

# CS131 - Week 6

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UCLA Winter 2019

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# Today

- Prolog
- Homework #4

# Declarative programming

- Describing *what* we want to achieve, not *how* to do it
- Examples: SQL, Prolog, Regular expressions, ...

# Prolog

- Logic programming language
- Programs defined using *Facts*, *Rules*, and *Queries*
- This course uses GNU Prolog: <http://www.gprolog.org>
  - Make sure you are not using SWI-Prolog, they have lots of differences
  - You can run it in SEASnet servers with command ***gprolog***

# Prolog - How to run code

- Facts and Rules written into a file, e.g. **myrules.pl**
- In interactive Prolog environment, you *consult* the rule file: **[myrules]**.
- After that, you can run *queries* in the interactive environment

# Facts

- Facts define what is true in our *database*
- Always start with a lowercase letter
- Closed-world assumption
- For example:

## Prolog file:

```
raining.  
john_is_cold.  
john_forgot_his_raincoat.
```

## Queries:

```
?- raining.  
yes
```

```
?- john_is_cold.  
yes
```

```
?- john_is_tired.  
exception
```

# Relations

- Facts consisting of one or more terms
- For example:

## Prolog file:

```
student(fred).  
eats(fred, oranges).  
eats(fred, bananas).  
eats(tony, apples).
```

## Queries:

```
?- eats(fred, oranges).  
yes.
```

```
?- eats(fred, apples).  
no.
```

```
?- student(fred).  
yes.
```

# Variables and Unification

- Unification tries to find a way to fill the missing values
- Introduce variables: any string that starts with a capital letter
  - E.g. My\_variable, What, Who
- Unification binds *variables* to *atoms*

## Prolog file:

```
eats(fred, oranges).  
eats(fred, bananas).  
eats(tony, apples).
```

## Queries:

```
?- eats(fred, What).  
What = oranges ? a  
What = bananas  
yes
```

```
?- eats(Who, apples).  
Who = tony  
yes
```



# Rules

- Rules allow us to make conditional statement
- Syntax: *conclusion :- premises.*
  - Conclusion is true if the premises are true
- Consider the statement “All men are mortal”:

```
mortal(X) :-  
    human(X).  
  
human(socrates).
```

## Queries:

```
?- mortal(socrates).  
yes
```

```
?- mortal(Who)  
Who = socrates  
yes
```

# Rules

- Rules can contain multiple statements
  - Comma (,) is the AND operator, semi-colon (;) is the OR operator

```
red_car(X) :-  
    red(X),  
    car(X).
```

```
red(ford_escort).  
car(ford_escort).
```

## Queries:

```
?- red_car(ford_escort).  
yes
```

```
?- red_car(What)  
What = ford_escort  
yes
```

# Rules

- Rules can contain multiple statements
  - Comma (,) is the AND operator, semi-colon (;) is the OR operator

```
red_or_blue_car(X) :-  
    (red(X);blue(X)),  
    car(X).
```

```
red(ford_escort).  
car(ford_escort).
```

## Queries:

```
?- red_or_blue_car(ford_escort).  
yes
```

```
?- red_or_blue_car(What)  
What = ford_escort  
yes
```

# Recap

- Facts: Start with a lowercase letter
  - e.g. raining. john\_hungry.
- Variables: Start with a uppercase letter:
  - e.g. What, Who, AnYtHinG123
- Relations: Like facts, but including multiple *atoms* inside parentheses
  - e.g. likes(steve,dancing). hungry(john).
- Rules: conclusion :- premises.
  - e.g. parent(X,Y) :- father(X,Y); mother(X,Y).

# Equality

- Three equality operators: **=** , **is** , **==**
  - **=** tries unification directly, **is** evaluates the right side and unifies, **==** evaluates both sides

?- 7 = 5 + 2.

**no**

?- X = 5 + 2.

**X = 5+2**

**yes**

?- A + B = 5 + 2.

**A = 5**

**B = 2**

?- X is 5 + 2.

**X = 7**

**yes**

?- 7 is 5 + 2.

**yes**

?- 5 + 2 is 7.

