Homework 3

Due: Friday, Feb.. 21, by 11:59pm,
via Gradescope

A. Refer to the Gradescope videos on YouTube if you have questions on how to submit homework.

B. I refer you to the syllabus for the rubric that is used on the homework.

1. (6 points) Section 8: 8.7(a), 8.9. Explain your answers.

2. 8.7(a) We can simplify the question as if there were three photos. If we only post the three photos on the friends page, then there would be 3! possible arrangements (6 total arrangements). In addition, if we only post the three photos on the family page, then the amount of possible arrangements would still be 3!. In addition, if we were to post 2 photos on the friends page and 1 photo on the family page, then there would also be 3! possible arrangements. Also, if we were to post 2 photos on the family page and 1 photo on the friends page, then the possible arrangements would be 3!, giving us a total of 4 combinations of 3!, thus giving us 4 * 3!. This example of recursion can be used if there are 30 photos, giving us 31*30! possible ways the photos can appear on the pages.

3. 8.9) Since a chess board is 8*8, there are 64 total squares on the chess board. In addition, we can use the multiplication principle to find out the total ways the black rook and the white rook can be in sequence with each other using m and n. M can represent the amount of possible ways the black rook can move, which is 64 because the chess board is 8*8. Since the white rook cannot move in the same way as the black rook, we need to deduct the squares that are both tangent to the column and the row, which are 15 squares. As a result, the total amount of ways the white rook can move is 64-15 = 49 ways. We then multiply m * n, 64 * 49, giving us 3136 ways.

4. (6 points) Section 8: 8.16, 8.19. Explain your answers.

5. 8.16) Since there are 10 digits in the lock (0-9), the first digit in the combination is free to use any lock which is 10 possible combinations. The second digit in the lock is limited to 7 choices as the digit cannot be next to the first digit. Following this same procedure, the third and fourth digit of the lock combination can only be 7 choices as well. The total number of combinations gives us 10 * 7^3, giving us 3430 total lock combinations, given that there are no consecutive or adjacent numbers.

6. 8.19) Since there are 52 cards in a deck, 26 cards are red and the other 26 cards are black. In addition, there are 4 suits, so 13 cards in the deck represent one of the suits. Since we are drawing 4 cards, the first card drawn has a combination of 4 * 13, since there are 4 suits and 13 denominations of each suit. The second card has a combination of 3 * 12, since we cannot draw the same suit and the same denomination. The third

card has a combination of 2 * 11 and the fourth card has a combination of 1 * 10. Using the multiplication principle, we get 4 * 13 * 3 * 12 * 2 * 11 * 1 * 10, giving us 411,840 ways.

7. (3 points) List 2655 consecutive composite numbers. Show that they are indeed composite.
8. (3 points) Let $A = 2^{\emptyset}$, $B = 2^A, C = 2^B$, and $D = 2^C$. Construct D.
9. A = 2^{θ} = θ
10. B = 2^A = θ,θ
11. C = 2^B= θ,θ,θ,θ,θ
12. (18 points) Section 10: 10.1(g), 10.3(c)(d)(e)(f)(g).
13. (21 points) Section 10: 10.4,
14. (24 points) Section 10: 10.5, 10.6
15. (9 points) 10.12, 10.14, 10.15.
16. (6 points) Let
$$A = \{x \in \mathbb{Z} : x = 6k - 5 \text{ for some } k \in \mathbb{Z}\}$$
$$B = \{x \in \mathbb{Z} : x = 3m + 1 \text{ for some } m \in \mathbb{Z}\}$$
   (a) Is $A \subseteq B$? If so, prove it. If not, provide an appropriate counter-example. Show that your counter-example works, i.e. show that your counterexample belongs to A however does not belong to B.
   (b) Is $B \subseteq A$? If so, prove it. If not, provide an appropriate counter-example. Show that your counter-example works, i.e. show that your counterexample belongs to B however does not belong to A.