Assignment 1

AI1110:Probability and random variables
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Question 12.13.6.14 : If each element of a second order determinant is either zero or one,what is the probability that the value of the determinant is positive?(Assume that the individual entries of the determinant are chosen independently,each value being assumed with probability $\frac{1}{2}$.)

Solution: We know that a second order determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$has four entries.It is given that each entry is either 0 or 1 i.e,each entry can be filled in two ways. The determinant of the second order matrix is ad-bc.For the determinant to be positive,

$$ad - bc > 0 \implies a > bc/d$$

$$\Pr(a > bc/d) = 1 - \Pr(a \leqslant bc/d)$$

$$\Pr(a \leqslant bc/d) = F_A(bc/d)$$

F_A is the cdf of a.

$$F_A(x) = \begin{cases} 0 & \text{if } x < 0, \\ 1/2 & \text{if } 0 \leqslant x < 1, \\ 1 & \text{if } 1 \leqslant x < \infty \end{cases}$$

a) Taking expectation of $F_A\left(\frac{bc}{d}\right)$ with respect to d we get,

$$E_d\left(F_A\left(\frac{bc}{d}\right)\right) = \frac{1}{2}F_A(bc) + \frac{1}{2}F_A(\infty) \tag{1}$$

$$= \frac{1}{2}F_A(bc) + \frac{1}{2} \tag{2}$$

b) Expectation of the above with respect to b we get,

$$E_b\left(\frac{1}{2}F_A(bc) + \frac{1}{2}\right) = \frac{1}{2}E_b(F_A(bc)) + \frac{1}{2} \tag{3}$$

$$= \frac{1}{2}\left(\frac{1}{2}F_A(0) + \frac{1}{2}F_A(c)\right) \tag{4}$$

$$= \frac{1}{2} + \frac{1}{8} + \frac{1}{4}F_A(c) \tag{5}$$

$$= \frac{5}{8} + \frac{1}{4}F_A(c) \tag{6}$$

c) Expectation of this with respect to c will be,

$$E_c\left(\frac{5}{8}+\frac{1}{4}F_A(c)\right)=\frac{5}{8}+\frac{1}{4}E_c\left(F_A(c)\right) \tag{7}$$

$$=\frac{5}{8}+\frac{1}{4}\left(\frac{1}{2}F_A(0)+\frac{1}{2}F_A(1)\right) \tag{8}$$

$$=\frac{5}{8}+\frac{1}{4}\left(\frac{1}{4}+\frac{1}{2}\right) \tag{9}$$

$$=\frac{5}{8}+\frac{3}{16} \tag{10}$$

$$=\frac{13}{16} \tag{11}$$

$\therefore$ Required probability is

$$\begin{aligned}\Pr\left(a>\frac{bc}{d}\right)&=1-E_b,c,d\left(F_A\left(\frac{bc}{d}\right)\right)\\&=1-\frac{13}{16}\\&=\frac{3}{16}\end{aligned}$$