**no**

?- X is 5 + Y.

**uncaught exception:**  
**error(instantiation\_error,(is)/2)**

?- 5 + 2 == 4 + 3.

**yes**

?- X == 4 + 3.

**uncaught exception:**  
**error(instantiation\_error,(==)/2)**

?- X = 5, Y = 5, X == Y.

**X = 5**

**Y = 5**

**yes**

# Arithmetic comparisons

## Arithmetic examples Prolog Notation

$x < y$

$X < Y.$

$x \leq y$

$X \leq Y.$

$x = y$

$X == Y.$

$x \neq y$

$X \neq Y.$

$x \geq y$

$X \geq Y$

$x > y$

$X > Y$

# Backtracking

```
hold_party(X) :-  
    birthday(X),  
    happy(X).
```

```
birthday(tom).  
birthday(fred).  
birthday(helen).
```

```
happy(mary).  
happy(jane).  
happy(helen).
```

- To understand the performance of Prolog, we need to understand how it solves queries
- Prolog goes through facts/rules one-by-one in order
- If one choice of variables fails, it backtracks and tries the next one
- E.g. consider query *hold\_party(Who)*.

Prolog visualizer: <http://www.cdglabs.org/prolog/#/>

# Backtracking

```
hold_party(X) :-  
    birthday(X),  
    happy(X).
```

```
birthday(tom).  
birthday(fred).  
birthday(helen).
```

```
happy(mary).  
happy(jane).  
happy(helen).
```

1. Ask query *hold\_party(Who)*
2. Our database does not have relations *hold\_party(name)*, so try applying the rule
3. Choose first person who matches *birthday(X)* (tom)
4. Try relation *happy(tom)*
5. Relation is not found, so backtrack to next *birthday*
6. Try again with fred
7. Relation *happy(fred)* fails too
8. Backtrack and try with *helen*
9. This time relation is found, so X=helen



# Recursion

```
busroute(culvercity,westwood).  
busroute(westwood,santamonica).  
busroute(santamonica,venice).
```

```
can_reach(X,Y) :- busroute(X,Y).  
can_reach(X,Y) :- busroute(X,Z),  
                  can_reach(Z,Y).
```

## Queries:

```
?- can_reach(culvercity,venice).  
yes
```

```
?- can_reach(venice,culvercity)  
no
```

# Recursion

- How can we define ancestor(X,Y) to test whether Y is X's parent, grandparent, grandgrandparent, etc?

```
father(john,paul).  
father(paul,henry).  
mother(paul,mary).  
mother(mary,susan).
```

# Recursion

- How can we define ancestor(X,Y) to test whether Y is X's parent, grandparent, grandgrandparent, etc?

```
father(john,paul).  
father(paul,henry).  
mother(paul,mary).  
mother(mary,susan).
```

```
ancestor(X,Y) :- father(X,Y); mother(X,Y).  
ancestor(X,Y) :- (father(X,Z), ancestor(Z,Y));  
                  (mother(X,Z), ancestor(Z,Y)).
```

```
?- ancestor(john,Who).  
Who = paul  
Who = henry  
Who = mary  
Who = susan
```

# Lists

- Important data structure in Prolog
  - Especially in your homework
- Syntax: ***[val1, val2, val3, ..., valn]***
- Similar to OCaml, we can do pattern matching to head and tail:
  - ***[1,2,3,4] = [A | B]*** -> A is bound to 1, B is [2, 3, 4]
  - ***[1,2,3,4] = [A, B | C]*** -> A = 1, B = 2, C = [3, 4]

# Lists - Examples

- Consider the following relation:

$p([H \mid T], H, T).$

- What is the result of the following queries?

