

ALGORITHMS

Homework 9

In our study of history at the University of Arkansas, we noticed that before the money or currency was introduced, barter systems were used. For instance, during a certain time window, one kg (kilogram) of corn can be exchanged for 0.9 kg of wheat, 0.8 kg of wheat can be exchanged for 1.2 kg millet (0.9 kg of wheat can be exchanged for 1.35 kg of millet), one kg millet can be exchanged for 1.5 kg of sorghum, and one kg of sorghum can be exchanged for 0.5 kg of corn. Observe that in this example, through a sequence of transactions, one kg corn may be exchanged for 1.0125 kg of corn! That is, 1 kg of corn gets 0.9 kg of wheat, which gets 1.35 kg of millet, which gets 2.025 kg of sorghum, which gets 1.0125 kg of corn. We notice the possibility that in a barter system one may be able to generate a profit by simply going through a sequence of exchanges! A barter system with this phenomenon is called **inefficient**. Note that these conversion values may change over time and in this example, not all the conversion values between agriculture products are shown and not all the products are shown (for example, rice and barley).

As a student of algorithms, we would like to abstract a barter system as follows. We use integer 1 to n to name the products of the system. If x kg of product i can be exchanged for y kg of product j , we will have an entry $i\ j\ x\ y$. If no market for or no one wants to exchange from product i to product j then no such entry exists in the system. As we have learned many algorithms and gained skills of develop them, we would like to challenge ourselves to develop an **efficient algorithm** to determine if a given barter system is **inefficient** or not. If the system is inefficient, we need to give a sequence of exchanges that demonstrates a profit.

Argue the correctness of your algorithm, analyze the running time of your algorithm for barter system efficiency analysis. Please do the same for the algorithm that discovers a sequence of exchanges that results a profit if such a sequence exists.

Input file format for a barter system

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n          :n is the number of products and products are 1 to n
i j x y    :i and j are products and x y kg floating point value
...        :each unique i and j appear at most once
...        :the quadruples end by EOF
```

Output file format for a barter system analysis

yes or no :yes for inefficient system, and no otherwise
i j x y :start of a profit sequence for an inefficient system
j k u v :
... :
p i w z :end of the profit sequence
one kg of i gets x ($x > 1$) kg of i from the above sequence