_{i=1}^{m} x_{ij} = 1, \qquad j = 1, \ldots, m$$

$$\sum_{j=1}^{m} x_{ij} = 1, \qquad \iota = 1, \ldots, m$$

where $x_{ij} = 1$ if j has been assigned to i and $x_{ij} = 0$ otherwise.

Group Analytic Hierarchy Process (AHP)

Hierarchy comes from the Hellenic compound word ***ιεραρχία***. The component words are ***ίερά*** (holy) and ***άρχή*** (beginning, origin or rule). Hierarchy, in general, as the Oxford English Dictionary reports, means a body of persons or objects ranked in grades, orders, or classes, one above another. A hierarchy of knowledge means a body of concepts and conclusions ranked in order of logical dependence, one upon another, according to each item's distance from the base of the structure. The base is the perceptual data with which cognition begins. [104]



"*Any ordering of forms, any hierarchy which is actual; will derive from how objects themselves are ordered; the most fundamental such ordering is: natural vs. artificial. Since everything which is artificial is based upon what is natural, our most basic understanding of the world derives from an understanding of 'nature' (physis) and the 'natural'.*" - Aristotle (384-322 B.C.) [105]

Aristotle's discussion of Plato's Republic gives insights to the direct and indirect proportionalities of social hierarchies with respect to the actual decision maker's order of knowledge in the hierarchy [98] and Maslow's "Hierarchy of Needs" even proposes a behavioural analysis to decision making [100]. All these ideas are nowadays very similarly applied to a great extend to businesses and organizations composed of hierarchies themselves, in their decision making processes, and have yet to be implemented and applied in full scale, as research is still new to this area of study [101, 102].

"*When first faced with a complex problem, we may be overwhelmed by its size and by the amount of detail involved. Our first instinct is to decompose the problem into smaller and more manageable parts, and so on. This, in essence, gives rise to a hierarchy. Hierarchies are thus a consequence of the effort of the human mind to seek understanding.*" - Thomas L. Saaty [106].

Saaty's Analytic Hierarchy Process (AHP) [1, 2, 3, 4, 5, 6, 7, 8, 9] is a basic approach to decision making, coping with both the qualitative and the quantitative measurements of decision elements arranged in a hierarchy. A defined hierarchy is called hierarchon [2]. The decision maker carries out pair wise comparisons (judgments), using a fundamental scale, to develop overall priorities for the relevant decision elements. [1, 3, 55]. Although this methodology has been criticized [10, 12] on inherent paradigms and weaknesses it possesses, further research on the AHP methodology, is deemed to reduce problem size on the decision maker's time frame even further.

The AHP [1, 3] comprises a general theory of measurement, deriving normalized ratio scales from pairwise comparisons in hierarchic structures. These measurements can be either objective or subjective in nature. In the objective case, measurement is quantitative, of tangible goods in the physical domain, like costs, consumption, price and other similar percepted scales, where the value of the measurement bears no direct relationship with the individual conducting decisions. On the other hand, in the subjective case, measurement is qualitative, of intangible goods in the psychological domain, making either absolute or relative comparisons between the objects of interest. Here, the value of the measurement bears direct relationship with the ideas, conceptions and beliefs of the decision maker with respect to the level of hierarchy at hand. If absolute comparison are needed a scoring scale may used, while for relative comparisons the fundamental scale seems most appropriate.

Overall, the AHP provides the objective mathematics to process the objective (quantitative) and subjective (qualitative) measurements of an individual or a group of individuals in a decision making process. In both cases, it develops priorities of alternatives and the criteria are used to weight the overall alternatives that contribute to the objective in mind. Prioritization solves for the different types of scales used in measuring the objective by direct normalization and the subjective by using the eigenvalue for consistency and the eigenvector to obtain normalized priorities. In this way, a multi dimensional scaling problem reduces to a one-dimensional one.

This project will allow for both objective and subjective measurements, but will concentrate mainly with the fundamental scale for relative comparisons rather than use a scoring scale, which would require questionnaire input. The particular scale is further discussed and suggestions of alternative scales are proposed. In short, the AHP provides the eigenvector function that allows to obtain individual priorities of the all the alternatives under multiple criteria. The group choice is thence obtained using Borda's function on the aggregation of all individual priorities.

The software tool in mind will demonstrate group decision making using the AHP which utilizes the eigenvector function to obtain individual priorities and the Borda function to obtain the group choice (rank), by a simple five-step method:

1. *The decision analyst (select committee member) is constrained to structure hierarchons based on three level hierarchies (objective / criteria / alternatives), where each level is kept to no more than 3 or 4 decision elements. The decision objectives are assigned to each and every decision maker (committee member) so as to form group (committee) decision with respect to the decision objective (hierarchon).*

2. *Each committee member inputs pairwise comparisons of qualitative criteria and alternatives for a particular hierarchon using basically the fundamental scale. For quantitative measurements the values are defaulted by the select committee members.*

3. *The class defined, calculates and outputs the relative weights (priorities) of the criteria and alternatives, for each committee member using the eigenvector and estimating for consistency using the principal maximum eigenvalue, in each case.*

4. *The relative weights of the alternatives are aggregated with respect to the criteria, for each committee member, to obtain individual priorities, by multiplying the alternatives weight vector with the criteria weight vector.*

5. *The individual priorities of the committee are aggregated in order to obtain the group choice, using Borda's positional and summing method.*

Step 1. - Structure the hierarchy into interrelated decision elements.

This is the most important step of decision-making and that is the structuring of the hierarchy itself. The decision analyst develops the hierarchical representation of the problem. At the top of the hierarchy is the overall objective and the decision alternatives are at the bottom. Between the top and bottom levels are the relevant criteria [1, 9]. Each hierarchon can then be assigned to specific decision maker or group of decision makers.

A simple three level hierarchy is shown in **Figure 1:**. *Three level Hierarchy*

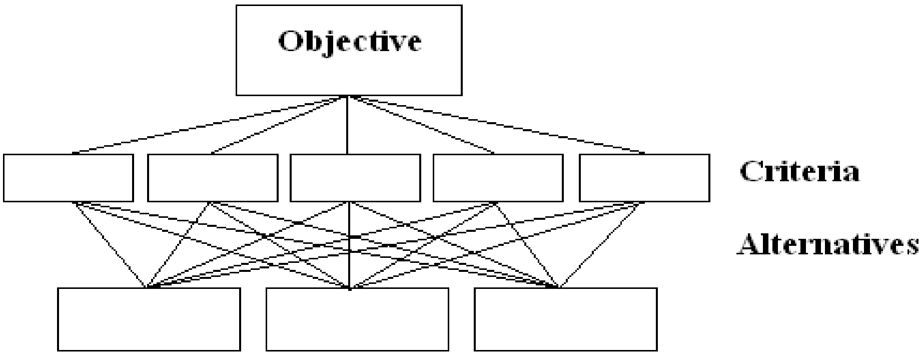


Figure 1: *Three level Hierarchy*

e.g.

Defining the hierarchy:

Objective: Select new car

Criteria: Style, Reliability, Economy

Alternatives: Civic, Saturn, Escort, Miata

Yields a hierarchon of selecting a new car (objective), as shown in **Figure 2:** Hierarchon

i.e.

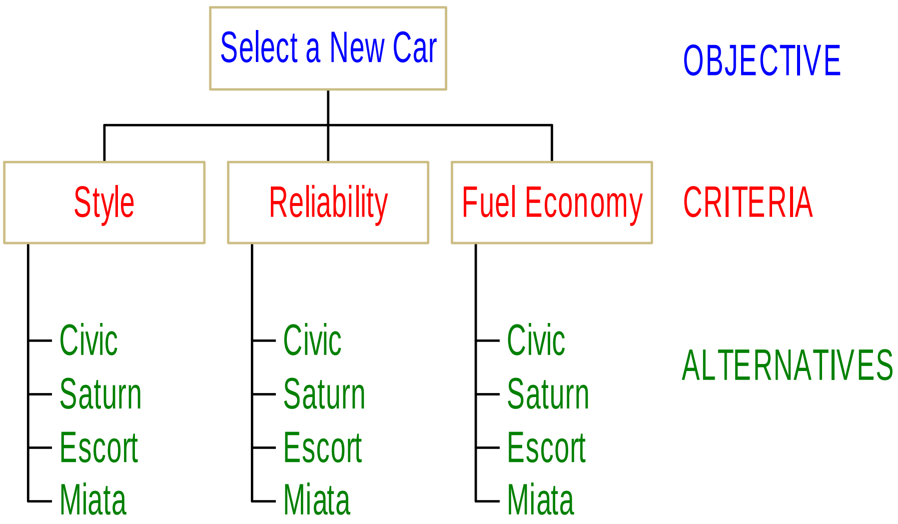


Figure 2: Hierarchon

See: AHP Example - Hierarchon

Step 2. - Collect pairwise comparisons of the decision elements.

This step generates relational data for comparing the criteria and alternatives. This requires the decision-maker to make pairwise comparisons of the decision elements for all alternatives under each criterion. In the AHP, the fundamental scale of real numbers from 1 to 9 is used to systematically assign preferences in terms of scale judgements as shown in **Table 1**: *The Fundamental Scale*. This paper makes extensive use of this scale. Indeed, here, it seems quite appropriate bear the famous quote:

"*Not everything that counts can be counted and not everything that can be counted, counts*" - Albert Einstein.

Intensity of importance	Definition	Explanation
1	**Equal importance**	**Two elements contribute equally**
2	Weak	
3	**Moderate importance**	**Experience and judgment slightly favor one element over another**
4	Moderate plus	
5	**Strong importance**	**An element is strongly favored over another**
6	Strong plus	
7	**Very strong or demonstrated importance**	**An element is strongly dominant**
8	Very, very strong	
9	**Extreme importance**	**The evidence favored one element over another is of the higher order of affirmation**
Reciprocals of above	**If element *i* has one of the above nonzero numbers assigned to it when compared with activity j, the *j* has the reciprocal value when compared to *i***	**A reasonable assumption**
Rationales	**Ratio arising from the scale**	**If consistency were to be forced by obtaining n numerical values to span the matrix**

Table 1. The Fundamental Scale

See: AHP Example – Pairwise Comparison Matrix (PCM)

Step 3. - Calculate the relative weights of the decision elements using the eigenvector and check for consistency.

Utilizing the pair wise comparisons of step 2 an eigenvector method is used to determine the relative priority of each criteria and alternatives in the hierarchy. In addition, a consistency ratio is calculated, using the Eigenvalue and displayed. According to Saaty, small consistency ratios (less than 0.1, that is, 10%) does not drastically affect the rankings. The user has the option of redoing the comparison matrix should the consistency ratio be larger than 10%.

If the decision maker knows the actual relative weights of n decision elements, the pair wise comparison matrix would be depicted as shown for matrix A. Assuming we are given **n** stones, $\mathbf{A_1,\ldots,A_n}$, whith weights $\mathbf{w_1,\ldots,w_n}$, respectively, the matrix will contain the ratios of the weights of each stone with respect to all others. The smaller of each pair of stones is used as the unit and the larger one is measured in terms of multiples of that unit:

$$
A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ . \\ . \\ . \\ n \end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & \ldots & n \end{array} \\
\begin{bmatrix}
w_1/w_1 & w_1/w_2 & w_1/w_3 & \ldots & w_1/w_n \\
w_2/w_1 & w_2/w_2 & w_2/w_3 & \ldots & w_2/w_n \\
w_3/w_1 & w_3/w_2 & w_3/w_3 & \ldots & w_3/w_n \\
. & . & . & . & . \\
. & . & . & . & . \\
. & . & . & . & . \\
w_n/w_1 & w_n/w_2 & w_n/w_3 & \ldots & w_n/w_n
\end{bmatrix}
\end{array}
$$

This leads to the eigenvalue equation:

$$
\begin{bmatrix}
w_1/w_1 & w_1/w_2 & w_1/w_3 & \ldots & w_1/w_n \\
w_2/w_1 & w_2/w_2 & w_2/w_3 & \ldots & w_2/w_n \\
w_3/w_1 & w_3/w_2 & w_3/w_3 & \ldots & w_3/w_n \\
. & . & . & & \\
. & . & . & & \\
. & . & . & & \\
w_n/w_1 & w_n/w_2 & w_n/w_3 & \ldots & w_n/w_n
\end{bmatrix}
*
\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ . \\ . \\ . \\ w_n \end{bmatrix}
= n *
\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ . \\ . \\ . \\ w_n \end{bmatrix}
$$

$$\boldsymbol{A} \cdot \boldsymbol{W} = n \cdot \boldsymbol{W} \tag{1}$$

The solution of the above is called the principal eigenvector of **A** which is the normalization of any column of **A**. Where,

$$\sum_{i=1}^{n} w_i = 1$$

A → vector comparison matrix
W → $(w_1,w_2,w_3,\ldots,w_n)^{\mathbf{T}}$ the transpose vector of relative weights. The right eigenvector of matrix **A**
n → scalar eigenvalue (principal maximum)of matrix **A**

The matrix $\boldsymbol{A = a_{ij}, a_{ij} = w_i/w_j, i,j = 1,\ldots,n}$, satisfies the reciprocal property if *a* is twice more preferable than *b*, *b* is twice more preferable than *c*, then *a* is four times more preferable than *c* (*cardinal scale*).

$$\boldsymbol{a_{ji} = 1/a_{ij}} \tag{2}$$

and is *consistent* provided the following condition is satisfied:

$$\boldsymbol{a_{jk} = a_{ik}/a_{ij},\ i, j, k = 1,\ldots,n.} \tag{3}$$

Example 1: *n=2, always consistent*

In a chess competition, behavioural capacity may be twice as important as technical capacity. The input matrix in this case would look like:

$$\begin{array}{ccc} & Technical\_capacity & Behavioural\_capacity \\ Technical\_capacity & 1 & 1/2 \\ Behavioural\_capacity & 2 & 1 \end{array}$$

applying (1) produces the vector of relative weights (0,67, 0.33) depicting that the above matrix does not contain inconsistencies due to its small size.

Example 2: *n=3, consistent*

A melon, a pineapple and an orange weigh 1kg, 0.25 kg and 0.125 kg respectively. For example, the weight comparison between a melon and a pineapple is $\mathbf{a_{12}} = 1/4$. The complete pairwise comparison *nxn consistent* matrix is consistent if, from (3):

i.e.

$$\begin{bmatrix} 1 & 4 & 8 \\ 1/4 & 1 & 2 \\ 1/8 & 1/2 & 1 \end{bmatrix}$$

Converting to decimals,

$$\begin{bmatrix} 1 & 4 & 8 \\ 0.25 & 1 & 2 \\ 0.125 & 0.5 & 1 \end{bmatrix}$$

hence,

$$\vec{A} \cdot \vec{w} = \begin{bmatrix} 1 & 4 & 8 \\ 0.25 & 1 & 2 \\ 0.125 & 0.5 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0.25 \\ 0.125 \end{bmatrix} = \begin{bmatrix} 3 \\ 0.75 \\ 0.375 \end{bmatrix}$$

i.e.

$$\vec{A} \cdot \vec{w} = \begin{bmatrix} 1 & 4 & 8 \\ 0.25 & 1 & 2 \\ 0.125 & 0.5 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0.25 \\ 0.125 \end{bmatrix} = \begin{bmatrix} 3 \\ 0.75 \\ 0.375 \end{bmatrix} = 3 \bullet \begin{bmatrix} 1 \\ 0.25 \\ 0.125 \end{bmatrix} = n \cdot \vec{w}$$

Hence,

$$n = 3$$

i.e.

$$\frac{1}{3} * \begin{bmatrix} 3 \\ 0.75 \\ 0.375 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.25 \\ 0.125 \end{bmatrix}$$

See: AHP Example – consistent Pairwise Comparison Matrix

For $n=3$ a consistent pairwise comparisons matrix can be generated by filling in two elements of the diagonal of the matrix and then computing the third element (using the transitive rule, as described in the proceeding section) and then calculating the reciprocals of all other entries. It can be shown that the principal eigenvalue λ_{max} of such a matrix will be ***n*** (number of items compared). If the third element of the diagonal is filled in manually, some inconsistency is usually observed. Deviation of λ_{max} from n is a measure of inconsistency in the pairwise comparisons matrix.

In a decision making process the ratios of $\mathbf{w_i/w_j}$ are not precise values but estimates. Thus the observed matrix **A** contains inconsistencies. This leads to an eigenvalue equation of the form:

$$\bar{A} * \hat{W} = \lambda_{max} * \hat{W} \tag{4}$$

Where:

$\bar{A}$ → vector comparison matrix
$\hat{W}$ → the right eigenvector of matrix $\bar{A}$
λ_{max}→ largest eigenvalue of matrix $\bar{A}$

Inconsistency throughout the matrix is given by $\boldsymbol{\lambda_{max} - n}$ which measures the deviation of the weights from the consistent approximation. Saaty has shown that $\boldsymbol{\lambda_{max}}$ is greater than or equal to n. The closer $\boldsymbol{\lambda_{max}}$ is to n, the more consistent are the observed values of $\bar{\mathbf{A}}$.

This leads to devise the Consistency Index (***CI***) as:

$$CI = (\lambda_{max} - n)/(n - 1) \tag{5}$$

and the Consistency Ratio (***CR***) as:

$$CR = (CI / RI) * 100: \tag{6}$$

Where

Random Index (***RI***) is the average consistency index of 100 randomly generated (inconsistent) pairwise comparisons matrices. These values have been tabulated for different values of n:

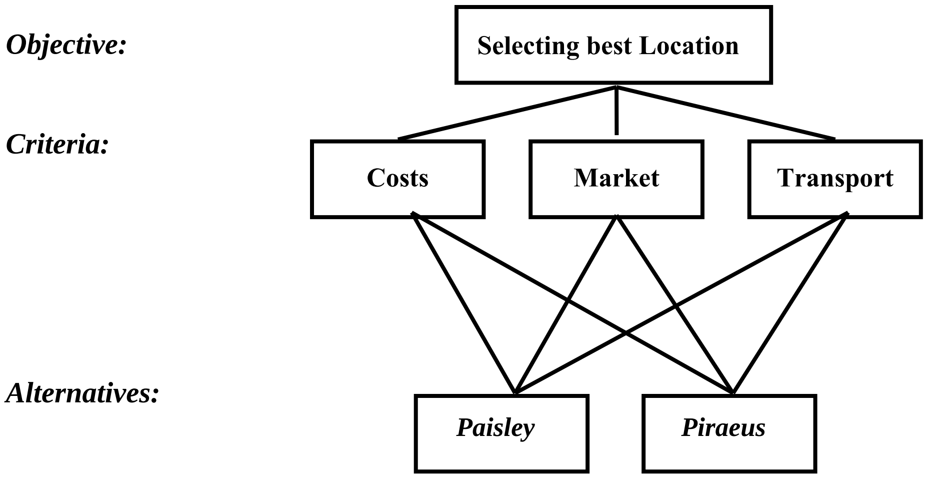
n	3	4	5	6	7	8	9
RI(n)	0.58	0.90	1.12	1.24	1.32	1.41	1.45

To obtain the priorities, one should compute the principal (maximum) eigenvalue and the corresponding eigenvector of the pairwise comparisons matrix. It can be shown that the (normalised) principal eigenvector is the priorities vector. The principal eigenvalue is used to estimate the degree of consistency of the data. In practice, one can compute both using approximation. To compute a good approximation of the principal eigenvector of a pairwise comparisons matrix, one can normalise each column and then average over each row (see **Example 3:** Approximation method). As an alternative to the approximation method, as illustrated from the example below, the project will use Saaty's Eigenvector method to obtain the Priorities Matrix (PM) – see AHP Example: Ranking Matrix (RM).

See: AHP Example – inconsistent Pairwise Comparison Matrix

Example 3: *n=3, inconsistent, Approximation Method*

Considering the following hierarchon [13, 15]:



Which may yield, at the ***criteria level***:

	costs	market	transport
costs	1	1/2	1/3
market	2	1	1/3
transport	3	3	1

To compute an estimate of λ_{max} for a pairwise comparison matrix: multiply the normalised matrix with the priorities vector, (principal eigenvector of the matrix), i.e., obtain $A*W$; divide the elements in the resulting vector by the corresponding elements of the vector of priorities and take the average, i.e.,from the equivalence $A*W=\lambda_{max}*W$ calculate an approximate value of scalar λ_{max}

$$\begin{array}{c|ccc} & c & m & t \\ \hline c & 1 & 1/2 & 1/3 \\ m & 2 & 1 & 1/3 \\ t & 3 & 3 & 1 \\ \hline sum & 6.00 & 4.50 & 1.67 \end{array} \rightarrow \begin{array}{c|ccc} & c & m & t \\ \hline c & 0.17 & 0.11 & 0.20 \\ m & 0.33 & 0.22 & 0.20 \\ t & 0.50 & 0.67 & 0.60 \\ & \multicolumn{3}{l}{(normalised)} \end{array} \rightarrow \begin{array}{c} c \\ m \\ t \end{array}\begin{bmatrix} 0.16 \\ 0.25 \\ 0.59 \end{bmatrix}$$

For the matrix from the example:

$$\begin{bmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 1/3 \\ 3 & 3 & 1 \end{bmatrix} * \begin{bmatrix} 0.16 \\ 0.25 \\ 0.59 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{0.48}{0.16} & \frac{0.77}{0.25} & \frac{1.82}{0.59} \end{bmatrix} = \begin{bmatrix} 3.00 & 3.08 & 3.08 \end{bmatrix}$$

λmax*=3.05*, ***CI****=0.025*, ***CR****=0.025 / 0.58=0.043* (acceptable).