- 1)  $p([a, b, c], a, [b, c]).$
- 2)  $p([a, b, c], X, Y).$
- 3)  $p([a], X, Y).$
- 4)  $p([], X, Y).$

# List searching

- How can we check if a specific element is in a list?
- Write a rule ***exists(X, List)*** which is true if ***X*** is in ***List***

# List searching

- How can we check if a specific element is in a list?
- Write a rule ***exists(X, List)*** which is true if ***X*** is in ***List***

```
exists(X, [X|_]).  
exists(X, [_|T]) :-  
    exists(X, T).
```

## Queries:

```
?- exists(a, [a,b,c]).  
yes.
```

```
?- exists(a, [x,y,z]).  
no.
```

# How can I debug my program?

- **trace.** shows all the evaluations (turn off using **notrace.**)

```
| ?- exists(2, [1,2,3]).  
    1    1  Call: exists(2,[1,2,3]) ?  
    2    2  Call: exists(2,[2,3]) ?  
    2    2  Exit: exists(2,[2,3]) ?  
    1    1  Exit: exists(2,[1,2,3]) ?  
  
true ?  
  
yes
```

```
| ?- exists(a, [1,2,3]).  
    1    1  Call: exists(a,[1,2,3]) ?  
    2    2  Call: exists(a,[2,3]) ?  
    3    3  Call: exists(a,[3]) ?  
    4    4  Call: exists(a,[]) ?  
    4    4  Fail: exists(a,[]) ?  
    3    3  Fail: exists(a,[3]) ?  
    2    2  Fail: exists(a,[2,3]) ?  
    1    1  Fail: exists(a,[1,2,3]) ?  
  
(1 ms) no
```



# List construction (Covered in lecture)

- Write a function ***append(X,Y,Result)*** which sets ***Result*** as ***X*** followed by ***Y***
- Start with the easy case:

```
append([], Y, Y).
```

- Then the more complicated case:

```
append([XH|XT], Y, [XH|RT]) :-  
    append(XT, Y, RT).
```

- Consider e.g. ***append([1,2,3], [a,b,c], [1,2,3,a,b,c])***

# List construction

- Write a function **append**(*X,Y,Result*) which sets **Result** as *X* followed by *Y*
- Start with the easy case:

```
append([], Y, Y).
```

- Then the more complicated case:

```
append([XH|XT], Y, [XH|RT]) :-  
    append(XT, Y, RT).
```

```
[ | ?- myappend([1,2,3],[a,b,c],Result).  
[   1   1   Call: myappend([1,2,3],[a,b,c],_291) ?  
[   2   2   Call: myappend([2,3],[a,b,c],_324) ?  
[   3   3   Call: myappend([3],[a,b,c],_351) ?  
[   4   4   Call: myappend([], [a,b,c], _378) ?  
[   4   4   Exit: myappend([], [a,b,c], [a,b,c]) ?  
[   3   3   Exit: myappend([3], [a,b,c], [3,a,b,c]) ?  
[   2   2   Exit: myappend([2,3], [a,b,c], [2,3,a,b,c]) ?  
[   1   1   Exit: myappend([1,2,3], [a,b,c], [1,2,3,a,b,c]) ?  
  
Result = [1,2,3,a,b,c]
```

- Consider e.g. **append**([1,2,3], [a,b,c], [1,2,3,a,b,c])

# List removal

- How to write function ***remove(X, List, Result)*** that sets ***Result*** to be otherwise the same as ***List*** but removing occurrences of ***X***?
  - E.g. ***remove(1, [1,2,3,1,2,3], Result)*** would bind ***Result*** to ***[2,3,2,3]***

# List removal

- How to write function ***remove(X, List, Result)*** that sets ***Result*** to be otherwise the same as ***List*** but removing occurrences of ***X***?
  - E.g. ***remove(1, [1,2,3,1,2,3], Result)*** would bind ***Result*** to ***[2,3,2,3]***

```
remove(X, [], []).
```

```
remove(X, [X|L1t], Result) :- remove(X, L1t, Result).
```

```
remove(X, [H|L1t], [H|Result]) :- remove(X, L1t, Result).
```

# List - append

- **From the manual:** “append(List1, List2, List12) succeeds if the concatenation of the list List1 and the list List2 is the list List12.”

```
?- append([1,2], [3,4], Result).
```

