Lambda Calculus

For the following forms apply β-reduction and α-substitution to reduce to lowest form. Indicate at each step the rule you are applying.

1. (λx.x)(λx.x)

   - (λx.x)(λx.x) → (λx.x)(λy.y) [α-substitution]
   - (λx.x)(λy.y) →(λy.y) [β-reduction] [Identity]

2. (λx.x x) (λx. λy.x x)

   - (λx.x x) (λx. λy.x x) → (λx.x x) (λt. λy.t t) [α-substitution]
   - (λx.x x) (λt. λy.t t) → (λt. λy.t t) (λt. λy.t t) [β-reduction]
   - (λt. λy.t t) (λt. λy.t t) → λy.(λt. λy.t t) (λt. λy.t t)
   - No final form available – reduction never ends

3. ((λx.(x y))(λz.z))

   - ((λx.(x y))(λz.z)) → (λz.z y) [β-reduction]
   - (λz.z) y) → y

4. (λz.z) (λy.y y) (λx.x a)

   - (λz.z) (λy.y y) (λx.x a) → (λy.y y) (λx.x a) [β-reduction]
   - (λy.y y) (λx.x a) → (λx.x a) (λx.x a) [β-reduction]
   - (λx.x a) (λx.x a) → (λx.x a) a [β-reduction]
   - (λx.x a) a → a a [β-reduction]

5. (λz.z) (λz.z z) (λz.z y)

   - (λz.z) (λz.z z) (λz.z y) → (λz.z) (λt.t t) (λz.z y) [α-substitution]
   - (λz.z) (λt.t t) (λz.z y) → (λz.z) (λt.t t) (λs.s y) [α-substitution]
   - (λz.z) (λt.t t) (λs.s y) → (λt.t t) (λs.s y) [β-reduction]
   - (λt.t t) (λs.s y) → (λs.s y) (λs.s y) [β-reduction]
   - (λs.s y) (λs.s y) → (λs.s y) y [β-reduction]
   - (λs.s y) y → y y [β-reduction]

6. (λx.λy.x y y) (λa.a) b

- (λx.λy.x y y) (λa.a) b → (λx.(λy.x y y)) (λa.a) b
- (λx.(λy.x y y)) (λa.a) b → (λy.(λa.a) y y) b [β-reduction]
- (λy.(λa.a) y y) b → (λy.y y) b [β-reduction]
- (λy.y y) b → b b [β-reduction]

7. (λx.x x) (λy.y x) z

- (λx.x x) (λy.y x) z → (λt.t t) (λy.y x) z [α-substitution]
- (λt.t t) (λy.y x) z → (λy.y x)(λy.y x) z [β-reduction]
- (λy.y x)(λy.y x) z → ((λy.y x) x) z [β-reduction]
- ((λy.y x) x) z → x x z [β-reduction]

8. (λx. (λy. (x y)) y) z

- (λx. (λy. (x y)) y) z → (λx. (λt. (x t)) y) z [α-substitution]
- (λx. (λt. (x t)) y) z → (λx. (x y)) z [β-reduction]
- (λx. (x y)) z → z y [β-reduction]

9. ((λx.x x) (λy.y)) (λy.y)

- ((λx.x x) (λy.y)) (λy.y) → ((λx.x x) (λy.y)) (λt.t) [α-substitution]
- ((λx.x x) (λy.y)) (λt.t) → ((λy.y)(λy.y)) (λt.t) [β-reduction]
- ((λy.y)(λy.y)) (λt.t) → (λy.y) (λt.t) [β-reduction]
- (λy.y) (λt.t) → (λt.t) [β-reduction] [Identity]

10. (((λx. λy.(x y))(λy.y)) w)

- ((((λx. λy.(x y))(λy.y)) w) → ((((λx. λy.(x y))(λt.t)) w) [α-substitution]
- ((((λx. λy.(x y))(λt.t)) w) → (((λy.(λt.t) y) w) [β-reduction]
- (((λy.(λt.t) y) w) → (λt.t) w [β-reduction]
- (λt.t) w → w [β-reduction] [Identity]