Example 4: $n \geq 3$

For $n \geq 3$, the transitivity property of the matrix needs also to be taken into account. Considering Ishizaka's description of a method which checks the consistency of each comparison entered in four phases which correspond to the following types of comparisons, as seen in **Figure 3**:

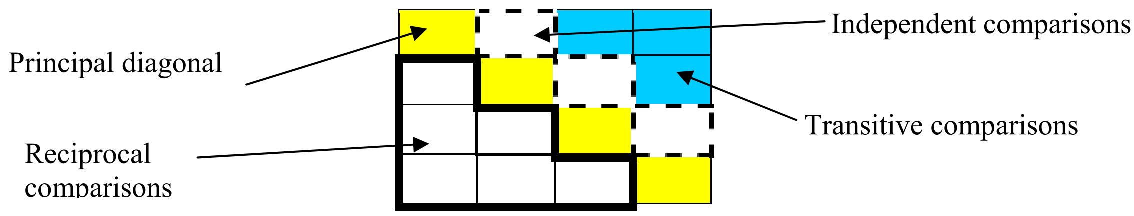


Figure 3: Phases of comparisons

1. comparisons on the *principal diagonal* compare decision elements with itself. The resulting comparison values are equal.

$$\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}$$

2. *Independent comparisons* are not linked to other comparisons by the transitivity or the reciprocity property. i.e. actual pairwise comparisons of the decision elements entered by the decision maker.

$$\begin{bmatrix} 1 & 0.5 & & & \\ & 1 & 2 & & \\ & & 1 & 2 & \\ & & & 1 & 0.25 \\ & & & & 1 \end{bmatrix}$$

3. *Transitive comparisons* are deduced with the transitivity property from the first diagonal entered. (e.g., in the utility theory): if *a* is preferred to *b*, *b* is preferred to *c*, then *a* is preferred to *c* (ordinal scale).

$$\begin{bmatrix} 1 & 0.5 & 1 & 1 & 0.25 \\ & 1 & 2 & 2 & 0.5 \\ & & 1 & 2 & 0.25 \\ & & & 1 & 0.25 \\ & & & & 1 \end{bmatrix}$$

Transitivity property: $\mathbf{a_{i,j} = a_{i,i+1} \times a_{i+1,j+2} \times , \ldots , a_{j-1,j}}$ (7)

i.e.

$$a_{13} = a_{12} \text{ x } a_{23} = 0.5 \text{ x } 2 = 1$$

$$a_{14} = a_{12} \text{ x } a_{23} \text{ x } a_{34} = 0.5 \text{ x } 2 \text{ x } 1 = 1$$

$$a_{15} = a_{12} \text{ x } a_{23} \text{ x } a_{34} \text{ x } a_{45} = 0.5 \text{ x } 2 \text{ x } 1 \text{ x } 1 = 0.25$$

$$a_{24} = a_{23} \text{ x } a_{34} = 2 \text{ x } 1 = 2$$

$$a_{25} = a_{24} \text{ x } a_{34} \text{ x } = 2 \text{ x } 0.25 = 0.5$$

$$a_{35} = a_{34} \text{ x } a_{45} = 1 \text{ x } 0.25 = 0.25$$

To allow for a certain degree of inconsistency, the transitivity rule (7) is supplemented by en error term, *e*.

$$\boldsymbol{a_{i,j}} = \boldsymbol{a_{i,i+1}} \cdot \boldsymbol{a_{i+1,i+2}} \cdot \ldots \cdot \boldsymbol{a_{j-1,j+1}} \pm \frac{e \cdot h}{100} \tag{8}$$

where,

$\boldsymbol{a_{ij}}$ = comparison between alternative i and j

$\boldsymbol{e}$ = tolerated error

$\boldsymbol{h}$ = highest value of the comparison scale

4. *Reciprocal comparisons* can be deduced using the reciprocity property (cardinal scale) as shown in (2). We obtain:

$$\begin{bmatrix} 1 & 0.5 & 1 & 1 & 0.25 \\ 2 & 1 & 2 & 2 & 0.5 \\ 1 & 0.5 & 1 & 2 & 0.25 \\ 1 & 0.5 & 1 & 1 & 0.25 \\ 4 & 2 & 4 & 4 & 1 \end{bmatrix}$$

It is apparent that a square matrix *(n x n)*, where $n \geq 3$ must also take in account the transitivity property, otherwise the eigenvector calculation will obtain wrong results.

Example 5: *n=4, consistent, full consistency correction*

Suppose we have a pairwise comparison matrix composed of four alternatives:

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

If we wish to obtain a consistent matrix, then we would require *n – 1* comparisons. That means that the matrix will be filled with the 3 independent ratios:

$$\begin{bmatrix} 1 & 1/4 & & \\ & 1 & 4 & \\ & & 1 & 1/5 \\ & & & 1 \end{bmatrix}$$

3 transitive:

$a_{13} = a_{12} \text{ x } a_{23} = 1/4 \text{ x } 4 = 1$

$a_{24} = a_{23} \text{ x } a_{34} = 4 \text{ x } 1/5 = 4/5$

$a_{14} = a_{12} \text{ x } a_{23} \text{ x } a_{34} = \frac{1}{4} \text{ x } 4 \text{ x } 1/5 = 1/5$

$$\begin{bmatrix} 1 & 1/4 & 1 & 1/5 \\ & 1 & 4 & 4/5 \\ & & 1 & 1/5 \\ & & & 1 \end{bmatrix}$$

the remaining 6 redundant elements are filled with the reciprocals

$$\begin{bmatrix} 1 & 1/4 & 1 & 1/5 \\ 4 & 1 & 4 & 4/5 \\ 1 & 1/4 & 1 & 1/5 \\ 5 & 5/4 & 5 & 1 \end{bmatrix}$$

Example 6: *n=4, inconsistent, partial consistency correction*

On the othe hand, if we had *n* comparisons, the matrix will be filled with the 4 independent ratios:

$$\begin{bmatrix} 1 & 1/4 & & \\ & 1 & 4 & \\ & & 1 & 1/5 \\ 6 & & & 1 \end{bmatrix}$$

2 transitive:

$a_{13} = a_{12} \times a_{23} = 1/4 \times 4 = 1$

$a_{24} = a_{23} \times a_{34} = 4 \times 1/5 = 4/5$

$$\begin{bmatrix} 1 & 1/4 & 1 & \\ & 1 & 4 & 4/5 \\ & & 1 & 1/5 \\ 6 & & & 1 \end{bmatrix}$$

the remaining 6 redundant elements are filled with the reciprocals:

$$\begin{bmatrix} 1 & 1/4 & 1 & 1/6 \\ 4 & 1 & 4 & 4/5 \\ 1 & 1/4 & 1 & 1/5 \\ 6 & 5/4 & 5 & 1 \end{bmatrix}$$

Example 7: *n=4, inconsistent, no consistency correction*

In this case, we would simply require *n + 2* comparisons to be made:

$$\begin{bmatrix} 1 & 1/4 & 4 & \\ & 1 & 4 & 1/4 \\ & & 1 & 1/5 \\ 6 & & & 1 \end{bmatrix}$$

and the empty elements filled with the reciprocals:

$$\begin{bmatrix} 1 & 1/4 & 4 & 1/6 \\ 4 & 1 & 4 & 1/4 \\ 1/4 & 1/4 & 1 & 1/5 \\ 6 & 4 & 5 & 1 \end{bmatrix}$$

For the purpose of the project, if the decision elements (criteria, alternatives) are three (3) we ignore the transitive element as to check for inconsistency. Whereas, if we have four (4) decision elements, then we have four independent elements and one transitive (see Example 6), for consistency check, otherwise the principal maximum eigenvalue can become smaller than the number of decision elements and produce results that are ill behaved, that is obtain eigenvalues that are smaller the the number of decision elements.

For larger decision elements , n, the number of pairwise comparisons (PC), independent, transitive and reciprocal elements for a full, partial and no consistency correction pairwise comparison matrix (PCM) are show in **Table 2**, **Table 3** and **Table 4** respectively.

FULL CONSISTENCY CHECK						
n	PCM	PC	principal	independent	transitive	reciprocal
1	1	0	0	1	0	0
2	4	1	2	1	0	1
3	9	2	3	2	1	3
4	16	3	4	3	3	6
5	25	4	5	4	6	10
6	36	5	6	5	10	15
7	49	6	7	6	15	21
8	64	7	8	7	21	28
9	81	8	9	8	28	36
10	100	9	10	9	36	45
11	121	10	11	10	45	55
12	144	11	12	11	55	66
13	169	12	13	12	66	78
14	196	13	14	13	78	91
15	225	14	15	14	91	105
16	256	15	16	15	105	120
17	289	16	17	16	120	136
18	324	17	18	17	136	153
19	361	18	19	18	153	171

Table 2: *Full Consistency Correction*

PARTIAL CONSISTENCY CHECK						
n	PCM	PC	principal	independent	transitive	reciprocal
1	1	0	0	1	0	0
2	4	1	2	1	0	1
3	9	3	3	3	0	3
4	16	4	4	4	2	6
5	25	5	5	5	5	10
6	36	6	6	6	9	15
7	49	7	7	7	14	21
8	64	8	8	8	20	28
9	81	9	9	9	27	36
10	100	10	10	10	35	45
11	121	11	11	11	44	55
12	144	12	12	12	54	66
13	169	13	13	13	65	78
14	196	14	14	14	77	91
15	225	15	15	15	90	105

Table 3: *Partial Consistency Correction*

NO CONSISTENCY CHECK						
n	PCM	PC	principal	independent	transitive	reciprocal
1	1	0	0	1	0	0
2	4	1	2	1	0	1
3	9	3	3	3	0	3
4	16	6	4	4	0	6
5	25	10	5	5	0	10
6	36	15	6	6	0	15
7	49	21	7	7	0	21
8	64	28	8	8	0	28
9	81	36	9	9	0	36
10	100	45	10	10	0	45
11	121	55	11	11	0	55
12	144	66	12	12	0	66
13	169	78	13	13	0	78
14	196	91	14	14	0	91
15	225	105	15	15	0	105

Table 4: *No Consistency Correction*

The last table shows that as the pairwise comparison matrix grows so does the pairwise comparisons and rather markedly too. This imposes the decision maker with the burden of 105 comparisons for 15 decision elements, whereas for the partial consistency correction only requires 15 comparison compared with the 14 for the full consistency correction method.

Step 4. - Aggregate the relative weights of the decision elements to arrive at an overall individual ranking.

In this step, the priorities of the lowest level alternatives relative to the top most objective are determined and displayed, which serves as a ratings of decision alternatives in achieving the objective. The final output from is the relative priorities of the bottom most (in the hierarchy) alternatives relative to the overall objective (top level of hierarchy). The composite relative weight vector of elements at the k^{th} level w.r.t. the first is computed by [3, 10]:

$$C[1,k] = \prod_{i=2}^{k} B_i \tag{9}$$

Where:

C[1,k] → vector of composite weights of elements at level k w.r.t. element on level 1.

B_i → $\mathbf{n_{i-1} \; x \; n_i}$ matrix of vector ***W***

Example 8: *Overall Individual Priorities*

Suppose we have a hierarchy defined by 3 criteria and under each criterion we have 4 alternatives, the priorities of the criteria could be:

$$\begin{bmatrix} 0,26 \\ 0.44 \\ 0,30 \end{bmatrix}$$

For the alternatives, their priorities under three criterions could be:

$$\begin{bmatrix} 0,12 \\ 0,31 \\ 0,19 \\ 0,38 \end{bmatrix} \qquad \begin{bmatrix} 0,22 \\ 0,11 \\ 0,42 \\ 0,09 \end{bmatrix} \qquad \begin{bmatrix} 0,32 \\ 0,21 \\ 0,14 \\ 0,33 \end{bmatrix}$$

This means that in order to obtain the overall individual priorities we simply concatenate the alternatives rankings matrix and multiply with the criteria ranking matrix:

i.e.

$$\begin{bmatrix} 0,12 & 0,22 & 0,32 \\ 0,31 & 0,11 & 0,21 \\ 0,19 & 0,42 & 0,14 \\ 0,38 & 0,09 & 0,33 \end{bmatrix} * \begin{bmatrix} 0,26 \\ 0,44 \\ 0,30 \end{bmatrix} = \begin{bmatrix} 0,2240 \\ 0,1920 \\ 0.2762 \\ 0,3078 \end{bmatrix}$$

Step 5. - Aggregate the overall individual priorities for each hierarchon to obtain group choice (ranking).

After having obtained the individual priorities for a particular hierarchon, we can then aggregate these results for all decision makers to obtain the group ranking. The simplest way to do this is to use Borda's [107, 108] positional method. The basic idea is to add the individual ranking matrix for every decision maker. For each individual ranking we assign one point for lowest ranked, two for the second lowest, three for the third an so forth. The Group Ranking, is obtained by adding all the points assigned for each of the individual rankings and ranking highest, this time, the rank with the most points, second with the next most points and so forth.

Given k individual rankings we choose Borda's coefficients, $r_1, r_2, r_3, \ldots, r_k$ such that,

$r_1 > r_2 > r_3 > \ldots > r_k$, for each decision maker, $c \in S$ list r_i, score $B_i(c)$ with a total Borda Score, $B(c)$ defined as:

$$B(c) = \sum_{i=1}^{k} B_i(c) \tag{10}$$

Example 10: *Group ranking*

Suppose we have 4 decision makers each with 4 individual priorities, as shown on **Table 5:** *Group rankings*:

	DM1	DM2	DM3	DM4
A1	0,1483	0,1413	0,2522	0,2083
A2	0,2480	0,2175	0,1974	0,1463
A3	0,3841	0,4159	0,3078	0,3135
A4	0,1741	0,1868	0,2030	0,2893

Table 5: *Group Rankings*

Borda's positional method, by sorting the lowest rank to the highest, would yield, as shown on **Table 6**: *Borda Ranking*:

	DM1	DM2	DM3	DM4
A1	1	1	3	2
A2	3	3	1	1
A3	4	4	4	4
A4	2	2	2	3

Table 6: *Borda Ranking*

Summing rows we get:

$$\begin{bmatrix} 7 \\ 8 \\ 16 \\ 9 \end{bmatrix}$$

Whereas, A3 is the highest ranked, A4 is the second highest, A2 is the third highest and A1 is the least highest in the Borda score.

Group AHP Example

Considering the well - known example [9, AHP Demo], which illustrate the method for selecting a new car; the decision analyst designs the particular hierarchon, to all decision makers to assign weights for the criteria and alternatives on the fundamerntal scale to compute individual rankings so as to obtain the group ranking.

Step 1 .- Hierarchon

A starting point is to consider the simplest of hierarchons and that is the selection of a new car, [9, AHP Demo] as illustrated from ExpertChoice. A brief summary is given below:

- **STATE THE OBJECTIVE**

i.e. Select a new car

- **DEFINE THE CRITERIA**

i.e. Style, Reliability, Fuel Economy (consumption in liters/km)

- **PICK THE ALTERNATIVES**

i.e. Civic, Saturn , Escort, Miata

A tree structure of the hierarchy is shown in **Figure 4:** *Hierarchon*

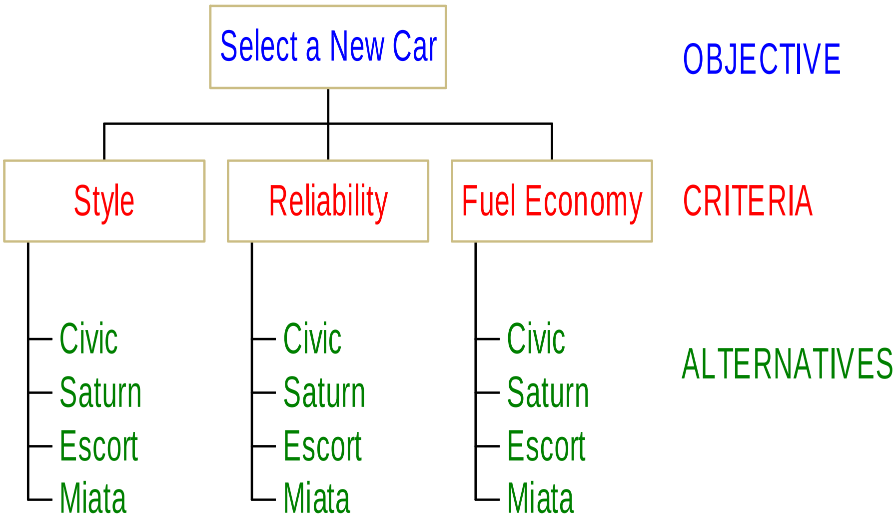


Figure 4: *Hierarchon*

We note the simplicity of the method and the clarity of thought it offers. Provided that the number of criteria and alternatives are kept to the aforementioned minimum, we avoid the main disadvantages of the method, as will be discussed in the proceeding section. Other hierarchons included in the project are:

1. Select House
2. Select Employee
3. Select College
4. Select Database

Step 2. - Pairwise Comparison Matrix (PCM) and Consistency Ratio (CR)

Here, we determine the ranking of criteria, using the fundamental scale.

e.g.

1. Reliability is 2 times more important than style
2. Style is 3 times more important than fuel economy
3. Reliability is 4 times more important than fuel economy

Which is **not** consistent. If we assume 1. and 2. are correct, then *Reliability should be 6 times more important than fuel economy*, to obtain a consistent matrix (for a proof see example described after the inconsistent determination).

i.e. for the *inconsistent* determination,

$$\begin{array}{cccc} & Style & \mathrm{Re}liability & Economy \\ Style & & 1/2 & \\ \mathrm{Re}liability & & & 4 \\ Economy & 1/3 & & \end{array}$$

Comparing Reliability with Reliability, Style with Style and Economy with Economy, we obtain:

$$\begin{array}{cccc} & Style & \mathrm{Re}liability & Economy \\ Style & 1 & 1/2 & \\ \mathrm{Re}liability & & 1 & 4 \\ Economy & 1/3 & & 1 \end{array}$$

Taking the Reciprocals, we have:

$$\begin{array}{cccc} & Style & \mathrm{Re}liability & Economy \\ Style & 1 & 1/2 & 3 \\ \mathrm{Re}liability & 2 & 1 & 4 \\ Economy & 1/3 & 1/4 & 1 \end{array}$$

i.e. The Pairwise Comparison Matrix:

$$\begin{bmatrix} 1/1 & 1/2 & 3/1 \\ 2/1 & 1/1 & 4/1 \\ 1/3 & 1/4 & 1/1 \end{bmatrix}$$

It is important to determine the Consistency Ratio (***CR***) which is obtained from the approximation method: i.e. the *inconsistent* matrix,

$$\begin{bmatrix} 1/1 & 1/2 & 3/1 \\ 2/1 & 1/1 & 4/1 \\ 1/3 & 1/4 & 1/1 \end{bmatrix}$$

Convert to decimals, normalize and obtaining sum of rows and average of rows:

$$\begin{bmatrix} 1.0000 & 0.50000 & 3.0000 \\ 2.0000 & 1.0000 & 4.0000 \\ 0.3333 & 0.2500 & 1.0000 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.3 & 0.29 & 0.375 \\ 0.6 & 0.57 & 0.5 \\ 0.1 & 0.14 & 0.125 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.965 \\ 1.67 \\ 0.365 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.32 \\ 0.56 \\ 0.12 \end{bmatrix}$$

Gives:

$$\frac{1}{3} * \begin{bmatrix} 0.965 \\ 1.67 \\ 0.365 \end{bmatrix} \neq \begin{bmatrix} 0.32 \\ 0.56 \\ 0.12 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 1/1 & 1/2 & 3/1 \\ 2/1 & 1/1 & 4/1 \\ 1/3 & 1/4 & 1/1 \end{bmatrix} * \begin{bmatrix} 0.32 \\ 0.56 \\ 0.12 \end{bmatrix} = \begin{bmatrix} 0.96 \\ 1.68 \\ 0.37 \end{bmatrix}$$

Dividing,

$$\begin{bmatrix} 0.96/0.32 \\ 1.68/0.56 \\ 0.37/0.12 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3.0833 \end{bmatrix}$$

λmax*= (3+3+3.083) / 3 =3.028*

CI*=(3.028 -3)/ 2 = 0.014*

CR*=0.014 / 0.58=0.024*, *2.4 % inconsistent*. (acceptable).