**Result = [1,2,3,4]**

```
?- append(A, [3,4], [1,2,3,4]).
```

**A = [1,2]**

```
?- append(A, B, [1,2,3]).
```

**A = []**

**B = [1,2,3] ? a**

**A = [1]**

**B = [2,3]**

**A = [1,2]**

**B = [3]**

**A = [1,2,3]**

**B = []**

# List - member

- **From the manual:** “member(Element, List) succeeds if Element belongs to the List. This predicate is re-executable on backtracking and can be thus used to enumerate the elements of List.

```
?- member(3, [1,2,3,4,5]).
```

```
true
```

```
?- member(X, [1,2,3]).
```

```
X = 1 ? a
```

```
X = 2
```

```
X = 3
```

# List - permutation

- **From the manual:** “permutation(List1, List2) succeeds if List2 is a permutation of the elements of List1.”

```
?- permutation([3,2,1], [1,2,3]).  
true
```

```
?- permutation([1,2,3], X).  
X = [1,2,3] ? ;  
X = [1,3,2] ? ;  
X = [2,1,3] ? ;  
X = [2,3,1] ? ;  
X = [3,1,2] ? ;  
X = [3,2,1] ?
```

# List - prefix / suffix

- **From the manual:** “prefix(Prefix, List) succeeds if Prefix is a prefix of List.  
suffix(Suffix, List) succeeds if Suffix is a suffix of List.”

?- prefix([1,2,3], [1,2,3,4,5]).

**yes**

?- suffix([4,5], [1,2,3,4,5]).

**yes**



# List - length

- **From the manual:** “length(List, Length) succeeds if Length is the length of List.”

```
?- length([1,2,3,4], 4).
```

```
yes
```

```
?- length([1,2,3,4], Len).
```

```
Len = 4
```

```
yes
```

```
?- length(List, 5).
```

```
List = [_,_,_,_,_]
```

```
yes
```

# List - nth

- **From the manual:** “nth(N, List, Element) succeeds if the Nth argument of List is Element.

```
?- nth(5, [1,2,3,4,5,6], Element).
```

```
Element = 5
```

```
yes
```

```
?- nth(N, [1,2,3,4,5,6], 3).
```

```
N = 3
```

```
yes
```

```
?- nth(3, L, 5).
```

```
L = [_,_,5|_]
```

```
yes
```

# List - maplist

- **From the manual:** “maplist(Goal, List) succeeds if Goal can successfully be applied on all elements of List.”

```
?- maplist(>(5), [1,2,3]).
```

**yes**

```
?- maplist(=(1), [1,1,1]).
```

**yes**

# Generating a List with Constraints

- Problem: Generate a list of length N where each element is a unique integer between 1..N
- We can start by outlining what we need:

```
unique_list(List, N) :-  
    length(List, N),  
    elements_between(List, 1, N),  
    all_unique(List).
```

# Generating a List with Constraints

- Problem: Generate a list of length N where each element is a unique integer between 1..N
- We can start by outlining what we need:

```
unique_list(List, N) :-
```

```
    length(List, N),
```

```
    elements_between(List, 1, N),
```

```
    all_unique(List).
```

← Provided by Prolog

← Prolog provides only ***between(Min, Max, X)***

← Not provided by Prolog

# Generating a List with Constraints

```
unique_list(List, N) :-  
    length(List, N),  
    elements_between(List, 1, N),  
    all_unique(List).
```

```
elements_between([], _, _).  
elements_between([H|T], Min, Max) :-  
    between(Min, Max, H),  
    elements_between(T, Min, Max).
```

Is there an easier way to write this?



# Generating a List with Constraints

```
unique_list(List, N) :-  
    length(List, N),  
    elements_between(List, 1, N),  
    all_unique(List).
```

```
elements_between(List, Min, Max) :-  
    maplist(between(Min,Max), List).
```

# Generating a List with Constraints

```
unique_list(List, N) :-  
    length(List, N),  
    elements_between(List, 1, N),  
    all_unique(List).
```



```
all_unique([]).  
all_unique([H|T]) :- exists(H, T), !, fail.  
all_unique([H|T]) :- all_unique(T).
```

```
elements_between(List, Min, Max) :-  
    maplist(between(Min,Max), List).
```



## Query:

```
?- unique_list(List, 6).  
List = [1,2,3,4,5,6]
```



# Generating a List with Constraints

```
unique_list(List, N) :-  
    length(List, N),  
    elements_between(List, 1, N),  
    all_unique(List).
```





```
elements_between(List, Min, Max) :-  
    maplist(between(Min,Max), List).
```

```
all_unique([]).  
all_unique([H|T]) :- exists(H, T), !, fail.  
all_unique([H|T]) :- all_unique(T).
```

Is this efficient?