For the *consistent* matrix, likewise, we can obtain from the approximation method:

i.e. for the *consistent* matrix,

$$\begin{bmatrix} 1/1 & 1/2 & 3/1 \\ 2/1 & 1/1 & 6/1 \\ 1/3 & 1/6 & 1/1 \end{bmatrix}$$

Convert to decimals, normalizing and obtaining sum of rows and average of rows:

$$\begin{bmatrix} 1.0000 & 0.50000 & 3.0000 \\ 2.0000 & 1.0000 & 6.0000 \\ 0.3333 & 0.1667 & 1.0000 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.3 & 0.3 & 0.3 \\ 0.6 & 0.6 & 0.6 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.9 \\ 1.8 \\ 0.3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.3 \\ 0.6 \\ 0.1 \end{bmatrix}$$

Gives:

$$\frac{1}{3} * \begin{bmatrix} 0.9 \\ 1.8 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 0.3 \\ 0.6 \\ 0.1 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 1/1 & 1/2 & 3/1 \\ 2/1 & 1/1 & 6/1 \\ 1/3 & 1/6 & 1/1 \end{bmatrix} * \begin{bmatrix} 0.3 \\ 0.6 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 1.8 \\ 0.3 \end{bmatrix}$$

Dividing,

$$\begin{bmatrix} 0.9/0.3 \\ 1.8/0.6 \\ 0.3/0.1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

λmax= *(3+3+3) / 3 =3*

CI=*(3 -3)/ 2 = 0*

CR=*0 / 0.58=0, 0 % inconsistent*. (*consistent*) *Q.E.D.*

Step 3. - Priorities Matrix - Eigenvector

Using pairwise comparison, from Step 2., the relative importance of one criterion over another is expressed, having obtained the pairwise comparison matrix:

$$\begin{bmatrix} 1/1 & 1/2 & 3/1 \\ 2/1 & 1/1 & 4/1 \\ 1/3 & 1/4 & 1/1 \end{bmatrix}$$

Convert to decimals:

$$\begin{bmatrix} 1.0000 & 0.50000 & 3.0000 \\ 2.0000 & 1.0000 & 4.0000 \\ 0.3333 & 0.2500 & 1.0000 \end{bmatrix}$$

To get the priorities vector from a pairwise matrix we use the Eigenvector:

1. Square the pair wise matrix, **A*A**

$$\begin{bmatrix} 1.0000 & 0.50000 & 3.0000 \\ 2.0000 & 1.0000 & 4.0000 \\ 0.3333 & 0.2500 & 1.0000 \end{bmatrix} \times \begin{bmatrix} 1.0000 & 0.50000 & 3.0000 \\ 2.0000 & 1.0000 & 4.0000 \\ 0.3333 & 0.2500 & 1.0000 \end{bmatrix}$$

gives, $\mathbf{A^2}$

$$\begin{bmatrix} 3.0000 & 1.7500 & 8.0000 \\ 5.3332 & 3.0000 & 14.0000 \\ 1.1666 & 0.6667 & 3.0000 \end{bmatrix}$$

2. Find the sum rows then the sum column and Normalize.

$$\begin{bmatrix} 3.0000 & 1.7500 & 8.0000 \\ 5.3332 & 3.0000 & 14.0000 \\ 1.1666 & 0.6667 & 3.0000 \end{bmatrix} \begin{matrix} = 12.7500 & 0.3194 \\ = 22.3332 & 0.5595 \\ = \underline{4.8333} & \underline{0.1211} \\ 39.9165 & 1.0000 \end{matrix}$$

3. Results to the Ranking Matrix or the **Eigenvector:**

$$\begin{bmatrix} 0.3194 \\ 0.5595 \\ 0.1211 \end{bmatrix}$$

The accuracy of the results obtained can be improved by iteration of step 1. over matrix $\mathbf{A^2}$. The number of iterations will depend on the desired accuracy.

Stated verbally,

$$\begin{array}{r} Style \\ \text{Re}\,liability \\ Economy \end{array}\begin{bmatrix} 0.3194 \\ 0.5595 \\ 0.1211 \end{bmatrix} \Rightarrow \begin{bmatrix} 32\% \\ 56\% \\ 12\% \end{bmatrix}$$

Reliability is the highest ranked criterion, with style in the second place and Economy in the third.

Using the above procedure the rankings of the alternatives can also be deduced for each criterion. The possible Pairwise Comparison Matrix (PCM) of the alternatives for the style criterion would yield:

$$\begin{bmatrix} 1 & 1/2 & 1 & 1/2 \\ 2 & 1 & 2 & 2/3 \\ 1 & 1/2 & 1 & 1/3 \\ 2 & 3/2 & 3 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0{,}1625 \\ 0{,}2922 \\ 0{,}1461 \\ 0{,}3941 \end{bmatrix}$$

Miata is the highest ranked

$\lambda_{max} = 4{,}0014$

$CR = 0{,}0005$

For the reliability criterion:

$$\begin{bmatrix} 1 & 1/5 & 1/25 & 1 \\ 5 & 1 & 1/5 & 3/5 \\ 25 & 5 & 1 & 3 \\ 1 & 5/3 & 1/3 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0{,}0466 \\ 0{,}1353 \\ 0{,}6764 \\ 0{,}1438 \end{bmatrix}$$

Escort is the highest ranked

$\lambda_{max} = 4{,}2571$

$CR = 0{,}0952$

For the economy criterion, which could be a quantitative (objective) measurement, would yield by direct normalization:

$$\begin{array}{c} \begin{bmatrix} 34 \\ 24 \\ 27 \\ 29 \end{bmatrix} \\ \overline{114} \end{array} \Rightarrow \begin{bmatrix} 34/114 \\ 24/114 \\ 27/114 \\ 29/114 \end{bmatrix} \Rightarrow \begin{bmatrix} 0{,}2982 \\ 0{,}2105 \\ 0{,}2368 \\ 0{,}2544 \end{bmatrix}$$

Here, special care must be taken, as it seems Civic is the highest ranked car but it is actually Saturn as it has the lowest fuel consumption compared to the rest. In order to solve for this paradigm, we simply order the ranks accordingly. That is, the final ranking should look like:

$$\begin{bmatrix} 0{,}2105 \\ 0{,}2982 \\ 0{,}2544 \\ 0{,}2368 \end{bmatrix}$$

Step 4. - Individual Priorities

To obtain individual overall priorities, we concatenate the aternatives rankings and multiply with the criteria ranking:

$$\begin{bmatrix}0,1625\\0,2922\\0,1461\\0,3941\end{bmatrix} \quad \begin{bmatrix}0,0466\\0,1353\\0,6764\\0,1438\end{bmatrix} \quad \begin{bmatrix}0,2105\\0,2982\\0,2544\\0,2368\end{bmatrix} \Rightarrow \begin{bmatrix}0,1625 & 0,0466 & 0,2105\\0,2922 & 0,1353 & 0,2982\\0,1461 & 0,6764 & 0,2544\\0,3941 & 0,1438 & 0,2368\end{bmatrix}$$

$$\begin{bmatrix}0,1625 & 0,0466 & 0,2105\\0,2922 & 0,1353 & 0,2982\\0,1461 & 0,6764 & 0,2544\\0,3941 & 0,1438 & 0,2368\end{bmatrix} * \begin{bmatrix}0.3194\\0.5595\\0.1211\end{bmatrix} = \begin{bmatrix}0,0905\\0,1898\\0,4598\\0,2203\end{bmatrix}$$

i.e. the particular decision maker shows a strong preference for Escort.

Step 5. - Group Ranking

The overall group ranking is obtained by collecting all individual rankings, applying Borda's positional method and then summing the rows:

i.e. four possible individuals with four possible individual rankings,

$$\begin{bmatrix}0,0905\\0,1898\\0,4598\\0,2203\end{bmatrix} \quad \begin{bmatrix}0,0905\\0,1898\\0,4598\\0,2203\end{bmatrix} \quad \begin{bmatrix}0,1419\\0,9420\\0,2900\\0,4343\end{bmatrix} \quad \begin{bmatrix}0,1617\\0,1563\\0,3893\\0,2533\end{bmatrix}$$

Borda's positional method (from the lowest rank to the highest),

$$\begin{bmatrix}1\\2\\4\\3\end{bmatrix} \quad \begin{bmatrix}1\\2\\4\\3\end{bmatrix} \quad \begin{bmatrix}2\\1\\3\\4\end{bmatrix} \quad \begin{bmatrix}2\\1\\4\\3\end{bmatrix}$$

Summing rows,

$$\begin{bmatrix}6\\6\\15\\13\end{bmatrix}$$

This gives a third rank draw for Civic and Saturn, with Miata in the second place and Escort in the first. In other words, the group shows a strong preference for Escort. In the case of draw, the group would reconsider the draw alternatives to distinguish between the two.

Advantages

The main strengths of the AHP include: [10, 12]:

1. Uniqueness of the method.
2. Clarity is provided by the formal structuring of the hierarchons.
3. Priorities Elicitation using pairwise comparisons implements an easy to use verbal mathematical terminology.
4. Consistency tests are provided by requiring more pairwise comparisons to be made than required. i.e. To compare the importance of n criterias only n-1 comparisons are reuired, whereas the AHP makes n comparisons. For criteria A, B and C requires two comparisons; A with B, B with C. Consistency requires three comparisons; A with B, B with C and A with C.
5. Judgments can be derived where otherwise it may be difficult to obtain.

Disadvantages

The main criticisms of the AHP, summarized briefly, include: [10, 12].

Hierarchy

The failure to distinguish between criteria and alternatives reduces the clarity with which a problem is perceived.

1. Criteria must be independent entities to represent the decision problem as thoroughly as possible.
2. Alternatives must provide similar coverage over each criterion to be independent entities.

Fundamental Scale

The pairwise comparison matrix is not considered to provide accurate measurements

3. Verbal judgements impose a quantitative scale. i.e. If A is weakly more important than B, implies that A is three times more important than B.
4. If criteria result in different scale, false judgements arise. i.e. Pollution level measured in billionths, relative to costs measured in billions of dollars and potential impact on jobs measured in life expectancy in years e.t.c.
5. The nine-point scale does not always represent real world problems faced by decision makers
6. There are major questions as to whether AHP can cope appropriately with intensity measures due to rank reversal.
7. The concept of zero value is not possible with AHP.

Pairwise comparisons

8. No agreement in the academic community on how to aggregate paired ratios and convert them into weights.
9. The number of comparisons that need to be made can make the method extremely time consuming. i.e. 5 criteria compared with respect to 5 alternatives would need 60 pairwise comparisons to be made.
10. No more than 9 criteria should be included in any cluster due to the effort neede, as stated above, and due to the problem of rank reversal.
11. Consistency is obtained not from the direct application of judgment, but from the need to be consistent. Anyone using AHP is warned by practitioners in the field that consistency should not be used to drive judgments, otherwise inappropriate weights will result.
12. Pairwise comparison is, in the end, the prerogative of the decision maker, and not the result of a black box elicitation and calculation process.

Consistency

13. Inconsistency in AHP can be generated directly as a result of the limits of the ratio scale - and have nothing to do with human inconsistency.
14. Psychological evidence indicates humans cannot be consistent beyond 5-9 comparisons. AHP can then only be used for 4 or fewer criteria at time, if confidence is needed in the result.
15. *"The consistency ratio is based on a Random Index (RI) is calculated by generating random matrices for a normal distribution which is not the case fot the AHP. The following table shows the results differ by orders of magnitude, not simply a few points. At five criteria, it is an order of magnitude, at six nearly two orders of magnitude, and seven three orders of magnitude. The results clearly show these assumptions are flawed, and suggest that the AHP process governs the judgments, not judgments govern the AHP. The value for 8 criteria and above was not done simply because the computing effort to find a statistically significant number of matrices for the calculation was beyond computing resources"* [12].

Number of crtieria in cluster	**Theoretical Probability to get consistency** ***ratio <0.1***	**Experimental Results** **to get consistency** ***ratio <0.1***
3	2.52E-001 2.19E-	001
4	1.07E-001 2.88E-	002
5	2.61E-002 2.24E-	003
6	2.45E-003 8.90E-	005
7	1.29E-004 8.45E-	007
Comparison of theoretical and experimental results of probability of meeting consistency ratios <0.1 as the number of criteria increases		

Table 7: *Consistency Distribution*

Eigenvalue / Eigenvector

16. Despite being descriptive, the AHP is based on a normative interpretation of the human experience - that is, the mathematics are made to reflect human thinking processes.
17. The eigenvector method for obtaining weights will not be transparent to most decision makers.

Rank Reversal

18. Removal of an alternative leads to rank reversal. In performance related measures, this is not an acceptable result since performance is measured on an absolute scale relative to a ideal performance.
19. The addition of a new alternative to a decision problem can also lead to a reversal of the rankings of the original options. However, Forman argues that this is a strength of the method [9].
20. The ideal mode which is used to avoid rank reversal is still not well understood.

DSS Software Listings

A listing of software vendors, include :

Software	Vendor	Web site
AIM	Advanced Information Managment	http://www.aimworld.com
AIMMS 3	Paragon Decision Tech.	www.aimms.com
Analytica	Lumina Decision Systems	www.lumina.com
Aspen MIMI	Aspen Technology Inc.	www.aspentech.com
Criterium Decision Plus 3.0	InfoHarvest Inc.	www.infoharvest.com
Crytall Ball 2000 Pro. Edition	Decisioneering, Inc.	www.decisioneering.com
DATA 3.5	TreeAge Software, Inc.	www.treeage.com
Data Interactive	TreeAge Software, Inc.	www.treeage.com
Decision Explorer	Baxia Software Ltd	www.banxia.com
Decision Hosting	InfoHarvest Inc.	www.infoharvest.com
Decision Tools Suite Pro. 4.0	Palisade Corporation	www.palisade.com
DPL	Applied Decision Analysis	www.adainc.com
ELECTRE 3-4	LAMSADE Softwares	www.lamsade.dauphine.fr
ELECTRE IS	LAMSADE Softwares	www.lamsade.dauphine.fr
ELECTRE TRI	LAMSADE Softwares	www.lamsade.dauphine.fr
EQUITY 2 for Windows	Enterprise LSE Ltd.	www.enterprise-lse.co.uk
Expert Choice Enterprise	EXPERT CHOICE, Inc	www.expertchoice.com
EXSYS Developer & Runtime	EXSYS, Inc	www.exsys.com
Exys Corvid	EXSYS, Inc	www.exsys.com
Frontier Analyst	Baxia Software Ltd	www.banxia.com
High Priority	Krysalis, Ltd	www.krysalis.co.uk
HIVIEW 2 for Windows	Enterprise LSE Ltd.	www.enterprise-lse.co.uk
Hugin Professional	Hugin Expert	www.hugin.com
Impact Explorer	Baxia Software Ltd	www.banxia.com
Joint Gains	Systems Analysis Laboratory	www.decisionarium.hut.fi
Logical Decisions for Windows	Logical Decisions	www.logicaldecisions.com
MACBETH	MACBETH	http://www.m-macbeth.com/Msite.html
MARS	Netka Advanced Networking	http://www.netka.com/mars/default.asp
Mesa Vista	Mesa Systems Guild, Inc.	www.mesasys.com
MIIDAS	LAMSADE Softwares	www.lamsade.dauphine.fr
MINORA	LAMSADE Softwares	www.lamsade.dauphine.fr
Netica	Norsys Software Corp.	www.norsys.com
On Balance	Krysalis, Ltd	www.krysalis.co.uk
On Balance Runtime	Krysalis, Ltd	www.krysalis.co.uk
Opinions Online	100GEN Inc.	www.opinions-online.com
ORESTE	LAMSADE Softwares	www.lamsade.dauphine.fr
Policy PC Judgment Analysis	Executive Decision Services LLC	www.albany.net/~sschuman/PolicyPC
PREFCALC	LAMSADE Softwares	www.lamsade.dauphine.fr
PRIME Decisions	Systems Analysis Laboratory	www.decisionarium.hut.fi
PROMCALC	LAMSADE Softwares	www.lamsade.dauphine.fr
Risk Detective	Rhythm Technology, Inc.	www.riskdetective.com
TreePlan	Decision Support Services	www.treeplan.com
UTA, UTA II	FINCLAS	http://www.dpem.tuc.gr/fel/finclas.htm
VIMDA	VIMDA	http://www.numplan.fi/vimda
Web HIPRE	Systems Analysis Laboratory	www.decisionarium.hut.fi
WINPRE	Systems Analysis Laboratory	www.decisionarium.hut.fi

Table 8: *Software Vendors*

Research and development in multicriteria analysis since first introduced in 1970 [Roy, 1990] are classified into two streams the American and the European. The American stream is concerned with MultiCriteria Decision Making Systems (MCDMS) which is based on a value system and aims to construct a value function, aggregating the partial preferences on multiple criteria, [Keeney and Raiffa, 1976, Saaty, 1990] the decision maker is directed by this value system by her/his final decision. The project is based solely on this stream.

On the other hand, the European stream is concerned with MultiCriteria Decision Aiding Systems (MCDAS) which uses the Disaggregation – Aggregation approach, initially presented by Jacquet – Lagreze and Siskos in 1982. In the disaggregation phase a preference model is constructed based on the decision maker's judgement policy based on a limited set of reference actions that are well known to her/him. The information induced in the aggregation phase thence, constructs a value (utility) function as in the American stream. Initial development of such system was firstly proposed by Hammond in 1978 with POLICY DSS. In 1982 Jacquet – Lagreze and Siskos improved the methodological approach of POLICY using the UTA (Utilite Additive) method, which uses the parameters of an additive utility function from the multicriteria table and the decision maker's judgement policy (weak preference) on a reference set of alternatives using linear programming techniques. The variants of the UTA method include PREFCALC by Jacquet – Lagreze in 1984, MINORA by Siskos and Yannacopoulos in 1985, and MIIDAS by Spyridakos in 1996. Other researchers that have been working on this stream include: Siskos, 1986, Stewart, 1987, Siskos and Zoupounidis, 1987, Despotis, 1990, Hadzinakos, 1991, Siskos and Matsatsinis, 1992, Despotis and Zoupounidis, 1993, Spyridakos and Yannacopoulos, 1993, Husson and Zoupounidis, 1993, Spyridakos, 1996.

In the past twenty years the various methodologies for solving multicriteria decision making problems have now been classified into three theoretical approaches:

1. Value System Approach

e.g.

(MARS, VIMDA, MAUT, EXPERT CHOICE (AHP))

2. Outranking Relation Approach

e.g.

(ELECALC, ELECTRE, PROMETHEE, PROMCALC, GAIA, ORESTE)

3. Dissagregation – Aggregation Approach

e.g.

(PREFCALC, UTA, MINORA, MIIDAS)

When AHP is compared to ELECTRE (ELimination and ChoicE Translating REality), TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), MACBETH (Measuring Attractiveness by Categorical Based Ecaluation Technique) and ANP (Analytic Network Process) [8], the results [17] show that the AHP has several advantages over all other methods except ANP, which is an evolution of AHP. This paper will concentrate on the AHP methodology primarily for its' practical widespread use worldwide by business and government sectors, its' ease of use as a multicriteria decision analysis tool, despite the inherent criticisms it possesses and due to the mere fact, that the Analytic Hierarchy Process appears to open a window of opportunity in the area of group decision making. The most well known DSS tool, using the AHP, is ExperChoice [9,44] which has being analyzed and considered in detail.

Other interesting software vendors can be found at D. J. Powers site [58], Bitpipe IT Research [59], Centre of Information and Organization Studies [60], Decision Analysis Society [66], Technical University of Crete [65], and Science Direct [62].

Applications of the AHP

Since its introduction the AHP methodology has been applied in a wide range of areas Zahedi [10] and the Hierarchon [2] show these applications have included:

1. Non-profit organization Strategy.
2. Government/Public Strategy.
3. Business/Corporate Strategy.
4. Political Strategy.
5. Military Strategy.
6. Business/Corporate Policy.
7. Government/Public Policy.
8. Healthcare Industry.
9. Personal.
10. Planning.
11. Forecasting.
12. Benefits/Costs
13. Risk Assesment.

More unusual applications include prediction of the winner of the world chess championships, and horse race prediction (the winners of 4 out of 9 races were correctly predicted) [10]. Saaty also claims to have used the method to predict correctly the winner of every US Presidential election since 1976.

Other recent applications include:

1. Undergraduate curriculum evaluation [30].
2. Student peer evaluation [45].
3. Planning and Analysis [61].
4. Performance evaluation [64].
5. Employee ranking and selection. [11].
6. Natural resources [33].
7. Selecting best irrigation methods [32].
8. Geographical representation [34].
9. Health medicine [27, 28].
10. Risk Analysis [23].
11. Forecasting (see Belton V and Goodwin, International Journal of Forecasting, 1996 [22] for a critique) [41].

Actors

Decision making today has become more complicated and difficult, because the technological revolution, although it has improved communication and collaboration, has on the other hand, created an information overload as data and information is accumulated at an unprecedented rate, increasing uncertainty and ambiguity. Organizations, hence, use Decision Analysis – a methodology for facilitating high quality decisions - conscious and irrevocable allocation of resources with the purpose of achieving a desired objective [54].

A good decision is one that is logically consistent with our state of information and incorporates the possible alternatives with associated probabilities and potential outcomes in accordance with our risk attitude.

It is an iterative process of gaining insights and promoting creative alternatives to help decision makers to make better decision, by aiding them in comprehending the state of knowledge in which decision is being made with redundant data and/or information. Usually, many people are involved in making a decision on a single objective. In real world problems, there are multiple oblectives that a decision maker must consider; and as organizations evolve from hierarchical and dictatorical to a more decentralized one, the single decision maker is being replaced with multiple decision makers, where many decisions are made by consensus between group of managers and/or technical experts.

For simplicity we wil, consider the Decision Analyst (DA), that is, select committee member and the Decision Maker (DM), a member of the committee.

Decision Analyst (DA)

This actor is responsible for carrying out Decision Analysis, decomposing problems into smaller ones (divide and conquer rule) and provide tools and processes which help facilitate a dialogue with the decision maker about thei preferences and objectives. A ten step process called the Scalable Decision Process (SDP) incorporates three frameworks –Structuring, Evaluation and Agreement, can be used in the Decision Analysis phase.

For the purpose of the project, the decision analyst's main acrivities include:

1. Administrator / End-user of the relational database system:
2. Creates Hierarchon Trees, can even assign default weights for criteria and alternatives.
3. Assigns Objectives (hierarchons) to Decision Makers

Decision Maker (DM)

The decision maker is the key factor for decision. A decision maker is anyone with the authority to allocate the necessary resources for the decision being made. Identifying the correct decision maker ensures commitment to the chosen course of action. The decision maker is involved with the decision analyst to provide preferences and establish the decision criteria.

For the purpose of the project, the decision maker's main acrivities include:

1. Assigning criteria and alternatives weights for each objective (hierarchon) provided in consesus with the decision analyst.
2. This software project suggests each decision maker to login with a username and password and assign individually their corresponding weights, as to be able to aggregate individual judgements and/or construct a group choice
3. Once the decision maker logins she/he is immediately faced with a choice of objectives (hierarchons) as provided in consensus with the decision analyst:

DATABASE MODELLING

The whole idea is to be able to create a relational database and map data hierarchically, by successive use of one-to-many relationships of the decision elements assigned to a many-to-many relationship of the decision makers. Jablonsky's [69] hierarchy model is modified, in order that the relational database would accommodate individual judgements in a hierarchy and thus demonstrate group decision support.

Requirements

For the requirements of the database we devise a Logical Data Model, its Relational Schema and Data Dictionary accordingly. Hence, we proceed to the Physical Data Model and create the required Tables, Forms, Queries that binds the complete Database System for MCGDSS. Thereafter, the database is implemented using sample data (hierarchons) and group members (decision makers) which are input by the decision analyst and its ease of use is measured with reference to the operational instructions (see: GDSS Application) provided for each decision maker. Particular attention is paid to the security matters of such a system. Therefore, each decision maker is password authenticated to access the database according to their job function and most importantly to assure that each individual (decision maker) would produce pairwise comparisons (judgements) uninfluenced by the other decision makers.

Logical Design

The actors involved comprise of the:

1. Decision Makers (DM)
2. Admininistrators (Admin)
3. Decision Analysts (DA)

The primary role of the decision maker and her/his role in decision making includes:

1. Structure decision hierarchies (hierarchons) in consensus with the decision analyst.
2. Input pairwise comparisons of criteria and alternatives, considering Consistency Ratio (*CR*) in each case.
3. Obtain priorities of criteria and alternatives.
4. Obtain overall (individual) priorities.
5. Obtain the group choice (ranking)

The administrators role is more diverse, they are solely responsible, yet exluded from the decision making process, for:

1. Allocating decision makers and decision analysts in their respective job functions
2. Allocate hierarchons to appropriate decision makers in consensus with the decision analyst.

This is particularly important, as every decision maker should be able to make their individual judgement without being influenced by the judgements of other decision makers [68]. The role of the administrator is indicative implying their role should the application be deployed in a network environment (intranet / internet / extranet)

The decision analysts are responsible for the functional parts of the decision making process, applying methods of the SDP (Scalable Decision Process) [54]. Their main activities include:

1. Design hierarchies in consensus with the decision maker.
2. Allocate hierarchons to decision makers in consensus with the administrator.

As this is a prototype the database management network environment which is proposed goes beyond the scope of the project, hence, the administrators role here is, we repeat, indicative rather than being included in the requirements of this project. For simplicity we are only dealing with the decision analysts and the decision makers, whom are members of the committee and are directly involved with group decision making of alternatives under multiple criteria.

Summary of the job functions at each level is shown in **Table 9:** *Job Functions* :

member	***Function***
Decision Maker	Define hierarchon
	Pairwise comparisons and Consistency Ratio
	Ranking (Eigenvector) of criteria and alternatives
	Obtain overall ranking
	View group ranking
Administrator	Allocate decision analysts and makers.
	Allocate hierarchons to decision makerss
Decision Analyst	Define hierarchons
	Allocate hierarchons to decision makers.s

Table 9: *Job Functions* :

Each member should be authenticated in order that they may only have access only according to their specific job functions. Further security can be achieved by setting User Groups and Group Policies in the database. Although such setting is beyond the scope of this project, nevertheless, we should bear it in mind for future implementation.

Logical Data Model

The *Entity Relationship* diagram [46] below depicts the conceptual process where a decision maker, each time, can make a decision based on the hierarchon, at hand, composed of an objective, criteria and alternatives.

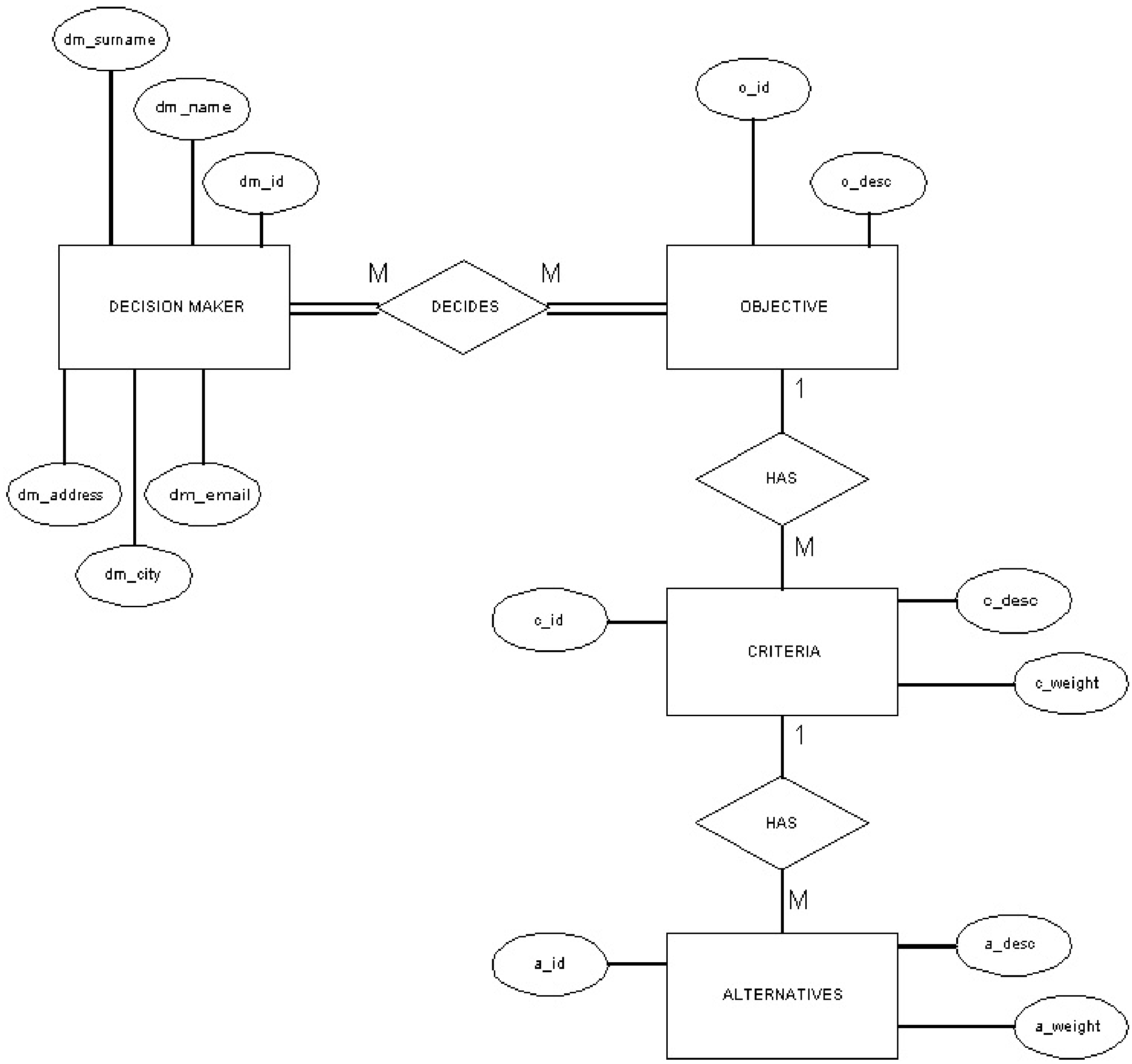


Figure 5: *Entity Relationship*

It can be seen that the hierarchon, which include the entities Objective, Criteria and Alternatives have a one to many relationship between them, that is, one objective can have many criteria and each criterion can have many alternatives. The hierarchon, on the other hand, is related to the decision maker with a many to many relationship, implying this time, that a decision maker can be assigned many hierarchons and that a hierarchon can belong to many decision makers.

Except for their identification number (id), the hierarchon is defined with a description and their weight for the criteria and alternatives. The decision maker is defined with an identification number, name, surname, address, city and e-mail.

Relational Schema

After Normalization, the following *Entity Relational Schema* [46] is produced. Because a decision maker may make many decisionsl and many decisions may belong to a specific decision maker, an intermediate UserObj table is used to create the many to many relationship.

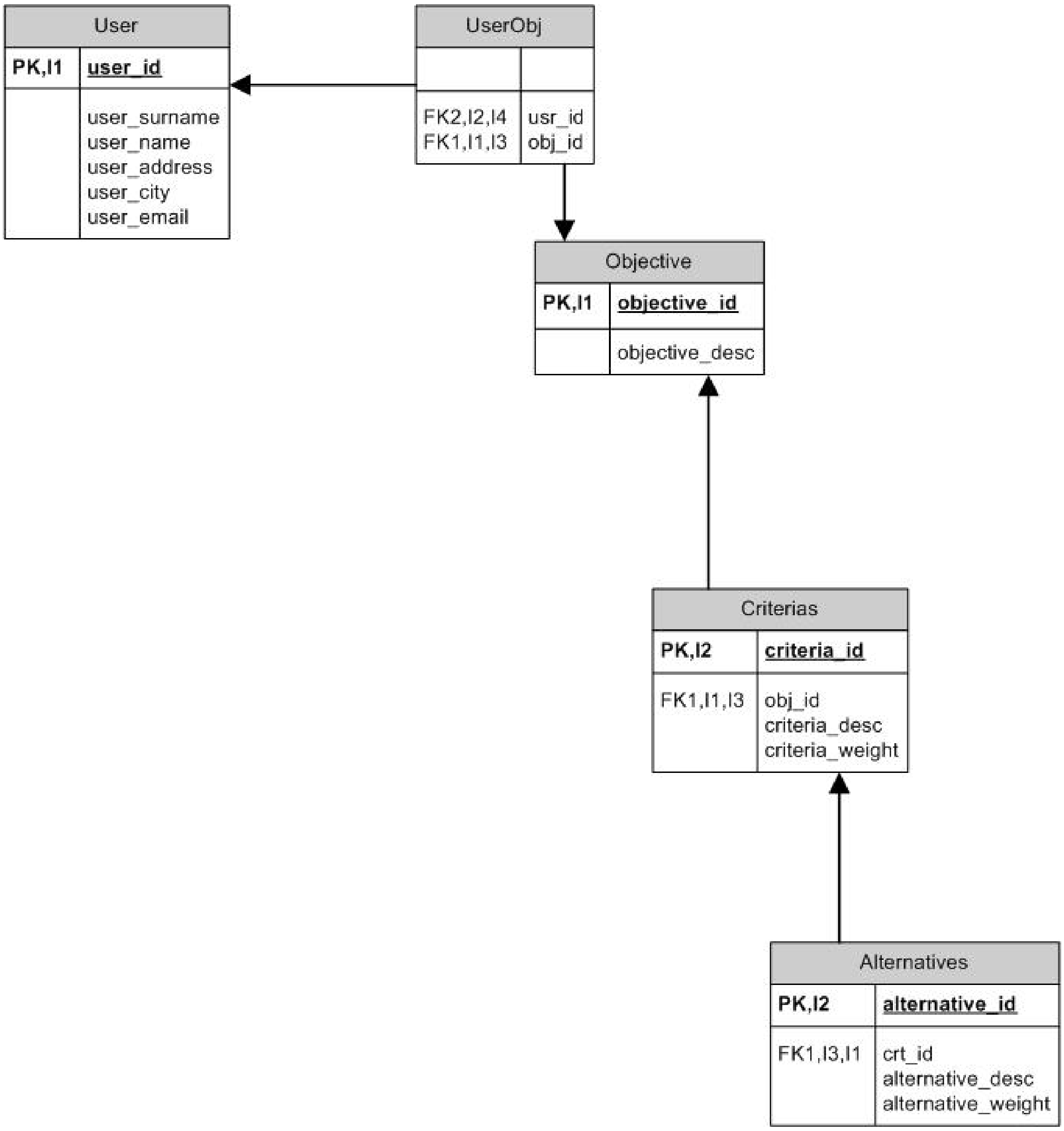


Figure 6: *Entity Relational Schema*

We observe that the foreign key usr_id of the UserObj table is related to the primary key user_id of the User table and the foreign key obj_id of the UserObj table is related to the Objective_id of the Objective table. Also, the foreign key obj_id of the Criteria is also related to the primary key objective_id of the Objective table. Finally, the foreign key crt_id of the Alternatives table is related to the primary key primary key criteria_id of the Criteria table.

Data Dictionary

Using Microsoft's Visio, we obtain:

Table Report:

Target DBMS:	**Microsoft Access 97**
Number of tables:	12
Number of columns:	93
Number of indexes:	12
Number of foreign keys:	4

Tables	Columns	Indexes	Foreign keys
UserObj	2	4	2
User	6	1	0
Objective	2	1	0
Criterias	4	3	1
Alternatives	4	3	1
qrytblMain	15	0	0
qryAlt	6	0	0
qryCount	4	0	0
qryMain	11	0	0

Table 10: *Data Dictionary*

The complete Data Dictionary continues at Appendix II: Data Dictionary

Physical Design

Tables

Once the Logical Design of the Database System is been determined, we thereafter proceed to the Physical Design of the Database. The tables created with Microsoft's Access 97 are:

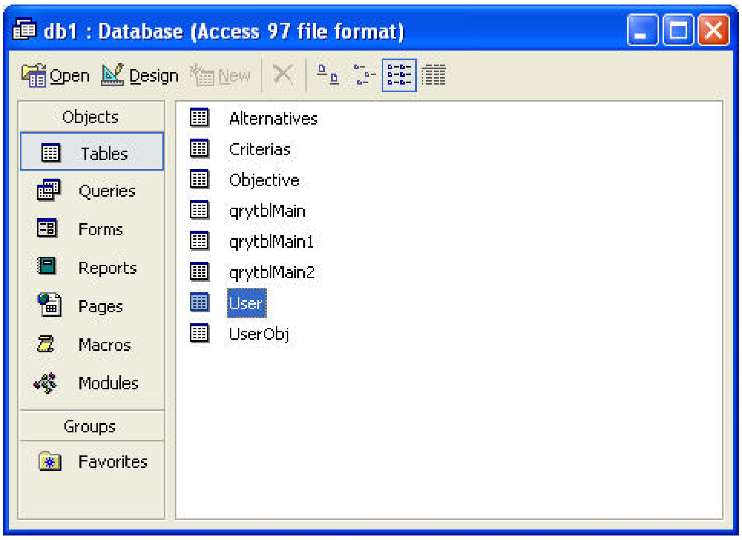


Figure 7: *Tables*

(P.S. The table qrytblMain1, qrytblMain2, qrytblMain3were created by make table queries.)

Relationships

The actual Relationships as created by MS Access 97 is shown below:

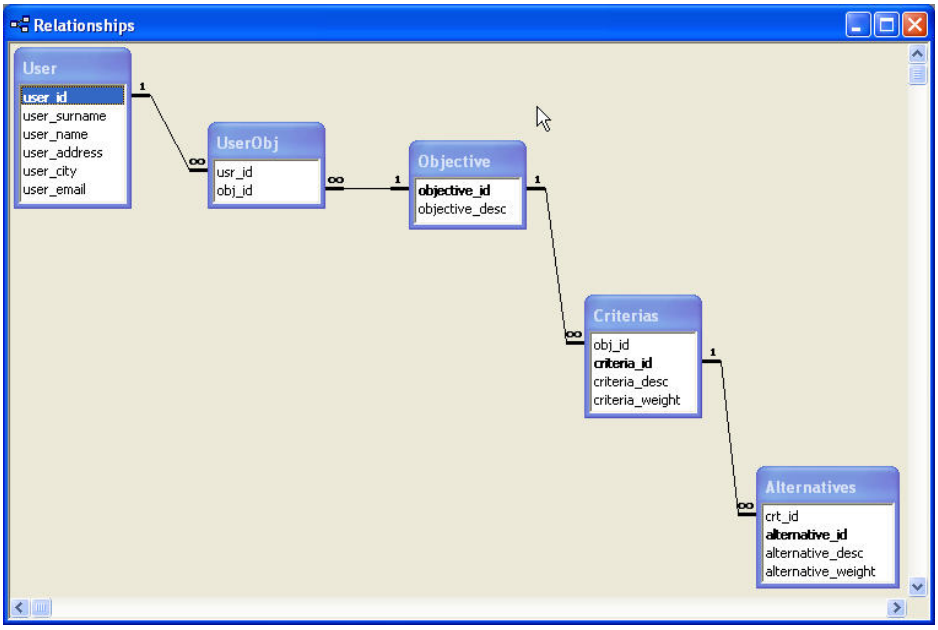


Figure 8: *Relationships*

Referential Integrity is imposed throughout, so that we will not be able to delete a decision maker unless we delete all of her/his hierarchons.

In Appendix III: Tables, we list each of the tables as been created:

Forms

The main forms for the decision analyst are Objective and User

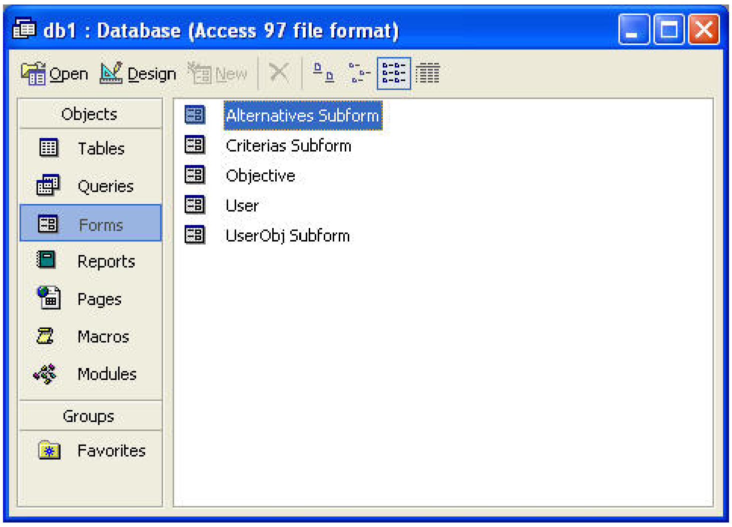


Figure 9: *Forms*

The Objective form, shown below, allows the decision analyst to define the hierarchons.

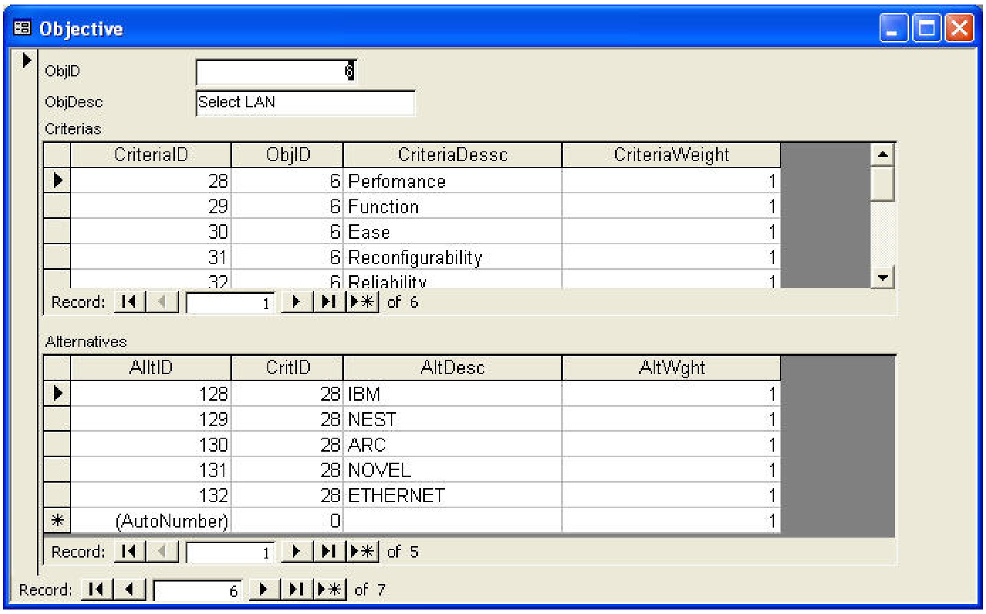
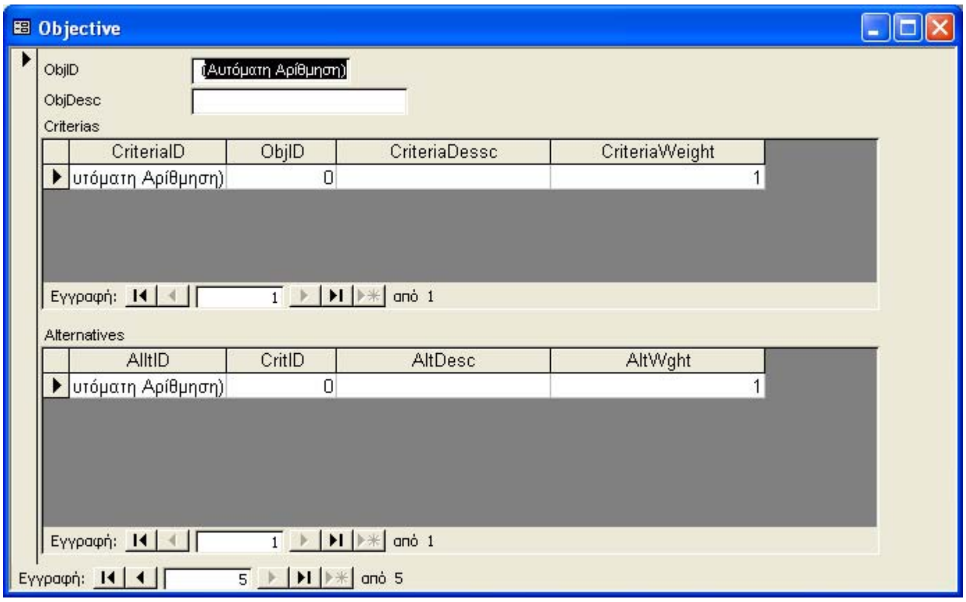