# Generating a List with Constraints

```
unique_list(List, N) :-  
    length(List, N),  
    elements_between(List, 1, N),  
    all_unique(List).
```



```
elements_between(List, Min, Max) :-  
    maplist(between(Min, Max), List).
```

```
all_unique([]).  
all_unique([H|T]) :- exists(H, T), !, fail.  
all_unique([H|T]) :- all_unique(T).
```

**Is this efficient?**  
 **$N^N$  possible lists to try**

# Finite Domain Solver

---

# Finite Domain Solver

- Finds assignments to variables that fulfill constraints
- Variable values are limited to a finite domain (e.g. integers between 0 and 10)
- Less code, optimized solution

# Finite Domain Solver

- Lets solve the earlier problem using the FD solver:

```
unique_list2(List, N) :-
```

```
    length(List, N),
```

```
    fd_domain(List, 1, N),
```

```
    fd_all_different(List),
```

```
    fd_labeling(List).
```

Create a list of length N with no bound values

Define all values in List to be between 1 and N

Define all values in List to be different

Find a solution (backtracking will generate a new solution)

- Optimized solution with a fraction of the code!

# FD Constraints

## Arithmetic Constraints:

- $\text{FdExpr1} \# = \text{FdExpr2}$  constrains  $\text{FdExpr1}$  to be equal to  $\text{FdExpr2}$ .
- $\text{FdExpr1} \# \neq \text{FdExpr2}$  constrains  $\text{FdExpr1}$  to be different from  $\text{FdExpr2}$ .
- $\text{FdExpr1} \# < \text{FdExpr2}$  constrains  $\text{FdExpr1}$  to be less than  $\text{FdExpr2}$ .
- $\text{FdExpr1} \# \leq \text{FdExpr2}$  constrains  $\text{FdExpr1}$  to be less than or equal to  $\text{FdExpr2}$ .
- $\text{FdExpr1} \# > \text{FdExpr2}$  constrains  $\text{FdExpr1}$  to be greater than  $\text{FdExpr2}$ .
- $\text{FdExpr1} \# \geq \text{FdExpr2}$  constrains  $\text{FdExpr1}$  to be greater than or equal to  $\text{FdExpr2}$ .

See [documentation](#) for more built-in constraints

# FD Constraints

- Note that constraints do not find a solution - they just limit the options:

```
?- X #= Y.
```

```
X = _#0(0..268435455)
```

```
Y = _#0(0..268435455)
```

```
?- X #< 5.
```

```
X = _#2(0..4)
```

```
?- X #< 5, fd_labeling(X).
```

```
X = 0 ? ;
```

```
X = 1 ? ;
```

```
X = 2 ? ;
```

```
X = 3 ? ;
```

```
X = 4
```

```
?- X #< 3, X*Y #= 6.
```

```
X = _#2(1..2)
```

```
Y = _#22(3..6)
```

# Sudoku with FD

How can you solve 4x4 Sudoku problem using FD solver?

Use FD constraints:

- `fd_domain(List, Min, Max)`
- `fd_all_different(List)`

	2		
1			
			4
		1	



# Sudoku with FD

sudoku4\_fd(L):-

```
L = [X11,X12,X13,X14,X21,X22,X23,X24,X31,X32,X33,X34,X41,X42,X43,X44],  
fd_domain(L, 1, 4),  
fd_all_different([X11,X12,X13,X14]) , fd_all_different([X21,X22,X23,X24]),  
fd_all_different([X31,X32,X33,X34]) , fd_all_different([X41,X42,X43,X44]),  
fd_all_different([X11,X21,X31,X41]) , fd_all_different([X14,X24,X34,X44]),  
fd_all_different([X12,X22,X32,X42]) , fd_all_different([X13,X23,X33,X43]),  
fd_all_different([X11,X12,X21,X22]) , fd_all_different([X13,X14,X23,X24]),  
fd_all_different([X31,X32,X41,X42]) , fd_all_different([X33,X34,X43,X44]),  
fd_labeling(L).
```