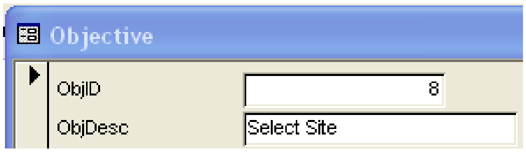
Figure 10: *Objectives (hierarchons)*

Suppose we wish to define the objective in a hierarchy for site selection of a branch office for an insurance company in Japan [2, pg 126]. The agreed criteria are: Price, Population, Growth, Image, Profit. The alternatives are: Tanashi, Yurigaok, Hashimot, Ginza, Katushi. The decision analyst create the following hierarchon by selecting a new objective on the new recordset button:

This yields an empty recordset for the form:



we type on the ObjDesc field Select Site, for example:



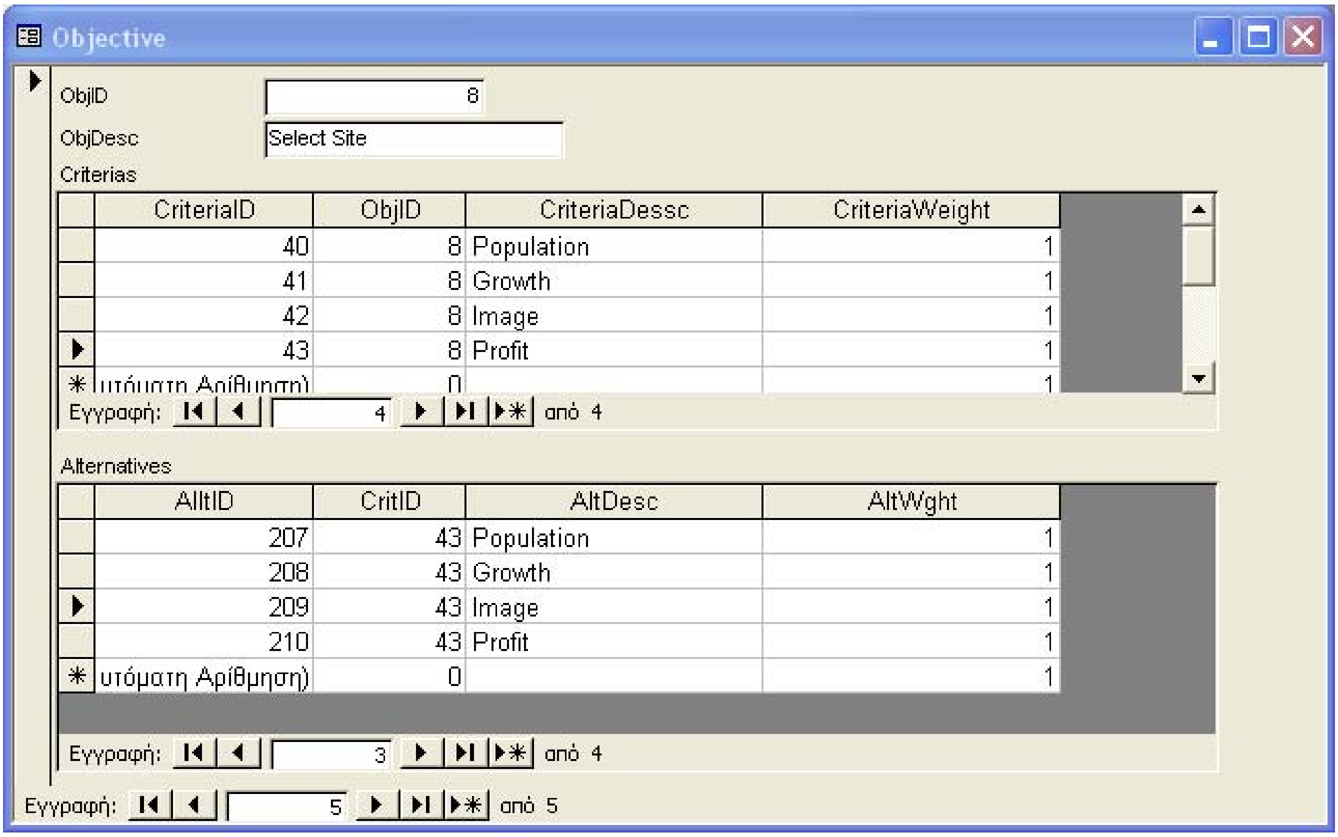
and for the CriteriaDessc, we enter the criteria:

CriteriaID	ObjID	CriteriaDessc
40	8	Population
41	8	Growth
42	8	Image
43	8	Profit

under each criterion we enter the alternatives on the AltDesc recordset

AltID	CritID	AltDesc
194	41	Tanashi
195	41	Yurigaok
196	41	Hashimot
197	41	Ginza
198	41	Katushi

The user form allows the decision analyst to allocate specific hierarchons to specific decision makers. The above figure shows how the alternatives are filled for the Growth criterion. The final form below, depicts the complete hierarchon:



Similarly, the User form, shown below, is used to assign the hierarchons to the decision makers.

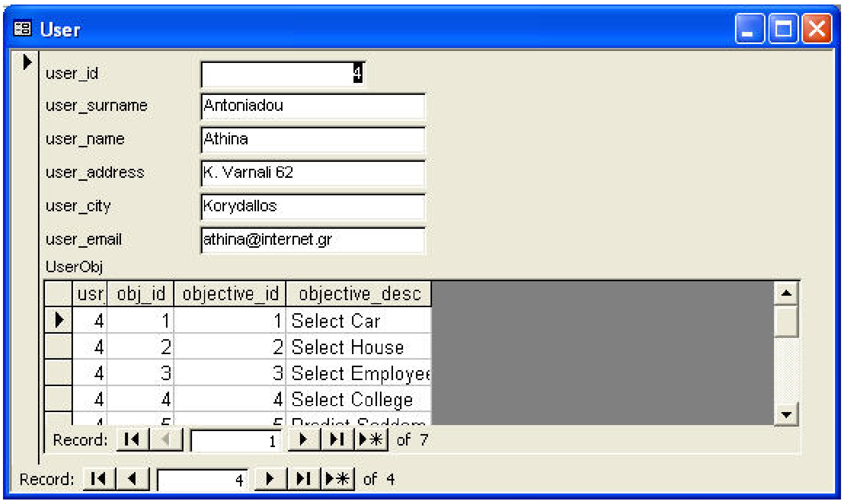
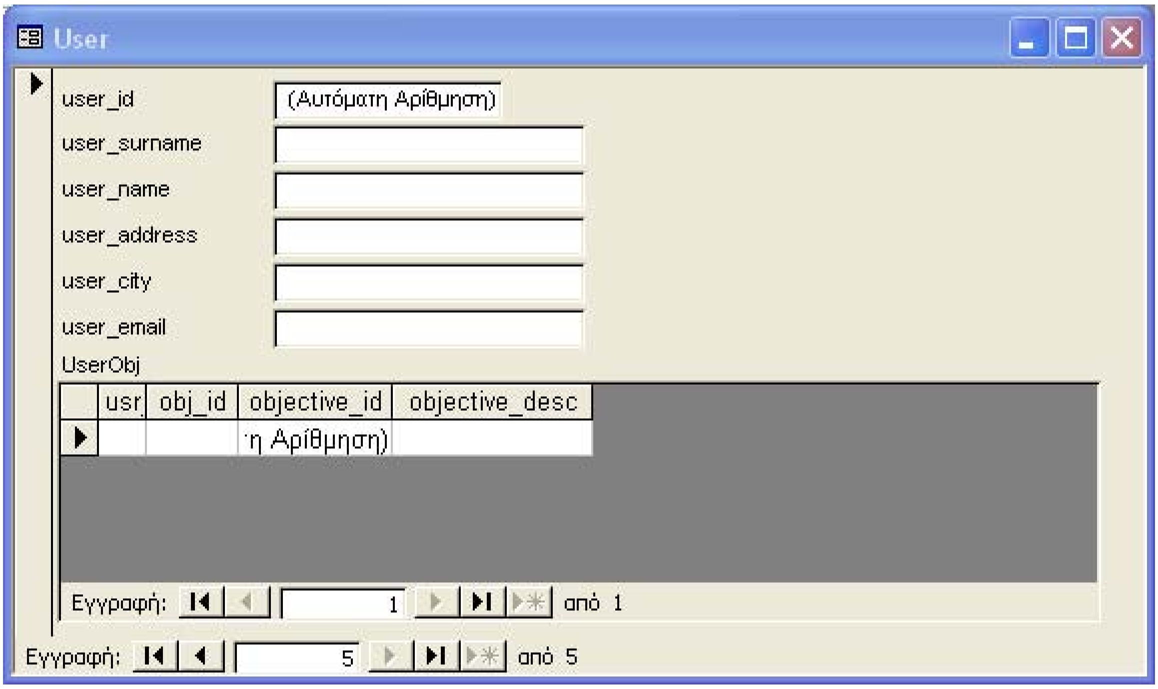
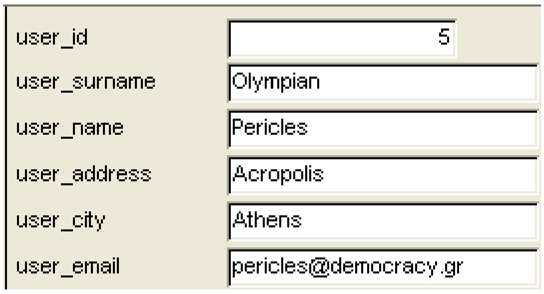


Figure 11: *Decision Makers*

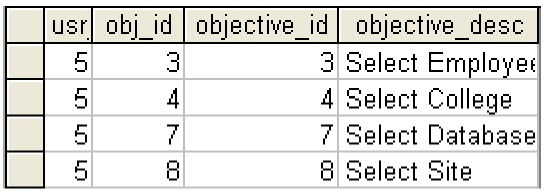
Suppose we wish to assign a new decision maker with all hierarchons created, we simply fill their personal details and add the number of the Obj_id to the recordset. The new recordset button gives:



the personal details are entered:



and assigned hierarchons:



usr	obj_id	objective_id	objective_desc
5	3	3	Select Employee
5	4	4	Select College
5	7	7	Select Database
5	8	8	Select Site

Thus, Pericles the Olympian (Xanthippus) is assigned the objectives of decision making to Select Employee, Select College, Select Database and Select Site for a Japanese insurance company, discussed previously.

Queries

For the project the project the following queries were created:

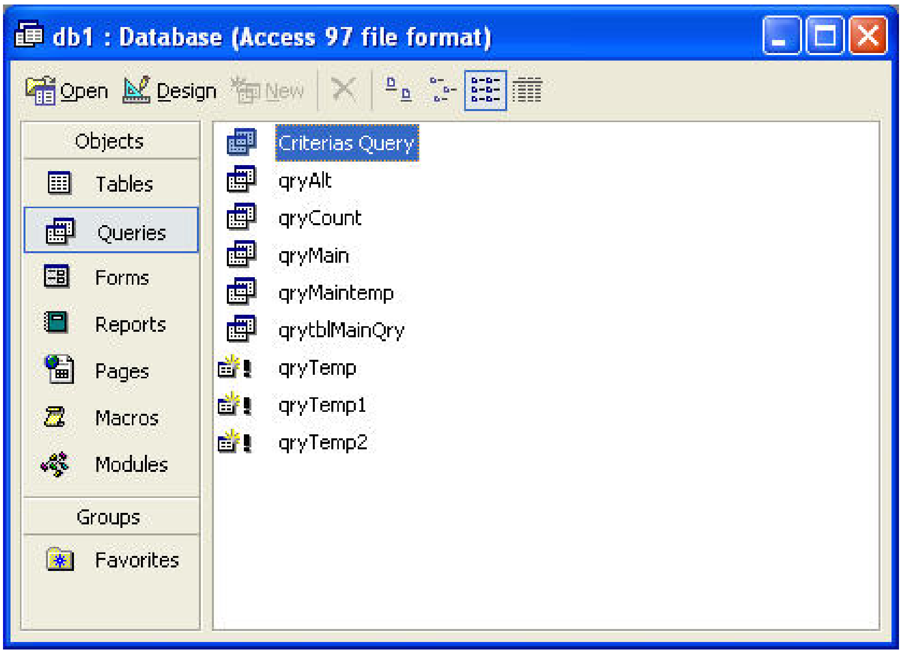


Figure 12: *Queries*

(PS. The qryTemp, qryTemp1, qryTemp2 is a create table query and is is discussed further in Appendix IV.)

(qryMainTemp qrytblMainQry are not used, but were created for testing purposes)

In Appendix VI: Queries, we list each of the Queries as been created:

OBJECT ORIENTED MODELLING

The purpose of this part of the project is to develop a part of a software system for group decision support. The project utilizes object orientated programming language, as to produce a prototype of a group decision making application for a business organization.

The application is interfaced with a relational database (Access 97), so the project's goal is to develop software using a set of rules that can be followed in any other future similar projects. Visual Basic 6.0 is being used as a Environment Development Interface (EDI), providing the object oriented programming principles and at the same time a modern Rapid Application Development (RAD) tool.

This project is also concerned in developing a user friendly Graphical User Interface (GUI) that would make entering pairwise comparison, using the slider control, as easy as possible for the decision maker. The hierarchon is been provided for by the decision analyst and presented to the decision maker by using simple array controls of text boxes, chosen from the list of a combo box. The result of the priorities obtained, using the eigenvector method should be as transparent as possible, so all the stages of the AHP are been presented to the decision maker, together with the ranking of criteria and alternatives under each criterion, correcting for consistency dynamically in each case. This results into a unique user friendly GUI which also gives the individual priorities and group choice (ranking) which is presented to each decision maker to allow her/him to see the "whole picture".

Having elicited the requirements of a prototype GDSS using the AHP, the input interface is designed, making extensive use of array controls, to obtain the independent diagonal of the pairwise comparison matrix and filling the remaining matrix namely, the principal diagonal, transitive diagonal (when required) and all the reciprocal elements. Thereafter, the Class developed, derives the eigenvector - to allow the decision maker to implement decision making by virtue of prioritization of her/his preferences, checking for consistency by calculating the principal maximum Eigenvalue. As this is a prototype, suggestions for further advancing the prototype is proposed, based on an efficient life cycle of the software development project.

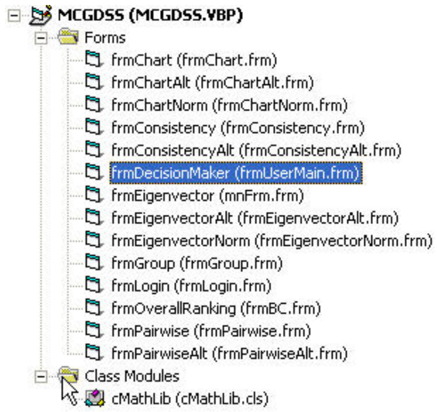
Requirements

The main software requirements, according to the requirements analysis for a prototype GDSS using the AHP, include:

1. Interface with a relational database (Access 97). i.e. use data control and apply DAO (Data Access Objects) programming techniques. The whole process, thus, could be easily migrated to ADO (Access Database Objects) programming techniques so as to be able to use this software tool in an intranet / internet / extranet environment.
2. Creation of text box control array to present the hierarchon structure from a listed choice of of a combo box.
3. Creation of slider control array, to represent the fundamental scale, and allow decision makers to enter the independent diagonal of the Pairwise Comparison Matrix (PCM). For quantitative (objective) measurements the slider control array is simply disabled.
4. Creation of a mathematical class for matrix operations to obtain the Eigenvector (Priorities Matrix), Eigenvalue (Consistency) and allow decision makers to also view the mathematics of the AHP on a multiline text box, as well as a graphical representation of the priorities using the MSChart control.
5. Using the above class the overall individual ranking and group ranking is obtained to be presented to the decision maker.

Design

The project contains the following form modules and one class module:



The main form is *frmDecisionMaker* (see **Figure 13**: Main Form) in which the decision maker can:

1. Select a hierarchon from the list of a combo box.
2. Obtain the Pairwise Comparison Matrix (PCM) on the fundamental scale – for each level of criteria and alternatives under each criterion by using the respective Preference buttons.
3. Obtain the Ranking Matrix (RM) and checking consistency, demonstrating the method both mathematically and graphically by the use of the class module, cMathLib:
4. Obtain overall individual priorities for every decision maker and aggregating the results to obtain the group ranking obtained by clicking on the Individual Priorities and Group Choice buttons respectively.

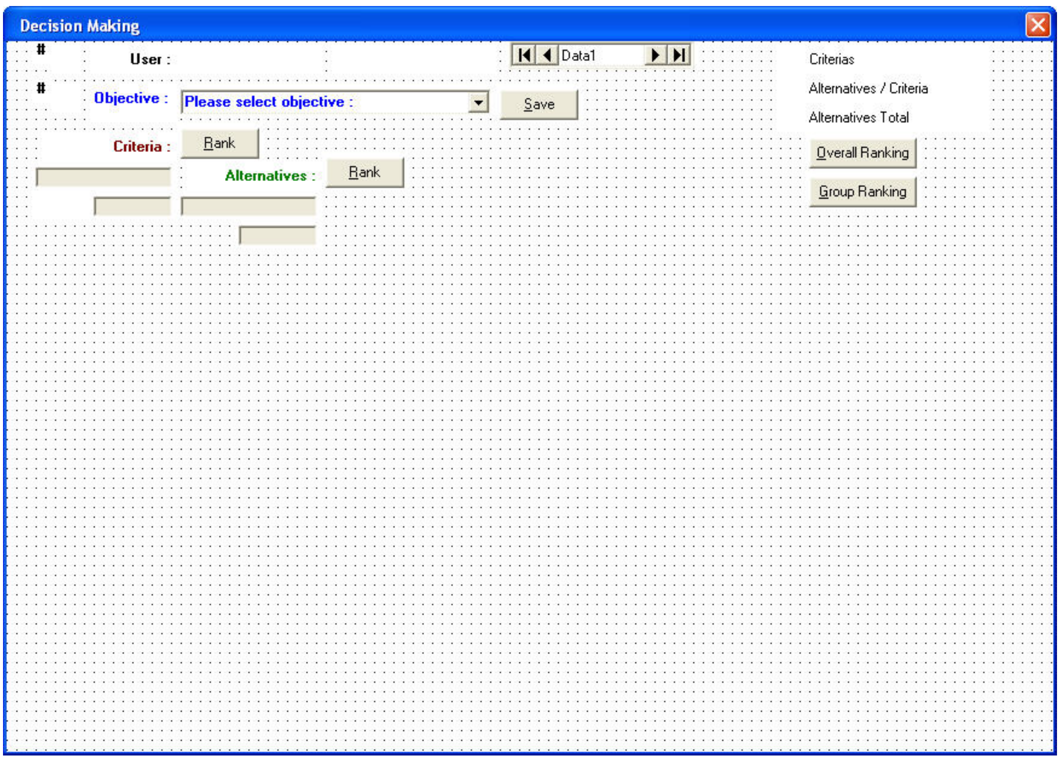


Figure 13: *Main Form*

In Appendix VII: Visual Basic-Modules describes all the forms in detail

Class

The *cMathLib* class contains functions of the following operations related to matrix operations:

Public

Add ( )	Add two matrixes
Subtract ( )	Subtracts two matrixes
Multiply ( )	Multiplies two matrixes
RSum ( )	Calculates the Row Sums of a matrix
CSum ( )	Calculates the Column Sums of a matrix
Det ( )	Calculates the Determinant of a matrix
Inv ( )	Calculates the Inverse of a matrix
MultiplyVectors ( )	Multiplies two Vectors
VectorMagnitude ( )	Calculates Vector Magnitude
Transpose ( )	Produces the Transpose of a matrix
ScalarMultiply ( )	Multiplies a matrix with a Scalar
ScalarDivide ( )	Divides a matrix with a Scalar
PrintMat ( )	Prints matrix to a multitext textbox.

Private

Find_R_C ( )	Checks dimensions of matrix
Mat_1D ( )	Checks if matrix has only one column
Mat_2D ( )	Checks if matrix has more than one column

Implementation

In order to use the class *cMathLib* we first define it with a variable e.g. ***Mat***. Below shows a demonstration on how to use the class on the frmEigenvector form module containing two multitext textboxes; txtDisplay and txtSolution (Multiline enabled) [89]:

```
Private Sub Form_Load()                                     '. . . of the frmEigenvector
Dim Mat As New cMathLib                                     'Define a variable as the matrix library class
Dim C() As Double                                           'Declare Result Matrix C ( )
Dim Column As Double                                        'Declare column Sum Matrix Column ( )
Dim i As Integer, j As Integer                              'Declare integer variables i and j
n = frmDecisionMaker.txtCriteria – 1                        'Obtain the dimension of the criteria matrix -1 (0→3)
ReDim A(n, n) As Double                                     'Declare Dynamically, input matrix A ( )
For i = 0 To n
   For j = 0 To n
      A(i, j) = frmPairwise.lblCriteriaWeight(i).Caption    'Input independent diagonal
      A(i, i) = 1                                           'Input principal diagonal
   Next j
Next i
For i = 1 To n
   A(i, i - 1) = 1 / A(i - 1, i)                            'Input reciprocal diagonal
Next i
A(0, n) = 1 / A(n, 0)                                       'Input first element of reciprocal diagonal !!!
 'Display Matrix A
txtDisplay = "  Matrix A = " & vbCrLf & vbCrLf              'Display literal characters Matrix A with two linefeeds
txtDisplay = txtDisplay + Mat.PrintMat(A) & vbCrLf & vbCrLf 'Pint the matrix A with two linefeeds
 'Display Matrix C
C = Mat.Multiply(A, A)                                      'Multiply matrix A with matrix A (C=A*A)
txtSolution = "  A x A " & vbCrLf & vbCrLf
txtSolution = txtSolution & Mat.PrintMat(C)                 'Show solution of C=A*A
C = Mat.RSum(C)                                             'Find Row Sum of C
txtSolution = txtSolution & vbCrLf & vbCrLf & "  Row Sum " & vbCrLf & vbCrLf
txtSolution = txtSolution & Mat.PrintMat(C)
Column = Mat.CSum(C)                                        'Find Column Sum of C
txtSolution = txtSolution & vbCrLf & vbCrLf & "  Col Sum " & vbCrLf & vbCrLf
txtSolution = txtSolution & "  " & Column
C = Mat.ScalarDivide(Column, C)                             'Divide Column Sum of C with Column
txtSolution = txtSolution & vbCrLf & vbCrLf & "  Normalized " & vbCrLf & vbCrLf
txtSolution = txtSolution & Mat.PrintMat(C)
C = Mat.ScalarMultiply(100, C)                              'Multiply by 100 to get percentage
txtSolution = txtSolution & vbCrLf & vbCrLf & "  % " & vbCrLf & vbCrLf
txtSolution = txtSolution & Mat.PrintMat(C)
frmChart.Show                                               'Load frmChart
End Sub
```

Life Cycle

The life cycle approach for the project used was the waterfall life cycle model, comprising of the all familiar iterative – feedback steps:

1. Planning
2. Searching
3. Design
4. Development
5. Integration
6. Test/Debugging
7. Evaluation
8. Presentation / Installation
9. Maintenance

This work suggest to use the Component Based Development model (CBM), which emphasizes the creation of classes that encapsulate both the data and the algorithms used to manipulate the data. Object Oriented Classes (OOC) are reusable across different applications and computer based system architectures [87, 88]. The CBM incorporates many of the characteristics of the WINWIN spiral model, requiring iteration and evolution as part of the process. A Unified Modelling Language (UML) e.g. VISIO, can be of great aid when it comes in defining the components that will be used to build the system and the interfaces that will connect the components. Ultimately, the Formal Methods Method (FMM) would lead to a defect free mathematical specification of the software developed.

In any case, further to enhancing "friendliness" of the user interface it is suggested to include comparative studies on the ease of use of the current development software tool with the software vendors listed in page 46, as part of the evaluation step of the waterfall life cycle. Weistroffer in 1992 describes such comparative studies on a group of students using the following decision support tools: EXPERT CHOICE, LOGICAL DECISIONS, CRITERIUM and VIMDA experimentally for a selection choice between various MSc programs.

Li and Goicoechea in the same year did comparative studies on MATS-PC, EXPERT CHOICE, ARIADNE, and ELECTRE on two groups of experts and non-experts on the selection of water management plant.

Sprague and Carlson in 1982 proposed quality assurance of a decision support system should include measures of:

1. Productivity
    a. Time and cost to obtain solution to the problem
    b. Reliability of the results obtained
    c. Cost to complete a decision
2. Processing
    a. Maximum number of criteria and alternatives for each objective
    b. Range of applications
    c. Maximum number of committee members involed in decision making
    d. Quantity of data
    e. Time taken in each decision phase
    f. Time limits of decision making in dynamic environments

3. Understanding
    a. Control procedures in decision making
    b. Usefulness of the system
    c. Ease of use and user friendliness
    d. Problem comprehension
    e. Ease of obtaining solutions
    f. Trust on the decision makers' choice of the solution
4. Reliability
    a. Cost and time of development, installation and training of decision makers
    b. Running cost of the system

It is clear that such comparative studies should include applications of such support systems in real situations implemented for real decision problems in measuring the quality of the decision result. Although such an undertaking is beyond the scope of the project such comparative studies would provide the means to design and develop a universal multicriteria group decision support system which is eliminated from the uncertainty of the Condorcet effect, inconsistency, rank reversal and most importantly, unsuitable hierarchy definition.

In short, multicriteria decision support systems are software systems the provides the decision maker with the necessary tools to manage the data/information of a percepted decision problem and aid her/him in reaching to a decision. Basically, MCDSS are applications of multicriteria analysis methodologies implemented on information systems. It generally comprise of a Graphical User Interface which is the communication channel between the decision maker, who inputs weights of decision elements based on particular models (social choice functions) which outputs the results, where the inputs and outputs are managed by a database.

GDSS APPLICATION

On execution of the program a login form appears requesting the name and surname of the decision maker:

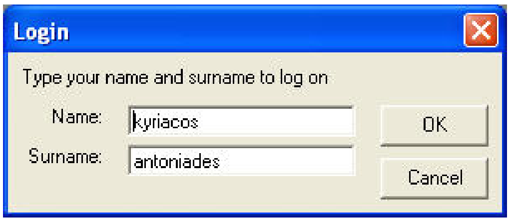


Figure 14: *Login*

The other sample decision makers include: *eleftheria grigoriadou, thanos antoniades and athina antoniadou*. On right clicking the mouse the OK button, the frmDecisionMaker form appears: The main form of the application is shown in **Figure n:** *Decision Making*:

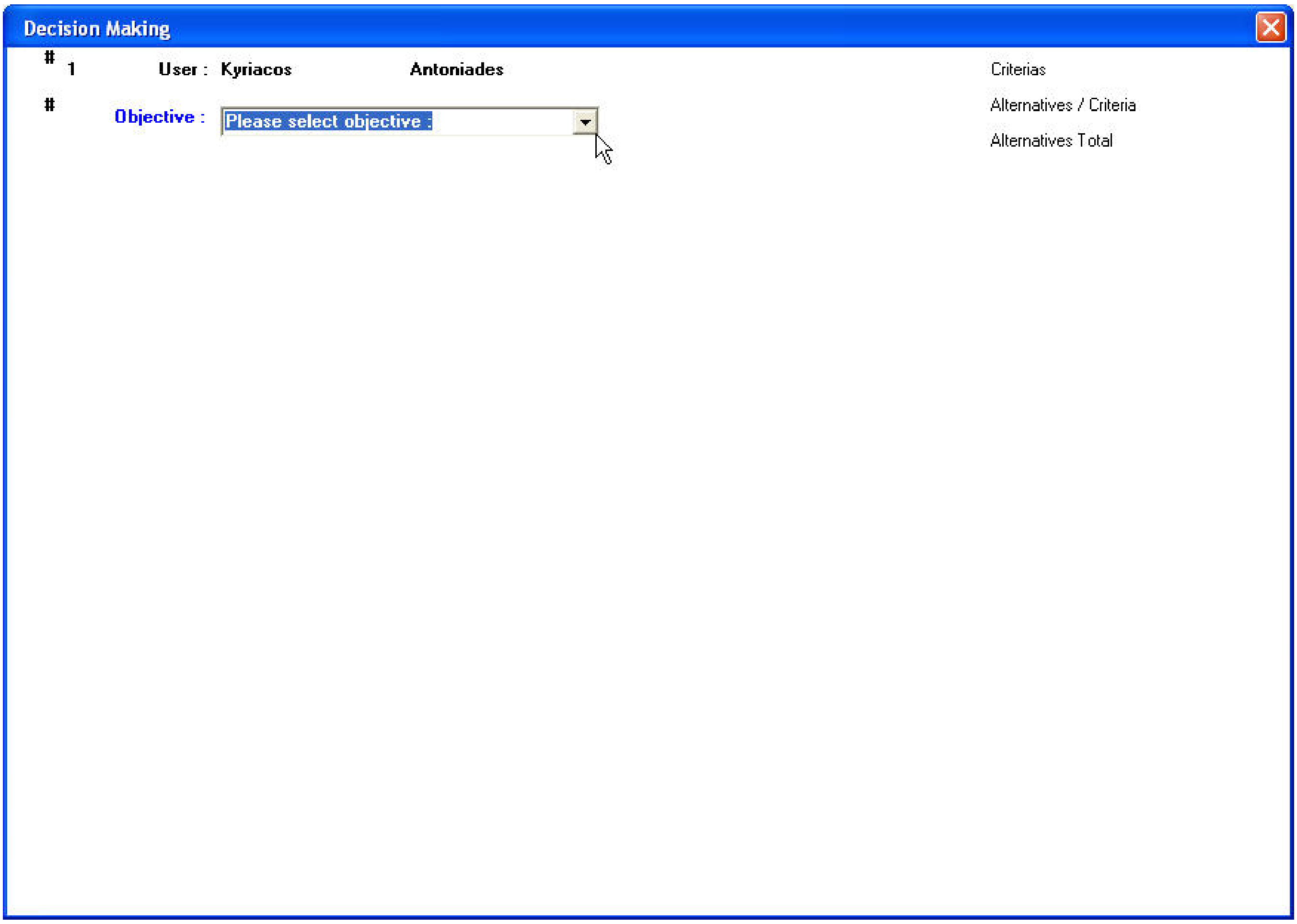


Figure 15: *Decision Making*.

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see: Appendix X: CD Contents and Setup

Hierarchy Trees

The decision makers chooses from a range of hierarchons. In our example, we consider from the choice of objectives, to Select Car.

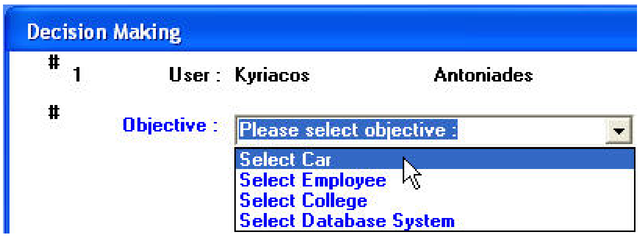

Figure 16: *Hierarchon Selection*

The hierarchon is then depicted, showing the hierarchy of criterias with their respective weights and alternatives under each criterion, again with their respective weights. Should the hierarchy had no comparison values, then all weights would default to 1.

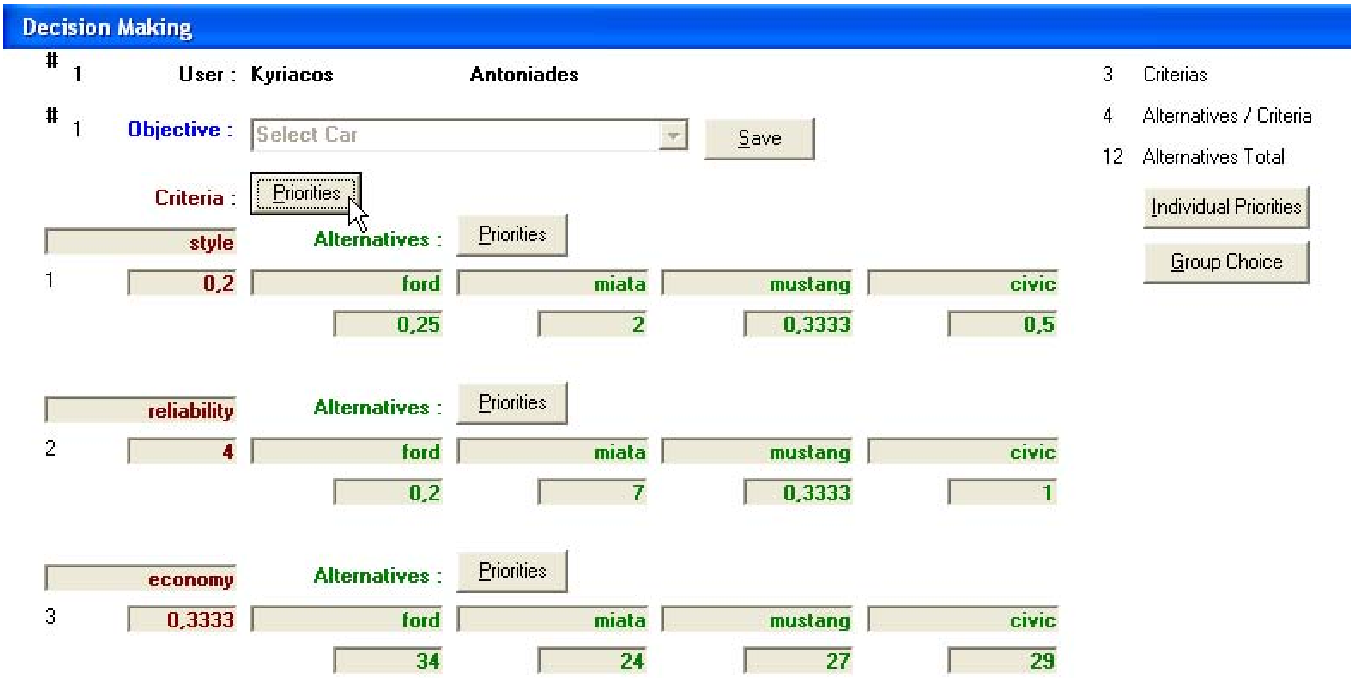

Figure 17: *Hiereachon View*

Criteria Ranking

The decision maker is able to enter the weights of the Pairwise Comparison Matrix (PCM) at the criteria level, by clicking on the respective Rank button.

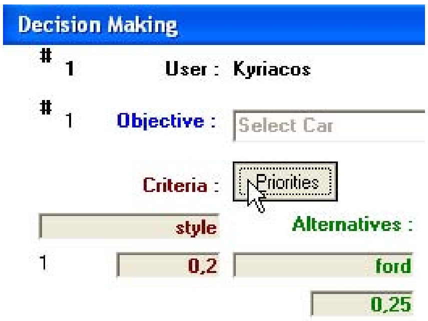

Figure 18: *Criteria Ranking*

Simultaneously, four forms appear, allowing the decision maker to alter the weights on the Pairwise Comparison form by the use of the slider control which represents the fundamental scale. The Criteria Eigenvector form shows dynamically, the new PCM (Matrix A) obtained and the mathematical steps to obtaining the Eigenvector as well as consistency ratio (CR) on the Criteria Consistency form. The graphical representation of the Eigenvector is shown on the Criteria Rankings form.

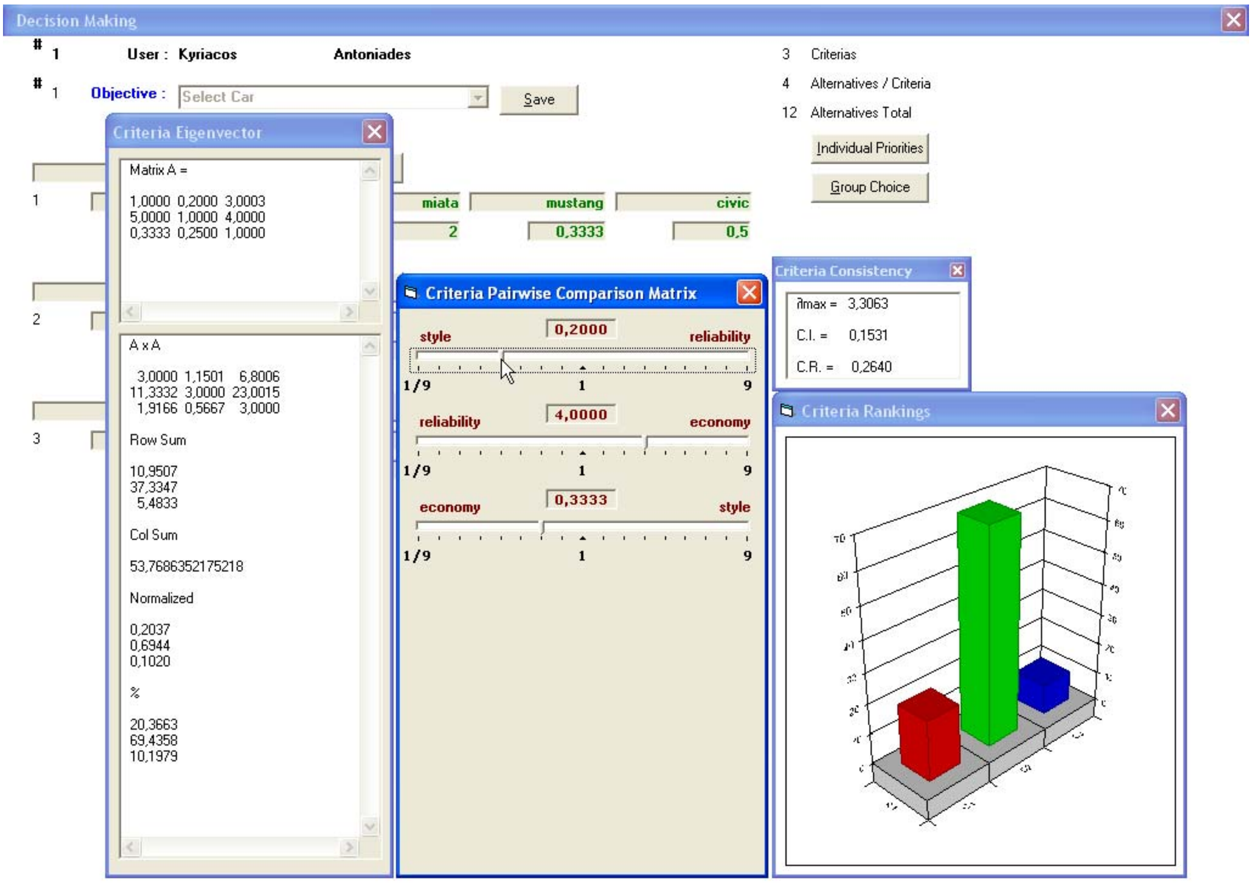

Figure 19: *Criteria Pairwise Comparisons*

Alternatives Ranking

Likewise, the decision maker is also able to alter the weights for the PCM at the alternatives level by clicking on the respective criterion Priorities button.

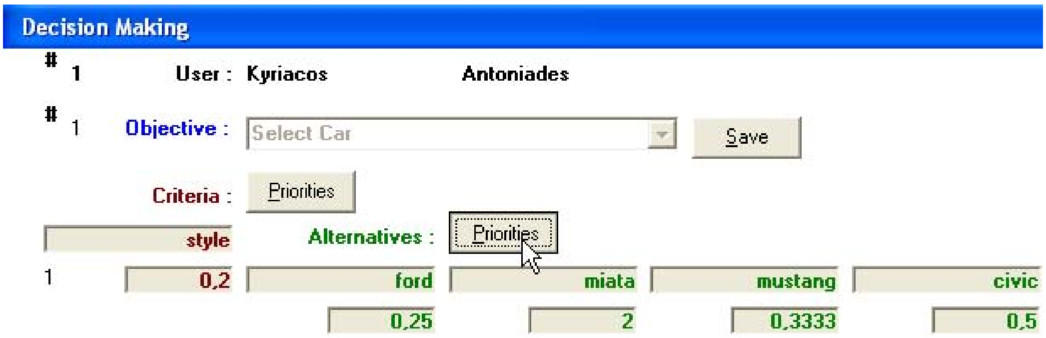

Figure 20: *Criterion Alternative Ranking*

As in the criteria ranking similar forms appear for each criterion.