?- *sudoku4\_fd([1,2,3,4,X21,X22,X23,X24,X31,X32,X33,X34,X41,X42,X43,X44]).*

X21 = 3, X22 = 4, X23 = 1, X24 = 2, X31 = 2, X32 = 1, X33 = 4, X34 = 3, X41 = 4, X42 = 3,  
X43 = 2, X44 = 1

# Homework #4

---

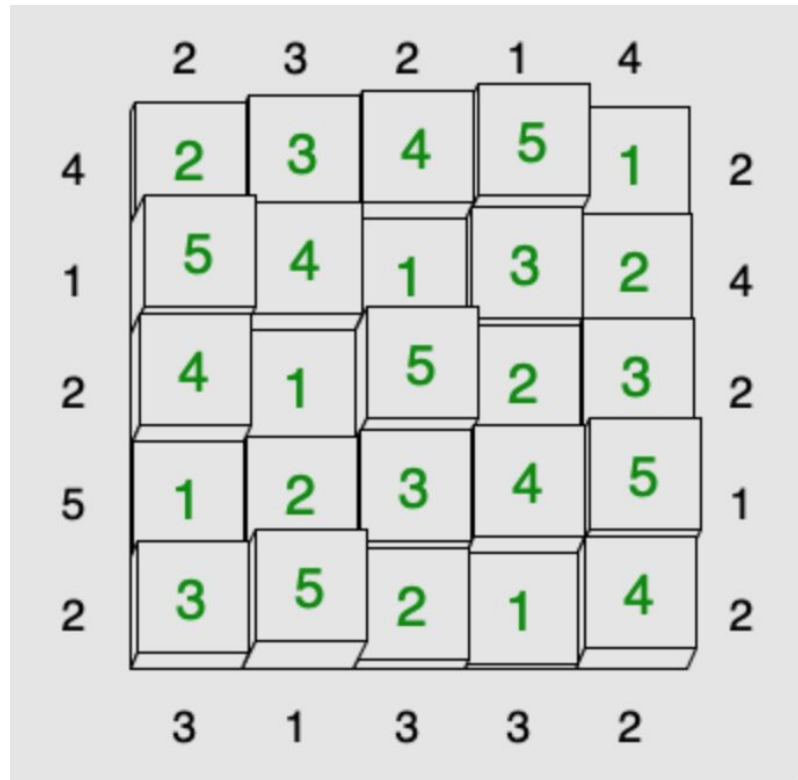
# Homework #4 - Towers Solver (DL Friday Feb 22)

$N \times N$  square is filled with numbers  $1..N$  so that values are not repeated in any row/column

Towers have different heights, can you determine the heights if you know how many can be seen from each position?

Try it online:

<https://www.chiark.greenend.org.uk/~sgtatham/puzzles/js/towers.html>



# Homework #4 - Towers Solver

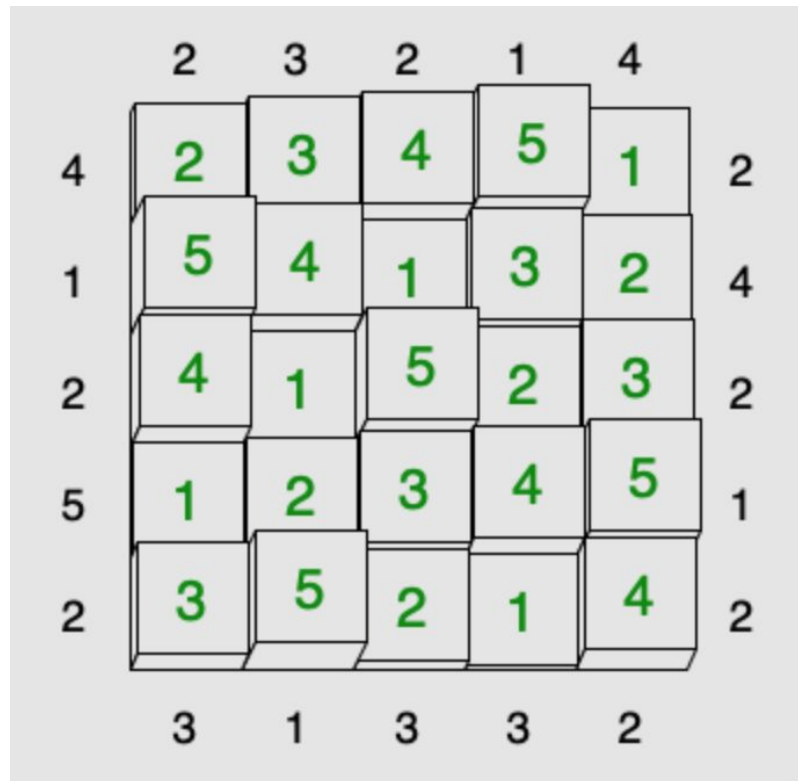
In your homework, write Prolog code for solving the heights based on how many can be seen and vice versa

Write two different implementations: One using FD solver and one without

- Provide comparison of their performance
- Note: Non-FD solver probably won't work with larger grids. Testing with 5x5 is enough

Write a solver that finds ambiguous rules

- i.e. Numbers on the side can be caused by multiple tower arrangements



# Homework #4 - Towers Solver

Examples:

```
?- tower(5,  
  [[2,3,4,5,1],  
   [5,4,1,3,2],  
   [4,1,5,2,3],  
   [1,2,3,4,5],  
   [3,5,2,1,4]],  
  C).
```

```
C = counts([2,3,2,1,4],  
  [3,1,3,3,2],  
  [4,1,2,5,2],  
  [2,4,2,1,2])
```

```
?- tower(5, T,  
  counts([2,3,2,1,4],  
         [3,1,3,3,2],  
         [4,1,2,5,2],  
         [2,4,2,1,2])).
```

```
T = [[2,3,4,5,1],  
  [5,4,1,3,2],  
  [4,1,5,2,3],  
  [1,2,3,4,5],  
  [3,5,2,1,4]]
```

# Homework #4 - Statistics

```
[1] ?- statistics.
```

Memory		limit	in use	free
trail	stack	16383 Kb	0 Kb	16383 Kb
cstr	stack	16384 Kb	0 Kb	16384 Kb
global	stack	32767 Kb	4 Kb	32763 Kb
local	stack	16383 Kb	0 Kb	16383 Kb
atom	table	32768 atoms	1775 atoms	30993 atoms

Times		since start	since last
-------	--	-------------	------------

user	time	0.010 sec	0.010 sec
system	time	0.027 sec	0.027 sec
cpu	time	0.037 sec	0.037 sec
real	time	69547.313 sec	69547.313 sec

```
(1 ms) yes
```

```
| ?- statistics(cpu_time, [SinceStart, SinceLast]).
```

```
SinceLast = 1  
SinceStart = 42
```

```
yes
```

# Homework #4

- Make sure everything compiles without warnings or errors on SEASnet servers
  - Before submitting, restart gprolog and try your solution once more - you might be calling rules that you renamed/deleted!
- Try to make your solution reasonably efficient
  - No need to put a lot of effort on this, but if your solution takes >10mins, it can be optimized...
  - Consider how the code can be reordered to avoid unnecessary backtracking
- Don't use FD solver in your plain solution
  - E.g. comparisons with `#<` are not allowed
- **Do not use SWI-Prolog!**

# Prolog Resources

- GNU Prolog manual: <http://www.gprolog.org/manual/gprolog.html>
- Prolog Wikibook: <https://en.wikibooks.org/wiki/Prolog>
- Prolog Visualizer: <http://www.cdglabs.org/prolog/#/>

When looking for resources, make sure to check they are for GNU Prolog, not SWI-Prolog!



# Questions?

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