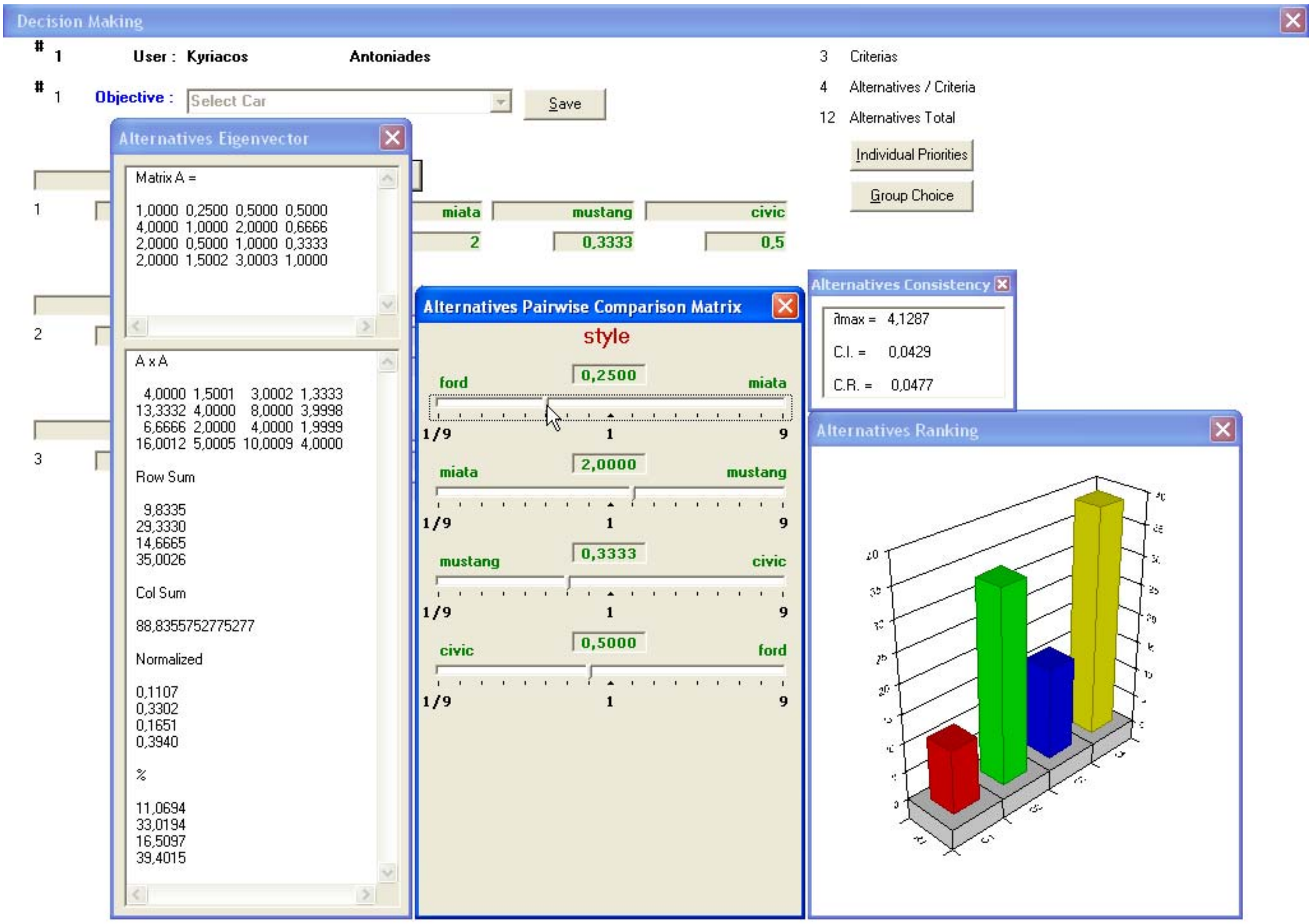

Figure 21: *Alternative Pairwise Comparison*

Quantitative Alternative Ranking

Suppose measurement is of quantitative nature (objective), then the Pairwise Comparison form ignores the slider control and the Consistency form is not shown as quantitative measurements require no consistency check. Considering the alternatives for the criterion Economy and clicking on the relevant Priorities button:

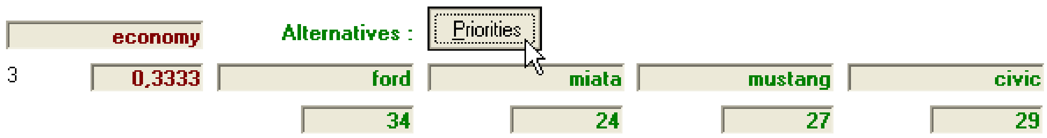

Figure 22: *Quantitative Criterion*

We obtain:

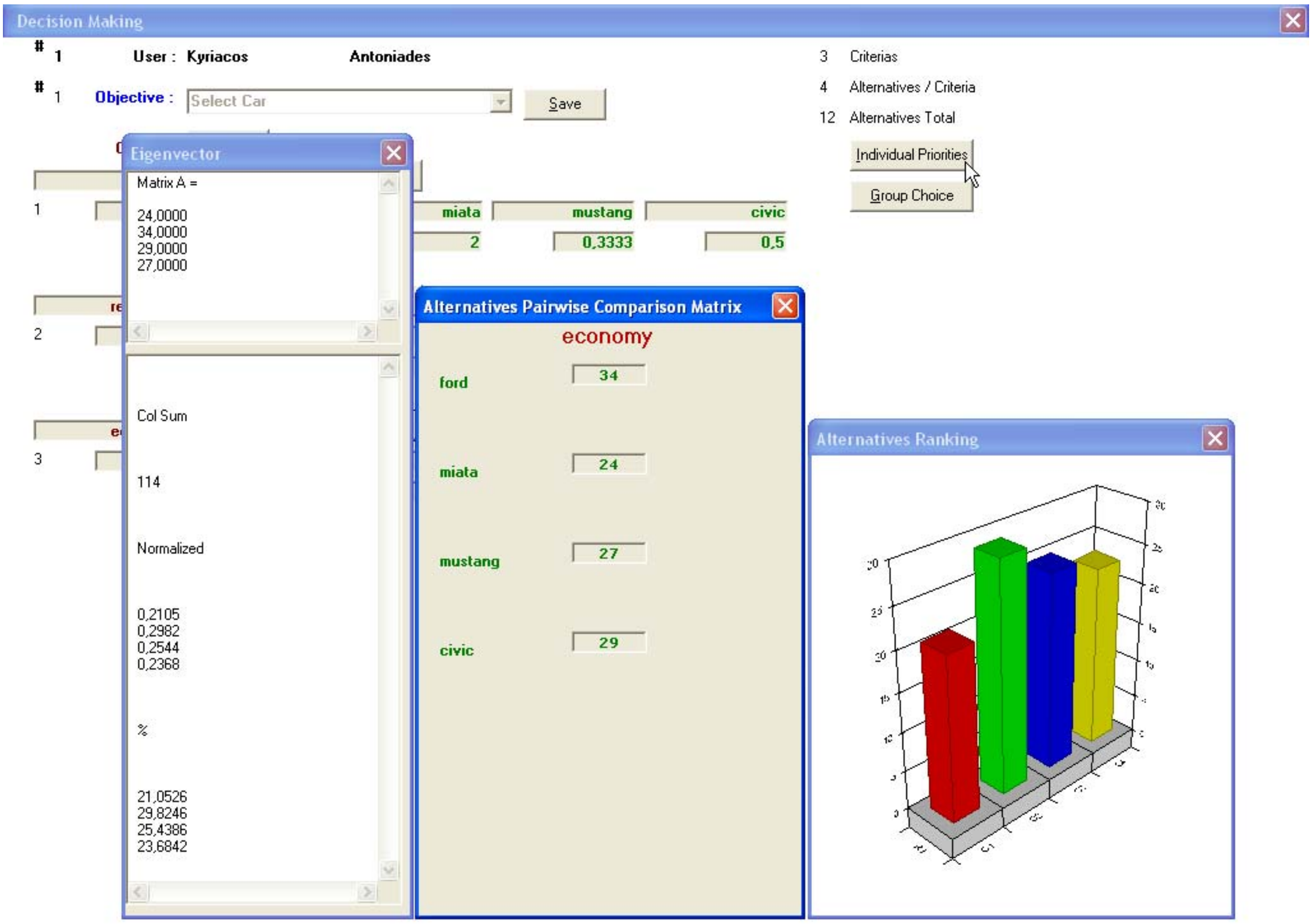

Figure 23: *Quantitative Normalization*

Observing that the rankings are depicted in the right order as direct normalization would have yielded that ford which has the highest fuel consumption as the highest ranked alternative. Here we simply switch the highest fuel consumption with the lowest.

Individual Priorities

After the decision maker has entered her/his pairwise comparisons (judgements) an overall ranking of the particular decision maker is depicted by clicking on the Overall Ranking button:

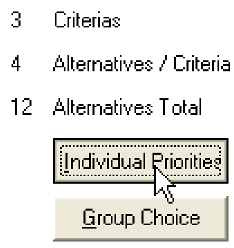

Figure 24: *Individual Ranking*

Showing the Overall Ranking form for the particular decision maker:

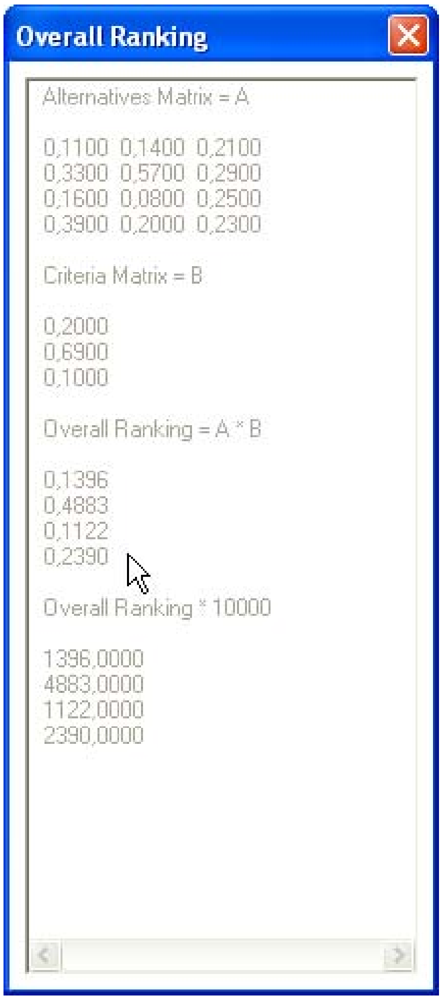

Figure 25: *Individual Ranking Form*

The mathematical procedure is shown for clarity and conformity.

Group Ranking

Group decision making is demonstrated here basically with the use of the make table query: qrtTemp2, which contain all the PCM of criteria and alternatives of all decision makers in table qrytblMain3. Borda's positional method ranks the alternatives for each decision maker from the lowest to the highest, showing the group ranking from the highest to the lowest. Clicking on the Group Ranking button,

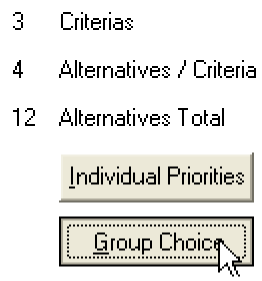

Figure 26: *Group Ranking*

Depicts the group ranking showing the mathematical procedure, again for clarity and conformity:

Figure 27: *Group Ranking Form*

Sample Data

The application is tested on four decision makers , each having to weight four hierarchons containing no more than three of four decision elements.

The four decision makers which are allowed access to the system are:

1. kyriacos antoniades
2. eleftheria grigoriadou
3. thanos antoniades
4. athina antoniadou

The hierarchons include:

1. Select car
   Criteria: style, reliability, economy
   Alternatives: ford, miata, mustang, civic

2. Select Employee
   Criteria: Attitude, Education, Experience, Leadership
   Alternatives: Susan, John, Michelle, Jim

3. Select College
   Criteria: Spiritual, Social, Vocation, Education
   Alternatives: CollegeA, CollegeB, CollegeC, CollegeD

4. Select Database System
   Criteria:Performance, Support, Growth, Compatibility
   Alternatives: DB1, DB2, DB3, DB4

CONCLUSIONS

The characteristics of group decision making under multiple objectives/criteria/alternatives are studied for simple majority rule using the non-ranked and the preferential voting method. We observe that the non-ranked voting method which is most commonly used in political elections today works perfectly well for a choice of two candidates (alternatives) but becomes ambiquous when the number of candidates are increased. The method lacks information of the relative merits of the other candidates producing results which are incomplete, does not represent the true will of the majority, prone to yield contradictory outcomes that depend on the counting method used. The preferential voting method is proposed, which includes the relative merits of all the respective candidates and observe Condorcet's paradox of voting comes into effect, producing a small percentage of nontransient majority. The Condorcet effect is studied extensively both mathematically and systematically, to determine when inconsistencies occur with respect to the number of committee members, with respect to the alternatives and with respect to both the committee members and the number of alternatives. The results show that as the number of alternatives are increased, the probability of nontransient majority increase towards 1, with little sensitivity to the number of voters for a given number of alternatives. The social choice theory defines the necessary social functions to solve for the Condorcet effect, which determines the counting method used, considered as an aggregation procedure based on the preferential voting sytem. The relational properties and the properties of group decision are defined for the Condorcet function and represented mathematically to give the group choice. From the study of the available social choice functions, we select Saaty's Eigenvector function to obtain individual priorities of alternatives under certain agreed criteria. The process of evaluating the alternatives is represented mathematically by the ordinal case of the agreed criteria approach which obtains the Borda score (ranking) for each alternative evaluated by a number of committee members.

Figure 28: Summary of procedure used, depicts briefly the whole framework of the methodology used.

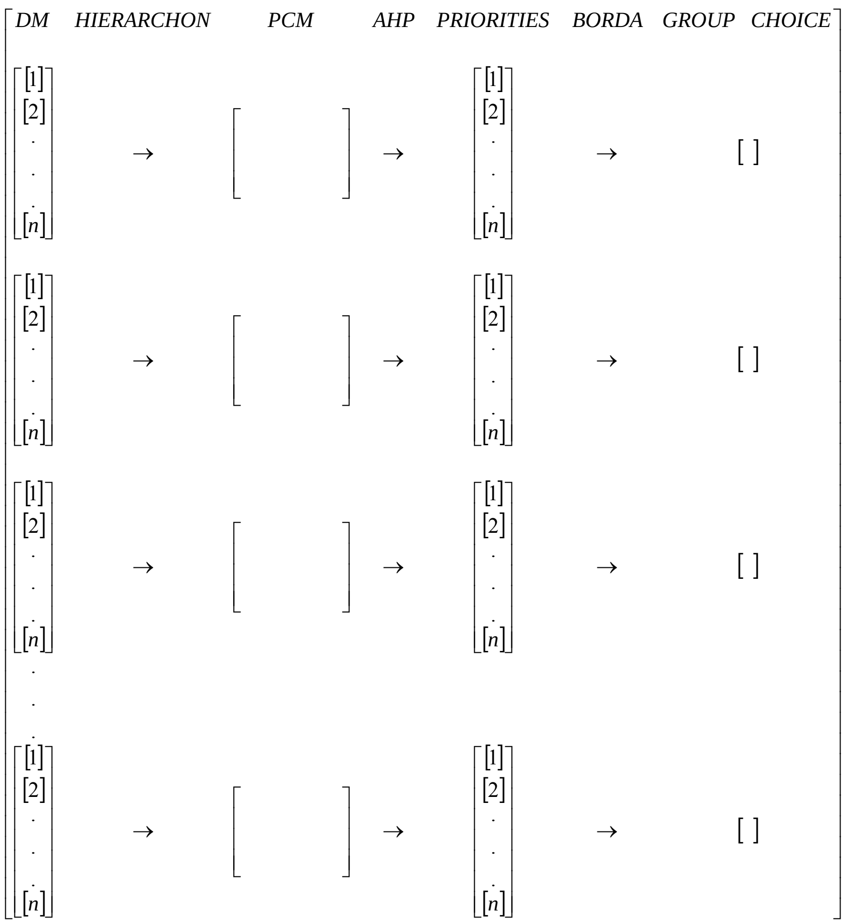

Figure 28: Summary of procedure used.

The group of decision makers (DM) for a particular hierarchon (defined objective), enter the pairwise comparisons for the criteria and alternatives under each criterion, thence, using the AHP which utilizes the Eigenvector function, we produce the individual priorities for each decision maker. These individual priorities are aggregated and using Borda'positional method, we obtain the collective group choice or ranking.

More descriptively, we see from **Figure 29:** Details of the procedure used, the complete workings of the project's methodology:

$$
\begin{bmatrix}\left[m_1\right]\\ \left[m_2\right]\\ \left[m_3\right]\\ \left[m_4\right]\end{bmatrix}
\rightarrow
\begin{bmatrix}
\begin{array}{ll}
\mathit{criteria} & \\
\begin{bmatrix}1 & c_{12} & \\ & 1 & c_{23}\\ c_{31} & & 1\end{bmatrix} & \begin{array}{l}\xrightarrow{EIGENVECTOR}\begin{bmatrix}c_1\\ c_2\\ c_3\end{bmatrix}\\ \xrightarrow{EIGENVALUE}[CR]\end{array}\\
\mathit{alternatives/criterion}\ 1 & \\
\begin{bmatrix}1 & a_{12} & & \\ & 1 & a_{23} & \\ & & 1 & a_{34}\\ a_{41} & & & 1\end{bmatrix} & \begin{array}{l}\xrightarrow{EIGENVECTOR}\begin{bmatrix}a_1\\ a_2\\ a_3\\ a_4\end{bmatrix}\\ \xrightarrow{EIGENVALUE}[CR]\end{array}\\
\mathit{alternatives/criterion}\ 2 & \\
\begin{bmatrix}1 & a'_{12} & & \\ & 1 & a'_{23} & \\ & & 1 & a'_{34}\\ a'_{41} & & & 1\end{bmatrix} & \begin{array}{l}\xrightarrow{EIGENVECTOR}\begin{bmatrix}a'_1\\ a'_2\\ a'_3\\ a'_4\end{bmatrix}\\ \xrightarrow{EIGENVALUE}[CR]\end{array}\\
\mathit{alternatives/criterion}\ 3 & \\
\begin{bmatrix}1 & a''_{12} & & \\ & 1 & a''_{23} & \\ & & 1 & a''_{34}\\ a''_{41} & & & 1\end{bmatrix} & \begin{array}{l}\xrightarrow{EIGENVECTOR}\begin{bmatrix}a''_1\\ a''_2\\ a''_3\\ a''_4\end{bmatrix}\\ \xrightarrow{EIGENVALUE}[CR]\end{array}
\end{array}
\end{bmatrix}
$$

$$
\rightarrow
\begin{bmatrix}a_1 & a'_1 & a''_1\\ a_2 & a'_2 & a''_2\\ a_3 & a'_3 & a''_3\\ a_4 & a'_4 & a''_4\end{bmatrix} * \begin{bmatrix}c_1\\ c_2\\ c_3\end{bmatrix}
\rightarrow
\begin{bmatrix} m_1 \\ \begin{bmatrix}a_1\\ a_2\\ a_4\\ a_3\end{bmatrix}\\ m_2 \\ \begin{bmatrix}a_1\\ a_3\\ a_2\\ a_4\end{bmatrix}\\ m_3 \\ \begin{bmatrix}a_2\\ a_1\\ a_3\\ a_4\end{bmatrix}\\ m_4 \\ \begin{bmatrix}a_2\\ a_1\\ a_4\\ a_3\end{bmatrix}\end{bmatrix}
\rightarrow
\begin{bmatrix}1 & 1 & 2 & 2\\ 2 & 3 & 1 & 1\\ 4 & 2 & 3 & 4\\ 3 & 4 & 4 & 3\end{bmatrix}
\rightarrow
\begin{bmatrix}7\\ 7\\ 13\\ 14\end{bmatrix}
$$

Figure 29: Details of the procedure used

Considering four (4) committee members, who have to evaluate an objective (hierarchon) containing four (4) alternatives under three (3) criteria, the members: m_1, m_2, m_3, m_4 enter the pairwise comparison for both the criteria decision elements, c_{12}, c_{23}, and c_{34}, as well as for the alternatives, under each criterion, are annotated as, a_{12}, a_{23}, a_{34}, a_{41}, a'_{12}, a'_{23}, a'_{34}, a'_{41}, a''_{12}, a''_{23}, a''_{34}, a''_{41}. In each case, we check the consistency ratio and the eigenvector to obtain the priorities of the decision elements. Thereafter, we collect all the alternatives priorities vectors and multiply this matrix with the criteria priorities. The result gives the individual priorities for each member of the committee. The Borda positional method is applied to obtain the priorities of all members by ranking the lowest of the individual preferences of alternatives under the agreed criteria, with mark 1, the second lowest with mark 2, the third lowest with mark 3 and the fourth lowest with mark 4. The row sum obtained from this matrix gives the group ranking, with the highest alternative ranked first, the second highest ranked second, the third highest third and the fourth highest ranked fourth.

Operational research in multi criteria decision support systems analysis [22, 55, 56] using the AHP [1, 2, 3, 4, 5, 6, 7, 8, 9] provided the requirements elicitation and analysis in order to design and develop a decision support system (DSS) that is further developed and enabled for groups (GDSS). This is achieved by creating a simplified environment using the basic principles of the AHP and applying this methodology to obtain the group choice under multiple criteria and alternatives by using the Borda social choice function. The project discussed the methodology of the AHP with its inherent advantages and disadvantages, the various software vendors involved, areas of applications and the role of the decision analysts and decision makers in group decision making. Thence, the database is developed to accommodate group decision by assigning decision elements arranged in a hierarchy to each and every individual of the group. Thereafter, at the heart of the project, the software tool developed demonstrates group decision making, bearing in mind for a friendly graphical user interface, by allowing each user to log in the system and simply assign individual pairwise comparisons of only the qualitative decision elements. An overall ranking is thus obtained for each individual and using Borda's positional method, [106, 107] we aggregate these results further to obtain the group choice. Coupled with the literature review and secondary research which revealed the theoretical background and methodologies that are used for a plethora of Decision Support Systems (DSS). The Analytic Hierarchy Process (AHP), belonging to the American stream, which utilizes the eigenvector social choice function was founded by Thomas L. Saaty [1, 2, 3, 4, 5, 6, 7, 8] back in 1970, and becomes a most popular methodology, due to its ease of use, for decision support systems and yet one of the most criticized methodology as well. In 1980 Ernest H. Forman [9] using the AHP develops a DSS computer software, patented as Expert Choice. Since 1983 Expert Choice has been stirring an ever increasing interest for an increasingly large number of private and public sectors worldwide, finding applications into industry, business, education, medicine, science, engineering, transportation, philosophy, psychology, social sciences, politics and many others. At the same time, the European stream are involved primarily with disaggregating individual preferences first, then aggregating to obtain the value function with DSSs such as the ELECTRE, UTA, MINORA, MIIDAS family of systems. Despite that the European stream have developed decision support systems that lack the ambiguities posed by the AHP, their popularity remained in the academic circles due to the fact that such systems still require high level expert operators who are specific to specific decision making problems. It is noteworthy to mention that the analogous software vendors (see DSS Software Listings) have had their success made possible only during the last decade or so, during which computing power was been made available. Also, it is only very recently that the aforementioned software vendors are considering to include group decision support either by using a network environment (intranet/internet/extranet) or with the use of add-on components. This project recognized a window of opportunity in the realms of group decision making as no DSS as such offers group decision making that is group user friendly, multithreaded, scalable and extensible. Realizing the need for group decision making in a multithreaded environment the evaluating and decision process of group decision making have been represented mathematically and systematically under multiple criteria. The software and database requirements are elicited and a multiple criteria group decision support system is proposed and implemented, under a server / client environment, providing a new approach to group decision making and a vast repositoty for further research in this diverse field.

Such support tools, thus, should be able to accommodate in the future, even more wider range of decision making, for even more wider range of individuals or group of individuals. The concept of the AHP however, requires pre determination of the decision elements at hand. A different pre determination normally yields completely different decision results. Here, it seems quite appropriate quote:

"*The essence of knowledge is, having it, to apply it; not having it, to confess your ignorance.*". - Confucius

The aim of this project is thus achieved by designing and developing a prototype multi criteria group decision support system with an interactive and intelligent graphical user interface that is unique in its ease of use compared to those at present. Requirements analysis and elicitation through literarure review and secondary research provided the theoretical background of multcriteria analysis, the analytic hierarchy process and the framework of the methodology for group decision making. We considered the actors involved, namely the decision analyst and decision maker and their role in group decision making is being proposed on a business organization.

Group decision is thus put forward on a simple five step method:

1. Hierarchon

   Initially, a database is developed for the decision analysts as a tool for defining hierarchies, the actual decision or hierarchon to be assigned to a group of decision makers. Particular attention is paid to the independence and actual scale of the decision elements. The simplified hierarchons are considered to satisfy the independence and homogeneity property of the decision elements. The advantages and disadvantages of the AHP method are discussed minimizing most by setting constraints on the levels of the hierarchon and the number of decision elements (criteria and alternatives) it could contain.

2. Paiwise Comparison Matrix.

   A decision maker thence, chooses a hierarchon and makes judgements (pairwise comparisons) of the decision elements using the fundamental scale (considered to be appropriate for the decision elements). Her/his sole concern is to keep the consistency ratio below 10%.

   The graphical user interface developed merits the advantage of being user friendly to the degree of being self descriptive, transparent to the user, as the methodology of the AHP is depicted both mathematically and graphically, dynamically interarctive as the values of the Eigenvalue and Eigenvector are updated either on change or scroll of the slider control, scalable as the prototype can easily be deployed in a multithreaded environment with group settings and policies, adaptable as to representing real life decisions involving a large number of decision makers for different application areas, functional as it can be a time saver, and at the same time a training tool of the AHP methodology, thus reducing the learning curve of operation, and finally and most importantly, it provide grounds for further research.

3. Eigenvector/Eigenvalue of criteria and alternatives.

   The calculations of the relative weights of the criteria and alternatives, consistency check, individual priorities and the group ranking are achieved by defining a class dealing with the main matrix operations.

   We observe that as the number of decision elements increase the transitive property of the pairwise comparison needs to be taken into account. We find that there are different ways to consistency correction and show that partial consistency correction provides an optimum consistency check as far as performance and reliability of the software tool are concerned.

4. Individual Priorities.

   Aided with the graphical user interface and the matrix class, from the aggregation of the relative weights for a given decision maker we obtain her/his overall individual priorities of the alternatives by multiplying the alternatives vectors with the criteria vector.

5. Group Choice.

   The values of the individual priorities for a group of decision makers are further aggregated to produce group ranking by using Borda's positional method. Each committee member can view the group choice obtained so as to reconsider in case of draw (tie) of the alternatives.

Further secondary research based on the thematic entities of ratio scales, eigenvalue (consistency), eigenvector (ranking of priorities) and rank reversal leads to a discussion of the disadvantages discussed previously. Hamalainen [18] showed that two competitive schools of thought, namely the AHP and the MAVT (multi Attribute Value Theory) have their similarities and relationships in defining new scales, whereas the value questioning mode of the AHP could give wights that have MAVT interpretration. Specifically, he states:

"In this light and knowing the widespread use of the AHP, the lack of behavioural research of the AHP elicitation procedures should be of great concern for the research community"

On Saaty's ratio comparisons (w_1/w_2), due to the fact that the 1-9 scale cannot contain zero, in the context of the MAVT, it lacks meaning from the standpoint of theoretical validity. Nevertheless, Saaty has shown that the value difference question gives the right answer, no matter what the unknowns are. Considering three identical containers, with weights 1, 3 and 6 and wish to estimate the weight of substance. Relative to the lightest object, the heaviest object contains (6-1)/(3-1) = 2.5 times more substance than the second heaviest object. Other scales [20], yet, can yield more accurate estimates. Rank reversal has also been debated extensively in the last ten years, despite the mathematics been well understood and can be easily eliminated [7]. Concerning group decision making, Saaty claims that it would not have been considered in the MAVT framework, whereas Keeney [54] already provides a set of sufficient and necessary conditions under which member's preferences may be aggregated into an additive joint representation of preferences. On the verbal comparisons [20], Hamalainen states:

"A major finding of the experiment is that the numerical counterparts of the verbal expressions vary accordingly to the set of elements involved in the comparison"

This dependence means that verbal comparisons do not lead to normatively correct numerical results. To solve this problem, he recommends that a decision maker should be asked to make verbal pairwise comparisons, so that the reference point is well defined. This would lead to making pairwise comparisons of value differences. Additionaly, the decision makers could enter intervals of numbers to represent their own understanding of numerical counterparts of the verbal expressions. To solve for the lack of statistical theory behind the AHP, to account for the fact that the ratios of the relative importance of entities contain random fluctuations, Hamalainen proposes a robust regression method [19] which shows no considerable bias compared to the Saaty's eigenvector method which does not gives unbiased weights if there are random variations and outliers of the judgement. Zahir [24] in 1991 used a non statistical approach to process imprecise non-consistent judgements into intervals and then optimizing.

Also, Choirat [26] showed that the pathological behaviour of the AHP corresponds to phenomena already encountered in Psychometric Choice Theory. The experiments show that the AHP is biomimicking to Neural Networks and Genetic Algorithms. Human sensatiuons on the basis of empirical observation show a logarithmic like relationship between the stimulus on the sense organs and sensation. i.e. Weber-Fechner Law. That is, it produces a bias (systematic error) on the decision makers evaluation of ratio scales (w_i/w_j) through their personal estimate of $a_{i,\,j}$. The choice of scale has a strong impact on the calculation, determining that the 1-5 linear scale behaves better than the 1-9 scale which, however, still gives acceptable results. Individual variability on weight calculation shows that systematic distortion of the weights is a typical feature of decision making and is not a linear relation but an exponential one.

Finally, Tuomala [21] showed that in cases were not all the pairwise comparisons are made, it is important to know which comparisons to leave out and which not, to make the experiment statistically optimal. The D-optimality criterion is devised for this purpose. Camerer [25], introduces a one-parameter (τ – mean number of thinking steps and variances) Cognitive Hierarchy (CH) model to predict behaviour in one shot games. The idea is that most players do some strategic thinking but is constrained by working memory based on the axiom for a Poisson distribution of thinking steps, τ. Further research would endogenize τ from some kind of benefits/costs analysis in which players weight marginal benefits of thinking against cognitive constraints.

Concentrating on group decision making, according to Saaty [1], in group decision two issues are raised:

1. Aggregation of individual judgements

   The function for synthesizing the judgements must satisfy:

   a. Separability condition (S)
   b. Unanimity condition (U)
   c. Homogeneity condition (H)
   d. Power condition (P_p)

2. Construction of a group choice from individual choices

   Using the ratio scale approach of the AHP, if the individual preferences are cardinal rather than ordinal it is possible to derive a rational group choice, which satisfy:

   a. Decisiveness condition
   b. Unanimity condition
   c. Independence of irrelevant alternatives
   d. No dictator

Which are in agreement with the theoretical background on the defined properties of group decision making.

Indeed, Raith observes [68], after analysis of the harmonization versus aggregation methods for applying group decision with the AHP requires a utilitarian solution concept, where the groups can achieve better outcomes when group preferences are aggregated at the end of individual assessments. Practically, this implies that each group member, who seeks to quantify preferences should do so independently and uninfluenced by the others. Rather than satisfying an average opinion, group decision should take into account the diverging preferences of its individual members.

In addition, Torkkeli concludes that GDSS provide an effective way of evaluating different technologies [67]. It has also been shown that GDSS improve communication with participants from different organizational levels and across functional barriers. When effective cooperation is achieved, the group of managers / experts:

1. Define different requirements each focuses on their expertise area comprehending requirements stated by other experts in different areas.
2. If necessary dynamically vote for the proper categorization of requirements.
3. Manage schedule discussion in the different phases of evaluation.
4. Anonymously enter preferences providing democratic participation of all members.

Further proposals to use NGTHP (Nominal Group Technique Hierarchy Process), which includes NGT (Nominal Group Technique) and AHP allows expert judgements to be entered efficiently and decision making be conducted objectively [70]. Is is shown that NGTHP outperforms DHP (Delphi Hierarchy Process] for solving Multi Criteria Decision Making (MCDM) problems. Jablonsky shows that after hierarchical modeling, the matrix of the individual preferences of the alternatives of all decision makers is obtained [69]. An index of concordance, using the PROMETHE Class method, is defined of the i_{th} decision maker with the j_{th} scenario to obtain a global threshold that expresses a necessary majority to accept the given scenario as a candidate for the global compromise scenario.

Summarizing, for the needs of the project concerning the hierarchons, the criteria and alternatives are chosen to be simple qualitative measurements, such that they can be considered as independent entities. As far as the fundamental scale is concerned, the scale range of the criteria are kept to the same range defined as fundamental (1-9) and agreed by all decision makers. The pairwise comparisons are kept to a minimum, lessening the burden on the decision maker. Inconsistency due to rank reversal as such is reduced to a minimum, as the decision maker cannot add or remove decision alternatives. The group choice is thus constructed using the aggregation method at the end of individual assessment independently and uninfluenced by the other decision makers (democratically). Simply put, we satisfy the properties of group choice as defined, for obtaining a social choice function that nearly eliminates the Condorcet paradox for group decision making.

Critical Evaluation

Studying the theoretical background of group decision making under multiple criteria and choosing the decision support systems based on the AHP of the American stream methodology, compared to basically those of the UTA, MINORA, MIIDAS systems, of the European stream proved to be appropriate to develop a prototype multicriteria group decision support system.

The Rapid Application Development (RAD) Object Oriented Modelling (OOM) language used, namely Visual Basic 6 and the Relational DataBase Management System (RDBMS), Access in this case, provided the necessary minimum tools to build the application. The database developed provided a tool for the decision analyst to be able to define the hierarchies and assign them to a group of decision makers. The software tool which is interfaced to the database provided the decision maker, who is authenticated to the system, to pass judgements (pairwise comparisons) of the decision problem or objective (hierarchon) from a list of a combo box. The hierarchon chosen is thus displayed by the use of control arrays of typical text boxes representing the description of the criteria together with their respective weights and the description of the alternatives together with their respective weights under each criterion. Pairwise comparisons are entered by the use of a slider control, showing dynamically the rankings mathematically and graphically together with consistency in each case. Individual priorities and group ranking is also computed. Overall the support system is characterized by its innovative ease of group user friendliness and multithreaded capabilities as the application is deployed on the web.

Testing and debugging the prototype software tool shows that the final version produced is free of either syntax and logical errors and installs perfectly well for a wide range of Windows versions (Windows 98, Windows Me, Windows 2000, Windows XP) with no apparent differences.

Confidence is gained knowing that after countless hours of literature review, secondary research, database modeling and code writing the project meets all the areas of the original specification and yielded new directions in this area of study which were not least imaginable at first. As such, further plans include in preparing a paper of this project for the summer of 2004 proceedings at Rhodes island and at Delphi next September and finally, this project has been a great inspiration that has already provided the motivation to seek and apply for further research on group decision support systems at a PhD level.

AHP is a multiple objective/criteria/alternatives decision making tool consolidating information, using pairwise comparisons, on qualitative and quantitative criterias and alternatives. Further research on the criticisms of this methodology should make this method even more suitable for solving complicated decision problems. It is here that the project recognizes the need for a group decision support system and amalgamting the AHP methodology with the social choice theory we propose and implement a new perspective to the AHP, namely Group AHP. This software project should demonstrate, after the recommended further improvements on the life – cycle of this software tool, that the proposed direct and interactive GUI applied would improve the ease of use as an aid to group decision making. It is hoped that the conclusions or recommendations drawn from this project will be of value as to further aid prospective research in this field of study.

Personal Knowledge Gained Through Conducting Project

Firstly, this project taught the virtue of patience and perseverance. As Thomas Edison had said:

"*Genius is one per cent inspiration and ninety-nine per cent perspiration.*"

Initially, the project gave the opportunity to gain insights into the mathematics and methodologies of the preferential voting system for group decision making and the Condorcet effect is understood with respect to the number of committee members and the number of alternatives under an aggreed set of criteria. Social choice theory provided the methods to solve for the Condorcet effect and we select the eigenvector method to obtain the individual priorities, which is the basis of the AHP and Borda's positional method to obtain the group choice, for its simplicity and democraticity it provides. The project also raised questions of the non ranked value system by which elections are run today and we conclude with a suprising absoluteness that this method is unsuitable in representing the will of the majority if a selection is to be made for a choice of more than two alternatives (candidates).

The mere fact that voting methods proposed in the eighteenth century are used in group decision making today comes into realizing of the importance of the social choice functions into producing a group choice. The project also provided the opportunity to learn the AHP methodology, realizing its simplicity and clarity it offers to decision making. The study shows that although this methodology is a newcomer in the scientific community it has already found applications in a vast number of areas and will tend to increase in the forthcoming years. Nevertheless, group decision support systems are just recently been proposed by the software vendors discussed, and here, we see a window of opportunity to propose our prototype. The necessity of operational research into group decision support sytems becomes vital so as to seek the requirements of such a system that could not have been made possible without an understanding of Condorcet's effect and Borda's social choice function.

Knowledge on databases was greatly enhanced and the parameter queries and make table queries created provided further insights into the applications of a relational database system. The software developed provided the techniques of interfacing with a relational database, creating control arrays, and utilizing a class for matrix operations. Having come from a scientific background based on utilizing quantitative measurements, this project endeavoured to apply this knowledge on qualitative measurements in the realms of business, management and information technology. Yet, despite the immense effort put into developing the software tool, comes the humble realization that this project has merely scratched the surface in the area of decision making. Nicolas Murray Butler's (Nobel Peace Prize) quote seems quite appropriate here:

"*An expert is someone who knows more and more about less and less, until eventually he knows everything about nothing*."

With all gratitude, Paisley University provided the high caliber of research methods and inspirational quests to keep at the frontiers of knowledge. This experience gained, has helped to become potentially more fluent in the realms of scientific thought, and taught the research methods that proved invaluable in the making of this project. It is hoped that this thesis should provide grounds for further research especially in the area of social choice theory and the analytic hierarchy process for group decision making.

Potential Areas for Future Work

Group decision making under a variety of multiple criteria for a large number of committee members and alternatives provides a large repository of potential areas for future work.. Initially, on the social choice functions, Cook and Seifords function provides perhaps improvement over Borda's. This function investigates a compromise (consensus ranking – minimization of disagreement), a distance function that measures a metric of agreement / disagreement and determining the distance that best agrees with all the committee's ranking. Denoting r_{ij} as the rank given to alternative j by individual i, where i=1, 2, 3, …, n and j = 1, 2, 3,…, m, that is for n committee members and m alternatives. The median, (consensus ranking) is given by r_j^c of alternative j, such that j = 1, 2, 3,…, m, minimizes the total absolute distance, disagreement of the committee.

The individual disagreement:

$$d_i = \sum_{j=1}^{m} \left| r_{ij} - r_j^c \right|, \qquad i = 1,2,3,\ldots,n$$

The overall measure of disagreement:

$$d = \sum_{i=1}^{n} d_i = \sum_{i=1}^{n} \sum_{j=1}^{m} \left| r_{ij} - r_j^c \right|$$

r_j^c can only equal to only one of the rank numbers, k = 1, 2, 3,…, m. If r_j^c = k, then:

$$d_{jk} = \sum_{i=1}^{n} \left| r_{ij} - \kappa \right|$$

and compute:

$$d = \sum_{i=1}^{m} d_{jk}, \qquad \textit{for all} \quad k = 1,2,3,\ldots m$$

The smallest sum of distances can be computed by the assignment problem of linear programming:

$$\min \sum_{j=1}^{m} \sum_{k=1}^{m} d_{jk} x_{jk}$$

subject to:

$$\sum_{j=1}^{m} x_{jk} = 1, \qquad k = 1,2,3,\ldots m$$

$$\sum_{k=1}^{m} x_{jk} = 1, \qquad j = 1,2,3,\ldots m$$

where, $x_{jk} = 1$ if k has been assigned to j and $x_{jk} = 0$ otherwise.

Taking Condorcet's example of nontransient majority,

23 votes:	a P b P c
17 votes:	b P c P a
2 votes:	b P a P c
10 votes:	c P a P b
8 votes:	c P b P a

In Cook and Seiford's notation:

i = 1, 2, 3,…, 60	committee members
j = a, b, c	alternatives
k = 1, 2, 3	rankings

distance of alternative a assigned rank 1:

$$d_{a1} = \sum_{i=1}^{60} \left| r_{ij} - \kappa \right| = \sum_{i=1}^{60} \left| r_{ia} - 1 \right|$$

= 23 |1-1| +17 |3-1| + 2| 2-1| + 10|2-1| +8|3-1| = 62

similarly for alternative a assigned rank 2:

$$d_{a2} = \sum_{i=1}^{60} \left| r_{ij} - \kappa \right| = \sum_{i=1}^{60} \left| r_{ia} - 2 \right|$$

= 23 |1-2| +17 |3-2| + 2| 2-2| + 10|2-2| +8|3-2| = 48

and for alternative a assigned rank 2:

$$d_{a3} = \sum_{i=1}^{60} \left| r_{ij} - \kappa \right| = \sum_{i=1}^{60} \left| r_{ia} - 3 \right|$$

= 23 |1-3| +17 |3-3| + 2| 2-3| + 10|2-3| +8|3-3| = 58

as for the other alternatives:

$$d_{b1} = \sum_{i=1}^{60} \left| r_{ib} - 1 \right| = 51$$

$$d_{b2} = \sum_{i=1}^{60} \left| r_{ib} - 2 \right| = 29$$

$$d_{b3} = \sum_{i=1}^{60} \left| r_{ib} - 3 = 69 \right.$$

$$d_{c1} = \sum_{i=1}^{60} \left| r_{ic} - 1 \right| = 67$$

$$d_{c2} = \sum_{i=1}^{60} \left| r_{ic} - 2 \right| = 43$$

$$d_{c3} = \sum_{i=1}^{60} \left| r_{ic} - 3 \right| = 53$$

The distance (disagreement) coefficients $d_{j\,k}$ can be summarized as:

j \ k	1	2	3
a	62	48	58
b	51	29	69
c	67	43	53

Subtracting the smallest number from each row:

j \ k	1	2	3
a	14	0	10
b	22	0	40
c	24	0	10

Subtracting the smallest from each column:

j \ k	1	2	3
a	**0**	0	0
b	8	**0**	30
c	10	0	**0**

The rankings are: a P b P c, and the corresponding minimum distance is, d = 62 + 29 + 53 = 144.

Cook and Seiford's axiomatic structure is similar to Kemeny's function, but the procedure to reach consensus is different. This approach is homogeneous and monotonic but not Paretan.

What's more, a good proposal for future work could include defining all the discussed social choice functions, into class code and compare the results obtained for each. This analysis could determine the behaviour of these functions with respect to one another and the range of applicability in various decision problems. Even so, the ordinal ranking method of the agreed criteria approach can be further developed to include ordinal ranking of individual criteria approach, whereby each member of the committee could determine their own set of criteria. On this basis, the cardinal method of either the agreed or individual approach would provide further information as in addition to ranking, their respective scores would also be included.

As decisions in society often affects groups of people instead of particular individuals, the general idea would be to define fair methods of aggregating the individual preferences into a social choice. The social welfare function is hence used to map the profile of individual preferences into one of the possible preference ordering for the society itself. If Arrow's conditions for social welfare function are satisfied on two axioms and five conditions then the preference ordering for society is the preference ordering for individuals within the society. Simply put, the counting method used is the social choice function, if we apply Arrow's impossibility theorem to the choice function it becomes a social welfare function. If we consider an infinite set of alternatives then the social welfare function may not equal a social choice function; however, with a finite set of alternatives, a social welfare function is always equal to the social choice function. It is clear then, that the social welfare function could be potential area for future work into moving from group decision support systems to social decision support systems.

Ultimately, the works of Kersten and Shakun describe a methodological framework that includes negotiation in group decision making, namely Negotiation Support Systems (NSS) which undertake to play the role of decision analysts (select committee members) capable of analyzing users'reasoning and consistencies and understanding of the negotiation problem and process. As both decision analysis, using the AHP and negotiation analysis are both prescriptive oriented their methodology can support each other, and models can be developed to aid both the decision analysts and the decision makers (that is, negotiators) to reach consensus in decision making problems. Negotiation, thus, could be amalgamated into a group decision process to observe sensitivity of the criteria and alternatives as they are altered with respect to each other, and determine the factors that may alter a decision maker's initial preferences in favour of others.

In short, it is deemed that the theoretical background of multicretiria analysis in group decision making provides an almost abysmal repository for further research and development into the design of multicriteria group decision support systems. It appears that although decision support systems that aid isolated decision makers are common practice today, group decision support systems, on the other hand, are just coming to surface both in the academic and business sectors. This interest in group decision systems is bound to increase in the near future and may be the cause for new innovations and ideas in this diverse field of study.

Lest not forget the AHP and its methodological framework is already been debated and discussed on the inherent advantages and disadvantages it posseses for the last twenty years, at least, and this debate would very likely continue for the next twenty years.

As far as the database is concerned the potential technical areas for future work could be to develop a more user friendly graphical user interface for the decision analyst. Perhaps using the Tree control and/or the FlexGrid control could be used to aid GUI and enable the decision analysts to create n- level hierarchies with as many criteria and alternatives under each criterion. The multithreading capabilities of the software development could be improved by converting analogous DAO code used to interface the application with the database, into ADO code with HTLM and ASP to transfer the application into the web. Application deployment, on the other hand, which was the method used in this project could provide similar coverage of functionality and remains to be seen which of the methods are more applicable in real life group decision making.

In the software development part of the project we identify ceratin areas that could be improved. Firstly, the number of class functions can be increased to cover broader range of matrix operations or even add a subroutine or function that could return the eigenvector and eigenvalue of a pairwise comparison matrix. Indeed, the accuracy of the eigenvector calculation could be increased by iterating the squares of the pairwise comparison matrix. Also, the approximation method for consistency calculation bears some improvements as well. In addition, classes of the social choice functions could also be defined and be used to aid further understanding of the group choice. Other improvements would include to convert repeat forms into classes as well as add repear code into a general module. Finally, a help file could also be included as to aid the decision analysts and decision makers in the operation of the multicretia group decision support system.

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